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Classical enhancement of quantum vacuum fluctuations

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Abstract

We propose a mechanism for the enhancement of vacuum fluctuations by means of a classical field. The basic idea is that if an observable quantity depends quadratically upon a quantum field, such as the electric field, then the application of a classical field produces a cross term between the classical and quantum fields. This cross term may be significantly larger than the purely quantum part, but also undergoes fluctuations driven by the quantum field. We illustrate this effect in a model for lightcone fluctuations involving pulses in a nonlinear dielectric. Vacuum electric field fluctuations produce fluctuations in the speed of a probe pulse, and form an analog model for quantum gravity effects. If the material has a nonzero third-order susceptibility, then the fractional light speed fluctuations are proportional to the square of the fluctuating electric field. Hence the application of a classical electric field can enhance the speed fluctuations. We give an example where this enhancement can be an increase of one order of magnitude, increasing the possibility of observing the effect.

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Quantum field fluctuations, such as vacuum fluctuations of the electromagnetic field, are responsible for a variety of physical effects, including the Lamb shift and the Casimir effect. In some cases, the effect can be expressed in terms of a time average of a field operator or of a square of the operator. For example, the one-loop QED vertex correction to quantum potential scattering can be interpreted as due to fluctuations of the averaged electric field [1]. Other effects, such as those associated with the stress tensor, are quadratic in the fields. Quantum fluctuations of the gravitational field can be of either variety, and may lead to lightcone fluctuations [2]. The active fluctuations of spacetime geometry due the quantum nature of gravity itself can be linear, whereas the passive fluctuations driven by quantum stress tensor fluctuations are quadratic in the matter fields. Both types of lightcone fluctuations can be modeled by nonlinear optical materials, where fluctuations of either the electric field or the squared electric field can produce fluctuations in the speed of a probe pulse [3–6].

In systems where the observable effect depends nonlinearly upon the field, it may be possible to enhance the fluctuations by the application of a classical field. Consider the square of the electric field, E^2, and suppose that $\mathbf{E} = \mathbf{E}_C + \mathbf{E}_Q$, where $\mathbf{E}_C$ is a classical electric field, and $\mathbf{E}_Q$ is the fluctuating quantum field. Then $E^2 = E_C^2 + 2\mathbf{E}_C \cdot \mathbf{E}_Q + E_Q^2$. The square of the classical field does not fluctuate, and E_Q^2 describes the effect in the absence of the classical field. If we can arrange that $2|\mathbf{E}_C \cdot \mathbf{E}_Q| > E_Q^2$, then the quantum fluctuation effects can be enhanced by the presence of the classical field. We can make this statement more precise by relating the observable quantities to expectation values of time averages of the quantum fields. Note that in the vacuum state, the expectation values of the quantum field, and hence of the cross term vanish, $\langle \mathbf{E}_Q \rangle = \langle \mathbf{E}_C \cdot \mathbf{E}_Q \rangle = 0$. However, the classical field does give a nonzero contribution to the variance of E^2. Here we treat the explicit example of the nonlinear optics model for lightcone fluctuations.

In a nonlinear material, the presence of a background field, $\mathbf{E}^0$, can alter the effective index of refraction and hence the speed of propagation of a probe pulse through the material. The change in the effective index of refraction can be linear in the background field (Pockels effect), or quadratic in this field (Kerr effect). If the background field fluctuates, then the propagation speed will also be subject to fluctuations. The optical properties of a material are described by the various susceptibilities which appear in the induced polarization, or dipole moment density. Unless stated otherwise, we use Lorentz-Heaviside units with $\hbar =$

$c = 1$. The conversion between these units and SI units can be facilitated by noting that in our units, $\epsilon_0 \approx 8.85 \times 10^{-12}\,\mathrm{C}^2/(\mathrm{N\,m}^2) = 1$, implying that $1V \approx 1.67 \times 10^7 \mathrm{m}^{-1}$.

