

This is the accepted manuscript made available via CHORUS. The article has been published as:

Natural generalized mirage mediation

Howard Baer, Vernon Barger, Hasan Serce, and Xerxes Tata

Phys. Rev. D **94**, 115017 — Published 12 December 2016

DOI: [10.1103/PhysRevD.94.115017](https://doi.org/10.1103/PhysRevD.94.115017)

Natural generalized mirage mediation

Howard Baer^{1*}, Vernon Barger^{2†}, Hasan Serce^{1‡} and Xerxes Tata^{3‡}

¹*Department of Physics and Astronomy, University of Oklahoma, Norman, OK 73019, USA*

²*Department of Physics, University of Wisconsin, Madison, WI 53706, USA*

³*Department of Physics, University of Hawaii, Honolulu, HI 96822, USA*

Abstract

In the supersymmetric scenario known as mirage mediation (MM), the soft SUSY breaking terms receive comparable anomaly-mediation and moduli-mediation contributions leading to the phenomenon of *mirage unification*. The simplest MM SUSY breaking models which are consistent with the measured Higgs mass and sparticle mass constraints are strongly disfavoured by fine-tuning considerations. However, while MM makes robust predictions for gaugino masses, the scalar sector is quite sensitive to specific mechanisms for moduli stabilization and potential uplifting. We suggest here a broader setup of generalized mirage mediation (GMM), where heretofore discrete parameters are allowed as continuous to better parametrize these other schemes. We find that natural SUSY spectra consistent with both the measured value of m_h as well as LHC lower bounds on superpartner masses are then possible. We explicitly show that models generated from natural GMM may be beyond the reach of even high-luminosity LHC searches. In such a case, the proposed International Linear e^+e^- Collider (ILC) will be required for natural SUSY discovery via higgsino pair production reactions. We also outline prospects for detection of higgsino-like WIMPs from natural GMM.

*Email: baer@nhn.ou.edu

†Email: barger@pheno.wisc.edu

‡Email: serce@ou.edu

‡Email: tata@phys.hawaii.edu

1 Naturalness in Mirage Mediation

Superstring theory yields a consistent quantum theory of gravity and appears to have the required ingredients to potentially unify all four forces of nature. However, in order to gain predictivity, it is necessary to understand how the degeneracy associated with the many flat directions in the space of scalar fields (the moduli) is lifted to yield the true ground state, since many quantities relevant for physics at low energy are determined by the ground state values of these fields. The implementation of a class of compactifications where the extra spatial dimensions are curled up to small sizes with fluxes of additional fields trapped along these extra dimensions was used by Kachru, Kallosh, Linde and Trivedi (KKLT) [1] to construct models with a stable, calculable ground state with a positive cosmological constant and broken supersymmetry. The KKLT toy model is based on type-IIB superstrings including compactification with fluxes to a Calabi-Yau orientifold. While the background fluxes serve to stabilize the dilaton and the moduli that determine the shape of the compact manifold, it is necessary to invoke a non-perturbative mechanism such as gaugino condensation [2] on a $D7$ brane to stabilize the size of the compact manifold. Finally, a non-supersymmetric anti-brane ($\overline{D3}$) was included in order to break supersymmetry completely and obtain a de Sitter universe as required by observations. The resulting low energy theory thus has no unwanted light moduli, has a broken supersymmetry, and a positive cosmological constant. The existence of these flux compactifications with stable calculable minima having many desired properties may be viewed as a starting point for the program of discovering a string ground state that may lead to a phenomenologically viable low energy theory of SM particles and their superpartners, with $N = 1$ supersymmetry softly broken just above the weak scale.

The KKLT picture motivated several groups to analyze the structure of the soft SUSY breaking (SSB) terms in models based on a generalization of the KKLT set-up [3]. The key observation is that because of the mass hierarchy,

$$m_{\text{moduli}} \gg m_{3/2} \gg m_{\text{SUSY}}, \quad (1)$$

that develops in these models, the soft terms receive comparable contributions from both modulus (gravity) [4] and anomaly mediation of SUSY breaking [5], with their relative size parametrized by an additional parameter α . Moreover, the hierarchy (1) that leads to this mixed modulus-anomaly mediated SUSY breaking (also known as mirage-mediation or MM as discussed shortly) automatically alleviates phenomenological problems from late decaying moduli and gravitinos that could disrupt, for instance, the predictions of light element abundances from Big Bang nucleosynthesis. Upon integrating out the heavy dilaton field and the shape moduli, one is left with an effective broken supergravity theory of the observable sector fields denoted by \hat{Q} and the size modulus field \hat{T} . The Kähler potential depends on the location of matter and Higgs superfields in the extra dimensions via their modular weights $n_i = 0$ (1) for matter fields located on $D7$ ($D3$) branes, or $n_i = 1/2$ for chiral multiplets on brane intersections, while the gauge kinetic function $f_a = \hat{T}^{l_a}$, where a labels the gauge group, is determined by the corresponding location of the gauge supermultiplets, since the power $l_a = 1$ (0) for gauge fields on $D7$ ($D3$) branes [6].

Within the MM model, the SSB gaugino mass parameters, trilinear SSB parameters and sfermion mass parameters, all renormalized just below the unification scale (taken to be $Q =$

M_{GUT}), are given by,

$$M_a = M_s (l_a \alpha + b_a g_a^2), \quad (2)$$

$$A_{ijk} = M_s (-a_{ijk} \alpha + \gamma_i + \gamma_j + \gamma_k), \quad (3)$$

$$m_i^2 = M_s^2 (c_i \alpha^2 + 4\alpha \xi_i - \dot{\gamma}_i), \quad (4)$$

where $M_s \equiv \frac{m_{3/2}}{16\pi^2}$, b_a are the gauge β function coefficients for gauge group a and g_a are the corresponding gauge couplings. The coefficients that appear in (2)–(4) are given by $c_i = 1 - n_i$, $a_{ijk} = 3 - n_i - n_j - n_k$ and $\xi_i = \sum_{j,k} a_{ijk} \frac{y_{ijk}^2}{4} - \sum_a l_a g_a^2 C_2^a(f_i)$. Finally, y_{ijk} are the superpotential Yukawa couplings, C_2^a is the quadratic Casimir for the a^{th} gauge group corresponding to the representation to which the sfermion \tilde{f}_i belongs, γ_i is the anomalous dimension and $\dot{\gamma}_i = 8\pi^2 \frac{\partial \gamma_i}{\partial \log \mu}$. Expressions for the last two quantities involving the anomalous dimensions can be found in the Appendix of Ref. [7, 8].

The MM model is then specified by the parameters

$$m_{3/2}, \alpha, \tan \beta, \text{sign}(\mu), n_i, l_a. \quad (5)$$

The mass scale for the SSB parameters is dictated by the gravitino mass $m_{3/2}$. The phenomenological parameter α , which could be of either sign, determines the relative contributions of anomaly mediation and gravity mediation to the soft terms, and is expected to be $|\alpha| \sim \mathcal{O}(1)$. Grand Unification implies matter particles within the same GUT multiplet have common modular weights, and that the l_a are universal. We will assume here that all $l_a = 1$ and, for simplicity, there is a common modular weight for all matter scalars c_m but we will allow for different modular weights c_{H_u} and c_{H_d} for each of the two Higgs doublets of the MSSM. Such choices for the scalar field modular weights are motivated for instance by $SO(10)$ SUSY GUT models where the MSSM Higgs doublets may live in different **10**-dimensional Higgs reps.

Various aspects of MM phenomenology have been examined in Refs. [6, 7, 9, 10, 11]. The universality of the l_a leads to the phenomenon of *mirage unification* [6, 7] of gaugino mass parameters (and also corresponding matter scalar mass parameters of first and second generation sfermions whose Yukawa couplings are negligible). Here, for reasons that will become clear later, we focus on the gaugino mass parameters M_i : when extrapolated to high energies using one loop renormalization group equations (RGEs), these will unify at a scale $Q = \mu_{\text{mir}} \neq M_{\text{GUT}}$, where M_{GUT} is the unification scale for gauge couplings. Indeed, the observation of gaugino mass unification at the mirage unification scale,

$$\mu_{\text{mir}} = M_{\text{GUT}} e^{-8\pi^2/\alpha}, \quad (6)$$

is the smoking gun of such a scenario [12]. If $\alpha < 0$, then $\mu_{\text{mir}} > M_{\text{GUT}}$ and one finds *virtual* mirage unification at super-GUT energy scales. We stress that there is no physical threshold at $Q = \mu_{\text{mir}}$, and the evolution can be continued to $Q = M_{\text{GUT}}$ where the gaugino mass parameters would take on the values close to (2). The determination of the mirage unification scale also determines α , the parameter that governs the relative moduli- versus anomaly-mediation contribution to the soft SUSY breaking terms. Once α is known, then further extrapolation of the gaugino masses to $Q = M_{\text{GUT}}$ allows for a determination of the *gravitino mass* $m_{3/2}$.

