Holographic pair and charge density waves

Sera Cremonini, Li Li, and Jie Ren

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Holographic Pair and Charge Density Waves

Sera Cremonini* and Li Li†
Department of Physics, Lehigh University, Bethlehem, PA, 18018, USA.

Jie Ren‡
Racah Institute of Physics, The Hebrew University of Jerusalem, 91904, Israel.

We examine a holographic model in which a $U(1)$ symmetry and translational invariance are broken spontaneously at the same time. Our construction provides an example of a system with pair-density wave order, in which the superconducting order parameter is spatially modulated but has a zero average. In addition, the charge density oscillates at twice the frequency of the scalar condensate. Depending on the choice of parameters, the model also admits a state with co-existing superconducting and charge density wave orders, in which the scalar condensate has a uniform component.

I. INTRODUCTION AND DISCUSSION

Over recent years holographic techniques originating from the AdS/CFT duality, and first developed in string theory, have been used to analyze models that may be in the same universality class as many highly correlated systems. Thanks to such approaches, challenging questions about dynamics in quantum phases of matter at strong coupling can be mapped to processes in theories of gravity that are tractable. Thus, holography provides a window into the often unconventional physics of these systems.

Inhomogeneities, striped phases and competing orders are believed to play an important role in the rich phase structure of high T_c superconductors [1–3]. In certain regions of the phase diagram – such as the pseudo-gap regime – many of these orders appear to be intertwined and sometimes have comparable strengths and common origin. Here we focus on a particular broken-symmetry phase, the pair-density wave (PDW) [4, 5], in which charge density wave (CDW) and superconducting (SC) orders are intertwined in a very specific way, and in which spin density wave (SDW) order can also play a role. PDW phases seem to be a robust feature of models of strongly correlated electrons including high T_c superconductors, and there is experimental evidence that they appear at least in the cuprate $La_{2-x}Ba_xCuO_4$ [6–8].

In this paper we construct and study a holographic model which exhibits either PDW or co-existing SC+CDW orders, depending on the parameters in the theory. To our knowledge this is the first holographic setup to realize a PDW. While both PDW and SC+CDW break translational invariance and a $U(1)$ symmetry spontaneously, there is a key difference between them. In a PDW the superconducting order parameter varies periodically as a function of position, but does so with a zero average, *e.g.* $\langle O_\chi \rangle \propto \cos(k\,x)$. Moreover, in such a phase the charge density, which is also modulated, has a period which is *half* of that of the scalar condensate, *e.g.* $\rho(x) = \rho_0 + \rho_1 \cos(2k\,x)$. In contrary, a SC+CDW state has a uniform component to the condensate, which oscillates at the same frequency as the charge density.

In our construction the $U(1)$ and translational symmetries are broken spontaneously at the same time. The set-up we adopt includes, in addition to gravity, two real scalar fields χ and θ and two vector fields A_μ and B_μ. The couplings between the scalars and the gauge fields can be generated via the Stückelberg mechanism. Indeed, our theory is not of the form of the standard holographic superconductor [9, 10], but rather falls within the *generalized* class of models advocated for in [11]. The more general structure of the scalar couplings allows us to break the desired symmetries without the need to introduce additional fields.

Here the presence of two vector fields (and the interaction between them) is crucial for obtaining the symmetry breaking features we are after. The role of the gauge field A_μ is transparent, since it provides a finite charge density ρ_A whose modulations agree with the behavior of a PDW or CDW state. What distinguishes whether the system is described by a PDW or by SC+CDW is whether the scalar χ is charged or not under the second vector field B_μ. The physical interpretation of B_μ depends on details of the model. In particular, when the field is massless it can be associated with spin degrees of freedom, and the modulations in its density ρ_B could characterize SDW order.

Before discussing our model we should mention that striped orders in holographic superconductors have been studied in a variety of setups, starting with [12], in which an inhomogeneous phase was sourced by a modulated chemical potential. There have been many generalizations since then. In particular, a study of backreaction in the presence of a periodic potential was initiated in [13]. However, in these setups the breaking of transla-

*Electronic address: cremonini@lehigh.edu
†Electronic address: lil416@lehigh.edu
‡Electronic address: jie.ren@mail.huji.ac.il

tional invariance was explicit and not spontaneous. Holographic superconductors with spontaneously generated helical structure were reported in [14, 15]. The competition between superfluid and striped phases has been examined within the context of holography, see [16, 17] for top-down models. The spontaneous formation of striped order in a holographic model with a scalar coupled to two $U(1)$ gauge fields was first studied in [18] and more recently in [19–22] (note that these models preserve the $U(1)$ symmetry).

