Single spin asymmetry in forward pA collisions. II. Fragmentation contribution

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Single spin asymmetry in forward pA collisions II: Fragmentation contribution

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We compute the twist-three fragmentation contribution to the transverse single spin asymmetry (SSA) in light hadron production $p^\uparrow p \to hX$ and $p^\uparrow A \to hX$ including the gluon saturation effect in the unpolarized nucleon/nucleus. Together with the results in our previous paper, this completes the full evaluation of the SSA in this process in the "hybrid" formalism. We argue that the dependence of SSAs on the atomic mass number in the forward region can elucidate the relative importance of the soft gluon pole contribution from the twist-three quark-gluon-quark correlation in the polarized nucleon and the twist-three fragmentation contribution from the final state hadron.

I. INTRODUCTION

Single-transverse spin asymmetries (SSAs) in inclusive hadron production in nucleon-nucleon scattering, $p^\uparrow p \to hX$, remain one of the long standing puzzles in hadron physics. In recent years, the physicists at the Relativistic Heavy Ion Collider (RHIC) have planned and explored the SSAs in the forward hadron production in nucleon-nucleus collisions, $p^\uparrow A \to hX$ [1, 2]. This will not only provide additional information on the underlying mechanism for the SSA phenomena, but also help us understand the small-x saturation of the gluon distributions in large nuclei.

In a previous paper [3], we have computed the SSA of light hadrons in proton-nucleus collisions $p^\uparrow A \to hX$ including the small-x gluon saturation effect in the nucleus. We adopted the so-called hybrid approach [4, 5] where the collinear twist-three Efremov-Teryaev-Qiu-Sterman (ETQS) functions [6, 7] are used on the polarized proton side and the unintegrated (k_T-dependent) gluon distribution is used on the nucleus side. We find that leading terms in the forward region come from the soft-gluon pole contributions of the twist-three ETQS matrix elements in the transversely polarized nucleon. In particular, the so-called derivative term will dominate the SSA in the forward region. From this we concluded that the asymmetry A_N does not depend on the saturation scale of the nucleus. Of course, for a complete evaluation in this hybrid approach, we also have to take into account the twist-three fragmentation function contributions. (See, also, [8].) The goal of this paper is to carry out this part of the calculation.

In the purely collinear framework, the twist-three fragmentation function contribution has been first studied in [9] and completed in [10] (see a recent review [11]). The gauge and Lorentz invariance of the result has been recently established [12]. In the forward region of $p^\uparrow A$ collisions, the saturation effect in the nucleus becomes important. The effect of saturation on the fragmentation contribution has been so far considered only in the k_T-factorization approach [13] which involves the Collins function [14]. However, in the Sivers-type contribution, we have found [3] that the k_T-factorization approach [15] misses the dominant derivative term. Whether this happens also in the fragmentation contribution is phenomenologically important, especially in view of the recent claim [16] that the SSA in $p^\uparrow p \to hX$ is completely dominated by the 'genuine twist-three' fragmentation function, with both the Sivers and Collins contributions playing only a minor role. However, the assumption of a large genuine twist-three fragmentation function made in [16] has not been tested yet because there are no other available experimental data sensitive to this function. In this paper, we show that the dependence of SSA on the mass number of the nucleus, as recently measured at RHIC [2], can be such a test.

In the hybrid formalism,[1] the single transverse spin-dependent cross section can be schematically written as

$$E_h \frac{d^3\Delta\sigma(p^\uparrow A \to hX)}{d^3\vec{P}_h} = \epsilon^{ij} S_{Ti} P_{hj} \int_{x_F} \frac{dz}{z^2} \Big\{ D_{h/q}(z) G_F(x_p, x_p) \otimes F(x_g, P_{hT}/z) + h_1(x_p)\hat{H}(z) \otimes F(x_g, P_{hT}/z) \Big\} . \quad (1)$$

The first term is what we have calculated in Ref. [3], and the second term is the object of this paper. In the above equation, S_T represents the traverse polarization vector of the projectile, P_{hT} is the transverse momentum of the final state hadron. $h_1(x_p)$ is the collinear leading-twist quark transversity distribution function and $D(z)$ is the leading-twist fragmentation function, whereas $G_F(x_p, x_p)$ and $\hat{H}(z)$ represent the twist-three ETQS distribution from the polarized nucleon and the twist-three fragmentation function, respectively. The small-x saturation physics is encoded in the unintegrated gluon distribution (or the dipole gluon distribution) $F(x_g, k_T)$. Although both of the contributions in (1) are classified as twist-three in the collinear approach, the underlying mechanisms are different. The twist-three terms associated with the incoming polarized nucleon comes from the initial/final state interaction effects which are necessary to generate a phase from the pole contributions. On the other hand, the twist-three fragmentation function contributions do not need a phase from the scattering amplitudes as we will show in the following calculations. Because of this difference, we expect that the two contributions depend differently on the saturation scale (or the atomic mass number).

The rest of the paper is organized as the following. In Section II, we compute the twist-three fragmentation contribution in the hybrid approach without including the saturation effect in the target. We explicitly check that, at large-P_{hT}, our result agrees with the previous result obtained in the collinear factorization framework [10]. We then include the saturation effects and present the complete formula in Section III. Finally in Section IV, we discuss the phenomenological consequences of our result.

II. FRAGMENTATION CONTRIBUTION TO SSA

In this section we compute the fragmentation contribution to SSA in the hybrid approach in the 'dilute' limit, i.e., without including the saturation effect in the target. Our starting point is Eq. (54) of Ref. [17] which was derived for semi-inclusive DIS (SIDIS) $ep^\uparrow \to ehX$ but is valid also for $p^\uparrow p \to hX$. The spin-dependent part of the cross section is

$$E_h \frac{d\sigma^{frag}}{d^3\vec{P}_h} = \frac{1}{4s(2\pi)^3} \Bigg\{ \int \frac{dz}{z^2} \mathrm{Tr}[\Delta(z)S(z)] + \int \frac{dz}{z^2} \mathrm{ImTr}\left[\Delta_\partial^\alpha(z) \frac{\partial S(K)}{\partial K^\alpha}\right]_{K=\frac{P_h}{z}} - \int \frac{dz_1 dz_2}{z_1^2 z_2^2} P\left(\frac{1}{1/z_2 - 1/z_1}\right) \mathrm{ImTr}[\Delta_F^\alpha(z_1,z_2)(S_\alpha^L(z_1,z_2) + S_\alpha^R(z_1,z_2))] \Bigg\} , \quad (2)$$

