Inclusive single-spin asymmetries, quark-photon, and quark-quark correlations

Matthias Burkardt

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Inclusive Single-Spin Asymmetries, Quark-Photon, and Quark-Quark Correlations

Matthias Burkardt
Department of Physics, New Mexico State University, Las Cruces, NM 88003-0001, U.S.A.

We consider quark-photon correlations that have been proposed as a source for single-spin asymmetries in inclusive deep-inelastic scattering. A new sum rule for these correlators is derived and its phenomenological consequences are discussed. The results are interpreted within the context of an intuitive 'electrodynamic lensing' picture.

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I. INTRODUCTION

Recent inclusive Deep Inelastic Scattering (DIS) experiments on a transversely polarized target in Hall A at Jefferson Lab showed for the first time a (small) Single-Spin Asymmetry (SSA) for the scattered electron [1]. As such an asymmetry has to vanish in single photon exchange, these measurements potentially reveal important information about quark correlations in the nucleon.

Since the leading order (in α_{QED}) SSA arises from the interference between the one photon exchange and the two photon exchange amplitude, it is unlikely that both photons involve large momentum transfers with different quarks as this would lead to a more complex final state (e.g. two jets) that would not interfere much with a typical one photon exchange event.

Thus two hard photon exchanges would arise dominantly from events where both photons couple to the same quark line. The resulting effective interaction has been estimated in Ref. [2] and will not be considered in this work. The other possibility for a large momentum transfer on the electron are processes where one of the exchanged photons is hard and the other one is soft. In this work we will focus on the latter processes, which we not only believe are dominant for inclusive SSAs but also carry information about the spatial structure of the hadron as we will explore.

II. QUARK PHOTON QUARK CORRELATOR

In a DIS process, the transverse position of the scattered electron should be very close to that of the struck quark. One may thus estimate the effect from the initial and final state interactions of the electron by correlating the leading twist quark density with the electromagnetic field strength tensor at the same transverse position. This observation motivates to consider [2]

$$-M\epsilon_T^{ij}S_T^jF_{FT}^q \equiv \int \frac{d\xi^- d\zeta^-}{2(2\pi)^2} e^{ixP^+\xi^-} \langle P,S|\bar{\psi}^q(0)\gamma^+ eF_{QED}^{+i}(\zeta^-)\psi^q(\xi^-)|P,S\rangle, \tag{1}$$

where $e>0$ is the electric charge. x represents the quark momentum, which is not changed in this 'soft photon pole' matrix element. If the electromagnetic field strength tensor F_{QED}^{+i} is replaced by its QCD counterpart then (1) represents the soft gluon pole matrix element [3] for the single-spin asymmetry in Semi-Inclusive DIS (SIDIS). We will make use of this analogy several times.

First we note that although Eq. (1) represents (up to a minus sign) the average transverse momentum acquired by the electron due to ISI and FSI, it would yield the average transverse momentum of the active quark due to electromagnetic FSI if we were to multiply by $\frac{1}{2}e_q$, where $e_u=\frac{2}{3}$ and $e_d=-\frac{1}{3}$. The factor $\frac{1}{2}$ arises here since there is only FSI acting on the quark while the electron experiences both ISI and FSI.

In Ref. [4] the electromagnetic field appearing in Eq. (1) was related to the $\perp$ charge density as

$$\int dx^- F_{QED}^{+i}(x^-,\mathbf{x}_\perp) = -\int \frac{d^2\mathbf{y}_\perp}{2\pi} \frac{x^i-y^i}{|\mathbf{x}_\perp-\mathbf{y}_\perp|^2}\rho(\mathbf{y}_\perp), \tag{2}$$

where

$$\rho(\mathbf{y}_\perp) = e\sum_{q'} e_{q'} \int dy^- \bar{\psi}^{q'}(y^-,\mathbf{y}_\perp)\gamma^+\psi^{q'}(y^-,\mathbf{y}_\perp) \tag{3}$$

is the charge density (integrated over y^-). The average $\perp$ momentum for flavor q

$$F_{FT}^q \equiv \int dx F_{FT}^q(x,x) \tag{4}$$

can thus be expressed as

$$-M\epsilon_T^{ij}S_T^jF_{FT}^q \equiv \int \frac{d\xi^- d\zeta^-}{4\pi P^+}\int \frac{d^2\mathbf{y}_\perp}{2\pi}\frac{y^i}{\mathbf{y}_\perp^2}\langle P,S|\bar{\psi}^q(0)\psi^q(0)\rho(\mathbf{y}_\perp)|P,S\rangle. \tag{5}$$