Here we have extended such constructions by simultaneously breaking both symmetries spontaneously, and focusing on the differences between a scalar condensate with PDW vs. CDW order. Moreover, we have recently seen in a number of holographic models of strongly correlated electrons the advantage of using multiple vector fields, as they typically lead to richer physics, *e.g.* [22–25]. In particular, such a picture was used to construct phase diagrams that are similar to those of high T_c superconductors as well as other strange metal materials in [22]. Our construction provides a further example of this idea. Note that while in our analysis the mass of the vector B_μ does not affect any of the physics in a qualitative way, it is expected to play a role for applications to transport. It would be interesting to study the effects of disorder on the PDW state, as well as the consequences of stripe order on the conductive properties of the system and on fermion spectral functions. We leave these questions to future work. A more detailed analysis for this model will appear in [26].

II. HOLOGRAPHIC SETUP

We choose our theory $S = \int d^4x\sqrt{-g}\,\mathcal{L}$ to describe gravity coupled to two real scalar fields χ and θ, and two vector fields A_μ and B_μ,

$$\mathcal{L} = \mathcal{R} + \frac{6}{L^2} - \frac{1}{2}(\partial\chi)^2 - \frac{Z_A}{4}F^2 - \frac{Z_B}{4}\tilde{F}^2 - \frac{Z_{AB}}{2}F\tilde{F}$$
$$-\mathcal{K}(\chi)(\partial_\mu\theta - q_A A_\mu - q_B B_\mu)^2 - \frac{m_v^2}{2}B^2 - \frac{m^2}{2}\chi^2, \quad (1)$$

with $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $\tilde{F}_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ denoting the field strengths of the two vectors, and $F\tilde{F} = F_{\mu\nu}\tilde{F}^{\mu\nu}$ for short. We take the gauge field couplings Z_A, Z_B, Z_{AB} to depend on χ, and in particular chose them so that in the limit $\chi \to 0$ they take the form

$$Z_A = 1 + \frac{a}{2}\chi^2 + \mathcal{O}(\chi^3)\,, \quad Z_B = 1 + \mathcal{O}(\chi^2)\,,$$
$$Z_{AB} = c\,\chi + \mathcal{O}(\chi^2)\,, \quad \mathcal{K} = \frac{1}{2}\chi^2 + \mathcal{O}(\chi^3)\,, \quad (2)$$

with (a, c) constants. We note that the c parameter which controls the interaction $\sim Z_{AB}$ between the two fluxes will play a crucial role in the breaking of translational invariance.

While in general we will assume that χ is charged under both $U(1)$ fields, we will see that the $q_B = 0$ case plays a special role, as it is associated with a PDW condensate. On the other hand, $q_B \neq 0$ will describe a state with SC+CDW order. Finally, note that while the current dual to A_μ is conserved, the same is not always true for the current dual to B_μ, because of the mass term m_v^2. Although in this paper we consider both massless and massive cases for the sake of completeness, they lead to the same qualitative results. On the other hand the mass parameter m_v^2 is expected to affect *e.g.* the transport properties of the system, which we plan to study in future work.

We are interested in considering two classes of background solutions to this system. The first one is the electrically charged AdS Reissner-Nordström (AdS-RN) black brane only supported by A_μ,

$$ds^2 = \frac{1}{f(r)}dr^2 - f(r)dt^2 + \frac{r^2}{L^2}(dx^2 + dy^2)\,,$$
$$f(r) = \frac{r^2}{L^2}\left(1 - \frac{r_h^3}{r^3}\right) + \frac{\mu^2 r_h^2}{4r^2}\left(1 - \frac{r}{r_h}\right)\,, \quad (3)$$
$$A_t = \mu\left(1 - \frac{r_h}{r}\right)\,,$$

where r_h is the horizon, μ the chemical potential and other fields are trivial. This background will describe the high temperature phase in which the dual theory possesses a global $U(1)$ symmetry, associated with the gauge field A_μ. The black brane temperature reads $T = \frac{12r_h^2 - \mu^2 L^2}{16\pi L^2 r_h}$, and in the extremal limit $T = 0$ the near horizon geometry becomes that of $AdS_2 \times R^2$,

$$ds^2 = \frac{L^2}{6\tilde{r}^2}d\tilde{r}^2 - \frac{6\tilde{r}^2}{L^2}dt^2 + \frac{r_h^2}{L^2}\,d\vec{x}^2\,, \quad A_t = \frac{2\sqrt{3}}{L}\tilde{r}\,, \quad (4)$$

with $\tilde{r} = r - r_h$ and the AdS_2 radius $L_{(2)} = L/\sqrt{6}$.

