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de Sitter Space is Unstable in Quantum Gravity

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Graviton loop corrections to observables in de Sitter space often lead to infrared divergences. We show that these infrared divergences are resolved by the spontaneous breaking of de Sitter invariance.

I. INTRODUCTION

Quantum effects in de Sitter (dS) space have become central to particle physics and cosmology since the measurement of the cosmic microwave background [1] and its interpretation as being produced from zero point fluctuations around the vacuum [2]. Much effort has been expended in trying to extend this to higher precision by including loop corrections to this process. In particular, loop corrections coming from gravitons have been considered [3–8], with mixed success. The major impediment is the presence of infrared (IR) divergences in the loop calculation. It turns out that the graviton propagator in de Sitter is more singular at low momenta than the corresponding propagator in flat space; loop corrections with an internal graviton propagator then become divergent at low graviton momenta [9–11]. In particular, graviton corrections to the CMB are divergent, naively invalidating the successful

tree level calculation.

Many approaches have been taken to resolve this issue. We review several of these below, before moving to our own attempt at a resolution.

(1) One general approach has been to consider loop corrections involving massless scalars. It happens to be the case that massless scalars in de Sitter space display the same type of infrared divergence as gravitons; indeed, the graviton propagator in a suitable gauge is identical to the propagator of two massless scalars [12, 13]. An understanding of the IR divergences for a massless scalar might then be hoped to shed light on the divergences for a graviton.

Unfortunately this turns out not to be the case. Multiple analyses using multiple approaches e.g. Euclidean continuation [14–18], the stochastic formalism [19–21], the dynamical RG [22, 23], truncated Schwinger-Dyson equations [24–29] and others [30–35] have conclusively resolved the physics of a massless scalar in de Sitter space. The scalar develops a dynamical mass; for example, an apparently massless scalar with a $\lambda\phi^4$ interaction develops a dynamical mass proportional to $\sqrt{\lambda}$. The IR divergence is then absent, but the perturbation expansion becomes an expansion in $\sqrt{\lambda}$ rather than λ .

Now this resolution, which is perfectly satisfactory for scalars, is no help at all for the graviton case. The gauge invariance of gravitational perturbations around de Sitter space precludes the development of a mass, dynamical or otherwise, for the graviton. Any corrections to the graviton propagator which preserve de Sitter invariance are necessarily suppressed by powers of the invariant de Sitter momentum, and are therefore irrelevant for solving the problem of the infrared divergences. We must therefore seek the resolution elsewhere.

(2) Since the primary difference between the scalar and the graviton is the existence of a gauge symmetry, one may wonder if the gauge symmetry itself removes the divergence. This possibility has been developed by [36], who argue that the IR divergence is in fact a gauge artifact (see also [37]). The evidence for this comes from an analysis of the graviton propagator. The IR divergence translates into a growth of the propagator at large separations; the authors show that this growth can be canceled by a gauge transformation, albeit one that also grows at large distances. This suggests that a suitable limiting procedure can remove the infrared divergences.

The problems with this approach have been elaborated in detail in [38]. In addition

to the points raised there, we note that if the IR divergence was indeed a gauge artifact, then loop corrections to gauge invariant quantities would not have this divergence. This is explicitly contradicted by calculations, and hence it does not seem possible to gauge away the divergence.

(3) Woodard and collaborators have suggested that the correct approach is to modify the graviton propagator *ab initio* (a few of these papers are [39–41]). Since the de Sitter invariant propagator has the issues described above, they use a non invariant propagator (which satisfies the same differential equation as the de Sitter propagator, but does not have the full de Sitter invariance). The analysis [41] however shows that an explicit non-invariant counterterm is required to cancel a loop divergence. Whether such a term is allowed in a consistent theory is unclear— explicit symmetry breaking in a gauge theory typically leads to violations of renormalizability and unitarity. It is also hard to see how the standard sum over metrics in the path integral would lead to such a propagator. Finally, at least in string theory, calculations of the effective action show no sign of counterterms breaking gauge invariance (see e.g. [42]). Lacking a quantum theory of gravity in de Sitter space, we cannot rule out the possibility that this is a consistent approach, but clearly other approaches should be considered.