Alas, this attractive MM scenario has recently been confronted by the twin constraints of LHC searches on the one hand and a clarified understanding of SUSY naturalness on the other. One important LHC constraint comes from the new-found Higgs mass $m_h \simeq 125$ GeV which in the context of the MSSM requires highly mixed TeV-scale top-squarks [13]. The other LHC constraint is that the gluino mass, based on LHC13 searches with $\sim 10 \text{ fb}^{-1}$ of data, require $m_{\tilde{g}} \gtrsim 1.9$ TeV (within the context of various simplified models) [14].

For the case of naturalness, it has been emphasized [15, 16, 17] that previous studies— that lead to the conclusion that naturalness requires light top squarks— neglect the fact that one must evaluate the sensitivity of m_h or m_Z only with respect to the *independent* parameters of the theory, as embodied for instance in the frequently used EENZ/BG measure [18], $\Delta_{\text{BG}} \equiv \max_i |\frac{\partial \log m_Z^2}{\partial \log p_i}|$. Here i labels the various independent, fundamental parameters p_i of the theory. Historically, this measure has been applied to multi-soft-parameter effective SUSY theories where the additional parameters are introduced to parametrize our ignorance of the source of soft terms. However, in any more fundamental theory the various soft terms are derived in terms of more fundamental entities, such as the gravitino mass in gravity mediation [19], or via Eq. (2)-(4) for mirage-mediation. In this case, the soft-SUSY breaking parameters are *correlated* and not independent: then, neglecting these correlations will lead to an *over-estimate* of the fine-tuning in these theories [15, 16, 17]. In MM, where α takes on a pre-determined value, the soft parameters are all determined by $m_{3/2}$ and Δ_{BG} reduces to the model-independent electroweak measure Δ_{EW} .¹

The electroweak fine-tuning parameter [20, 21], Δ_{EW} , is a measure of the degree of cancellation between various contributions on the right-hand-side (RHS) in the well-known expression for the Z mass:

$$\frac{m_Z^2}{2} = \frac{m_{H_d}^2 + \Sigma_d^d - (m_{H_u}^2 + \Sigma_u^u) \tan^2 \beta}{\tan^2 \beta - 1} - \mu^2 \simeq -m_{H_u}^2 - \Sigma_u^u - \mu^2 \quad (7)$$

which results from the minimization of the Higgs potential in the MSSM. Here, $\tan \beta = v_u/v_d$ is the ratio of Higgs field vacuum-expectation-values and the Σ_u^u and Σ_d^d contain an assortment of radiative corrections, the largest of which typically arise from the top squarks. Expressions for the Σ_u^u and Σ_d^d are given in the Appendix of Ref. [21]. If the RHS terms in Eq. (7) are individually comparable to $m_Z^2/2$, then no unnatural fine-tunings are required to generate $m_Z = 91.2$ GeV. Δ_{EW} is defined to be the largest of these terms, scaled by $m_Z^2/2$. Clearly, low electroweak fine-tuning requires that μ be close to m_Z^2 and that $m_{H_u}^2$ be radiatively driven to *small* negative values close to the weak scale. This scenario has been dubbed radiatively-driven natural supersymmetry or RNS [20, 21].

The main requirements for low electroweak fine-tuning ($\Delta_{\text{EW}} \lesssim 30$)² are the following.

- $|\mu| \sim 100 - 300$ GeV [23] where $\mu \gtrsim 100$ GeV is required to accommodate LEP2 limits from chargino pair production searches.

¹More generally, we advocate the use of Δ_{EW} in the discussion of naturalness of models with a given superpartner spectrum since discarding any high scale model with a (seemingly) large value of Δ_{BG} and a low value of Δ_{EW} may be prematurely discarding an effective theory because (unincorporated) correlations among the high scale parameters could well lower the value of Δ_{BG} all the way to Δ_{EW} : *das Kind mit dem Bade ausschütten!*

² The onset of fine-tuning for $\Delta_{\text{EW}} \gtrsim 30$ is visually displayed in Ref. [22].

- $m_{H_u}^2$ is driven radiatively to small, and not large, negative values at the weak scale [20, 21].
- The top squark contributions to the radiative corrections $\Sigma_u^u(\tilde{t}_{1,2})$ are minimized for TeV-scale highly mixed top squarks [20]. This latter condition also lifts the Higgs mass to $m_h \sim 125$ GeV. For $\Delta_{EW} \lesssim 30$, the lighter top squarks are bounded by $m_{\tilde{t}_1} \lesssim 3$ TeV [21, 22].
- The gluino mass, which feeds into the stop masses and hence the $\Sigma_u^u(\tilde{t}_{1,2})$, is bounded by $m_{\tilde{g}} \lesssim 4$ TeV [21, 22].

Detailed scans over MM parameter space for various choices of matter and Higgs field modular weights found all models consistent with LHC8 sparticle and Higgs mass constraints were in fact highly fine-tuned with $\Delta_{EW} > 100$ (for a summary, see Fig. 13 of Ref. [16]). This means these models give a poor prediction for the weak scale as typified by $m_{\text{weak}} \sim m_{W,Z,h} \sim 100$ GeV, *i.e.* the weak scale of 100 GeV is only generated by excessive fine-tuning of the μ parameter. One may thus ask: are mirage mediation models on their way to the dustbin of failed SUSY models?^{3 4}

2 Natural Generalized Mirage Mediation

The evident failure of naturalness in MM mentioned at the end of the last section leads us to re-examine the phenomenological implications of moving from discrete choices of the parameters a_{ijk} and c_i in Eqs. (3) and (4) to a continuous range, and also to allow c_i values greater than 1. While the discrete parameter choices occur in a wide range of KKLT-type compactifications (for some discussion, see Ref. [27]), a continuous range of these parameters may be expected if one allows for more generic methods of moduli stabilization and potential uplifting. For instance, if the soft terms scan as in the string landscape picture, then their moduli-mediated contributions may be expected to be parametrized by a continuous value. For models which generate a small μ term ~ 100 GeV from multi-TeV soft terms, such as radiative Peccei-Quinn breaking [28], it has been suggested that the statistical pull by the landscape towards large soft terms, coupled with the anthropic requirement of $m_{\text{weak}} \sim 100$ GeV, acts as an attractor towards natural SUSY soft term boundary conditions [29].

Note that the phenomenological modification that we suggest will not affect the result (2) for gaugino mass parameters, which has been stressed [12] to be the most robust prediction of the MM mechanism. In this paper, we allow for the more *general* mirage mediation (GMM) parameters, thus adopting a parameter space given by

$$m_{3/2}, \alpha, \tan \beta, a_3, c_m, c_{H_u}, c_{H_d} \quad (GMM), \quad (8)$$

where a_3 is short for $a_{Q_3 H_u U_3}$. The independent values of c_{H_u} and c_{H_d} which set the moduli-mediated contribution to the soft Higgs mass terms may conveniently be traded for weak scale values of μ and m_A as is done in the two-parameter non-universal Higgs model [30]:

$$m_{3/2}, \alpha, \tan \beta, a_3, c_m, \mu, m_A \quad (GMM'). \quad (9)$$

³The models of deflected mirage mediation [24] which combine gauge-, moduli- and anomaly-mediation, still seem viable and may allow for naturalness [25].

⁴A phenomenological AMSB model has been proposed which can reconcile $(g-2)_\mu$ with the value $m_h \simeq 125$ GeV [26].