The induced polarization vector $\mathbf{P}$ of a nonlinear optical material, can be expanded as a power series in the electric field as

$$P_i = \chi^{(1)}_{ij} E_j + \chi^{(2)}_{ijk} E_j E_k + \chi^{(3)}_{ijkl} E_j E_k E_l + \cdots \,, \tag{1}$$

where $\chi^{(1)}_{ij}$ are the components of the linear susceptibility tensor, while $\chi^{(2)}_{ijk}$ and $\chi^{(3)}_{ijkl}$, are the second- and third-order nonlinear optical susceptibilities of the medium [7], respectively. We use the convention that Latin indices i, j, k, ... run from 1 to 3, and repeated indices are summed upon. The susceptibilities are generally dependent on frequency. However most materials exhibit approximately constant susceptibilities within a certain range of frequencies, defining a dispersionless regime, which will be assumed here. We follow the procedure in Refs. [3–6], in which the electric field is written as a superposition of a background field $\mathbf{E}^0$ and a smaller, but more rapidly varying probe field $\mathbf{E}^{\mathrm{P}}$. To leading order, the probe field satisfies a linear wave equation. If we take this field to be propagating in the x-direction, but have linear polarization in the z-direction, $\mathbf{E}^{\mathrm{P}} = E^{\mathrm{P}}(x,t)\hat{\mathbf{z}}$, the equation is

$$\frac{\partial^2 E^{\mathrm{P}}}{\partial x^2} - \frac{1}{v_{ph}^2}\frac{\partial^2 E^{\mathrm{P}}}{\partial t^2} = 0\,. \tag{2}$$

Here

$$v_{ph}^2 = \frac{1}{{n_p}^2}\left[1 + 2\gamma_i E^0_i + 3\gamma_{ij} E^0_i E^0_j\right]^{-1}\,, \tag{3}$$

where

$$\gamma_i = \frac{\chi^{(2)}_{z(zi)}}{{n_p}^2}\,, \tag{4}$$

and

$$\gamma_{ij} = \frac{1}{{n_p}^2}\left(\frac{\chi^{(3)}_{zzij} + \chi^{(3)}_{zizj} + \chi^{(3)}_{zijz}}{3}\right)\,, \tag{5}$$

with $\chi^{(2)}_{z(zi)} = (\chi^{(2)}_{zzi} + \chi^{(2)}_{ziz})/2$. That is, the parentheses denote symmetrization on the pair of enclosed indices.

In a dispersionless regime, the phase velocity v_{ph} is also approximately the group velocity of wave packets, and the flight time is proportional to $\int dx/v_{ph}$. Following Refs. [5, 6] we introduce a sampling function $F(x)$ which describes the density profile of a slab of material and also acts as a switching function for the electric field fluctuations. It has the normalization

$$\int_{-\infty}^{\infty} F(x)\, dx = d\,, \tag{6}$$

where d is the effective width of the slab. The flight time operator is given by

$$t_d = n_p \int_{-\infty}^{\infty} \left[1 + \gamma_i E^0_i(\mathbf{x},t) + \mu_{ij} : E^0_i(\mathbf{x},t) E^0_j(\mathbf{x},t) :\right] F(x)\, dx\,, \tag{7}$$

where an expansion to second order in E^0 has been performed. Here we take quadratic operators to be normal ordered, and define

$$\mu_{ij} = \frac{1}{2}\left(3\gamma_{(ij)} - \gamma_i \gamma_j\right)\,. \tag{8}$$

Thus the probe pulse flight time can depend nonlinearly upon the background field.

In Refs. [5, 6], $\mathbf{E}^0$ was taken to be the quantized electric field operator, and the state to be the vacuum, so the background field arises from vacuum fluctuations. Now we wish to add a classical electric field $\mathbf{E}^C$ and write

$$\mathbf{E}^0 = \mathbf{E}^C + \mathbf{E}^Q\,, \tag{9}$$

where now $\mathbf{E}^Q$ is the quantized electric field operator

The fight time operator can be written as $t_d = t^C_d + t^Q_d + t^{Cross}_d$, where