(4) If it is not possible to have a propagator which is de Sitter invariant, and if we do not wish to explicitly break the symmetry, then that leaves only one option — de Sitter invariance must be *spontaneously* broken. We will explore this possibility in this paper, and argue that this is indeed the appropriate resolution.

The idea that de Sitter space is unstable has been forcefully advocated by Polyakov [43–46] (for related ideas, see [47–54]). In these papers, Polyakov has argued that a loop calculation in scalar field theory in de Sitter space already shows IR divergences which can be interpreted as catastrophic particle production, leading to a decay of de Sitter. However, other calculations show no signs of an instability e.g [30]. Furthermore, it has been argued [16] that the scalar field theory can be formulated in an Euclideanized version of the theory, in which the finiteness properties can be proven [17].

The difference between these results appears to be due to the choice of formalism. While Polyakov has argued that one should use the in-out formalism, which calculates the transition amplitudes from the earliest times to the latest times, most calculations use the in-in (or Schwinger-Keldysh or CTP) formalism [55–61] which calculates transitions effectively from

the earliest times to a finite time in de Sitter in a particular choice of time slicing (we shall elaborate further on this below). This may suggest that the instability found by Polyakov in scalar field theory is an artifact of the choice of the in-out formalism. This is worrying because perturbation theory in the in-out formalism is known to fail for other theories in de Sitter space. For example, the exact solution for the massive scalar propagator is known, but cannot be reproduced in the in-out formalism using the mass term as a perturbation [62]. This failure can be attributed to the out-vacuum not being close to the in-vacuum due to particle production in de Sitter space [63]. The stability of scalar field theory in de Sitter is therefore an open issue.

In this paper, we shall apply the in-in formalism, not to scalar field theory, but to gravitational perturbations around de Sitter space. As we have already argued, the infrared divergences point to a spontaneous breakdown of the de Sitter symmetry. We show that indeed loop corrections lead to an instability of the de Sitter metric and a deformation of the metric. We therefore conclude that the instability argued for by Polyakov definitely does occur when gravitational perturbations are considered.

We note that results similar to ours have also been arrived at in the papers [9–11]. In these papers the fluctuations of geodesics are considered. The authors find that (to quote from the abstract of [11]) ”metric perturbations produce significant and growing corrections to the lengths of such geodesics.. These become large, signaling breakdown of a perturbative description of the geometry via such observables, and consistent with perturbative instability of de Sitter space”. This is qualitatively similar to our result, but appears to disagree quantitatively, because we find a perturbatively calculable corrections to the metric, while the authors of [11] appear to find a nonperturbative effect. It would be very interesting to see if these results are consistent with each other.

In the following section, we set out some well-known properties of de Sitter space and gravitational perturbations around it (this section also serves to establish our notation and to review the in-in formalism). We show that indeed the gravitational perturbations satisfy the same equations as a massless scalar, and that this leads to divergences in loop calculations.

We then proceed, in the subsequent section, to regulate this divergence. We do this by deforming the metric slightly away from de Sitter (we choose a deformation which is spatially homogeneous and rotationally invariant). As one might expect, this modifies the equations for the gravitational perturbations slightly, and they no longer satisfy the same

equation as a massless scalar. In such a background, the loop calculations are well defined, and the effective action can be computed in the usual way. Crucially, if the deformation is small, traces of the IR divergence are still visible in the enhancement of certain correlation functions. We explicitly show that if the deformation is parametrized by a small parameter ϵ , then the propagator of the gravitational perturbations is enhanced by a factor $\frac{1}{\epsilon}$. The effective action and the effective equations of motion then have terms which are enhanced by this factor.