where P_h^μ is the momentum of the measured hadron species h whose mass is neglected $P_h^2 = 2P_h^+ P_h^- - P_{hT}^2 = M_h^2 \approx 0$. The momenta of the polarized and unpolarized protons are denoted by p^μ and q^μ, respectively. The center-of-mass energy is then $s \approx 2p^+q^-$. Δ's describe the fragmentation process into h, and S's represent the rest of the cross section. We shall be interested in the forward region $P_h^+ \gg P_{hT} \gg P_h^-$ and keep only the leading contributions in P_{hT}/P_h^+. In this kinematics, S and S^L are depicted in the first and the last two diagrams of Fig. 1, respectively. (S^R is the mirror image of S^L.) In our approach, the transverse momentum of the final state hadron P_{hT} comes from the intrinsic transverse momentum of the small-x gluon from the unpolarized target. This is why we only consider $2 \to 1$ scattering instead of $2 \to 2$ scattering.

The twist-three fragmentation functions are contained in Δ's as

$$\Delta(z) = \frac{M}{2z} \sigma_{\lambda\alpha} i\gamma_5 \epsilon^{\lambda\alpha w P_h} \hat{e}_{\bar{1}}(z) + \cdots , \quad (3)$$

$$\Delta_\partial^\alpha = \frac{M}{2}\gamma_5 \frac{\not{P}_h}{z} \gamma_\lambda \epsilon^{\lambda\alpha w P_h} \tilde{e}(z) + \cdots , \quad (4)$$

[1] The twist-three contribution from the unpolarized nucleon/nucleus in the current kinematics is suppressed in the small-x calculations, and neglected in this paper.

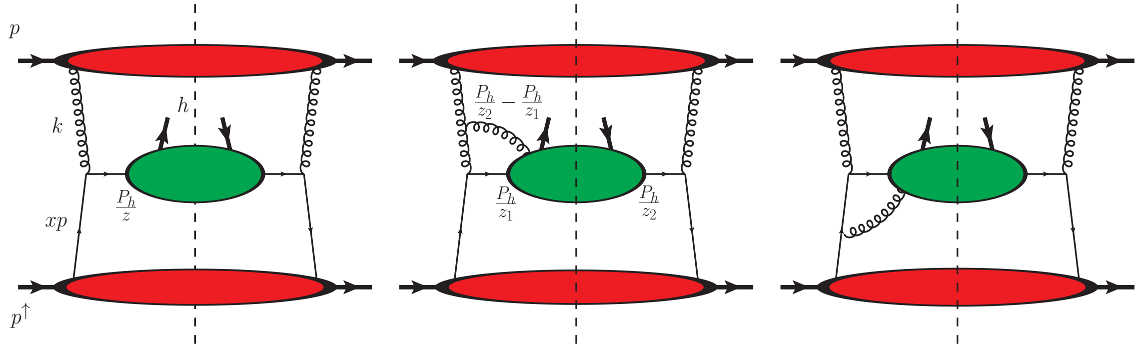

FIG. 1. Fragmentation contribution to single spin asymmetry in the hybrid approach. The left diagram represents the first two terms in (2). The middle and the right diagrams represent the last term in (2).

$$\Delta_F^\alpha(z_1, z_2) = \frac{M}{2}\gamma_5 \frac{\not{P}_h}{z_2}\gamma_\lambda \epsilon^{\lambda\alpha w P_h} \hat{E}_F(z_1, z_2) + \cdots , \tag{5}$$

where M is the proton mass. We use the conventions $D^\mu = \partial^\mu - igA^\mu$, $\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3$ and $\epsilon^{\lambda\alpha w P_h} \equiv \epsilon^{\lambda\alpha\rho\sigma} w_\rho P_{h\sigma}$ with $\epsilon_{0123} = +1$. The two-dimensional antisymmetric tensor ϵ^{ij} is defined as $\epsilon^{12} = -\epsilon^{21} = 1$ so that $\epsilon^{+-ij} = \epsilon^{ij}$. (We use Latin letters $i, j, l = 1, 2$ for transverse indices.) w^μ is a vector which satisfies the conditions $P_h \cdot w = 1$ and $w^2 = 0$. Explicitly,

$$(w^+, w^-, w^i) = \frac{1}{2E_h^2}(P_h^-, P_h^+, -P_h^i) \approx \frac{1}{(P_h^+)^2}(P_h^-, P_h^+, -P_h^i) \tag{6}$$

The largest component is $w^- \approx 1/P_h^+$. The three functions in (3)-(5) are not totally independent. They satisfy the relation

$$\frac{\hat{e}_{\bar{1}}(z)}{z} - \mathrm{Im}\tilde{e}(z) = \int \frac{dz'}{z'^2} P \frac{1}{1/z' - 1/z} \mathrm{Im}\hat{E}_F(z', z) \,. \tag{7}$$

The relevant distribution function for the transversely polarized proton is the transversity distribution $h_1(x)$

$$\langle p|\psi\bar{\psi}|p\rangle = \frac{1}{8}\langle \bar{\psi} i\gamma_5 \sigma^{\mu\nu}\psi\rangle i\gamma_5\sigma_{\mu\nu} + \cdots = -\frac{p^+ S_{Ti}}{2}\int dx h_1(x) i\gamma_5 \sigma^{-i} + \cdots , \tag{8}$$

where $\vec{S}_T$ is the transverse spin vector normalized as $\vec{S}_T^2 = 1$.

A. First term

Let us calculate the three terms in (2) one by one. The integrand of the first term reads

$$\begin{aligned}
\mathrm{Tr}[\Delta(z)S(z)] &= -g^2 C_F \frac{Mp^+ S_{Ti}}{4z}\epsilon^{\lambda\alpha w P_h}\hat{e}_{\bar{1}}(z)\int dx h_1(x)\int d^3k \mathrm{Tr}[i\gamma_5\sigma^{-i}\gamma^\nu\sigma_{\lambda\alpha}i\gamma_5\gamma^\mu] \\
&\quad \times \frac{\langle q|A_\mu(k)A_\nu(-k)|q\rangle}{N_c^2-1}(2\pi)^4\delta^{(4)}\left(xp + k - \frac{P_h}{z}\right) \\
&= -(2\pi)^4 g^2 \frac{MS_{Ti}}{8N_c}\epsilon^{\lambda\alpha w P_h}\frac{\hat{e}_{\bar{1}}(z)}{z}h_1(x)\mathrm{Tr}[i\gamma_5\sigma^{-i}\gamma^\nu\sigma_{\lambda\alpha}i\gamma_5\gamma^\mu]\langle q|A_\mu(k)A_\nu(-k)|q\rangle \,,
\end{aligned} \tag{9}$$

where $C_F = \frac{N_c^2-1}{2N_c}$ and $k^\mu = (0, k^-, \vec{k}_T)$. The momentum conserving delta function fixes the components of P_h^μ as

$$P_h^+ = xzp^+ \equiv x_F p^+ \,, \qquad P_h^- = zk^- \,, \qquad \vec{P}_{hT} = z\vec{k}_T \,. \tag{10}$$