Note that $q' = q$ does not contribute in (5) after integration over $d^2\mathbf{y}_\perp$, i.e. when for example a u quark is struck, the average transverse momentum due to electromagnetic ISI/FSI is only from fields caused by d (or s and heavier) quarks and *vice versa*. Furthermore, the average $\perp$ momentum of u quarks due to electromagnetic FSI with d quarks is equal and opposite to the average $\perp$ momentum of d quarks due to electromagnetic FSI with u quarks. As a corollary, one finds the 'sum rule' [4]

$$\frac{2}{3}F_{FT}^u - \frac{1}{3}F_{FT}^d + ... = 0 \tag{6}$$

regardless whether the target is a proton or a neutron. Thus similar to the case for QCD [5], the average transverse momentum due to the FSI also vanishes in the abelian case, provided one sums over all charged constituents. Note that if one neglects strange or heavier quarks, then the sum rule implies that F_{FT}^u and F_{FT}^d must have the same sign so that they can sum to zero after weighting with e_q

$$F_{FT}^d = 2F_{FT}^u. \tag{7}$$

This result should apply to any target, i.e. the transverse momentum asymmetry on the electron should receive contributions from electromagnetic interactions with u and d quarks that are of the same sign for any given target. The model asumptions made in Ref. [2], lead to $F_{FT}^{u/p}$ and $F_{FT}^{d/p}$ having opposite signs, so that the sum rule cannot be satisfied. For the neutron, $F_{FT}^{u/p}$ and $F_{FT}^{d/p}$ have the same sign, consistent with (6). However, with the choice of coefficients made in Ref. [2] the sum rule does not seem to be satisfied for the neutron either.

III. A MODEL FOR QUARK PHOTON CORRELATORS

In Ref. [2] it was proposed to estimate $F_{FT}^q(x,x)$ by taking phenomenological fits [6] of its QCD counterpart $T_F^q(x,x)$ and rescale those as

$$F_{FT}^q(x,x) = -e_{q_s}\frac{\alpha_{em}}{2\pi C_F\alpha_s M}gT_F^q(x,x), \tag{8}$$

where e_{q_s} is the charge of the spectators: for example, if the active quark is a u quark in a proton then $e_{q_s} = \frac{1}{3}$ in the model of Ref. [2]. This results in quark-photon correlators

$$F_{FT}^{u/p}(x,x) = -\frac{\alpha_{em}}{6\pi C_F\alpha_s M}gT_F^{u/p}(x,x) \qquad F_{FT}^{d/p}(x,x) = -\frac{2\alpha_{em}}{3\pi C_F\alpha_s M}gT_F^{d/p}(x,x) \tag{9}$$

$$F_{FT}^{u/n}(x,x) = \frac{\alpha_{em}}{3\pi C_F\alpha_s M}gT_F^{d/p}(x,x) \qquad F_{FT}^{d/n}(x,x) = -\frac{\alpha_{em}}{6\pi C_F\alpha_s M}gT_F^{u,p}(x,x). \tag{10}$$

Here charge symmetry has been used to relate neutron matrix elements to those in the proton It is evident that (10) violates the above 'sum-rule' (6).

The root of this problem is the assumption that both spectator quarks contribute equally to the gauge fields in the matrix elements for $\int dxF_{FT}(x,x)$ and $\int dxT_F(x,x)$. However, that is not the case as FSI with quarks from the same flavor as the active quark do not contribute to either one of them [4, 5].

x-averaged contributions only arise from quarks with flavor other than the active quark due to symmetry. To account for that in the model from Ref. [2] one should thus replace (8) by

$$F_{FT}^q(x,x) = -e_{\tilde{q}}\frac{\alpha_{em}}{\pi C_F\alpha_s M}gT_F^q(x,x), \tag{11}$$

for the majority flavor (u in p and d in n) which is almost identical to (8), except that $e_{\tilde{q}}$ would be the charge of the spectator flavor in the proton/neutron. For example, in the above example where the active quark is a u quark in a proton $e_{\tilde{q}} = -\frac{1}{3}$, or for a d quark in a proton $e_{\tilde{q}} = \frac{4}{3}$ This would result in modified inclusive quark-photon correlators as

$$F_{FT}^{u/p}(x,x) = \frac{\alpha_{em}}{3\pi C_F\alpha_s M}gT_F^{u/p}(x,x) \qquad F_{FT}^{d/p}(x,x) = -\frac{2\alpha_{em}}{3\pi C_F\alpha_s M}gT_F^{d/p}(x,x) \tag{12}$$

$$F_{FT}^{u/n}(x,x) = \frac{\alpha_{em}}{3\pi C_F\alpha_s M}gT_F^{d/p}(x,x) \qquad F_{FT}^{d/n}(x,x) = -\frac{2\alpha_{em}}{3\pi C_F\alpha_s M}gT_F^{u,p}(x,x). \tag{13}$$