We then look for solutions to the effective equations of motion. Classically, the only solution to the equations of motion is de Sitter space. This is seen by the fact that we find a tadpole for perturbations around the deformed space. These tadpoles vanish classically only when $\epsilon = 0$; that is, for de Sitter space. However, we find that the quantum effective equations of motion has new tadpoles, some of which are even enhanced by a factor $\frac{1}{\epsilon}$. The tadpole cancellation now occurs when ϵ is nonzero. That is, de Sitter space, corresponding to $\epsilon = 0$, is *not* a solution to the quantum corrected equations of motion. This explicitly shows that the de Sitter symmetry is spontaneously broken.

As part of this calculation, we are also able to estimate the value of ϵ at which the geometry is stabilized in the quantum theory. We find that ϵ scales as $\sqrt{\kappa}$, where $\kappa = 8\pi G$ is the coupling constant in gravity. The propagators are then enhanced by a factor $\frac{1}{\epsilon}$ leading to a perturbation expansion in powers of $\sqrt{\kappa}$ rather than the expected κ . This behavior has similarities to the massless scalar.

We finally close with a discussion of our results.

II. GRAVITATIONAL PERTURBATIONS AROUND DE SITTER SPACE

A. Notation and Overview

We shall work in the mostly minus signature. Indices μ, ν will run over 0,1,2,3 while indices i, j run over 1,2,3. The time coordinate $x^0 \equiv \tau$. We follow the conventions

$$R^\rho_{\sigma\mu\nu} = -\partial_\mu\Gamma^\rho_{\nu\sigma} + \partial_\nu\Gamma^\rho_{\mu\sigma} - \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} + \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma} \quad R_{\mu\nu} = R^\rho_{\sigma\mu\rho} \quad (1)$$

The Einstein action coupled to a cosmological constant is

$$\mathcal{L} = \frac{1}{2\kappa}\sqrt{g}(R + 2\Lambda) \quad (2)$$

where $\kappa = 8\pi G$. The equation of motion admits the de Sitter solution, written in Poincare patch coordinates as

$$ds^2 = \frac{1}{H^2\tau^2}(d\tau^2 - dx_i dx^i) \quad (3)$$

where

$$H^2 = \frac{\Lambda}{3} \quad (4)$$

Equivalently, the de Sitter metric is

$$\bar{g}_{00} = \frac{1}{H^2\tau^2} \quad \bar{g}_{ij} = -\frac{1}{H^2\tau^2}\delta_{ij} \quad (5)$$

Here the spatial coordinates x^i run from $-\infty$ to ∞ . The coordinate τ runs from $-\infty$ corresponding to early times, till $\tau = 0$ which corresponds to late times.

B. Scalar perturbations around de Sitter and the in-in formalism

As we have said above, calculations in de Sitter space need to be done using the in-in formalism. We shall here summarize a few important details of this formalism which are necessary for us; more details can be found in [55, 57]. We shall use the scalar field theory as an example.

The in-in formalism requires us to double the number of fields. For a scalar field, we go from ϕ to ϕ^+, ϕ^- . The two fields are constrained to be equal at a time τ_0 ; i.e. $\phi^+(\tau_0) = \phi^-(\tau_0)$. t_0 is the intermediate time at which correlations are calculated.

The Lagrangian $\mathcal{L}(\phi)$ is replaced by $\mathcal{L}^{in-in} = \mathcal{L}(\phi^+) - \mathcal{L}(\phi^-)$. This formalism therefore has two independent propagators; in addition to the standard propagator, one must introduce the Schwinger-Keldysh propagator:

$$F(x, y) = \frac{1}{2}(\langle\phi(x)\phi(y)\rangle + \langle\phi(y)\phi(x)\rangle) \quad (6)$$

We can find this propagator explicitly for a free scalar. Such a scalar satisfies

$$\partial_\mu(\sqrt{g}g^{\mu\nu}\partial_\nu)\phi_{m^2} + \frac{m^2}{H^4\tau^4}\phi_{m^2} = 0 \quad (7)$$