Spin-dependent cross sections are often measured at fixed x_F. In the forward region in which we are interested, $x_F \approx 1$. Working out the trace of gamma matrices, we get

$$\begin{aligned}
&\epsilon^{\lambda\alpha w P_h}\mathrm{Tr}[i\gamma_5\sigma^{-i}\gamma^\nu\sigma_{\lambda\alpha}i\gamma_5\gamma^\mu]A_\mu(k)A_\nu(-k) \\
&= -8\Big(-\epsilon^{\alpha-wP_h}(A^iA_\alpha + A_\alpha A^i) + \epsilon^{\lambda iwP_h}(A^-A_\lambda + A_\lambda A^-) - \epsilon^{-iwP_h}A^\mu A_\mu\Big) \\
&\approx \frac{-8}{P_h^+}\left(\epsilon^{lj}P_{hj}(A^iA_l + A_lA^i) + \epsilon^{li}\Big(P_{hl}(2A^-A^- + A^jA_j) + P_h^+(A^-A_l + A_lA^-)\Big)\right). \qquad (11)
\end{aligned}$$

One might be puzzled by this complicated expression which cannot be rewritten as a gauge invariant combination of $F^{\mu\nu}(k) = i(k^\mu A^\nu - k^\nu A^\mu) + \mathcal{O}(g)$. In fact, the other terms in (2) also give similar, non-gauge-invariant terms, and the identity (7) is needed to check whether the sum is gauge invariant [12]. However, this is beyond the scope of this work. A simple counting argument $A^- \sim q^- \sim P_h^+ \gg A^i \sim P_{hT}$ shows that the whole expression (11) is subleading by a factor $(P_{hT}/P_h^+)^2$ compared to what we shall keep in the end, and at this subleading level diagrams other than those in Fig. 1 will come into play. We thus simply ignore (11) for the present purpose.

B. Second term

The second term in (2) is evaluated as

$$\begin{aligned}
\mathrm{ImTr}\left[\Delta_\partial^\alpha(z)\frac{\partial S(K)}{\partial K^\alpha}\right]_{K=\frac{P_h}{z}} &= -g^2(2\pi)^4\frac{MS_{Ti}}{2N_c}\epsilon^{\lambda\alpha wP_h}\,\mathrm{Im}\,\tilde{e}(z) \\
&\times\frac{\partial}{\partial K^\alpha}\left[h_1(K^+/p^+)\frac{1}{4}\mathrm{Tr}[i\gamma_5\sigma^{-i}\gamma^\nu\gamma_5\frac{\not{P}_h}{z}\gamma_\lambda\gamma^\mu]\langle A_\mu(\tilde{K})A_\nu(-\tilde{K})\rangle\right]_{K=\frac{P_h}{z}}, \qquad (12)
\end{aligned}$$

where we introduced the notation $\tilde{K}^\mu = (0, K^-, \vec{K}_T)$. We use the trick

$$\begin{aligned}
&\frac{\partial}{\partial K^\alpha}\left[h_1(K^+/p^+)\mathrm{Tr}[i\gamma_5\sigma^{-i}\gamma^\nu\gamma_5\frac{\not{P}_h}{z}\gamma_\lambda\gamma^\mu]A_\mu(\tilde{K})A_\nu(-\tilde{K})\right]_{K=\frac{P_h}{z}} \\
&= \frac{\partial}{\partial K^\alpha}\Big[h_1(K^+/p^+)\mathrm{Tr}[i\gamma_5\sigma^{-i}\gamma^\nu\gamma_5\not{K}\gamma_\lambda\gamma^\mu]A_\mu(\tilde{K})A_\nu(-\tilde{K})\Big]_{K=\frac{P_h}{z}} \\
&\quad -h_1(x)\mathrm{Tr}[i\gamma_5\sigma^{-i}\gamma^\nu\gamma_5\gamma_\alpha\gamma_\lambda\gamma^\mu]A_\mu(k)A_\nu(-k)\,. \qquad (13)
\end{aligned}$$

The second term on the right hand side has exactly the same γ-matrix structure as in (9).[2] It is thus subleading in energy and can be dropped.[3] As for the first term in (13), we find

$$\begin{aligned}
&\frac{1}{4}\mathrm{Tr}[i\gamma_5\sigma^{-i}\gamma^\nu\gamma_5\not{K}\gamma_\lambda\gamma^\mu]A_\mu A_\nu \\
&= \delta^i_\lambda(K\cdot AA^- + A^-K\cdot A - K^-A^\mu A_\mu) - \delta^-_\lambda(K\cdot AA^i + A^iK\cdot A - K^iA^\mu A_\mu) \\
&\quad +K_\lambda(A^-A^i - A^iA^-) + A_\lambda(K^-A^i - K^iA^-) + (K^-A^i - K^iA^-)A_\lambda\,. \qquad (14)
\end{aligned}$$

The dominant term is $\sim \delta^i_\lambda K^+A^-A^-$ which combines with other terms to form the gauge invariant operator[4]

$$\begin{aligned}
F^{-\mu}F^-_{\ \mu} &= K^-\Big(K^-A^\mu A_\mu - (\tilde{K}\cdot AA^- + A^-\tilde{K}\cdot A)\Big) + \tilde{K}^2A^-A^- + \mathcal{O}(g) \\
&= K^-\Big(K^-A^\mu A_\mu - (K\cdot AA^- + A^-K\cdot A)\Big) + K^2A^-A^- + \mathcal{O}(g)\,. \qquad (15)
\end{aligned}$$

To twist-two accuracy, we only keep this term and use

$$\frac{\langle F^{-i}F^{-j}\rangle}{K^-} = \frac{K^iK^j}{K_T^2}G(x_g, K_T)\,, \qquad \frac{\langle F^{-\mu}F^-_{\ \mu}\rangle}{K^-} = -G(x_g, K_T)\,, \qquad (x_g = K^-/q^-) \qquad (16)$$

---

[2] One can replace $\gamma_\alpha\gamma_\lambda \to \frac{1}{2}[\gamma_\alpha, \gamma_\lambda]$ due to the presence of $\epsilon^{\lambda\alpha wP_h}$.