This result agrees with Ref. [2] in the case of minority flavor ($F_{FT}^{d/p}$ & $F_{FT}^{u/n}$) correlators, but differs for the majority flavor, where Ref. [2] multiplies by the net charge of all spectators, but we only multiply by the net charge of the spectator flavor. Furthermore, we divide for the majority flavor by $\frac{1}{2}C_F\alpha_s$ rather than $C_F\alpha_s$ The latter step is to account for the fact that the QCD FSI interaction between one of the u quarks in the proton with the d quark has a color factor that is only $\frac{1}{2}$ the color factor for the d quark in a proton to interact with the u-diquark pair.

It is easy to verify that these F_{FT} satisfy the above 'sum rule' (6), i.e.

$$\frac{2}{3}\int dx F^{u/N}(x,x) - \frac{1}{3}\int dx F^{d/N}(x,x) = 0, \tag{14}$$

for both $N = n, p$, provided the $T_F^{q/p}$ saturate the corresponding sum rule $\int dx T_F^{u,p}(x,x) + \int dx T_F^{d,p}(x,x) = 0$.

IV. DISCUSSION

In the model proposed in Ref. [2] the inclusive SSA is proportional to

$$\sigma_{UT} \propto 4F_{FT}^u + F_{FT}^d. \tag{15}$$

Using the correlators from Eq. (9) one thus finds

$$\sigma_{UT}^p \propto -\frac{2\alpha}{3\pi C_F\alpha_s M} g\left(T_F^{u/p} + T_F^{d/p}\right). \tag{16}$$

Recent extractions of the Sivers function from SIDIS data indicates that $T_F^{d/p} \approx -T_F^{u/p}$, which is also consistent with a very small Sivers function on a deuterium target. In combination with the correlators from Eq. (9) this yields a very small result for σ_{UT}^p .

In contradistinction, with the new quark-photon-quark correlators (12) one finds

$$\sigma_{UT}^p \propto \frac{2\alpha}{3\pi C_F\alpha_s M} g\left(2T_F^{u/p} - T_F^{d/p}\right), \tag{17}$$

where no cancellation occurs provided $T_F^{u/p}$ and $T_F^{d/p}$ have opposite signs.

For the neutron, the resulting change of the asymmetry with the new F_{FT} is small, since only $F_{FT}^{d/n}$ has changed (increased by factor 4) compared to Ref. [2]. In the asymmetry, $F_{FT}^{d/n}$ gets multiplied by the charge squared of the down quark and thus the asymmetry increases by only about 50%. With the values from Eq. (13) one finds

$$\sigma_{UT}^n \propto \frac{2\alpha}{3\pi C_F\alpha_s M} g\left(2T_F^{d/p} - T_F^{u/p}\right), \tag{18}$$

which is equal and opposite to that of the proton provided one makes the additional assumption that $T_F^{d/p} \approx T_F^{u/p}$. Of course, the asymmetries would still be numerically larger for the neutron but only because one divides by the total cross section. Nevertheless, our results predict a significant cancellation for deuterium as

$$\sigma_{UT}^d \propto \frac{2\alpha}{3\pi C_F\alpha_s M} g\left(T_F^{d/p} + T_F^{u/p}\right). \tag{19}$$

While the above relation $\sigma_{UT}^p \approx -\sigma_{UT}^n$ was derived on the basis of what we believe is an improved version of the model from Ref. [2], it can also be derived using only charge symmetry and Eq. (15), i.e. neglecting s and heavier quarks and assuming that the cross section asymmetry is described by the quark photon correlator. The key observation is the fact that (after integrating over x) photons that contribute to F_{FT}^u cannot originate from u quarks and similarly for d quarks. Combining this result with the approximate assumption that only u and d quarks play a role in these matrix elements this implies that photons that contribute to F_{FT}^u can only originate from d quarks and *vice versa*. In combination with charge symmetry this allows us to relate proton and neutron matrix elements after rescaling by the charge of the quark flavor from which the photons originated as

$$\frac{1}{-\frac{1}{3}} F_{FT}^{u/p} = \frac{1}{\frac{2}{3}} F_{FT}^{d/n} \tag{20}$$

$$\frac{1}{\frac{2}{3}} F_{FT}^{d/p} = \frac{1}{-\frac{1}{3}} F_{FT}^{u/n} \tag{21}$$