We perform a Fourier transform in the spatial directions; we then find [57]

$$\phi_{m^2}(t, \vec{k}) = -\frac{H\tau\sqrt{-\pi\tau}}{2}H_\nu(-k\tau) \quad (8)$$

where $H_\nu(-k\tau)$ is a Hankel function, and $\nu^2 = \frac{9}{4} - \frac{m^2}{H^2}$. The Schwinger-Keldysh propagator is then

$$F(k, \tau_1, \tau_2) = \frac{\pi H^2 \tau_1^{3/2} \tau_2^{3/2}}{4} \text{Re}(H_\nu(-k\tau_1) H_\nu^*(-k\tau_2)) \quad (9)$$

An important special case is the massless scalar. This sets $m^2 = 0$, $\nu = \frac{3}{2}$. We have then

$$\phi_{m^2=0}(t, \vec{k}) = i \frac{H}{\sqrt{2k^3}} (1 + ik\tau) e^{-ik\tau} \quad (10)$$

and

$$F(k, \tau_1, \tau_2) = \frac{H^2}{2k^3} [(1 + k^2 \tau_1 \tau_2) \cos(k(\tau_1 - \tau_2)) + k(\tau_1 - \tau_2) \sin(k(\tau_1 - \tau_2))] \quad (11)$$

The final important point is that for very small k , the massless propagator limits to

$$F(k, \tau_1, \tau_2) \rightarrow \frac{H^2}{2k^3} \quad (12)$$

while the massive propagator limits to

$$F(k, \tau_1, \tau_2) \rightarrow \frac{H^2}{2k^3} (k^2 \tau_1 \tau_2)^{\frac{m^2}{3H^2}} \quad (13)$$

Note that $F(x, y)$ is not well defined for a massless scalar; the inverse Fourier transform does not exist. This is a manifestation of the well known fact that a massless minimally coupled scalar in de Sitter space does not have a de Sitter invariant propagator [12, 13].

C. Linearized gravitational perturbations

Gravitational fluctuations around the background metric are parametrized as

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \quad (14)$$

Up to second order, the Lagrangian for these perturbations is found to be [64]

$$\frac{1}{2\kappa H^4 \tau^4} \left(\frac{1}{8} (\bar{D}_\mu h)^2 - \frac{1}{4} (\bar{D}_\nu h_{AB})^2 + \frac{1}{2} (\bar{D}_B h_A^B - \frac{1}{2} \bar{D}_A h)^2 - \frac{1}{2} h^{A\nu} \bar{R}_{AF\nu B} h^{FB} + \frac{1}{4} \Lambda h^2 \right) \quad (15)$$

where $h = \bar{g}^{\mu\nu} h_{\mu\nu}$, barred covariant derivatives are taken with respect to the background de Sitter metric, and indices are raised and lowered with the background metric.

We can solve the corresponding equations of motion in a gauge $h_{00} = h_{0i} = 0$. The equations with indices $0i, 00$ set $\partial_i h_{ij} = h = 0$. A perturbation is then characterized by the transverse traceless polarizations. These satisfy [65]

$$\partial_\mu(\sqrt{\bar{g}}\bar{g}^{\mu\nu}\partial_\nu(\tau^2 h_{ij}(\tau, \vec{x}))) = 0 \quad (16)$$

It is convenient to define

$$\gamma_{ij} = H^2 \tau^2 h_{ij} \quad (17)$$

Then γ_{ij} satisfies the same equation as for a massless scalar field.