[3] Incidentally, if we add this term to (9), we get the combination $\frac{\hat{e}_{\bar{1}}}{z} - \mathrm{Im}\,\tilde{e}(z)$ which appears in the identity (7).

[4] Note that terms proportional to K^2 and K_λ can be omitted. If the K^α-derivative in (13) acts on K_λ, it gives $g_{\alpha\lambda}$ and vanishes when contracted with $\epsilon^{\lambda\alpha wP_h}$. If the derivative does not act on K_λ, then after setting $K_\lambda = P_{h\lambda}/z$ we get zero $P_{h\lambda}\epsilon^{\lambda\alpha wP_h} = 0$. Similarly, if the derivative acts on K^2, it gives K_α and vanishes after replacing $K_\alpha \to P_{h\alpha}/z$. If the derivative does not act on K^2, then again it vanishes because $K^2 \to P_h^2/z^2 = 0$.

where G is the unintegrated gluon distribution of the unpolarized proton. The K-derivative can be decomposed as

$$\epsilon^{i+wP_h}\frac{\partial}{\partial K^+}+\epsilon^{i-wP_h}\frac{\partial}{\partial K^-}+\epsilon^{ijwP_h}\frac{\partial}{\partial K^j}\approx\epsilon^{ij}\left[\frac{P_{hj}}{P_h^+}\left(\frac{\partial}{\partial K^+}-\frac{\partial}{\partial K^-}\right)+\frac{\partial}{\partial K^j}\right]. \tag{17}$$

The K^+-derivative can be safely neglected. However, the K^--derivative should be kept since $1/K^-\sim z/P_h^-$ is large. This can be combined with the K_T-derivative as

$$\left(-\frac{P_{hj}}{P_h^+}\frac{\partial}{\partial K^-}+\frac{\partial}{\partial K^j}\right)G\left(x_g=\frac{K^-}{q^-},K_T\right)\Bigg|_{K=\frac{P_h}{z}}=\frac{d}{d(P_h^j/z)}G\left(x_g=\frac{P_{hT}^2}{xz^2s},\frac{P_{hT}}{z}\right). \tag{18}$$

We thus arrive at

$$\mathrm{Im}\mathrm{Tr}\left[\Delta_\partial^\alpha(z)\frac{\partial S(K)}{\partial K^\alpha}\right]_{K=\frac{P_h}{z}}=-\frac{g^2M}{2N_c}(2\pi)^4h_1(x)\mathrm{Im}\,\tilde{e}(z)S_{Ti}\epsilon^{ij}\frac{d}{d(P_h^j/z)}G\left(x_g=\frac{P_{hT}^2}{xz^2s},\frac{P_{hT}}{z}\right). \tag{19}$$

C. Third term

The last term in (2) is the 'genuine twist-three' contribution

$$\begin{aligned}&\mathrm{Im}\mathrm{Tr}[\Delta_F^\alpha(z_1,z_2)(S_\alpha^L(z_1,z_2)+S_\alpha^R(z_1,z_2))]=g^2M\frac{(2\pi)^4}{4z_2}h_1(x)S_{Ti}\epsilon^{\lambda\alpha wP_h}\mathrm{Im}\hat{E}_F(z_1,z_2)\\&\times\Bigg\{\frac{N_c}{2}\frac{\mathrm{Tr}\left[i\gamma_5\sigma^{-i}\gamma^\nu\gamma_5\not{P}_h\gamma_\lambda\gamma^\beta\right]}{(k+P_h(1/z_1-1/z_2))^2}\Big(\delta_\alpha^\mu(\frac{P_h}{z_1}-\frac{P_h}{z_2}-k)_\beta-g_{\alpha\beta}(k+2\frac{P_h}{z_1}-2\frac{P_h}{z_2})^\mu+\delta_\beta^\mu(2k+\frac{P_h}{z_1}-\frac{P_h}{z_2})_\alpha\Big)\\&\quad-\frac{1}{2N_c}\mathrm{Tr}\left[i\gamma_5\sigma^{-i}\gamma^\nu\gamma_5\not{P}_h\gamma_\lambda\gamma^\mu\frac{x\not{p}+\not{P}_h(1/z_1-1/z_2)}{(xp+P_h(1/z_1-1/z_2))^2}\gamma_\alpha\right]+(\mu\leftrightarrow\nu)\Bigg\}\frac{\langle A_\mu(k)A_\nu(-k)\rangle}{N_c^2-1}.\end{aligned} \tag{20}$$

The two terms correspond to the middle and right diagrams of Fig. 2 and have different dependence on N_c. Let us first look at the $\mathcal{O}(1/N_c)$ contribution. The quark propagator contains two terms, $x\not{p}$ and $\not{P}_h(1/z_1-1/z_2)$. The former gives

$$\begin{aligned}&xp^+\epsilon^{\lambda\alpha wP_h}\mathrm{Tr}\left[i\gamma_5\sigma^{-i}\gamma^\nu\gamma_5\not{P}_h\gamma_\lambda\gamma^\mu\gamma^-\gamma_\alpha+(\mu\leftrightarrow\nu)\right]\langle A_\mu A_\nu\rangle\\&=\frac{16P_h^+}{z_2}\epsilon^{\lambda-wP_h}\left\langle A_\lambda(P_h^-A^i-P_h^iA^-)+(P_h^-A^i-P_h^iA^-)A_\lambda+\delta_\lambda^i(A^-P_h\cdot A+P_h\cdot AA^--P_h^-A^\mu A_\mu)\right\rangle.\end{aligned} \tag{21}$$

Using relations such as

$$\begin{aligned}A^-P_h\cdot A+P_h\cdot AA^--P_h^-A^\mu A_\mu&=\frac{z_2}{k^-}\left(k_T^2A^-A^-+k^-k_i(A^-A^i+A^iA^-)-(k^-)^2A^iA_i\right)\\&=-\frac{z_2}{k^-}F^{-\mu}F^-_{\ \mu},\end{aligned} \tag{22}$$

and (16), we can rewrite (21) in the form

$$\begin{aligned}16\frac{P_h^+}{k^-}\epsilon^{\lambda-wP_h}\left\langle F^-_{\ \lambda}F^{-i}+F^{-i}F^-_{\ \lambda}-\delta_\lambda^iF^{-\mu}F^-_{\ \mu}\right\rangle&\approx16P_h^+\epsilon^{j-+l}w^-P_{hl}\left(\frac{2k_jk^i}{k_T^2}+\delta_j^i\right)G(x_g,k_T)\\&=-16\epsilon^{ij}P_{hj}G(x_g,k_T).\end{aligned} \tag{23}$$

