Coherent observations of gravitational radiation with LISA and gLISA

Massimo Tinto and José C. N. de Araujo

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Massimo Tinto[*]

Jet Propulsion Laboratory,

California Institute of Technology,

4800 Oak Grove Drive, Pasadena,

California, 91109, USA

José C.N. de Araujo[†]

Divisão de Astrofísica,

Instituto Nacional de Pesquisas Espaciais

Avenida dos Astronautas 1758,

São José dos Campos,

São Paulo, 12227-010, Brasil

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Abstract

The geosynchronous Laser Interferometer Space Antenna (gLISA) is a space-based gravitational wave (GW) mission that, for the past five years, has been under joint study at the Jet Propulsion Laboratory, Stanford University, the National Institute for Space Research (I.N.P.E., Brazil), and Space Systems Loral. If flown at the same time as the LISA mission, the two arrays will deliver a joint sensitivity that accounts for the best performance of both missions in their respective parts of the milliHertz band. This simultaneous operation will result in an optimally combined sensitivity curve that is white from about 3×10^{-3} Hz to 1 Hz, making the two antennas capable of detecting, with high signal-to-noise ratios (SNRs), coalescing black-hole binaries (BHBs) with masses in the range $(10 - 10^8) M_\odot$. Their ability of jointly tracking, with enhanced SNR, signals similar to that observed by the Advanced Laser Interferometer Gravitational Wave Observatory (aLIGO) on September 14, 2015 (the GW150914 event) will result in a larger number of observable small-mass binary black-holes and an improved precision of the parameters characterizing these sources. Together, LISA, gLISA and aLIGO will cover, with good sensitivity, the $(10^{-4} - 10^3)$ Hz frequency band.

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[*] massimo.tinto@jpl.nasa.gov

[†] jcarlos.dearaujo@inpe.br

The first direct observation of a GW signal announced by the LIGO project [1] on February 11, 2016 [2], represents one of the most important achievements in experimental physics today, and marks the beginning of GW astronomy [3]. By simultaneously measuring and recording strain data with two interferometers at Hanford (Washington) and Livingston (Louisiana), scientists were able to reach an extremely high level of detection confidence and infer unequivocally the GW source of the observed signal to be a coalescing binary system containing two black-holes of masses $M_1 = 36^{+5}_{-4} \ M_\odot$ and $M_2 = 29^{+4}_{-4} \ M_\odot$ out to a luminosity distance of $410^{+160}_{-180} \ Mpc$ corresponding to a red-shift $z = 0.09^{+0.03}_{-0.04}$ (the above uncertainties being at the 90 percent confidence level). The network of the two LIGO interferometers could constrain the direction to the binary system only to a broad region of the sky because the French-Italian VIRGO detector [4] was not operational at the time of the detection and no electromagnetic counterparts could be uniquely associated with the observed signal.

Ground-based observations inherently require use of multiple detectors widely separated on Earth and operating in coincidence. This is because a network of GW interferometers operating at the same time can (i) very effectively discriminate a GW signal from random noise and (ii) provide enough information for reconstructing the parameters characterizing the wave's astrophysical source (such as its sky-location, luminosity distance, mass(es), dynamical time scale, etc.) [5, 6]. Space-based interferometers instead, with their six links along their three-arms, have enough data redundancy to validate their measurements and uniquely reconstruct an observed signal [7]. A mission such as LISA [8], with its three operating arms, will be able to assess its measurements' noise level and statistical properties over its entire observational frequency band. By relying on a time-delay interferometric (TDI) measurement that is insensitive to GWs [9], LISA will assess its in-flight noise characteristics in the lower part of the band, i.e. at frequencies smaller than the inverse of the round-trip-light time. At higher frequencies instead, where it can synthesize three independent interferometric measurements, it will be able to perform a data consistency test by relying on the null-stream technique [6, 10, 11], i.e. a parametric non-linear combination of the TDI measurements that achieves a pronounced minimum at a unique point in the search parameter space when a signal is present. In addition, by taking advantage of the Doppler and amplitude modulations introduced by the motion of the array around the Sun on long-lived GW signals, LISA will measure the values of the parameters associated with the GW source of the observed signal [7].