The Schwinger-Keldysh propagator can now be determined. We will particularly be interested in the coincident limit, where we find [9]

$$\langle \gamma_{ij}(x) \gamma_{kl}(x) \rangle = \int \frac{d^3 q}{(2\pi)^3} \frac{H^2}{q^3} (1 + q^2 \tau^2) P_{ijkl} \quad (18)$$

where the last factor is a projection operator

$$P_{ijkl} = \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl} + \delta_{ij}\hat{q}_k\hat{q}_l + \delta_{kl}\hat{q}_i\hat{q}_j - \delta_{ik}\hat{q}_j\hat{q}_l - \delta_{il}\hat{q}_j\hat{q}_k - \delta_{jk}\hat{q}_i\hat{q}_l - \delta_{jl}\hat{q}_i\hat{q}_k + \hat{q}_i\hat{q}_j\hat{q}_k\hat{q}_l \quad (19)$$

where \hat{q} is the unit vector in the direction of q .

For a graviton (or a massless scalar) this propagator goes as $\frac{1}{q^3}$ at small q . Any loop diagram with an internal F propagator then diverges (as $\int \frac{d^3 q}{q^3}$) when the momentum flowing through this propagator goes to zero. (Examples of such calculations can be found in e.g. [9].) This makes essentially all loop calculations ill-defined, and in particular any computation of the gravitational effective action around de Sitter is impossible. Just as for a massless scalar, $F(x, y)$ is not well defined.

We note that for a massive scalar, the divergent integral is regulated by the mass and becomes of the form $\int \frac{d^3 q}{q^{3-2\epsilon}}$ which is finite. For the graviton, gauge invariance prevents such a mass from being generated.

III. DEFORMING AWAY FROM DE SITTER

A. Gravitational Perturbations

Since our calculation of the quantum effects around de Sitter have been derailed by the infrared divergences, we move slightly away from de Sitter and attempt to calculate the

effective action around a slightly deformed metric. While we can in principle consider any deformation, we will restrict ourselves to a deformation that preserves the spatial rotation and spatial translational symmetry. We therefore consider a metric of the form

$$g_{00}^{(ddS)} = \frac{1}{H^2\tau^2} \quad g_{ij}^{(ddS)} = -\frac{1}{H^2\tau^2} f(\tau)\delta_{ij} \quad (20)$$

Here the superscript ddS stands for deformed de Sitter. The function $f(\tau)$ parametrizes the deformation. We will assume that $f(\tau)$ is close to 1.

We can now consider perturbations around this metric. This is hard to do for a general background metric, but is facilitated here by the fact that this metric is close to the de Sitter solution. Accordingly, the equations of motion for the perturbations will also be close to the de Sitter equations of the previous section. We continue to use the gauge $h_{00} = h_{0i} = 0$.

We found in the previous section that the transverse traceless perturbations around de Sitter satisfy the same equation as a massless scalar. This led to the solution being singular at small momenta, and so we are interested in whether the deformation can modify the equation at low momenta. One possibility is that the deformation causes the transverse traceless perturbations to mix with the other perturbations. This however does not happen; the rotational symmetry requires any such mixing to come with derivatives, and this mixing is then suppressed at low momenta.

Direct evaluation of the equations (using Mathematica [66]) shows that the transverse traceless perturbations now have the action (this is for a perturbation with momentum along the z-direction)

$$\mathcal{L} = \frac{H^2}{8\kappa f^{3/2}} \left(h_{ij}^2 (-2f - 3\tau f' + \tau^2 f'') + \tau^2 (\partial_z h_{ij})^2 - f\tau^2 (\partial_\tau h_{ij})^2 \right) \quad (21)$$

Here primes indicate derivatives with respect to τ .

To bring this to the form of a scalar action, we define

$$\gamma_{ij} = \frac{H^2}{f^{1/4}} \tau^2 h_{ij} \quad (22)$$

This field satisfies the equation of motion

$$\frac{\partial}{\partial \tau} \left(\frac{1}{\tau^2} \frac{\partial}{\partial \tau} \gamma_{ij} \right) + \frac{1}{f\tau^2} k^2 \gamma_{ij} + \frac{5}{4\tau^3 f} \gamma_{ij} (-2f' + \tau f'') = 0 \quad (23)$$