The other term $\not{P}_h(1/z_1-1/z_2)$ can be evaluated as

$$\begin{aligned}\epsilon^{\lambda\alpha wP_h}\mathrm{Tr}\left[i\gamma_5\sigma^{-i}\gamma^\nu\gamma_5\not{P}_h\gamma_\lambda\gamma^\mu\not{P}_h\gamma_\alpha\right]&=-P_h^\mu\epsilon^{\lambda\alpha wP_h}\mathrm{Tr}\left[\gamma^-\gamma^i\gamma^\nu(\gamma_\lambda\not{P}_h\gamma_\alpha-\gamma_\alpha\not{P}_h\gamma_\lambda)\right]\\&=-2iP_h^\mu P_h^\rho\epsilon^{\lambda\alpha wP_h}\epsilon_{\lambda\alpha\rho\sigma}\mathrm{Tr}[\gamma^-\gamma^i\gamma^\nu\gamma^\sigma\gamma_5]\\&=-16P_h^\mu P_{h\sigma}\epsilon^{-i\nu\sigma}.\end{aligned} \tag{24}$$

Multiplying by $A_\mu A_\nu$ and adding the $\mu\leftrightarrow\nu$ terms, we get

$$\begin{aligned}&-16\epsilon^{ij}\Big(P_{hj}(P_h\cdot AA^-+A^-P_h\cdot A)-P_h^-(P_h\cdot AA_j+A_jP_h\cdot A)\Big)\\&=-8\epsilon^{ij}z_2^2\left(-2\frac{k_j}{k^-}F^{-\mu}F^-_{\ \mu}+F^{-\mu}F_{j\mu}+F_{j\mu}F^{-\mu}-F^-_{\ j}\partial_\mu A^\mu-\partial_\mu A^\mu F^-_{\ j}\right).\end{aligned} \tag{25}$$

The first term gives the gluon distribution G, while the other terms are subleading. The factor in the denominator is simplified as

$$\left(xp + \left(\frac{1}{z_1} - \frac{1}{z_2}\right) P_h\right)^2 = \frac{\vec{P}_{hT}^2}{z_2}\left(\frac{1}{z_1} - \frac{1}{z_2}\right) . \tag{26}$$

Next we compute the $\mathcal{O}(N_c)$ contribution. It is easy to see that the terms proportional to δ^μ_α in the three-gluon vertex do not contribute. (Note that $\not{k}\gamma^- = \frac{\not{P}_h}{z_2}\gamma^-$.) The terms proportional to $g_{\alpha\beta}$ can be evaluated similarly to (24)

$$\begin{aligned} &8P_{h\sigma}\Big(\epsilon^{-i\nu\sigma}(k^\mu + 2P_h^\mu(\frac{1}{z_1} - \frac{1}{z_2})) + \epsilon^{-i\mu\sigma}(k^\nu + 2P_h^\nu(\frac{1}{z_1} - \frac{1}{z_2}))\Big) A_\mu A_\nu \\ &= 16z_2^2\left(\frac{1}{z_1} - \frac{1}{z_2}\right)\frac{\epsilon^{ij}}{k^-}\left\{-k_j F^{-\mu}F^-_{\ \mu} + \frac{k^-}{2}(F_{j\mu}F^{-\mu} + F^{-\mu}F_{j\mu})\right\} - 8\frac{z_2^2}{z_1}\epsilon^{ij}\left(\partial_\mu A^\mu F^-_{\ j} + F^-_{\ j}\partial_\mu \cdot A^\mu\right) . \end{aligned} \tag{27}$$

Among the terms proportional to δ^μ_β, only the term $2k_\alpha$ gives a nonvanishing contribution. Using $k_\alpha = (P_{h\alpha} - \delta^-_\alpha P_h^+)/z_2$, we get

$$\begin{aligned} &-\frac{16P_h^+}{z_2}\Big[\epsilon^{j-wP_h}\left(P_h^-(A_jA^i + A^iA_j) - P_h^i(A_jA^- + A^-A_j)\right) + \epsilon^{i-wP_h}(P_h\cdot AA^- + A^-P_h\cdot A - P_h^- A^\mu A_\mu)\Big] \\ &\approx -\frac{16}{z_2}\left(\frac{1}{2}\epsilon^{jl}(F_{jl}F^{-i} + F^{-i}F_{jl}) + \epsilon^{ij}P_{hj}\frac{z_2}{k^-}F^{-\mu}F^-_{\ \mu}\right) . \end{aligned} \tag{28}$$

The factor in the denominator is

$$\left(k + \left(\frac{1}{z_1} - \frac{1}{z_2}\right) P_h\right)^2 = -\frac{\vec{P}_{hT}^2}{z_1 z_2} . \tag{29}$$

All in all, (20) becomes

$$\begin{aligned} \mathrm{ImTr}[\Delta^i_F(z_1,z_2)(S^L_\alpha(z_1,z_2) + S^R_\alpha(z_1,z_2))] &= -g^2 M_N\frac{(2\pi)^4}{4z_2}h_1(x)S_{Ti}\frac{\mathrm{Im}\hat{E}_F(z_1,z_2)}{N_c} \\ \times\Bigg\{\frac{8z_2^2}{P_T^2}\epsilon^{ij}P_{hj}\frac{G(x_g,k_T)}{N_c^2-1}&\left(N_c^2 + \frac{1}{z_1\left(\frac{1}{z_2} - \frac{1}{z_1}\right)}\right) - \frac{4z_2^3}{P_T^2}\epsilon^{ij}\langle\partial\cdot AF^-_{\ j} + F^-_{\ j}\partial\cdot A\rangle\Bigg\} . \end{aligned} \tag{30}$$

where we omitted higher twist terms. We kept the gauge-dependent terms just to note that the prefactor $1/(N_c^2-1)$ has been canceled so that they have the same N_c-dependence as the other gauge dependent terms in (9). Below we shall omit them because they are also subleading.