We will then examine solutions with a non-trivial profile for χ and B_μ. These will describe the formation of a scalar condensate in the low temperature regime of the dual field theory, and provide holographic probes of phases with a broken $U(1)$ symmetry. Moreover, by allowing for modes which source spatial modulations, we will trigger instabilities to striped superconducting phases. The detailed structure of the modulations of the condensate and charge densities will be sensitive to q_B as well as the parameters in the theory, as we will see shortly.

III. STRIPED INSTABILITIES

To determine whether in this model we can spontaneously break translational invariance at the same time as the $U(1)$ symmetry, we need to examine the spatially modulated static mode in the spectrum of fluctuations around the unbroken phase. Our strategy will be to first consider instabilities arising from the IR $AdS_2 \times R^2$ geometry, and to construct analytically momentum-dependent modes which violate the IR AdS_2 BF bound. The presence of such modes is a strong indication that there should be a region in which one has superconducting order that is spatially modulated – a striped superconductor. We will then move on to studying numerically the behavior of the perturbations and of the condensate at finite temperature.

A. Instabilities of the IR $AdS_2 \times R^2$ geometry

We are now ready to examine instabilities of the electrically charged AdS-RN black blane (3). We start from the $AdS_2 \times R^2$ background (4) which arises as the IR limit of the zero temperature AdS-RN geometry, and turn on the following two spatially modulated perturbations,

$$\delta\chi = \varepsilon\, w(r)\cos(k\,x), \quad \delta B_t = \varepsilon\, b_t(r)\cos(k\,x)\,, \tag{5}$$

where we have relabeled $\tilde{r} \to r$ for convenience, and ε is a formal perturbative expansion parameter. By substituting into the equations of motion and working at linear level in ε, we obtain the two coupled equations

$$\frac{6}{L^2}(r^2 w')' - \frac{2\sqrt{3}\,c}{L} b_t' - \left(M_{(2)}^2 + \frac{k^2 L^2}{r_h^2}\right) w \;=\; 0\,, \tag{6}$$
$$\frac{6r^2}{L^2} b_t'' + \frac{12\sqrt{3}\,c\,r^2}{L^3} w' - \left(m_v^2 + \frac{k^2 L^2}{r_h^2}\right) b_t \;=\; 0\,, \tag{7}$$

with $M_{(2)}^2 = m^2 - \frac{6a}{L^2} - 2q_A^2$ and (a,c) as defined in equation (2). We make the further ansatz

$$w(r) = v_1\, r^{\lambda}\,, \quad b_t(r) = v_2\, r^{\lambda+1}\,, \tag{8}$$

where v_1, v_2 are constants and λ denotes the scaling dimension of an IR operator in the one-dimensional CFT dual to the AdS_2 geometry. The linearized equations can then be written in matrix form, solving which we find

$$\lambda_+^{\pm} = -\frac{1}{2} + \sqrt{\frac{1}{4} + m_\pm^2}\,, \quad \lambda_-^{\pm} = -\frac{1}{2} - \sqrt{\frac{1}{4} + m_\pm^2}\,, \tag{9}$$

with

$$m_\pm^2 = \frac{L^2}{12}\left[M_{(2)}^2 + m_v^2 + 12\frac{c^2}{L^2} + 24k^2\right]$$
$$\pm \frac{L^2}{12}\sqrt{(M_{(2)}^2 - m_v^2)^2 + 24\frac{c^2}{L^2}(M_{(2)}^2 + m_v^2 + 6\frac{c^2}{L^2} + 24k^2)}$$

where we have fixed the chemical potential to be $\mu = 1$.

The onset of the instability associated with the violation of the AdS_2 BF bound is linked to λ becoming imaginary, *i.e.* when $m_-^2 < -\frac{1}{4}$. For striped instabilities, one needs a non-zero wave number k at which the value of λ is imaginary, for a fixed choice of Lagrangian parameters. By inspecting the form of m_-^2, one can check explicitly that this is clearly possible for various parts of the parameter space. As a specific example, for the parameters chosen in the finite temperature analysis below (*e.g.* $m^2 = -8, q_A = 1, m_v^2 = 0, L = 1/2, a = 4, |c| = 2.34$), we find a momentum range $0.99 < |k| < 2.62$ in which the modes violate the BF bound, and are associated with spatially modulated phases. We come back to this point in greater detail in the numerical analysis below.