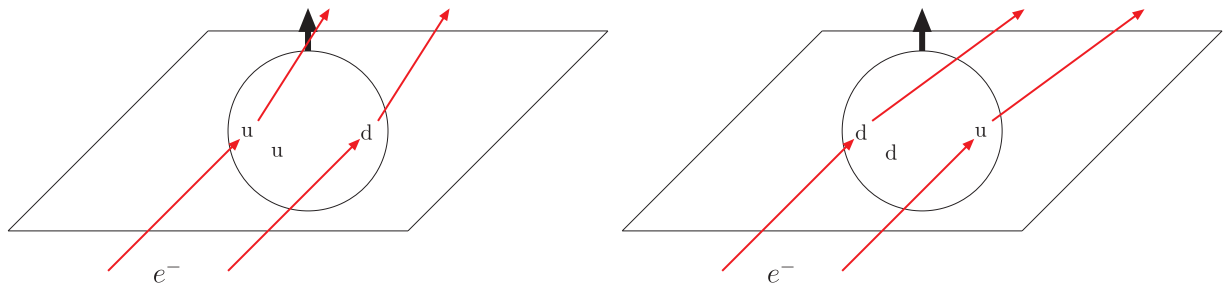

FIG. 1: Inclusive SSA in the lensing picture. When the e^- knocks out a u quark in a proton that is polarized up (left picture), this happens preferentially on the left side of the nucleon. The repulsive force from the spectator d quark on average excerts a force on the e^- to the left. When the e^- knocks out the d quark, the attractive force from the u quarks is on average also to the left. Fot a neutron target (right picture) the forces are preferentially to the right.

or

$$-2F_{FT}^{u/p} = F_{FT}^{d/n} \tag{22}$$

$$F_{FT}^{d/p} = -2F_{FT}^{u/n} \tag{23}$$

As illustrated in Fig. 1 the same sign contribution for scattering from u *vs.* d quarks can be understood from a mechanism similar to the 'lensing mechanism' proposed in Ref. [7]. For a transversely polarized nucleon the virtual hard photon sees u quarks shifted towards one side of the nucleon and d quarks to the other. When the e^- knocks out a u quark, it is repelled by the negatively charged d quarks on the other side of the nucleon. As explained above the average transverse momentum from interactions with spectator u quarks is zero. On the other hand, when the e^- knocks out a d quark it is attracted by the positively charged u quarks. However, since the u and d distributions in a transversely polarized nucleon are deformed in opposite directions, the net force from the spectators on the e^- is in both cases (knocking out u or d quarks) in the same direction, i.e. there should not be a cancellation between u and d quarks.

For the neutron, the resulting change of the asymmetry is small, since only $F_{FT}^{d/n}$ has changed (increased by factor 4) compared to Ref. [2]. In the asymmetry, $F_{FT}^{d/n}$ gets multiplied by the charge squared of the down quark and thus the asymmetry increases by only about 50%.

For the proton the change is more significant, as our result for $F_{FT}^{u/p}$ has a sign which differs from that in Ref.[2], and there is no longer an almost complete cancellation between u and d contributions to σ_{UT}^p. Moreover, since the latter gets multiplied by $e_u^2 = \frac{4}{9}$, the resulting change is quite significant. In fact, we now expect an asymmetry in the proton of the same order of magnitude as that in the neutron. To see this we consider the ratio

$$\frac{A_{UT}^p}{A_{UT}^n} = \frac{\sigma_{unp}^n}{\sigma_{unp}^p}\frac{4F_{FT}^{u/p} + F_{FT}^{d/p}}{4F_{FT}^{u/n} + F_{FT}^{d/n}} = \frac{\sigma_{unp}^n}{\sigma_{unp}^p}\frac{2T_F^{u/p} - T_F^{d/p}}{2T_F^{d/p} - T_F^{u/p}}. \tag{24}$$

For an order of magnitude estimate, we approximate $T_F^{d/p} \approx -T_F^{u/p}$, yielding

$$\frac{A_{UT}^p}{A_{UT}^n} \approx -\frac{\sigma_{unp}^n}{\sigma_{unp}^p}, \tag{25}$$

i.e. a suppression only due to the fact that the unpolarized cross section is larger for a proton target. More detailed estimates for the asymmetries using the revised F_{FT}^q can be found in Ref. [8].

The observation that $\sigma_{UT}^p \approx \sigma_{UT}^n$ should motivate to revisit A_{UT}^p with better statistics than obtained in the HERMES analysis [9] where the measured asymmetry is consistent with zero.