D. Comparison to the fully collinear result

Summing (19) and (30), we finally obtain, relabeling $z_2 \to z$,

$$\begin{aligned} E_h\frac{d\sigma^{frag}}{d^3P_h} = \frac{M\alpha_s\pi^2}{N_c s}S_{Ti}\epsilon^{ij}\int\frac{dz}{z^2}h_1(x)\Bigg\{&-\mathrm{Im}\,\tilde{e}(z)\frac{d}{d(P_h^j/z)}G\left(x_g = \frac{P_{hT}^2}{xz^2 s}, \frac{P_{hT}}{z}\right) \\ &+4P_{hj}\int\frac{dz_1}{z_1^2}\frac{z}{\frac{1}{z} - \frac{1}{z_1}}\frac{\mathrm{Im}\hat{E}_F(z_1,z)}{N_c^2-1}\frac{G(x_g,P_{hT}/z)}{P_{hT}^2}\left(N_c^2 + \frac{1}{z_1\left(\frac{1}{z} - \frac{1}{z_1}\right)}\right)\Bigg\} . \end{aligned} \tag{31}$$

Let us check if (31) is consistent with the result previously obtained in the collinear twist-three framework relevant in the high-P_{hT} region [10]. At large $P_{hT} \gg \Lambda_{QCD}$ and small-x, we can use (cf. [18])

$$G(x_g, P_{hT}/z) \approx \frac{\alpha_s}{2\pi^2}\frac{z^2}{P_{hT}^2}\int\frac{dx'}{x'}G(x')P_{gg}(x_g/x') \approx \frac{N_c\alpha_s}{\pi^2}\frac{xz^4 s}{P_{hT}^4}\int dx' G(x') , \tag{32}$$

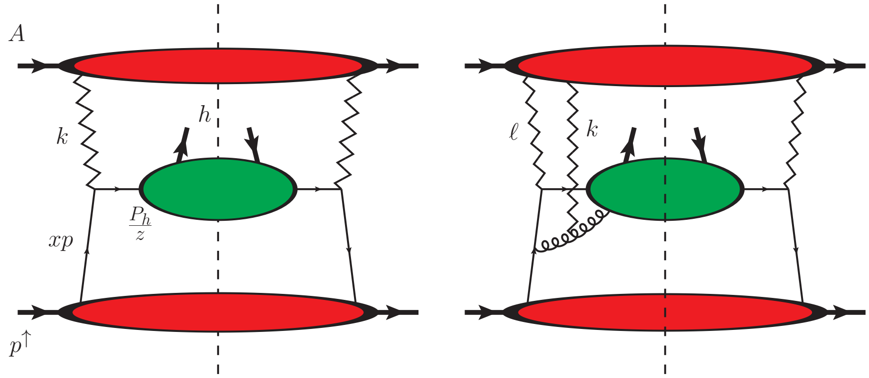

FIG. 2. Fragmentation contribution with saturation effects. The zigzag lines represent the multiple insertion of the A^- field in the eikonal approximation.

where $G(x)$ is the usual collinear gluon distribution and P_{gg} is the splitting function. (31) reduces to

$$E_h \frac{d\sigma^{frag}}{d^3P_h} = 4M\alpha_s^2 \int \frac{dz}{z^3} h_1(x)\epsilon^{ij} S_{Ti} P_{hj} \frac{xz^6}{(P_{hT}^2)^3} \int dx' G(x') \times \left\{ -\mathrm{Im}\,\tilde{e}(z) + \int \frac{dz_1}{z_1^2} \frac{1}{\frac{1}{z}-\frac{1}{z_1}} \frac{\mathrm{Im}\hat{E}_F(z_1,z)}{N_c^2-1} \left(N_c^2 + \frac{1}{z_1\left(\frac{1}{z}-\frac{1}{z_1}\right)} \right) \right\} . \tag{33}$$

This should be compared with Eq. (15) of [10] which uses different notations for the fragmentation functions.

$$M\hat{e}_{\bar{1}} = -M_h H\,, \qquad M_N \mathrm{Im}\tilde{e} = 2M_h \hat{H}\,, \qquad M\mathrm{Im}\hat{E}_F(z_1,z) = 2M_h z^2 \hat{H}_{FU}^{\Im}(z,z_1)\,. \tag{34}$$

Taking the limit $\hat{s} \gg |\hat{t}|$ in the quark-gluon channel ($\hat{s} = xx's$, $\hat{t} = -2\frac{x}{z}p^+P_h^- = -P_{hT}^2/z^2$ are the partonic Mandelstam variables), we find

$$\begin{aligned} E_h \frac{d\sigma^{frag}}{d^3\vec{P}_h} &= -\frac{2\alpha_s^2 M_h}{s} \epsilon^{ij} S_{Ti} P_{hj} \int \frac{dz}{z^3} \int \frac{dx'}{x'} \frac{1}{x\hat{s}^2} h_1(x) G(x') \\ &\quad \times \left(\frac{H(z)}{z} + \frac{2z}{N_c^2-1} \int \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{\Im}(z,z_1)}{\left(\frac{1}{z}-\frac{1}{z_1}\right)^2} \right) \frac{2x\hat{s}^3}{\hat{t}^3} \\ &= 4\alpha_s^2 M_h \epsilon^{ij} S_{Ti} P_{hj} \int \frac{dz}{z^3} \int dx' h_1(x) G(x') \\ &\quad \times \left(-2\hat{H}(z) + \frac{2z^2}{N_c^2-1} \int \frac{dz_1}{z_1^2} \frac{\hat{H}_{FU}^{\Im}(z,z_1)}{\frac{1}{z}-\frac{1}{z_1}} \left(N_c^2 + \frac{1}{z_1} \frac{1}{\frac{1}{z}-\frac{1}{z_1}} \right) \right) \frac{xz^6}{(P_{hT}^2)^3}\,, \end{aligned} \tag{35}$$

where we used (7). This agrees perfectly with (33).

III. INCLUDING SATURATION EFFECTS

We now include the gluon saturation effects. We closely follow the strategy used in [3]. The diagrams to be computed are shown in Fig. 2. The zigzag lines represent the Wilson line U arising from the eikonal exponentiation

$$ig\gamma^\mu A_\mu^a(k)t^a \to \gamma^+ \int \frac{d^2\vec{x}_T}{(2\pi)^3} e^{i\vec{x}_T\cdot\vec{k}_T} (U(\vec{x}_T)-1)\,, \qquad U(\vec{x}_T) = \exp\left(ig \int dx^+ A_a^-(x^+,\vec{x}_T) t^a \right) . \tag{36}$$

In the high energy limit, the unpolarized target can be viewed as a highly Lorentz contracted shockwave. The multiple scatterings (the zigzag lines) between the polarized proton and the target can only occur either before or after the collinear gluon splitting. This is why we only need to consider the two diagrams as shown in Fig. 2.