B. Critical Temperature

The instabilities of the IR AdS_2 solutions that we have just discussed occur at zero temperature. Nevertheless, they suggest that analogous instabilities should appear in the black brane background (3) at finite temperature. Next, we shall calculate the critical temperature T_c below which the AdS-RN geometry becomes unstable, as a function of wave number k. In particular, if the scalar field instabilities are associated with a finite value of k, we will have found a striped condensate. Note that to obtain T_c it is sufficient to work to linear order in perturbations.

Motivated by the AdS_2 analysis, we turn on the same fluctuations as in (5). By expanding around the AdS-RN background, one then obtains two coupled linear ODEs,

$$w'' + \left(\frac{2}{r} + \frac{f'}{f}\right) w' + \frac{c\mu r_h}{r^2 f} b_t'$$
$$-\frac{1}{f}\left(m^2 - \frac{\kappa\, q^2 \mu^2 (r - r_h)^2}{r^2 f} + \frac{k^2 L^2}{r^2} - \frac{a\mu^2 r_h^2}{2r^4}\right) w = 0\,,$$
$$b_t'' + \frac{2}{r} b_t' + \frac{c\mu r_h}{r^2} w' - \frac{1}{f}\left(m_v^2 + \frac{k^2 L^2}{r^2}\right) b_t = 0\,, \tag{10}$$

which can be solved numerically. We demand the fluctuations to be regular at the horizon at $r = r_h$, with

$$w(r) = w^h + \mathcal{O}(r - r_h)\,,\; b_t(r) = b_t^h (r - r_h) + \mathcal{O}(r - r_h)^2\,.$$

On the other hand their $r \to \infty$ UV expansion is

$$\begin{aligned} w(r) &= \frac{w_s}{r^{3-\Delta_\chi}}(1 + \cdots) + \frac{w_v}{r^{\Delta_\chi}}(1 + \cdots)\,, \\ b_t(r) &= \frac{b_s}{r^{2-\Delta_B}}(1 + \cdots) + \frac{b_v}{r^{\Delta_B - 1}}(1 + \cdots)\,, \end{aligned} \tag{11}$$

where the quantities $\Delta_\chi = \frac{1}{2}(3 + \sqrt{9 + 4m^2 L^2})$ and $\Delta_B = \frac{1}{2}(3 + \sqrt{1 + 4m_v^2 L^2})$ are, respectively, the scaling dimensions of the scalar operator dual to χ and vector operator dual to B_μ. Since we are only interested in breaking both symmetries spontaneously, we turn off the parameters w_s and b_s, which correspond to the sources for the operators in the dual field theory.

After fixing theory parameters, for a given wave number k we expect there to be a normalizable zero mode appearing at a particular temperature. We choose the χ mass term to be $m^2 L^2 = -2$, so that $\Delta_\chi = 2$, and consider two separate cases for the second vector field B_μ. We first take it to be massless, $m_v^2 = 0$, so that $\Delta_B = 2$ and the associated current is conserved. We then consider the case in which it is massive, choosing $m_v^2 L^2 = 0.11$ in our numerics, corresponding to

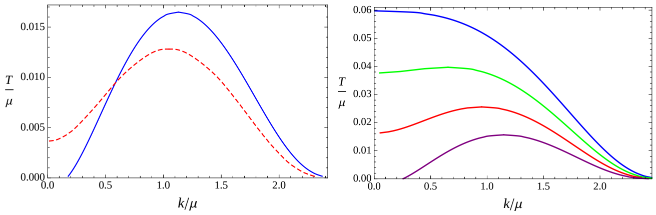

FIG. 1: Critical temperature as a function of wave number, for the onset of striped instabilities. Left panel: the solid blue line describes the massless case $m_v^2 = 0$ with $|c| = 2.34$, while the dashed red line the massive case $m_v^2 = 0.44$ with $|c| = 2.46$. Right panel: dependence on the coupling c. From top to bottom $|c| = 2.05, 2.15, 2.25, 2.35$. In both figures the remaining parameters are chosen to be $m^2 = -8, L = 1/2, a = 4, q_A = \mu = 1$.

$\Delta_B = 2.1$. For both scenarios we see the onset of a phase transition, as shown by the formation of a scalar condensate at T_c.