$$t^C_d = n_p \int_0^d \left(1 + \gamma_i E^C_i + \mu_{ij} E^C_i E^C_j\right) F(x) dx, \tag{10}$$

$$t^Q_d = n_p \int_0^d \left(\gamma_i E^Q_i + \mu_{ij} : E^Q_i E^Q_j :\right) F(x) dx, \tag{11}$$

$$t^{Cross}_d = 2n_p \int_0^d \mu_{ij} E^C_i E^Q_j F(x) dx. \tag{12}$$

Note that t^{Cross}_d is a cross term coupling classical and quantum contributions of the background electric field. The vacuum expectation value of t_d is just t^C_d, as $\langle t^Q_d \rangle = 0$ and $\langle t^{Cross}_d \rangle = 0$. However, as t^C_d is a c-number, only t^Q_d and t^{Cross}_d contribute to the variance of the flight time, i.e.,

$$\begin{aligned}(\Delta t_d)^2 &= \langle {t_d}^2 \rangle - \langle t_d \rangle^2 = \langle (t^Q_d + t^{Cross}_d)^2 \rangle = \\ &= {n_p}^2 \int_{-\infty}^{\infty} dx\, F(x) \int_{-\infty}^{\infty} dx'\, F(x') \Big[\Gamma_i\, \Gamma_j\, \langle E^Q_i(\mathbf{x},t) E^Q_j(\mathbf{x}',t') \rangle \\ &\quad + \mu_{ij}\mu_{lm} \langle : E^Q_i(\mathbf{x},t) E^Q_j(\mathbf{x},t) :: E^Q_l(\mathbf{x}',t') E^Q_m(\mathbf{x}',t') : \rangle\Big]. \end{aligned} \tag{13}$$

where we have defined

$$\Gamma_i = \gamma_i + 2\mu_{ki} E_k^C \,. \tag{14}$$

Next, we use Wick's theorem to simplify the last term in Eq. (13), and introduce the needed correlation functions of the electric field for a non-dispersive isotropic material with refractive index n_b [6]. We then obtain

$$(\Delta t_d)^2 = \int_{-\infty}^{\infty} dx\, F(x) \int_{-\infty}^{\infty} dx' F(x') \left[\frac{A}{(\Delta x)^4} + \frac{\alpha}{(\Delta x)^8}\right], \tag{15}$$

where we have defined the A and α parameters by

$$A = \frac{n_b {n_p}^2}{\pi^2 \left({n_p}^2 - {n_b}^2\right)^2} \left[\Gamma_x^2 + \left(\Gamma_y^2 + \Gamma_z^2\right) \frac{\left({n_p}^2 + {n_b}^2\right)}{\left({n_p}^2 - {n_b}^2\right)}\right], \tag{16}$$

$$\begin{aligned}\alpha = &\frac{2{n_b}^2 {n_p}^2}{\pi^4 \left({n_p}^2 - {n_b}^2\right)^4} \left[2\left(\mu_{xy}^2 + \mu_{xz}^2\right) \frac{\left({n_p}^2 + {n_b}^2\right)}{\left({n_p}^2 - {n_b}^2\right)}\right. \\ &\left. + \mu_{xx}^2 + \left(\mu_{yy}^2 + \mu_{zz}^2 + 2\mu_{zy}^2\right) \frac{\left({n_p}^2 + {n_b}^2\right)^2}{\left({n_p}^2 - {n_b}^2\right)^2}\right].\end{aligned} \tag{17}$$

Now, in order to describe the physical transition experienced by the probe pulse as it enters and exits the optical material, we choose the two parameter switching function [6] $F(x) = F_{b,d}(x)$, where

$$F_{b,d}(x) = \frac{1}{\pi}\left[\arctan\left(\frac{x}{b}\right) + \arctan\left(\frac{d-x}{b}\right)\right], \tag{18}$$

which satisfies Eq. (6). Here, parameter d describes the width of $F_{b,d}(x)$, while b determine how fast the function rises and falls. For instance when $b \to 0$ we recover a step function,