The unintegrated gluon distribution $G(x_g, k_T)$ is converted to the correlation function of Wilson lines

$$\frac{x_g G(x_g,k_T)}{k_T^2} \to \frac{N_c}{2\pi^2\alpha_s}\int \frac{d^2x_T d^2y_T}{(2\pi)^2} e^{i\vec{k}_T\cdot(\vec{x}_T-\vec{y}_T)} \frac{\langle q|\frac{1}{N_c}\mathrm{Tr}[U^\dagger(\vec{y}_T)U(\vec{x}_T)]|q\rangle}{\langle q|q\rangle}$$
$$\equiv \frac{N_c}{2\pi^2\alpha_s}F(x_g,k_T)\,, \tag{37}$$

where $\langle q|q\rangle = 2q^-(2\pi)^3\delta^{(3)}(0) = 2q^-\int dx^+ d^2\vec{x}_T$. Evaluated at $x_g = \frac{P_{hT}^2}{xz^2s}$, (37) becomes

$$G\left(x_g, \frac{P_{hT}}{z}\right) \to \frac{xsN_c}{2\pi^2\alpha_s}F\left(x_g,\frac{P_{hT}}{z}\right)\,. \tag{38}$$

In the derivative term of (31) which now comes from the left diagram of Fig. 2, it is enough to make this replacement. The genuine twist-three terms are more complicated because they involve an extra collinear gluon which can be dressed by the Wilson line as shown in the right diagram of Fig. 2. Still, the topology of the diagram is very similar to the one considered in [3]. We find that their color structures are exactly the same and read

$$\int \frac{d^2\vec{x}_T d^2\vec{y}_T d^2\vec{z}_T}{(2\pi)^6}(2\pi)^2\delta(\vec{k}_T+\vec{\ell}_T-\vec{P}_{hT}/z_2)e^{i\vec{k}_T\cdot\vec{z}+i\vec{\ell}_T\cdot\vec{x}_T-i\frac{\vec{P}_{hT}}{z_2}\cdot\vec{y}}$$
$$\times\left\langle \mathrm{Tr}[U^\dagger(\vec{y})U(\vec{z})]\mathrm{Tr}[U^\dagger(\vec{z})U(\vec{x})] - \frac{1}{N_c}\mathrm{Tr}[U^\dagger(\vec{y})U(\vec{x})]\right\rangle$$
$$\approx \langle q|q\rangle\delta^{(2)}(\vec{k}_T+\vec{\ell}_T-\vec{P}_{hT}/z_2)\left(\frac{N_c^2}{\int d^2\vec{x}}F(x_g,\ell_T)-\delta^{(2)}(\vec{k}_T)\right)F(x_g,P_{hT}/z_2)\,, \tag{39}$$

where we used the large-N_c approximation in the nonlinear term

$$\langle q|\mathrm{Tr}[U^\dagger(\vec{y})U(\vec{z})]\mathrm{Tr}[U^\dagger(\vec{z})U(\vec{x})]|q\rangle \approx \frac{\langle q|\mathrm{Tr}[U^\dagger(\vec{y})U(\vec{z})]|q\rangle\langle q|\mathrm{Tr}[U^\dagger(\vec{z})U(\vec{x})]|q\rangle}{\langle q|q\rangle}\,. \tag{40}$$

We now compute the hard part. There are two propagator denominators

$$\int d\ell^- \frac{1}{\left((\frac{P_h}{z_1}-\ell)^2+i\epsilon\right)\left((xp+\ell-\frac{P_h}{z_1})^2+i\epsilon\right)}\,. \tag{41}$$

The two poles in ℓ^- are located in the opposite sides of the real axis because $\hat{E}_F(z_1,z_2)$ has a support at $z_1>z_2$ [17]. We pick up the pole at $(\frac{P_h}{z_1}-\ell)^2=0$ at which

$$\frac{1}{\left(xp+\ell-\frac{P_h}{z_1}\right)^2} = -\frac{z_2}{z_1\left(\frac{\vec{P}_{hT}}{z_1}-\vec{\ell}_T\right)^2}\,. \tag{42}$$

As for the numerator, we only need to calculate the component $\mu=\nu=+$.

$$\epsilon^{\lambda\alpha w P_h}\mathrm{Tr}\left[i\gamma_5\sigma^{-i}\gamma^+\gamma_5 \not{P}_h\gamma_\lambda\gamma^+\left(\frac{\not{P}_h}{z_1}-\not{\ell}\right)\gamma^\beta\right]\left(-2g_{\alpha\beta}\left(\frac{1}{z_1}-\frac{1}{z_2}\right)P_h^+ + 2\delta_\beta^+k_\alpha\right) \approx \frac{32(P_h^+)^2}{z_1}\epsilon^{ij}\left(\frac{P_{hj}}{z_1}-\ell_j\right)\,.$$

We thus arrive at the product

$$\int d^2\vec{k}_T d^2\vec{\ell}_T \frac{\frac{P_{hj}}{z_1}-\ell_j}{\left(\frac{\vec{P}_{hT}}{z_1}-\vec{\ell}_T\right)^2}\delta^{(2)}\left(\vec{k}_T+\vec{\ell}_T-\frac{\vec{P}_{hT}}{z_2}\right)\left(\frac{N_c^2}{\int d^2\vec{x}_T}F(x_g,\ell_T)-\delta^{(2)}(\vec{k}_T)\right)F(x_g,P_{hT}/z_2)$$
$$=\int d^2\vec{\ell}_T\frac{\frac{P_{hj}}{z_1}-\ell_j}{\left(\frac{\vec{P}_{hT}}{z_1}-\vec{\ell}_T\right)^2}\left(\frac{N_c^2}{\int d^2\vec{x}_T}F(x_g,\ell_T)-\delta^{(2)}(\vec{\ell}_T-\frac{\vec{P}_{hT}}{z_2})\right)F(x_g,P_{hT}/z_2)\,. \tag{43}$$

In the dilute limit, $F(x_g,\ell_T)\to\delta^{(2)}(\vec{\ell}_T)\int d^2\vec{x}_T$, and (43) correctly reduces to the combination in (33)

$$z_1\frac{P_{hj}}{\vec{P}_{hT}^2}\left(N_c^2+\frac{1}{z_1\left(\frac{1}{z_2}-\frac{1}{z_1}\right)}\right)F(x_g,P_{hT}/z_2)\,. \tag{44}$$