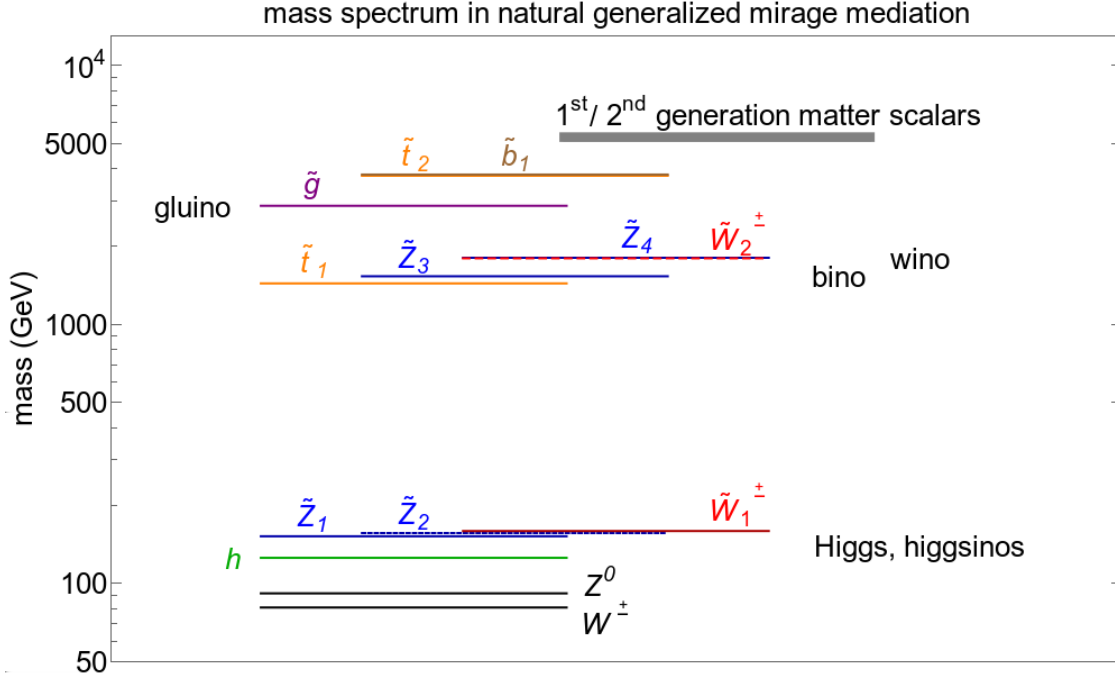


Figure 1: A typical superparticle mass spectrum generated from natural generalized mirage mediation (nGMM) as in Table 1.

This trick allows for more direct exploration of natural SUSY parameter space which requires $\mu \sim 100 - 300$ GeV.

In Fig. 1, we show the SUSY spectrum generated from one such parameter space point in the *natural* GMM model (nGMM), with the corresponding data shown in Table 1. This benchmark point was generated using the Isajet/Isasugra computer code [31] with non-universal soft term inputs. The specific input parameters are $m_{3/2} = 75$ TeV, $\alpha = 4$, $\tan \beta = 10$, $a_3 = 3$, $c_m = 6.9$ and with $\mu = 150$ GeV and $m_A = 2$ TeV. The latter two choices end up corresponding to $c_{H_u} = 11.3$ and $c_{H_d} = 1.15$. From Table 1, we see the gluino mass is $m_{\tilde{g}} = 2856$ GeV, which is just beyond the 5σ projected reach of HL-LHC with $\sqrt{s} = 14$ TeV and 3000 fb^{-1} of integrated luminosity [32], at least without tagged bs to further enhance the signal. The Higgs mass $m_h = 124.9$ GeV agrees well with measurements from LHC. The squarks and sleptons of the first/second generation lie in the 5 TeV range while third generation squarks can be lighter, with $m_{\tilde{t}_1} \simeq 1433$ GeV. This latter value appears beyond the reach of HL-LHC where a 95% exclusion reach with 3000 fb^{-1} extends out to $m_{\tilde{t}_1} \sim 1100$ GeV for $m_{\tilde{Z}_1} \sim 100$ GeV [33]. Note that this benchmark point has $\Delta_{\text{EW}} = 15.5$ and so relatively low electroweak fine-tuning. A high scale theory with $\alpha = 4$ which led to the assumed values of c_i and a_3 would have $\Delta_{\text{BG}} \simeq 15$ and would not be fine-tuned.

In Fig. 2, we show the running of the three gaugino masses for the nGMM benchmark model. In this case, we see the most robust feature of GMM: the celebrated mirage unification of gaugino masses at the intermediate scale $\mu_{\text{mir}} \sim 10^{7.5}$ GeV consistent with $\alpha = 4$, as can be seen from Eq. (6).

parameter	nGMM
$m_{3/2}$	75000
α	4
$\tan \beta$	10
c_{H_u}	11.3
c_{H_d}	1.15
c_m	6.9
μ	150
m_A	2000
$m_{\tilde{g}}$	2856.5
$m_{\tilde{u}_L}$	5266.7
$m_{\tilde{u}_R}$	5398.2
$m_{\tilde{e}_R}$	4824.6
$m_{\tilde{t}_1}$	1433.1
$m_{\tilde{t}_2}$	3732.0
$m_{\tilde{b}_1}$	3770.5
$m_{\tilde{b}_2}$	5124.5
$m_{\tilde{\tau}_1}$	4749.5
$m_{\tilde{\tau}_2}$	5093.9
$m_{\tilde{\nu}_\tau}$	5103.1
$m_{\tilde{W}_2}$	1791.6
$m_{\tilde{W}_1}$	158.7
$m_{\tilde{Z}_4}$	1799.4
$m_{\tilde{Z}_3}$	1526.9
$m_{\tilde{Z}_2}$	155.8
$m_{\tilde{Z}_1}$	151.4
m_h	124.9
$\Omega_{\tilde{Z}_1}^{std} h^2$	0.005
$BF(b \rightarrow s\gamma) \times 10^4$	3.1
$BF(B_s \rightarrow \mu^+\mu^-) \times 10^9$	3.9
$\sigma^{SI}(\tilde{Z}_1, p)$ (pb)	3.0×10^{-10}
$\sigma^{SD}(\tilde{Z}_1 p)$ (pb)	9.6×10^{-6}
$\langle \sigma v \rangle _{v \rightarrow 0}$ (cm ³ /sec)	3.1×10^{-25}
Δ_{EW}	15.5

Table 1: Input parameters and masses in GeV units for a natural generalized mirage mediation SUSY benchmark point with $m_t = 173.2$ GeV.

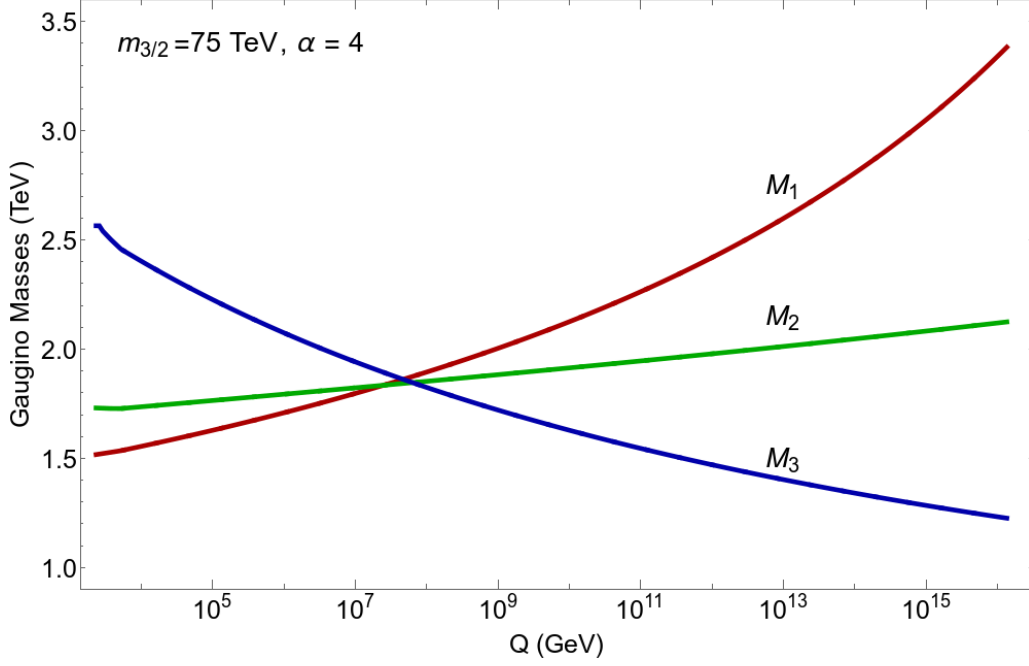


Figure 2: Evolution of gaugino masses from the nGMM benchmark point with $m_{3/2} = 75$ TeV, $\alpha = 4$.