Although a single space-based array such as LISA can synthesize the equivalent of four interferometric (TDI) combinations (such as the Sagnac TDI combinations $(\alpha, \beta, \gamma, \zeta)$) [12], its best sensitivity level is achieved only over a relatively narrow region of the mHz frequency band. At frequencies lower than the inverse of the round-trip light time, the sensitivity of a space-based GW interferometer is determined by the level of residual acceleration noise associated with the nearly free-floating proof-masses of the onboard gravitational reference sensor and the size of the arm-length. This is because in this region of the accessible frequency band the magnitude of a GW signal in the interferometer's data scales linearly with the arm-length. At frequencies higher than the inverse of the round-trip light time instead, the sensitivity is primarily determined by the photon-counting statistics [13]. Note that this noise grows linearly with the arm-length through its inverse proportionality to the square-root of the received optical power and because, in this region of the accessible frequency band, the GW signal no-longer scales with the arm-length. In summary, for a given performance of the onboard science instrument and optical configuration, the best sensitivity level and the corresponding wideness of the bandwidth over which it is reached are uniquely determined by the size of the array.

The best sensitivity level and the corresponding bandwidth over which it is reached by a space-based interferometer become particularly important when considering signals sweeping up-wards in frequency such as those emitted by coalescing binary systems containing black-holes. As recently pointed out by Sesana [14], it is theoretically expected that a very large ensemble of coalescing binary systems, with masses comparable to those of GW150914, will have characteristic wave's amplitudes that could be observable by LISA over an accessible frequency region from about 1.5×10^{-2} Hz to about 7.6×10^{-2} Hz. These frequency limits correspond to (i) the assumption of observing a GW150914-like signal for a period of five years (approximately equal to its coalescing time), and (ii) the value at which the signal's amplitude is equal to the LISA sensitivity. Although one could in principle increase the size of the LISA's optical telescopes and rely on more powerful lasers so as to increase the upper frequency cut-off to enlarge the observational bandwidth, in practice pointing accuracy and stability requirements together with the finiteness of the onboard available power would result in a negligible gain.

A natural way to broaden the mHz band, so as to fill the gap between the region accessible by LISA and that by the ground interferometers, is to simultaneously fly an additional

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interferometer with a smaller arm-length. The geosynchronous Laser Interferometer Space Antenna (gLISA) [13, 16, 17], which has been analyzed during the past five years by a collaboration of scientists and engineers at the Jet Propulsion Laboratory, Stanford University, the National Institute for Space Research (I.N.P.E., Brazil), and Space Systems Loral, would allow this [18]. With an arm-length of about 7.3×10^4 km, it could achieve a shot-noise limited sensitivity in the higher region of its accessible frequency band that is about a factor of 70 better than that of LISA [19]; gLISA will display a minimum of its sensitivity in a frequency region that perfectly complements those of LISA and aLIGO, resulting into an overall accessible GW frequency band equal to $(10^{-4} - 10^3)$ Hz.

Regarding the gLISA onboard science payload components (primirely the laser, the optical telescope, and the inertial reference sensor), we will assume them to have a similar noise performance as those of LISA. Other subsystems are regarded to have a noise performance that results in a high-frequency noise spectrum that is essentially determined by the photon-counting statistics. For further details we refer the reader to appendix A of Ref. [13].