In the general case, we can perform the angular integral

$$\int d^2\vec{\ell}_T \frac{\frac{P_{hj}}{z_1} - \ell_j}{\left(\frac{\vec{P}_{hT}}{z_1} - \vec{\ell}_T\right)^2} F(\ell_T) = 2\pi z_1 \frac{P_{hj}}{P_{hT}^2} \int_0^{P_{hT}/z_1} \ell_T d\ell_T F(\ell_T)\,, \tag{45}$$

and obtain

$$E_h \frac{d\sigma^{frag}}{d^3\vec{P}_h} = \frac{M}{2} S_{Ti}\epsilon^{ij} \int \frac{dz}{z^2} x h_1(x) \Bigg\{ -\mathrm{Im}\,\tilde{e}(z) \frac{d}{dP_h^j/z} F\left(x_g, \frac{P_{hT}}{z}\right) \tag{46}$$

$$+4\frac{P_{hj}}{P_{hT}^2} \int_z^\infty \frac{dz_1}{z_1^2} \frac{z}{\frac{1}{z} - \frac{1}{z_1}} \frac{\mathrm{Im}\hat{E}_F(z_1,z)}{N_c^2-1} \left(\frac{2\pi N_c^2}{\int d^2\vec{x}_T} \int_0^{P_{hT}/z_1} \ell_T d\ell_T F(x_g,\ell_T) + \frac{1}{z_1\left(\frac{1}{z} - \frac{1}{z_1}\right)} \right) F(x_g, P_{hT}/z) \Bigg\}.$$

This is the main result of this paper. If we assume the form

$$F(x_g,\ell_T) = \frac{\int d^2\vec{x}_T}{\pi Q_s^2} e^{-\ell_T^2/Q_s^2}\,, \tag{47}$$

which is a good approximation when $\ell_T^2 \leq Q_s^2$, we get

$$\frac{2\pi N_c^2}{\int d^2\vec{x}_T} \int_0^{P_{hT}/z_1} \ell_T d\ell_T F(\ell_T) = N_c^2 \left(1 - e^{-\frac{P_{hT}^2}{z_1^2 Q_s^2}}\right)\,. \tag{48}$$

Thus the effect of saturation is to reduce the $\mathcal{O}(N_c^2)$ contribution for $P_{hT} < z_1 Q_s$.

IV. DISCUSSION

The total spin-dependent cross section in the saturation regime is the sum of (46) and the soft gluon pole contribution calculated in [3]

$$E_h \frac{d\sigma^{SGP}}{d^3\vec{P}_h} = -\frac{\pi M x_F}{2(N_c^2-1)} \epsilon^{ij} S_{Ti} \int_{x_F}^1 \frac{dz}{z^3} D(z) \Bigg\{ -\frac{1}{(P_{hT}/z)^2} \frac{\partial}{\partial P_h^j/z} \left(\frac{P_{hT}^2}{z^2} F(x_g, P_{hT}/z) \right) G_F(x,x)$$

$$+ \frac{2P_{hj}/z}{(P_{hT}/z)^2} F(x_g, P_{hT}/z) x \frac{d}{dx} G_F(x,x) \Bigg\}\,, \tag{49}$$

where $G_F(x,x)$ is the Qiu-Sterman function [7]. (As shown in [3], the contribution from the soft fermionic pole vanishes in the saturation region.) Note that in (49) the P_h^j-derivative acts on P_{hT}^2 times F, not F itself as in (46).

Let us discuss the phenomenological implications of our result. Consider the dependence of the asymmetry A_N on the atomic mass number A. In the k_T-factorization approach, one only has the Collins-like term proportional to $\mathrm{Im}\tilde{e} \sim \hat{H}$ in (46). Assuming the form (47), one gets

$$\frac{\partial}{\partial P_h^j} F \sim \frac{P_h^j}{Q_s^2} F\,, \tag{50}$$

at low momentum $P_{hT} < Q_s$. Since $Q_s^2 \propto A^{1/3}$, one finds that $A_N \propto A^{-1/3}$, namely, the asymmetry is suppressed in pA collisions. This is essentially the result of [13]. Turning to the other terms in (46) proportional to $\mathrm{Im}\hat{E}_F$, we see that the $\mathcal{O}(N_c^2)$ term scales as

$$\frac{P_{hj}}{P_{hT}^2}\left(1 - e^{-\frac{P_{hT}^2}{z_1^2 Q_s^2}}\right) \sim \frac{P_{hj}}{Q_s^2} \tag{51}$$

for $\Lambda_{QCD} \ll P_{hT} \ll Q_s$. Therefore, this term also leads to the behavior $A_N \sim A^{-1/3}$. On the other hand, the $\mathcal{O}(N_c^0)$ term has a different P_{hT} dependence $\sim P_{hj}/P_{hT}^2$ which implies $A_N \sim A^0$. However, a recent study [16] suggests that this term is numerically small compared to the other terms in (46). We thus conclude that A_N from the twist-three fragmentation functions (46) scales as $A_N \sim A^{-1/3}$ in the forward region at low momentum $\Lambda_{QCD} \ll P_{hT} \ll Q_s$. This is in contrast to the observation in [3] that A_N from the ETQS function (49) is independent of A. Indeed, the dominant term in the forward region is expected to be the derivative term $x\frac{d}{dx}G_F(x,x)$. Since its coefficient is proportional to P_{hj}/P_{hT}^2, we get $A_N \sim A^0$.

Experimentally, the preliminary STAR data [2] show that A_N is almost independent of A at least up to $x_F = 0.7$. This favors the interpretation that SSA is dominated by the derivative term in (49). However, such an interpretation is inconsistent with the recent fit to the $p^\uparrow p \to hX$ data in [16]. There it was concluded that neither the Sivers nor Collins contribution extracted from the SIDIS data is sufficient to explain the observed asymmetry. To resolve this problem, the authors assumed that the genuine twist-three function $\mathrm{Im}\hat{E}_F \sim \hat{H}_{FU}^{\Im}$, not previously constrained by any data, is large. In particular, the term proportional to N_c^2 in (46) was found to be the dominant contribution. Yet, our result (48) shows that this term is most strongly affected by the saturation effect and, as we have just argued, gives rise to the scaling $A_N \sim A^{-1/3}$. We thus think more work and more data are needed to finally pin down the origin of SSA in QCD.

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