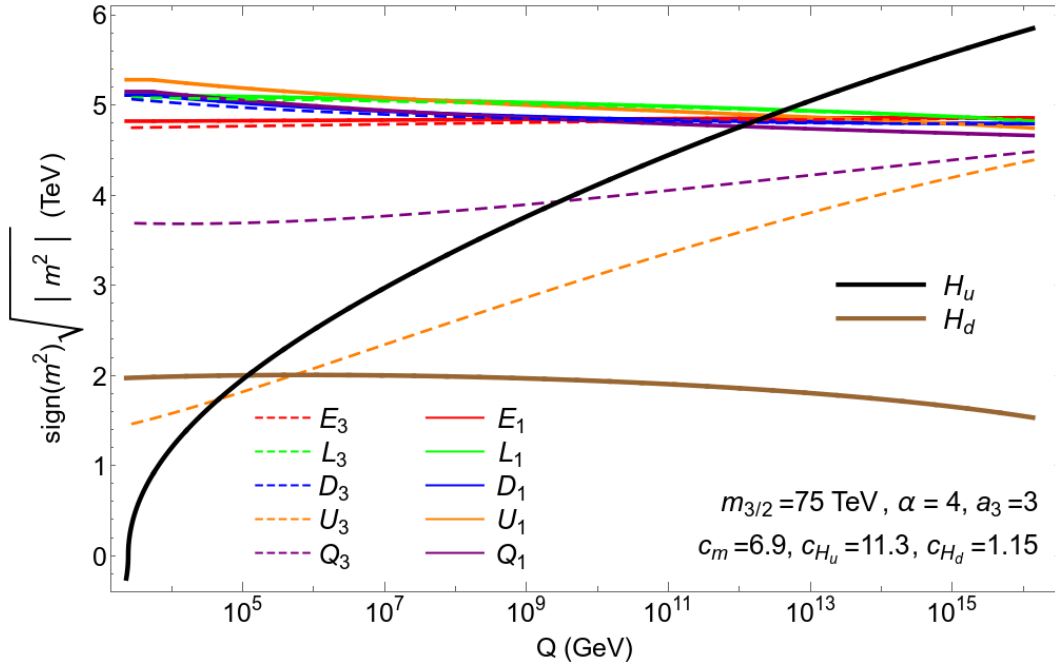


Figure 3: Plot of running scalar masses from the nGMM benchmark point with $m_{3/2} = 75$ TeV, $\alpha = 4$, $\tan \beta = 10$ and $c_m = 6.9$, $a_3 = 3$ with $c_{H_u} = 11.3$ and $c_{H_d} = 1.15$ (corresponding to $\mu = 150$ GeV and $m_A = 2$ TeV at the weak scale)

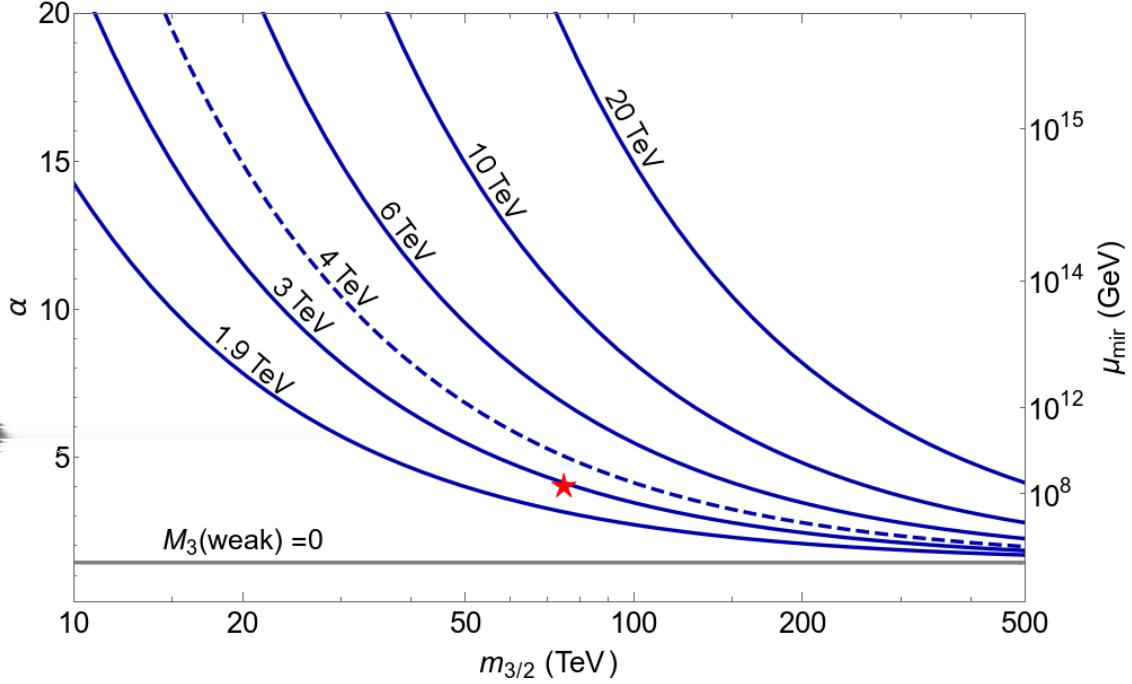


Figure 4: Contours of $M_3(weak)$ in the $m_{3/2}$ vs. α plane of the nGMM model with other parameters as fixed in Table 1. The region below $M_3 \sim 1.9$ is roughly excluded by LHC gluino pair searches. The location of our benchmark point is shown with a red star. The region below the dashed $m_{\tilde{g}} = 4$ TeV contour has the capacity to be natural. On the right side, some corresponding values of μ_{mir} are shown.

In Fig. 3, we show the renormalization group evolution of the various scalar soft mass terms for the nGMM benchmark model. First/second generation matter scalar mass parameters remain close to 5 TeV. Unlike for the model with a common modular weight for the two Higgs doublets, these do not unify at $Q = \mu_{mir}$ because for the nGMM model, the hypercharge D -term contribution to the evolution no longer vanishes. In contrast, third generation and Higgs mass square parameters evolve considerably more because of large Yukawa interactions. In particular, $m_{U_3}^2$, runs to much lower values ~ 1.5 TeV at the weak scale. The up-Higgs soft mass $m_{H_u}^2$ begins about 20% higher in value than matter scalar masses at $Q = M_{GUT}$, but then evolves to small negative values at the weak scale, so that the requirement for electroweak naturalness, $|m_{H_u}^2| \sim m_Z^2$ is satisfied. The soft term $m_{H_d}^2$ which sets the heavy Higgs mass scale can be adjusted up or down with not-to-much cost to naturalness Δ_{EW} . We remark here that because the matter scalars are essentially decoupled, our spectra for phenomenological purposes is similar to what may be derived from the NUHM2 model but with gaugino mass parameters fixed by the MM values rather than by universality.

In Fig. 4, we show a larger set of GMM parameter space by contours of gaugino mass $M_3(weak)$ in the $m_{3/2}$ vs. α plane. At tree level, then $m_{\tilde{g}} \sim M_3(weak)$. Thus, the region below $M_3(weak) \lesssim 1.9$ TeV is excluded by LHC13 gluino pair searches. The location of our benchmark point is noted with a red star. The region below the dashed $m_{\tilde{g}} = 4$ TeV contour

has the capacity to be natural. On the right side, some corresponding values of μ_{mir} are shown.