We show the dependence of the temperature on wave number in Fig. 1. In particular, the curves in the left panel exhibit clearly the bell curve behavior – the fact that they are peaked at non-zero values of k shows that the condensate is driven by the momentum-dependent spatial modulations. The right panel of Fig. 1 shows the dependence of T_c on the strength c of the coupling $Z_{AB} \sim c\,\chi$ between the two gauge fields. We would like to point out that as this coupling decreases, the effect of the spatial modulation also decreases – one may still have a superconducting instability, but not striped. Thus, in this model in order to ensure that the phase transition indeed occurs at finite values of k, the coupling must be non-zero and in fact sufficiently large. However, when $|c|$ becomes too large the instability once again disappears – the BF bound can no longer be violated.

IV. PAIR AND CHARGE DENSITY WAVES

In our model at low temperatures the scalar operator $\mathcal{O}_\chi$ dual to χ acquires a spatially modulated expectation value spontaneously, breaking the $U(1)$ symmetry. Thus, the spatially modulated phase is always associated with a non-vanishing superconducting condensate. Moreover, the "charge" density ρ_B associated with B_μ becomes spatially modulated, and this, in conjunction with $\langle \mathcal{O}_\chi \rangle$, induces a modulation in the charge density ρ_A dual to A_μ. While the second gauge field B_μ does not determine the type of order (PDW or SC+CDW) developed in the system, it can in principle be associated with spin degrees of freedom, with its modulated density ρ_B describing SDW order.

As we have already mentioned, in a system with PDW order

- the average value of the superconducting order parameter $\langle O_\chi \rangle$ vanishes
- the charge density oscillations have half the period of those of the scalar condensate

Thus, a PDW differs from a state with co-existing SC+CDW orders, in which the scalar condensate has a uniform component. In our holographic model both of these features can be reproduced, along with the spontaneous – and simultaneous – breaking of the $U(1)$ symmetry and of translational invariance. In particular, we find that when $q_B = 0$ the scalar condensate and the charge density ρ_A associated with the first vector field A_μ satisfy the conditions required for PDW order. On the other hand, when $q_B \neq 0$ we find a state with SC + CDW order.

We have studied backreaction in our system numerically, focusing on the behavior of the scalar condensate $\langle \mathcal{O}_\chi \rangle$ and of the two charge densities ρ_A and ρ_B. We work in the grand canonical ensemble by setting $\mu = 1$ and as an example, we choose the parameters in (2) to be $m^2 = -8, m_v^2 = 0, L = 1/2, c = -2.34, a = 4, q_A = 1$. We focus on the branch of solutions with $k = 1$ and find a second order phase transition at $T_c = 0.01608$. To gain intuition for our results, one can compare our numerics with a next-to-leading order perturbative analysis in ε, which in our case can be taken to be $\propto \sqrt{1 - T/T_c}$ and measures how close T is to T_c,

$$\begin{aligned}
\delta\chi &= \varepsilon\, w(r)\cos(k\,x) + \varepsilon^2[\chi^{(1)}(r) + \chi^{(2)}(r)\cos(2k\,x)]\,,\\
\delta B_t &= \varepsilon\, b_t(r)\cos(k\,x) + \varepsilon^2[b_t^{(1)}(r) + b_t^{(2)}(r)\cos(2k\,x)]\,,\\
\delta A_t &= \varepsilon^2[a_t^{(1)}(r) + a_t^{(2)}(r)\cos(2k\,x)]\,,
\end{aligned} \tag{12}$$

where we are singling out the perturbations of the scalar and vector fields for the sake of space.

We find that the order $\mathcal{O}(\varepsilon^2)$ components of $\delta\chi$ and δB_t are sourced by $\mathcal{O}(\varepsilon)$ terms proportional to $q_A\,q_B$, and therefore vanish when $q_B = 0$. In particular, note that this implies that the homogenous perturbations $\chi^{(1)}(r)$ and $b_t^{(1)}(r)$ both vanish when $q_B = 0$, causing the scalar condensate $\langle O_\chi \rangle$ modulations (which to leading order are $\propto \cos(k\,x)$) to average out to zero. Note that by the same argument the oscillations of the charge density ρ_B also average out to zero. Their period agrees with that of the scalar condensate, which is consistent with SDW order in a PDW. On the other hand, since we are working at finite charge density with respect to A_μ, the charge density ρ_A always has a uniform component in our model.