V. IMPLICATIONS FOR SPECTATOR MODELS

In Ref. [4] it was shown model independently that the average transverse momentum from the FSI using one photon/gluon exchange can be expressed in terms of transverse density-density correlations. For example, for the average transverse momentum for quarks with flavor q from QED FSI one finds

$$\langle k_\perp^q \rangle \propto \int d^2x_\perp \int d^2y_\perp \frac{\mathbf{x}_\perp - \mathbf{y}_\perp}{|\mathbf{x}_\perp - \mathbf{y}_\perp|^2} \sum_{q'} \langle \rho_q(\mathbf{x}_\perp)\rho_{q'}(\mathbf{y}_\perp)\rangle. \tag{26}$$

It is because of the symmetry of the integrand in Eq. (26) that the sum rule (6) is obtained as

$$\sum_q \langle k_\perp^q \rangle = 0 \tag{27}$$

. For QCD FSI the result is slightly more complicated, but if the nucleon state is approximated without explicit gluons present then a result similar to (26). Likewise, one finds model-independently that the average transverse momentum for a given flavor receives no contribution from interactions with the same flavor, e.g.

$$\langle k_\perp^u \rangle_{from\,u} \propto \int d^2x_\perp \int d^2y_\perp \frac{\mathbf{x}_\perp - \mathbf{y}_\perp}{|\mathbf{x}_\perp - \mathbf{y}_\perp|^2} \langle \rho_u(\mathbf{x}_\perp) \rho_u(\mathbf{y}_\perp) \rangle = 0. \tag{28}$$

In the derivation of these results it is important to consider the FSI on all constituents of the nucleon. For example in Ref. [10] it was found that in a scalar diquark model Eq. (27) is satisfied - but only if one adds up the Sivers functions for the 'lone quark' plus that of the scalar diquark.

In spectator models the two u quarks in a proton are usually treated very asymmetrically. One u quark is assumed to be tightly bound with the d quark and when calculating the u quark Sivers function then usually only the u that is not bound in the diquark is considered. It is due to this asymmetric treatment of the two u quarks that in general Eq. (27) - which is considered a fundamental result of QCD [10] - is violated.

Hypothetically, if one were to consider a model for the nucleon where one u quark is tightly bound to the d quark and the other u quark is more loosely bound to the diquark then Eqs. (27,28) would still be satisfied provided one adds up the Sivers functions of all three quarks: the loosely bound u quark plus the Sivers functions of the u quark and the d quark inside the diquark. And when calculating the Sivers function, the FSI with both spectators in each case needs to be included. Therefore even though the initial nucleon state in this hypothetical model would describe the two u quarks asymmetrically, Eqs. (27,28) would still be satisfied provided the Sivers functions were evaluated symmetrically.

However, in the standard treatment of spectator models Sivers functions are usually calculated by only considering the loosely bound quark and not including the Sivers functions of the quarks inside the diquark. Because of that, they violate Eqs. (27,28) as we demonstrated in this work.

VI. SUMMARY

We have developed a revised model for the quark-photon-quark correlator relevant for inclusive transverse single-spin asymmetries for scattering of unpolarized electrons from a transversely polarized target. In the model single-spin asymmetries from SIDIS experiments are used as an input and after rescaling by the appropriate coupling constants applied to inclusive SSAs. The novel feature in this work is that if an electron knocks out for example a u quark then the average transverse momentum from interactions of that electrons with u quarks vanishes (and similar for d quarks). As a result the quark-photon-quark correlators now observe sum rules analogous to similar sum rules for quark-gluon-quark correlators. The immediate consequence of this modification is that σ_{UT} for a proton is now predicted to be on the same order as that of the proton, and proton asymmetries are only suppressed by the larger total cross sections.

However, beyond this revised prediction for A_{UT}^p, our results have much more general implications for spectator models for ISI/FSI. More specifically, our analysis exhibited an issue for the ISI/FSI applied to the majority flavor. In diquark spectator models the spectators are lumped into a single diquark and the ISI/FSI are estimated typically by considering one-gluon exchange interactions with a diquark rather than the spectator quarks individually. For u quarks interacting with a ud diquark, this implies that the ISI/FSI from interactions with the spectator u quark is the same as that with the spectator d quark. However, from the symmetry of the interaction, the average transverse momentum acquired from interactions with the spectator u quark should be zero [4], which is consistent only if the net effect from interactions with the ud diquark vanish, i.e. no net SSA from FSI/ISI for the majority flavor. However, to satisfy the transverse momentum sum rule the net effect on the d quarks would then also have to be zero, which illustrates that diquark spectator models for the SSAs have an intrinsic conflict with the transverse momentum sum rule.

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