3 Consequences for Colliders

3.1 LHC

It has been pointed out in Ref. [34] that in natural SUSY models such as RNS with gaugino mass unification, additional signatures for SUSY with light higgsinos are present at the LHC even though gluinos and also top squarks may be too heavy to be detectable. The first of these, labeled same-sign diboson production [35] (SSdB), arises from wino pair production $pp \rightarrow \widetilde{W}_4^\pm \widetilde{Z}_4$ where, for instance, $\widetilde{W}_2^+ \rightarrow W^+ \widetilde{Z}_{1,2}$ while $\widetilde{Z}_4 \rightarrow W^+ \widetilde{W}_1^-$. This leads to a robust $W^\pm W^\pm + \cancel{E}_T$ signature consisting of two acollinear same-sign dilepton + \cancel{E}_T events with jet activity only from QCD radiation. These event topologies have very low backgrounds. For large integrated luminosity $\sim 300 - 3000 \text{ fb}^{-1}$ – anticipated at the high luminosity LHC – this channel yields the greatest LHC14 reach.

A second robust signature expected in RNS-type models is higgsino pair production $\widetilde{Z}_1 \widetilde{Z}_2 j$ in association with a hard monojet from QCD radiation, followed by $\widetilde{Z}_2 \rightarrow \widetilde{Z}_1 \ell^+ \ell^-$ decay. The leptons in the OS/SF pair emerging from \widetilde{Z}_2 decay are quite soft (due to the small $m_{\widetilde{Z}_2} - m_{\widetilde{Z}_1} \sim 10 - 20 \text{ GeV}$ mass gap expected in models with universal gaugino masses) and would frequently fail detector trigger requirements. However, the hard ISR jet or the associated large \cancel{E}_T could serve as a trigger. After suitable cuts, it appears this signature gives a good reach in the $\mu - m_{1/2}$ parameter plane of the model. The calculations of Ref. [34] indicate that essentially all of the RNS parameter space with $\Delta_{EW} \leq 30$ is covered by these two channels assuming $\sim 3000 \text{ fb}^{-1}$ of integrated luminosity at LHC14.

In contrast, for the nGMM model, both these signatures appear much more challenging for LHC SUSY searches. The reason is the compressed spectrum of gauginos which occurs in nGMM. In NUHM2 with gaugino mass unification at $Q = M_{GUT}$, then the weak scale gauginos after RG running are expected to occur in a ratio $M_3 : M_2 : M_1 \sim 7 : 2 : 1$. Naturalness considerations require gluinos not much heavier than $\sim 4 \text{ TeV}$ in NUHM2 for $\Delta_{EW} < 30$ [21, 22]; if they do become heavy, they increase the top-squark masses which increases the $\Sigma_u^u(\widetilde{t}_{1,2})$ contributions so that again one must fine-tune against these contributions. This naturalness condition, together with gaugino mass universality, then guarantees that the winos are almost always accessible to LHC14 searches for NUHM2 if $\Delta_{EW} \leq 30$. Also, in this case, the $\widetilde{Z}_2 - \widetilde{Z}_1$ mass gap is always larger than $\sim 10 \text{ GeV}$. In contrast, compressed gaugino spectra with $M_1 \sim M_2 \sim M_3$ at an intermediate scale are the hallmark of MM models with a low α and concomitantly low mirage unification scale. This means that – with $m_{\widetilde{g}} \sim 3 - 4 \text{ TeV}$ – wino pairs (with mass $m(wino) \sim m_{\widetilde{g}}$) may well be too heavy to be produced at detectable rates at LHC14. Moreover, these larger values of M_1 and M_2 from nGMM result in an even more compressed spectrum of neutral higgsinos, as exemplified by the benchmark in Table 1 for which the mass gap $m_{\widetilde{Z}_2} - m_{\widetilde{Z}_1} \sim 5.4 \text{ GeV}$. Such a small mass gap makes the LHC monojet plus soft dilepton search much more difficult – in fact, in a recent CMS search for this channel [36], they indeed required $m(\ell^+ \ell^-) > 4 \text{ GeV}$ to stay away from the J/ψ and γ^* poles with a cut around $9 - 10.5 \text{ GeV}$ to stay away from the Υ pole. Such cuts would veto much of the signal

region expected from our nGMM benchmark.

3.2 Linear electron-positron colliders

In Ref. [37], a variety of measurements were proposed for MM models at the LHC and International Linear e^+e^- Collider or ILC which could determine the modular weights associated with matter scalars and measure the relative moduli-/anomaly-mediated contributions to the soft terms and the gravitino mass $m_{3/2}$. The ILC would initially be operating with $\sqrt{s} = 0.5$ TeV but is upgradable to 1 TeV. In Ref. [38], it was pointed out that for SUSY models with radiatively-driven naturalness, the ILC would be a *higgsino factory* for $\sqrt{s} > 2m(\text{higgsino}) \sim 2\mu$. The two reactions $e^+e^- \rightarrow \widetilde{W}_1^+ \widetilde{W}_1^-$ and $\widetilde{Z}_1 \widetilde{Z}_2$ occur at rates comparable to muon pair production once the kinematic production threshold is passed. Moreover, the higgsino pair production cross section exceeds that for Higgs boson production unless higgsino production is kinematically suppressed. In Ref. [38], it was shown that the clean environment of ILC detector events and the adjustable beam energy and polarization can easily allow for both discovery as well as a suite of precision measurements, at least for $\widetilde{Z}_1 - \widetilde{Z}_2$ mass gaps expected in the RNS framework with $\Delta_{\text{EW}} < 30$. Direct measurement of the $E(\ell^+\ell^-)$ and $m(\ell^+\ell^-)$ distributions from $\widetilde{Z}_1 \widetilde{Z}_2$ production followed by $\widetilde{Z}_2 \rightarrow \widetilde{Z}_1 \ell^+ \ell^-$ decay allows for measurement of $m_{\widetilde{Z}_2}$ and $m_{\widetilde{Z}_1}$ to sub-percent precision [38, 39]. Measurement of the $E(jj)$ and $m(jj)$ distributions from $\widetilde{W}_1 \widetilde{W}_1 \rightarrow (q\bar{q}' \widetilde{Z}_1) + (\ell \nu_\ell \widetilde{Z}_1)$ production allow for sub-percent measurements of $m_{\widetilde{W}_1}$ and $m_{\widetilde{Z}_1}$ if the mass gap is sufficiently large. Moreover, the mass gaps are sensitive to $\tan\beta$ and gaugino masses M_1 and M_2 . In the RNS case with $m_{\widetilde{Z}_2} - m_{\widetilde{Z}_1} \sim 20$ GeV, it was shown that the gaugino mass parameters can be extracted with a precision of 5 – 10%, and examination of the more difficult case of the 10 GeV mass gap is in progress [39]. Clearly, prospects for the detection of the higgsinos of nGMM models (where the mass gap is even smaller) and corresponding measurements of gaugino masses will be even more challenging but worthy of investigation.⁵ A positive outcome would mean that the ILC would be a *discovery machine* for a scenario that would likely be beyond the reach of even a high luminosity LHC. We emphasize that if the extraction of gaugino masses turns out to be feasible, then extrapolation of these masses via RGEs to high energies would indicate mirage unification and allow extraction of the parameter α , and also the associated gravitino mass $m_{3/2}$.