B. A Special Case

Let us focus on a particular case. We take $f = \tau^\epsilon$ where ϵ is small. The equation then becomes (to order ϵ)

$$\frac{\partial}{\partial \tau} \left(\frac{1}{\tau^2} \frac{\partial}{\partial \tau} \gamma_{ij} \right) + \frac{\tau^{-\epsilon}}{\tau^2} k^2 \gamma_{ij} - \frac{15}{4\tau^4} \epsilon \gamma_{ij} = 0 \quad (24)$$

The perturbations no longer satisfy the equation for a massless de Sitter scalar. In particular, a term similar to the scalar mass term has appeared. We can solve this equation as we did for a massive scalar

$$\gamma_{ij}(k, \tau) = -\frac{H\tau\sqrt{-\pi\tau}}{2} H_\nu(-k\tau) \quad (25)$$

with $\nu^2 = \frac{9}{4} - \frac{15\epsilon}{4}$. The coincident limit of the propagator is then modified to

$$\langle \gamma_{ij}(x) \gamma_{kl}(x) \rangle = \int \frac{d^3 q}{(2\pi)^3} \frac{H^2}{q^3} (1 + q^2 \tau^2) (q^2 \tau^2)^{\frac{5\epsilon}{4}} P_{ijkl} \sim \frac{H^2}{5\epsilon\pi^2} P_{ijkl} \quad (26)$$

The coincident limit of the propagator, and in fact the propagator in general, is now finite, in contrast to the de Sitter case. The IR divergence is regulated, and loop calculations are now well defined. Note that the propagator is enhanced by a factor $\frac{1}{\epsilon}$, which is the vestige of the IR divergence.

(We note parenthetically that this is not exactly correct because the equation is not quite that of a massive scalar — there is a time dependence in the momentum dependent term. However, this can be treated as a slow variation of k with time. The solution to this mode equation is then approximately the solution above with k replaced by $k\tau^{-\epsilon/2}$. This does not affect the low momentum behavior of the solution.)

IV. TADPOLE CANCELLATION

We must however address another issue; the metric we are expanding around is not a solution to the classical equations of motion. Accordingly, if we were to consider the trace perturbations around the metric (20), we would find a tadpole. Specifically consider a fluctuation around the metric (20) of the form

$$g_{00} = g_{00}^{(ddS)} \quad g_{ij} = g_{ij}^{(ddS)} + h\delta_{ij} \quad (27)$$

Classically, the equation of motion for the perturbation h does not allow the solution $h = 0$. Indeed, to leading order in $(f - 1)$, the perturbation h satisfies the equation (after a suitable normalization of the kinetic term)

$$8h - 8\tau h' - 4\tau^2 h'' = \frac{(8f' - 4\tau f'')}{H^2 \tau \sqrt{\kappa}} \quad (28)$$

This tadpole vanishes when f is a constant, which is the statement that classically, the only solution to Einstein's equations of the form (20) is the de Sitter metric.

However, this does not have to be the case at the quantum level. There is a new tadpole generated at one loop that contributes to the equations. This can modify the equation for h . More precisely, there can be terms in the action roughly of the form hh_{ij}^2 . At one loop, we find a tadpole for h when h_{ij}^2 is replaced by its propagator.

These trilinear couplings can be very complicated [64], and it is useful to look for simplifications. In this case, we will use the fact that we are expanding around a metric which is close to de Sitter. As we have seen above, the propagator for the transverse traceless polarizations then receives an enhancement, and will dominate the tadpole. For this enhancement to occur, no derivative can act on the propagator. This means that we are only interested in a trilinear coupling hh_{ij}^2 where no derivative acts on the h_{ij} . It is straightforward to expand the action to find this term.