To derive the expression of the joint LISA-gLISA sensitivity, we first note that the noises in the TDI measurements made by the two arrays will be independent. This means that the joint signal-to-noise ratio squared, averaged over an ensemble of signals randomly distributed over the celestial sphere and random polarization states, $\langle SNR^2_{L+gL} \rangle$, can be written in the following form

$$\langle SNR^2_{L+gL} \rangle \equiv 4 \int_{f_1}^{f_2} \frac{|\widetilde{h}(f)|^2}{S_h^{L+gL}(f)} \, df \; = \langle SNR^2_L \rangle + \langle SNR^2_{gL} \rangle$$

$$= 4 \int_{f_1}^{f_2} \left\{ \frac{1}{S_h^L(f)} + \frac{1}{S_h^{gL}(f)} \right\} |\widetilde{h}(f)|^2 \, df \; , \tag{1}$$

where the functions $S_h^i(f)$, $i = L$, gL correspond to the squared optimal sensitivities (as defined through the A, E, and T modes [12, 22]) of the LISA and gLISA arrays respectively. Note that the factor 4 in front of the integral in Eq.(1) reflects the adopted convention throughout this article of using one-sided power spectral densities of the noises [23, 24].

From Eq. (1) it is straightforward to derive the following expression of the joint sensitivity squared, $S_h^{L+gL}(f)$, in terms of the individual ones, $S_h^L(f)$ and $S_h^{gL}(f)$

$$S_h^{L+gL}(f) \equiv \frac{S_h^L(f) \, S_h^{gL}(f)}{S_h^L(f) + S_h^{gL}(f)} \; . \tag{2}$$

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In Fig. 1 we plot the optimal sensitivities of LISA (blue), gLISA (red) averaged over sources randomly distributed over the sky and polarization states, together with the joint LISA-gLISA (black) given by Eq. (2). For completeness, and to visually exemplify the scientific advantages of flying simultaneously two space-based missions of different arm-lengths, we have included the anticipated sensitivity of the third-generation aLIGO detector (green) together with the amplitude of the GW signal GW150914 (magenta) as functions of the Fourier frequency f. We have also included a black horizontal line showing the frequency evolution of GW150914 over a 5 years period before coalescence.

Shortly before the announcement made by aLIGO of the detection of a second signal emitted by another black-hole binary system, GW151226, with masses roughly half of those of GW150914 [2, 25], Sesana estimated that a large number of such systems [14] could be observed by LISA while they are still spiraling around each other over periods as long as the entire five years duration of the mission. Because of these compelling astrophysical reasons, to quantify the scientific advantages of flying gLISA jointly with LISA we will focus our attention on signals emitted by coalescing black-hole binaries with chirp masses in the range $(10 - 100)M_{\odot}$.

In our analysis we will use the following expressions (valid for circular orbits) of the Fourier transform of the amplitude of the GW signal emitted by such systems, $\widetilde{h}(f)$, and the time, t_c, it takes them to coalesce [26]

$$\widetilde{h}(f) = \sqrt{\frac{5}{24}} \, \pi^{-2/3} \left[\frac{(1+z)GM_c}{c^3}\right]^{5/6} \frac{c}{D_L} \, f^{-7/6} \, , \tag{3}$$

$$t_c = \frac{5}{256} \left[\frac{(1+z)GM_c}{c^3}\right]^{-5/3} (\pi f_{gw})^{-8/3} \, . \tag{4}$$

In Eqs.(3, 4) G is the gravitational constant, c is the speed of light, z is the cosmological red-shift, D_L is the corresponding luminosity distance, $M_c \equiv (M_1 M_2)^{3/5}/(M_1 + M_2)^{1/5}$ is the chirp mass associated with the binary system whose components have masses M_1 and M_2, f is the Fourier frequency and f_{gw} the instantaneous frequency of the emitted GW.