4 WIMP signals from nGMM

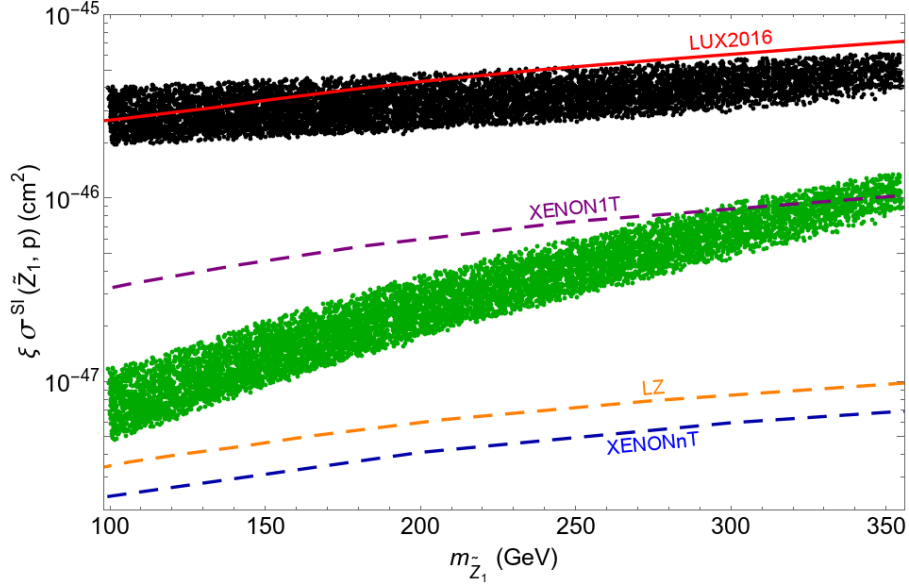
We now turn to prospects for dark matter detection in the natural generalized mirage mediation scenario. Since electroweak naturalness requires a low μ parameter, $\mu \sim 100 - 300$ GeV, the LSP is expected to be mainly higgsino-like with a non-negligible gaugino component. However,

⁵The nGMM model is not the only scenario with a compressed higgsino spectrum and very heavy gauginos that suggests that the ILC could be a discovery machine. If the vacuum-expectation-value of the auxiliary field that breaks supersymmetry transforms as a **75** dimensional representation of $SU(5)$ (rather than a singlet as is usually assumed), the resulting non-universal pattern of GUT scale gaugino masses leads to $M_3 : M_2 : M_1 = 6, 6, -5$ at the weak scale, so winos and binos would be even heavier than for our nGMM case study, and the higgsinos even more compressed. Such a scenario would be even more challenging to detect.

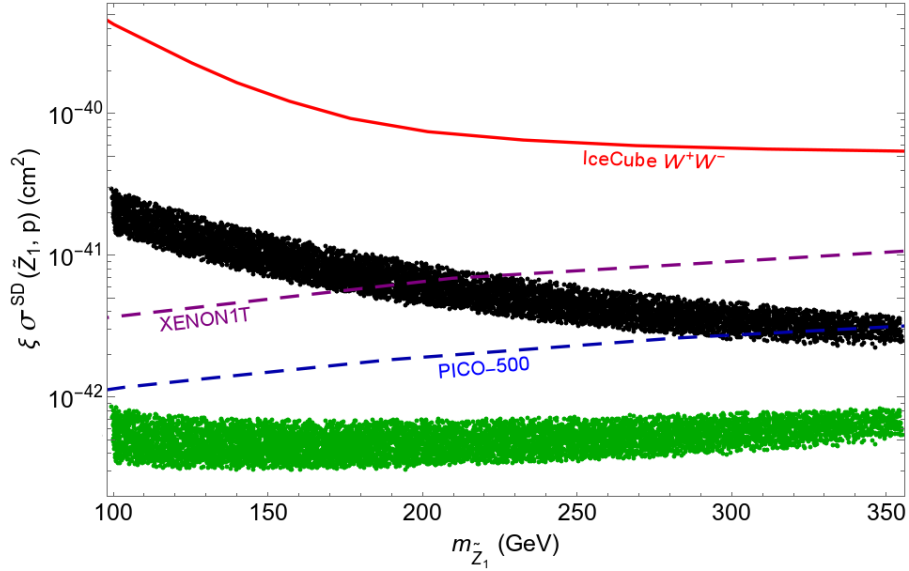
comparing nGMM to natural models with gaugino mass unification like RNS, it is clear that for nGMM, the electroweak gauginos are much heavier because the gaugino spectrum is more compressed. As a result, both \tilde{Z}_1 and \tilde{Z}_2 are considerably more higgsino-like than in RNS, and further, the inter-higgsino mass gaps are also smaller. This, in turn, means the higgsino co-annihilation rate is enhanced in nGMM relative to RNS. Consequently the thermally-produced higgsino-like neutralino abundance can be as low as $\Omega_{\tilde{Z}_1}^{TP} h^2 \sim 0.12/40$, *i.e.* thermally produced higgsinos make up just a few percent of the observed DM, an even lower relic abundance than in natural NUHM2 models. The possibility that the deficit in dark matter abundance is made up by non-thermal processes such as moduli production and subsequent decay to higgsinos is excluded as we will see below. In Ref. [40] it is suggested that if one insists on naturalness in the electroweak sector, one ought to have naturalness in the QCD sector: this brings into the discussion axion superfields, mixed axion-higgsino dark matter and production of neutralinos via axino/saxion production and decay. In this latter case, then axions may make up the bulk of dark matter with only a small fraction of the abundance consisting of higgsino-like WIMPs.

In Fig. 5a), we show the WIMP spin-independent (SI) direct detection rates expected from nGMM in the $m_{\tilde{Z}_1}$ vs. $\xi\sigma^{SI}(\tilde{Z}_1, p)$ plane. The vertical axis includes a factor of $\xi \equiv \Omega_{\tilde{Z}_1}^{TP} h^2 / 0.12$ to account for the possibility of a depleted local abundance of target WIMPs. Here, we adopt matter scalar soft terms ~ 5 TeV with $a_3 = 3$ and the $m_A = 2$ TeV and then scan over $m_{3/2} : 10 - 200$ TeV, $\alpha : 0 - 20$ and $\mu : 100 - 400$ GeV. We show only the points with $\Delta_{EW} < 30$. The upper black points assume that higgsinos produced by an additional non-thermal \tilde{Z}_1 production saturate the observed dark matter density, so $\xi = 1$, while for the lower green points we assume the higgsino abundance is given by its thermal value so that the bulk of dark matter is axions. Non-thermal production of higgsinos from axino/saxion decays would increase $\Omega_{\tilde{Z}_1} h^2$ resulting in an increase to ξ of the green points. Of course, the density of neutralinos could be diluted if there was additional entropy production [41] during the history of the Universe. The current reach of the LUX experiment [42] is shown as red-solid while the XENON1T reach [43] is purple-dashed. We see that the current LUX experiment has just started to probe the parameter space with $\xi = 1$ while all of this space will be probed by XENON1T. Multi-ton noble liquid detectors such as LZ [44], XENONnT (20tY exposure) [43], DarkSide-20K [45], DEAP-50T [46] and DARWIN [47] will be required to probe the entire parameter space with $\xi < 1$. We note these detection rates are lower than expected from natural NUHM2 models [48, 49] since both ξ is reduced and also with heavier electroweakino masses, the LSP is more pure higgsino-like in nGMM. In this case, the Higgs exchange amplitude, which depends on a product of higgsino times gaugino couplings, is reduced in nGMM compared to NUHM2.

In Fig. 5b), we show the spin-dependent cross sections for the same scan as in frame a) with $\xi = 1$ and $\xi < 1$ (fixed by the thermal abundance of higgsinos), along with the current bound from the IceCube experiment (red solid line) [50] and projected reaches of the XENON1T (dashed purple line) and PICO500 (dashed-blue line) [51]. We see that the nGMM points, even with $\xi = 1$, satisfy all current bounds. This situation is quite different from the case of the well-tempered neutralino where the higgsino-rich neutralino branch is solidly excluded by the IceCube data. The reason is that though higgsinos couple with full gauge strength to the Z , in the case of the (nearly) pure higgsino-LSP of the nGMM, the coupling of Z to



(a) SI direct detection rate



(b) SD direct detection rate

Figure 5: *a*) The spin-independent, and *b*) the spin-dependent neutralino-nucleon direct detection rates multiplied by fractional dark matter abundance $\xi \equiv \Omega_{\tilde{Z}_1}^{TP} h^2 / 0.12$ in the $m_{\tilde{Z}_1}$ vs. $\xi \sigma^{SI}(\tilde{Z}_1, p)$ plane from a scan over $m_{3/2}$, α and μ , with other parameters fixed as in the benchmark model. The black points have $\xi = 1$ while the green points have $\xi < 1$ corresponding to the fraction given by thermally produced higgsinos. The current LUX bound is denoted by the solid line, while the projected reaches of several noble liquid direct detection experiments are shown by the dashed lines in frame *a*). In frame *b*), we show the current IceCube limit by the red-solid line and projected reaches of future detectors XENON1T (dashed-purple) and PICO-500 (dashed-blue).