Thus, the perturbative analysis suggests that the system behaves like a PDW when $q_B = 0$ (no uniform component to $\langle O_\chi \rangle$), while when $q_B \neq 0$ it describes a SC+CDW state (the uniform contribution $\propto \chi^{(1)}(r)$ is sourced). This behavior is precisely confirmed by our numerics, as visible clearly in Fig. 2, which shows the oscillatory pattern of the scalar condensate $\langle O_\chi \rangle$ when $q_B = 0$ (solid line) versus $q_B \neq 0$ (dashed line). In the former case the average value of the order parameter vanishes, but not in the latter.

In order to have PDW order the period of the charge density must be one half of that of the scalar condensate.

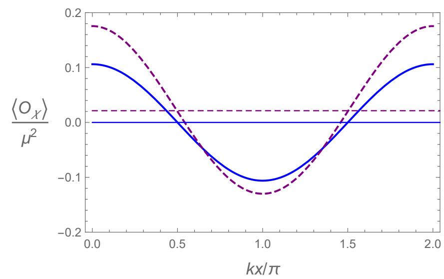

FIG. 2: The scalar condensate for $T = 0.01571$. The solid blue curve corresponds to $q_B = 0$, while the dashed purple line to $q_B = 1/2$. The two horizontal lines denote the average values of the condensate in each case. Note that the average is zero only for $q_B = 0$.

This is precisely what happens in our model when $q_B = 0$, as shown in Fig. 3, where we clearly see that ρ_A (dashed line) oscillates twice as fast as the scalar condensate (solid line).

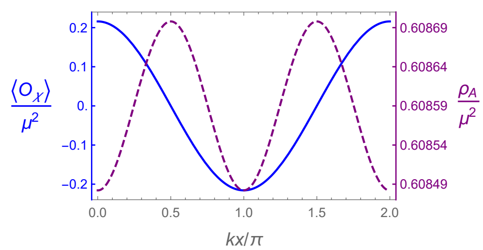

FIG. 3: The charge density ρ_A (dashed purple line) associated with the A_μ gauge field, plotted against the scalar condensate (solid blue line) for $q_B = 0$. The period associated with ρ_A is one half of that of the scalar condensate. We have chosen $T = 0.01427$.

This result, which we have found numerically, can also be understood by inspecting the $\varepsilon^2 \cos(2k\,x)$ term in the perturbation δA_t, which is sourced by the product of the two $\mathcal{O}(\varepsilon)$ terms in $\delta\chi$ and δB_t. Since the oscillation of ρ_A is a next-to-leading order effect, which is sourced by the leading order oscillations of χ and B_t, this particular feature of the PDW order is in some sense *induced*. We have also verified that for $q_B = 0$ the frequency of the oscillations of the density ρ_B is one half of that of ρ_A. We note that a similar doubling of frequencies was also seen in [21] in the behavior of the magnetization densities.

On the other hand, when $q_B \neq 0$ the frequency of the oscillations of ρ_A is the same as that of the condensate, which now has a uniform component. Thus, what we have is a co-existing SC+CDW state, and not a PDW. This is shown clearly in Fig. 4, in which ρ_A and $\langle O_\chi \rangle$ have the same period. From the next-to-leading order perturbative analysis this is not quite clear. However, a $\cos(k\,x)$ mode is expected to appear at $\mathcal{O}(\varepsilon^3)$ from the terms $\omega(r) b_t^{(1)}(r)\cos(k\,x)$ or $b_t(r)\chi^{(1)}(r)\cos(k\,x)$ which are present when $q_B \neq 0$. Indeed, we have confirmed this in our numerics.

A more detailed analysis of this system will appear in [26], where we will include the behavior of the background geometry and the thermodynamics. While in this paper the matter content was chosen for its simplicity, more complicated models with additional fields can in principle be constructed, in which the superconducting state is associated with the condensation of a *complex* scalar. Moreover, at very low temperatures the physics encoded in this model may be richer than what our preliminary analysis has shown. A very interesting question is that of the nature of the ground state once striped superconducting order develops. Finding the fully backreacted geometry at zero temperature remains a challenge.

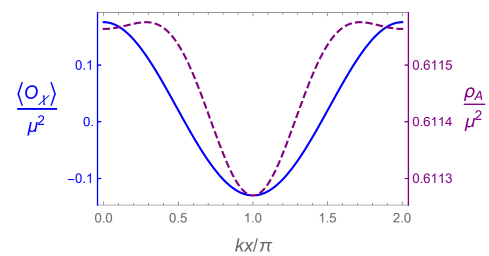

FIG. 4: The scalar condensate (solid blue line) and charge density ρ_A (dashed purple line) for $q_B = 1/2$ at $T = 0.01571$. The two share the same period.

V. ACKNOWLEDGMENTS

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