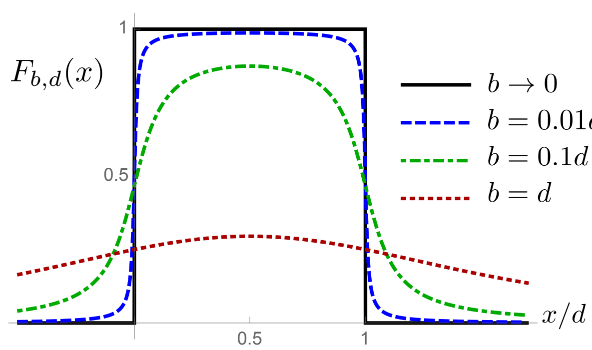

FIG. 1: Plots of $F_{b,d}(x)$ for few values of b/d. Notice that as $b/d \to 0$ the switching function approaches a step function.

which describes a sudden transition between the different regimes, and for $b \approx 0.9d$ we find a broad function similar to a Lorentzian. Figure 1 depicts the behavior of $F_{b,d}(x)$ for some representative values of the ratio b/d. We are especially interested in the case where $0 < b \ll d$, so the transition occurs over a finite region smaller than the width. The derivative of $F_{b,d}(x)$ with respect to x is a sum of two Lorentzian functions. Assuming that $\Delta x = x - x' - i\varepsilon$, with $\varepsilon > 0$, the integrals in Eq. (15) can be evaluated by means of residue theorem, which leads to

$$(\Delta t_d)^2 = \frac{d^2(d^2+12b^2)}{12b^2(d^2+4b^2)^2}\,A + \frac{d^2(21504b^{10}+1344b^6d^4+240b^4d^6+24b^2d^8+d^{10})}{1344b^6(4b^2+d^2)^6}\,\alpha, \quad (19)$$

where we assumed the classical field to be a constant. We now define the fractional fluctuations in the flight time, δ, by

$$\delta^2 = \frac{(\Delta t_d)^2}{\langle t_d\rangle^2}\,. \quad (20)$$

We will be interested in the regime of $b/d << 1$, for which

$$\delta^2 \approx \frac{A}{12{n_p}^2d^4}\left(\frac{d}{b}\right)^2 + \frac{\alpha}{1344{n_p}^2d^8}\left(\frac{d}{b}\right)^6\,. \quad (21)$$

In the case of a crystal possessing spatial inversion symmetry (a centrosymmetric material), the second order polarizability must vanish [7]. We can see this from Eq. (1), which must be invariant under change of the signs of the applied electric field and of the polarization vector for such a material, leading to $\chi^{(2)}_{ijk} = 0$. Then $\gamma_i = 0$, which leads to $\mu_{ij} = (3/2)\gamma_{(ij)}$ and $\Gamma_i = 3\gamma_{(ij)}E^C_j$. In this case, the first term in Eq. (21) is proportional to $(E^C)^2$, and describes the effect of the classical electric field on the enhanced vacuum fluctuations. The second term is independent of the classical field, and describes the vacuum fluctuations in the absence of the classical field.

We may estimate the magnitudes of both effects for the case of silicon. This material has a third order susceptibility $\chi^{(3)}_{zzzz} \approx 2.80\times10^{-19}\mathrm{m^2V^{-2}}$ and a refractive index $n_b = 3.418$, both at wavelength $\lambda_b = 11.8\mu\mathrm{m}$ [8–10]. Assume that the probe field wave packet is prepared so that its peak wavelength is $\lambda_p = 1.4\mu\mathrm{m}$, for which $n_p = 3.484$ [11], and set $E^C_j = E^C\delta_{jz}$. This leads to

$$A \approx\; 5.07\times10^{-36}(E^C)^2(\mathrm{m^4/V^4})\,, \quad (22)$$

and

$$\alpha \approx\; 2.21\times10^{-34}(\mathrm{m^4/V^4})\,. \quad (23)$$

Now Eq. (21) may be expressed as

$$\delta^2 \approx 1.24 \times 10^{-17} \left(\frac{10\mu\text{m}}{d}\right)^4 \left(\frac{d}{b}\right)^2 \left(\frac{E^C}{E_{_{Break}}}\right)^2 + 1.74 \times 10^{-27} \left(\frac{10\mu\text{m}}{d}\right)^8 \left(\frac{d}{b}\right)^6 . \tag{24}$$