We find that this new coupling modifies the equation of motion to

$$8h - 8\tau h' - 4\tau^2 h'' = \frac{(8f' - 4\tau f'')}{H^2 \tau \sqrt{\kappa}} + 2\sqrt{\kappa} H^2 h_{ij}^2 \tau^2 \quad (29)$$

At one loop, we should replace the term h_{ij}^2 by the coincident limit of the Schwinger-Keldysh propagator evaluated in the metric (20). While we cannot solve this in general, we can find the solution for the case $f = \tau^\epsilon$. Here we have from eqn (26)

$$\langle h_{ij}(x) h_{ij}(x) \rangle = \frac{4}{5H^2 \tau^4 \epsilon \pi^2} \quad (30)$$

The tadpole in (29) now cancels if

$$\epsilon^2 + \frac{2\kappa H^2}{15\pi^2} = 0 \quad (31)$$

indicating that the quantum equations of motion are indeed solved by a metric of the form (20) with $f = \tau^\epsilon$ where ϵ is proportional to $\sqrt{\kappa}$. (Note though that this solution is only

valid when τ^ϵ is small. It would be interesting to find an exact solution; we leave this for future work.)

However, we have established our main result: de Sitter space, corresponding to $\epsilon = 0$ is *not* a solution to the quantum corrected equations! Spontaneous symmetry breaking of the de Sitter symmetry has occurred.

V. DISCUSSION AND CONCLUSION

We have argued that de Sitter space is *not* a solution to gravity coupled to a cosmological constant when quantum effects are taken into account. The classical equations of motion receive quantum corrections which are singular if the solution is taken to be de Sitter. We have argued that the deviations from de Sitter are calculable, and that the true solution is deformed away from de Sitter by a parameter proportional to $\sqrt{\kappa}$.

The argument for this was straightforward. In the exact metric of de Sitter space, there are gravitational perturbations whose propagator is ill defined, and which caused infrared divergences. A small deviation from de Sitter parametrized by a small parameter ϵ allows these modes to have a well defined propagator. However, the quantum effective action computed around this new metric now generically has terms which go as $\frac{1}{\epsilon}$. The quantum equations of motion are singular as we take ϵ to zero, and cause de Sitter to not be a solution when quantum corrections are included. (Another way to say this is that the de Sitter metric has an infinite action when quantum effects are included.) The calculation in the previous section argues that these qualitative arguments can be made quantitative, and that the perturbation away from de Sitter can be computed in perturbation theory.

These arguments are related to previous arguments in the literature, for instance by Polyakov. While Polyakov has argued that scalar field theory in de Sitter (using the in-out formalism) already leads to an instability, we have shown that gravitons produce an instability in the more controlled in-in formalism. The in-in formalism is expected to asymptote to the in-out result when the intermediate time is taken to infinity; it would be interesting to see if this is the case.

The Schwinger-Keldysh propagator for the gravitational perturbations is enhanced by a factor proportional to $\frac{1}{\sqrt{\kappa}}$. This indicates that the gravitational perturbation series, which is normally in powers of κ , is modified. A 1-loop diagram with a Schwinger-Keldysh propagator

now scales as $\sqrt{\kappa}$ and in general, the perturbation series becomes an expansion in $\sqrt{\kappa}$.

We should also discuss the occasionally thorny issue of gauge invariance. It is well known that tadpoles of gravitons are not gauge invariant, and so one might wonder about the status of the tadpoles we have calculated. The resolution is that our intermediate steps have been performed in a fixed gauge, but our final result (that de Sitter is unstable) is a gauge invariant statement. It is therefore valid in any gauge. Similarly, the deformed metric is not a gauge invariant quantity, but it has been presented in a particular gauge, and can be transformed to any gauge of choice.

A more subtle issue is the question of observables in quantum gravity. It is often argued that correlation functions, even for a fixed geodesic distance, are not well defined; this is roughly because any pointlike sources are smeared into black holes. However, our corrections are of order $\sqrt{\kappa}$ and therefore scale faster than any perturbative effect in quantum gravity, including the size of black holes. They are hence dominant at weak coupling and will not be washed out by quantum gravity effects.

Finally we note that this solution to the issue of the IR divergences may potentially lead to observable effects, at least if the Hubble scale is large enough. We leave this issue for future work.

VI. ACKNOWLEDGMENTS

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