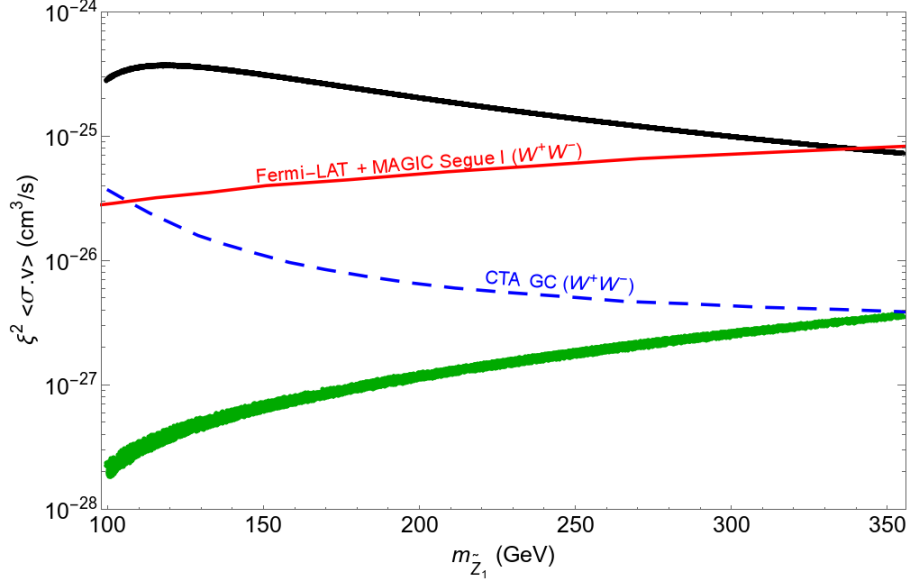


Figure 6: The neutralino annihilation cross section times velocity scaled by ξ^2 in the $m_{\tilde{Z}_1}$ vs. $\xi^2\langle\sigma v\rangle$ plane from a scan over $m_{3/2}$, α and μ . The black points have $\xi = 1$ while the green points have $\xi < 1$. The solid red-line shows the upper bound from FERMI-LAT collaboration (assuming neutralino annihilation to W^+W^- pairs) while the dashed blue line shows the corresponding projected reach of CTA.

identical neutralinos (which determines the SD cross section) vanishes when the $\tilde{Z}_i \simeq \frac{\tilde{h}_u \pm \tilde{h}_d}{\sqrt{2}}$. We see that the XENON1T experiment will detect a signal even via spin-dependent scattering for $m_{\tilde{Z}_1} \lesssim 200$ GeV if neutralinos make up all the local DM. Experiments like PICO-500 will be needed to probe yet higher mass values. Finally, we remark that if the neutralino density is determined by its thermal value, it will escape detection via SD neutralino-nucleon scattering in the case of the nGMM.

In Fig. 6 we plot the values of $\xi^2\langle\sigma v\rangle$, the thermally-averaged neutralino annihilation cross section times velocity, versus the lightest neutralino mass for the same scan as in Fig. 5. Higgsino-like neutralinos in the range of interest dominantly annihilate to W^+W^- pairs. As before, we show results for $\xi = 1$ by black dots, and for ξ determined assuming the neutralino relic density is given by its thermal value by green dots. The solid, red line shows the upper bound on the neutralino cross section, assuming annihilation to W boson pairs, obtained in Ref. [52] by combining the dwarf-spheroidal data from the Fermi-LAT collaboration and the MAGIC collaboration.⁶ Taken at face value, this analysis excludes the possibility that higgsino relics dominate the CDM density over almost the entire mass range favoured by electroweak naturalness.⁷ In contrast, if we assume that the higgsino contribution to the DM density is given by its thermal expectation, it appears that in nGMM dark matter indirect detection via

⁶For the mass range of our interest the limit is mainly dominated by FERMI-LAT observations.

⁷For the nGMM model scan that we are discussing, we have checked that the Fermi-LAT constraint restricts the higgsino component of the dark matter to no more than $\sim 35\%$ (50%) [85%] for $m_{\tilde{Z}_1} = 150$ GeV (200 GeV) [300 GeV].

the gamma ray signal would be very difficult even at the proposed ground-based Cherenkov Telescope Array, projections for which are shown by the dashed blue line in the figure [53].

5 Concluding Remarks

The simplest renditions of the very intriguing model of mirage mediation seem to be strongly disfavoured by naturalness considerations, when combined with the measured value of the Higgs boson mass and lower limits from the LHC on superparticle masses. However, several groups have observed that while MM gaugino mass predictions are very robust, the scalar sector is quite sensitive to the mechanisms for moduli stabilization and potential uplifting. Here, we advocated a generalized version of MM where discrete parameters depending on modular weights are elevated to continuous ones to parametrize more general possibilities for moduli stabilization and potential uplifting. The added flexibility of general mirage mediation allows for construction of natural GMM models which are consistent with LHC Higgs mass measurements and sparticle search constraints. We exhibit a benchmark point with a natural superpartner spectrum which maintains mirage unification in the gaugino sector. The resulting spectrum, while highly natural, will likely elude LHC searches even at very high luminosity. In the nGMM, prospects for dark matter detection are also modified significantly from expectations in natural scenarios with GUT scale gaugino mass unification. The possibility that (non-thermally produced) higgsinos comprise all the DM appears to be excluded by the combined FERMI-LAT-MAGIC analysis. If instead the WIMP density is given by its thermal value, with the remainder being composed for instance of axions, then multi-ton noble liquid detectors such as LZ or XENONnT or others will be required for detection. For the nGMM scenario, the resolving power of ILC may well offer the best hope to unearth the predicted light higgsinos signal. If ILC finds such a signal, it is possible that fits to the gaugino masses may allow for measurements of the relative moduli/anomaly mixing (α) parameter and the gravitino mass $m_{3/2}$.

Acknowledgments

We thank Kiwoon Choi and Jenny List for e-discussions. This work was supported in part by the US Department of Energy, Office of High Energy Physics. The computing for this project was performed at the OU Supercomputing Center for Education & Research (OSCER) at the University of Oklahoma (OU).

References

- [1] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, Phys. Rev. **D68**, 046005 (2003).
- [2] S. Ferrara, L. Girardello and H. P. Nilles, Phys. Lett. B **125** (1983) 457; H. P. Nilles, hep-th/0402022.