Here $E_{_{Break}} \approx 3.15 \times 10^7 \text{Vm}^{-1}$ is the breakdown electric field for silicon [12], and we need to require $E^C < E_{_{Break}}$. Now set $b = 0.01d$, which corresponds to the most rapid switching which is compatible with our neglect of dispersion [6]. Then we have the estimate

$$\delta^2 \approx 1.24 \times 10^{-13} \left(\frac{10\mu\text{m}}{d}\right)^4 \left(\frac{E^C}{E_{_{Break}}}\right)^2 + 1.74 \times 10^{-15} \left(\frac{10\mu\text{m}}{d}\right)^8 . \tag{25}$$

In the absence of the classical field, the vacuum effect gives $\delta \approx 4.2 \times 10^{-8}\,(10\mu\text{m}/d)^4$, as found in Ref. [6]. However, if $E^C \gtrsim 0.1\,E_{_{Break}}$, then the first term in Eq. (25) dominates, and we have

$$\delta \approx 3.52 \times 10^{-7} \left(\frac{10\mu\text{m}}{d}\right)^2 \left(\frac{E^C}{E_{_{Break}}}\right) . \tag{26}$$

Thus the presence of the classical electric field can enhance the fractional flight time variation due to vacuum fluctuation by close to one order of magnitude. This may aid experimental observation of vacuum driven lightcone fluctuations.

In this paper, we have been concerned with the effects of vacuum fluctuations, but have not explicitly considered the effects of finite temperature. Here we wish to give some estimates of when thermal fluctuations can be ignored. At temperature T, the thermal effects are characterized by the parameter $\beta = 1/(k_B T)$, where k_B is Boltzmann's constant, which has dimensions of time in units where $\hbar = 1$. Low temperature corresponds to large β, so if β is large compared to other time scales in a problem, then one can expect thermal effects to be small. This is seen, for example, in Ref. [13] where the combined thermal and vacuum effects on the Brownian motion of a charge near a mirror were calculated. These authors find that for motion on time scales less than β, vacuum fluctuations dominate over thermal fluctuations. In our problem, there are two relevant time scales. One is the time required for the probe pulse to pass through the slab of material, $t_d \approx n_p\, d$. If $b \ll d$, then there is shorter time, $t_b \approx n_p\, b$, required for the pulse to pass through the transition regions at either end of the slab. The effect of the classical electric field treated here is described by the first term on the right hand side of Eq. (25), which is proportional to $1/(b^2\, d^2)$. This suggests that the time scale associated with vacuum fluctuations is the geometric mean, $\sqrt{t_d\, t_b}$. A conservative estimate of the range of dominance of vacuum effects takes the longest of these scales, and estimates that vacuum effects dominate thermal ones if $\beta \gtrsim t_d$, or

$$T \lesssim 60K \left(\frac{3}{n_p}\right) \left(\frac{10\mu\text{m}}{d}\right) . \tag{27}$$

However, if the relevant criterion is $\beta \gtrsim \sqrt{t_d\, t_b}$, as the above argument suggests, then vacuum fluctuations can be dominant even at room temperature. This issue will be studied in more detail in future work.

In summary, we have constructed a specific model in which application of a classical field can enhance vacuum fluctuations effects. The extent to which this enhancement can be observed in an experiment remains to be seen. This model illustrates the general principle that fluctuations of a quantity which is quadratic (or higher power) in a quantum field can be enhanced by the presence of a classical field. This includes fluctuations of the stress tensor for a quantum field, which is typically quadratic in the field. Note that the probability distribution associated with the cross term between a quantum field and a classical field will be the Gaussian distribution for free field fluctuations. However, the probability distribution associated with quantities quadratic in quantum fields can be very different, and sensitive to the choice of sampling function. [14–16].

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