- [3] K. Choi, A. Falkowski, H. P. Nilles, M. Olechowski and S. Pokorski, J. High Energy Phys. **0411**, 076 (2004); K. Choi, A. Falkowski, H. P. Nilles and M. Olechowski, Nucl. Phys. **B718**, 113 (2005). J. P. Conlon, F. Quevedo and K. Suruliz, JHEP **0508**, 007 (2005) [arXiv:hep-th/0505076].
- [4] A. Chamseddine, R. Arnowitt and P. Nath, *Phys. Rev. Lett.* **49** (1982) 970; R. Barbieri, S. Ferrara and C. Savoy, *Phys. Lett. B* **119** (1982) 343; N. Ohta, *Prog. Theor. Phys.* **70** (1983) 542; L. Hall, J. Lykken and S. Weinberg, *Phys. Rev. D* **27** (1983) 2359
- [5] L. Randall and R. Sundrum, Nucl. Phys. **B557**, 79 (1999); G. F. Giudice, M. Luty, H. Murayama and R. Rattazzi, J. High Energy Phys. **9812**, 027 (1998); J. Bagger, T. Moroi and E. Poppitz, J. High Energy Phys. **0004**, 009 (2000); P. Binetruy, M. K. Gaillard and B. Nelson, Nucl. Phys. **B604**, 32 (2001).
- [6] K. Choi, K-S. Jeong and K. Okumura, J. High Energy Phys. **0509**, 039 (2005).
- [7] A. Falkowski, O. Lebedev and Y. Mambrini, J. High Energy Phys. **0511**, 034 (2005).
- [8] K. Choi, K. S. Jeong, T. Kobayashi and K. i. Okumura, Phys. Rev. D **75** (2007) 095012.
- [9] M. Endo, M. Yamaguchi and K. Yoshioka, Phys. Rev. **D72**, 015004 (2005).
- [10] R. Kitano and Y. Nomura, Phys. Lett. **B632**, 162 (2006) and hep-ph/0602096.
- [11] H. Baer, E. Park, X. Tata and T. T. Wang, hep-ph/0604253.
- [12] K. Choi and H. P. Nilles, JHEP **0704**, (2007) 006 stress the robustness of the gaugino mass relation in MM models. See also, O. Lebedev, V. Löwen, Y. Mambrini, H. P. Nilles and M. Ratz, JHEP **0702** (2007) 063; E. Dudas, C. Papineau and S. Pokorski, JHEP **0702** (2007) 028, and H. Abe, T. Higaki and Y. Omura, Phys. Rev. D **75** (2007) 025019 for earlier work leading up to this.
- [13] H. Baer, V. Barger and A. Mustafayev, Phys. Rev. D **85** (2012) 075010.
- [14] The ATLAS collaboration [ATLAS Collaboration], ATLAS-CONF-2016-052.
- [15] H. Baer, V. Barger and D. Mickelson, Phys. Rev. D **88** (2013) no.9, 095013.
- [16] H. Baer, V. Barger, D. Mickelson and M. Padeffke-Kirkland, Phys. Rev. D **89** (2014) 115019.
- [17] A. Mustafayev and X. Tata, Indian J. Phys. **88** (2014) 991.
- [18] J. Ellis, K. Enqvist, D. Nanopoulos and F. Zwirner, Mod. Phys. Lett. A **1** (1986) 57; R. Barbieri and G. Giudice, Nucl. Phys. B **306** (1988) 63.
- [19] S. K. Soni and H. A. Weldon, Phys. Lett. B **126** (1983) 215; V. S. Kaplunovsky and J. Louis, Phys. Lett. B **306** (1993) 269; A. Brignole, L. E. Ibanez and C. Munoz, Nucl. Phys. B **422** (1994) 125 [Erratum-ibid. B **436** (1995) 747].
- [20] H. Baer, V. Barger, P. Huang, A. Mustafayev and X. Tata, Phys. Rev. Lett. **109** (2012) 161802.
- [21] H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev and X. Tata, Phys. Rev. D **87** (2013) 11, 115028.

- [22] H. Baer, V. Barger and M. Savoy, arXiv:1509.02929 [hep-ph].
- [23] K. L. Chan, U. Chattopadhyay and P. Nath, Phys. Rev. D **58** (1998) 096004; R. Kitano and Y. Nomura, Phys. Rev. D **73** (2006) 095004; R. Barbieri and D. Pappadopulo, JHEP **0910**, 061 (2009); H. Baer, V. Barger and P. Huang, JHEP **1111**, 031 (2011); M. Papucci, J. T. Ruderman and A. Weiler, JHEP **1209** (2012) 035; C. Brust, A. Katz, S. Lawrence and R. Sundrum, JHEP **1203** (2012) 103.
- [24] L. L. Everett, I. W. Kim, P. Ouyang and K. M. Zurek, Phys. Rev. Lett. **101** (2008) 101803; L. L. Everett, I. W. Kim, P. Ouyang and K. M. Zurek, JHEP **0808** (2008) 102; K. Choi, K. S. Jeong, S. Nakamura, K. I. Okumura and M. Yamaguchi, JHEP **0904** (2009) 107; B. Altunkaynak, B. D. Nelson, L. L. Everett, I. W. Kim and Y. Rao, JHEP **1005** (2010) 054; B. Altunkaynak, B. D. Nelson, L. L. Everett, Y. Rao and I. W. Kim, Eur. Phys. J. Plus **127** (2012) 2; L. L. Everett, T. Garon, B. L. Kaufman and B. D. Nelson, Phys. Rev. D **93** (2016) no.5, 055031.
- [25] V. Barger, L. L. Everett and T. S. Garon, Phys. Rev. D **93** (2016) no.7, 075024.
- [26] D. Chowdhury and N. Yokozaki, JHEP **1508** (2015) 111.
- [27] K. Choi and K. S. Jeong, JHEP **0701** (2007) 103.
- [28] H. Murayama, H. Suzuki and T. Yanagida, Phys. Lett. B **291** (1992) 418; K. J. Bae, H. Baer and H. Serce, Phys. Rev. D **91** (2015) no.1, 015003.
- [29] H. Baer, V. Barger, M. Savoy and H. Serce, Phys. Lett. B **758** (2016) 113.
- [30] D. Matalliotakis and H. P. Nilles, Nucl. Phys. B **435** (1995) 115; P. Nath and R. L. Arnowitt, Phys. Rev. D **56** (1997) 2820; J. Ellis, K. Olive and Y. Santoso, Phys. Lett. B **539** (2002) 107; J. Ellis, T. Falk, K. Olive and Y. Santoso, Nucl. Phys. B **652** (2003) 259; H. Baer, A. Mustafayev, S. Profumo, A. Belyaev and X. Tata, J. High Energy Phys. **0507** (2005) 065.
- [31] ISAJET 7.85, by H. Baer, F. Paige, S. Protopopescu and X. Tata, hep-ph/0312045; Isasugra, by H. Baer, C. H. Chen, R. B. Munroe, F. E. Paige and X. Tata, Phys. Rev. D **51** (1995) 1046.
- [32] H. Baer, V. Barger, A. Lessa and X. Tata, Phys. Rev. D **86** (2012) 117701.
- [33] [ATLAS Collaboration], arXiv:1307.7292 [hep-ex].
- [34] H. Baer, V. Barger, M. Savoy and X. Tata, Phys. Rev. D **94** (2016) no.3, 035025
- [35] H. Baer, V. Barger, P. Huang, D. Mickelson, A. Mustafayev, W. Sreethawong and X. Tata, Phys. Rev. Lett. **110** (2013) no.15, 151801.
- [36] CMS Collaboration [CMS Collaboration], CMS-PAS-SUS-16-025.
- [37] H. Baer, E. K. Park, X. Tata and T. T. Wang, Phys. Lett. B **641** (2006) 447.
- [38] H. Baer, V. Barger, D. Mickelson, A. Mustafayev and X. Tata, JHEP **1406** (2014) 172.
- [39] J. List, talk at *38th International Conf. on High Energy Phys.* ICHEP2016, Chicago, USA (Aug. 2016); H. Baer, M. Berggren, S. L. Lehtinen, J. List, K. Fujii, T. Tanabe and J. Yan, to appear.

- [40] K. J. Bae, H. Baer and E. J. Chun, Phys. Rev. D **89** (2014) no.3, 031701.
- [41] K. J. Bae, H. Baer and A. Lessa, JCAP **1304** (2013) 041; K. J. Bae, H. Baer, A. Lessa and H. Serce, JCAP **1410** (2014) no.10, 082.
- [42] D. Akerib *et al.* [LUX Collaboration] arXiv:1608.07648 (2016).
- [43] E. Aprile *et al.* JCAP **04** (2016) 027.
- [44] D. S. Akerib *et al.* [LZ Collaboration], arXiv:1509.02910 [physics.ins-det].
- [45] P. Agnes *et al.* [DarkSide Collaboration], J. Phys. Conf. Ser. **650** (2015) no.1, 012006. doi:10.1088/1742-6596/650/1/012006.
- [46] P.-A. Amaudruz *et al.* [DEAP Collaboration], Nucl. Part. Phys. Proc. **273-275** 340 doi:10.1016/j.nuclphysbps.2015.09.048 [arXiv:1410.7673 [physics.ins-det]].
- [47] J. Aalbers *et al.* [DARWIN Collaboration], arXiv:1606.07001 [astro-ph.IM].
- [48] H. Baer, V. Barger and D. Mickelson, Phys. Lett. B **726** (2013) 330; K. J. Bae, H. Baer, V. Barger, M. R. Savoy and H. Serce, Symmetry **7** (2015) no.2, 788.
- [49] H. Baer, V. Barger and H. Serce, arXiv:1609.06735 [hep-ph].
- [50] M. Aartsen *et al.* JCAP **04** (2016) 022.
- [51] C. Kraus, talk at *38th International Conf. on High Energy Phys.* ICHEP2016, Chicago, USA (Aug. 2016)
- [52] M. Ahlen *et al.* JCAP **02** (2016) 039.
- [53] M. Wood, J. Buckley, S. Digel, D. Nieto and M. Sanchez-Conde, arXiv:1305.0302 (2013).