By using the above expression for the signal amplitude and a time to coalescence of 5 years (which, in the case of the GW150914 signal, defines the lower-limit of integration in the integral of the SNR to be equal to 0.0151 Hz), we have estimated the SNRs achievable by the two interferometers when operating either as stand-alone or jointly. We find LISA can observe a GW150914-like signal with a SNR equal to 10.7, while gLISA with an SNR
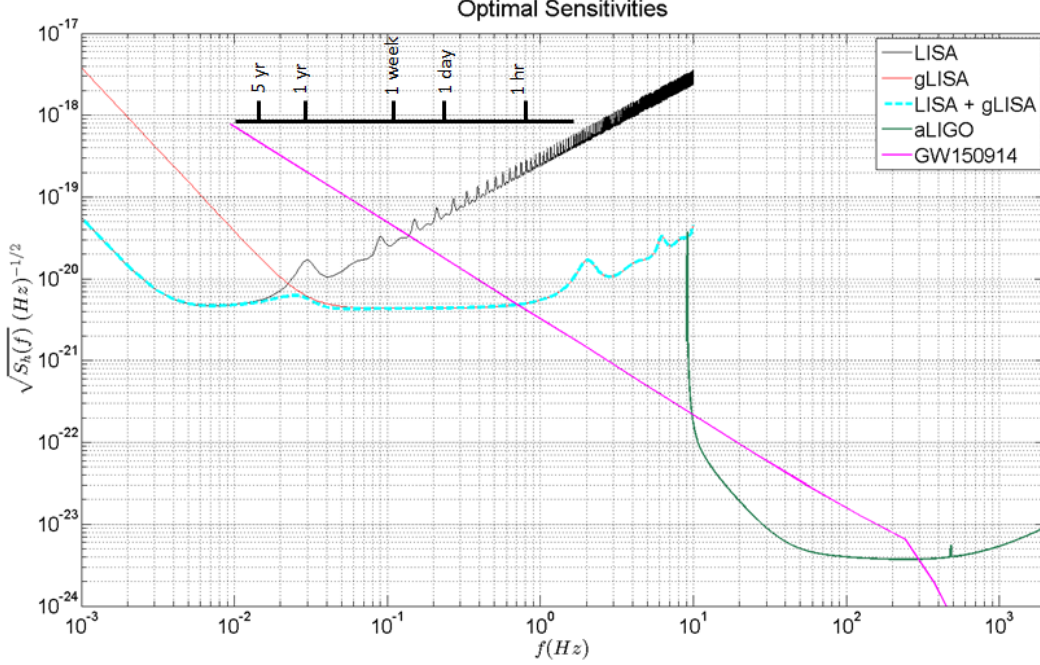
FIG. 1. The optimal sensitivities of LISA (blue) and gLISA (red) averaged over sources randomly distributed over the sky and polarization states. Each of these curves is uniquely determined by the TDI A, E, and T data combinations associated with the data from each mission. (see [22] for the derivation of the A, E, and T combinations and the resulting optimal sensitivity. Also refer to [12] for the revised expressions of their noise spectra). The dashed-line (black) is the result of optimally combining the sensitivities of LISA and gLISA. As expected, in the lower part of the mHz band, LISA defines the network sensitivity; in the overlapping region the two sensitivities smoothly blend, resulting in a maximum sensitivity gain of $\sqrt{2}$. In the higher part of the accessible band gLISA defines the joint LISA-gLISA sensitivity. For completeness we have included the projected aLIGO sensitivity (green) together with the amplitude of GW150914 (magenta) as a function of the Fourier frequency f. Also included is a black horizontal line showing the frequency evolution of GW150914 over a 5 years period before coalescence.

of 14.4 because of its better sensitivity over a larger part of its observable band (see Fig. 1). As expected, the joint LISA-gLISA network further improves upon the SNR of gLISA-alone by reaching a value of about 18.0. From these results we can further infer that gLISA will achieve a sufficiently high SNR to warrant the detection of a GW150914-like signal by integrating for a shorter time. We find that, by integrating for a period that corresponds to

135 days prior to the moment of coalescense of a GW150914-like system, gLISA can achieve a SNR of 10.7.

The level of SNR achievable by the LISA-gLISA network over an integration time which corresponds to five years before coalescense is about 80 percent higher than that of LISA alone. This implies, from the estimated parameter precisions derived by Sesana [14] and their dependence on the value of the SNR [15], that the LISA-gLISA network will estimate the parameters associated with the GW source of the observed signal with a precision that is 1.8 better than that obtainable by LISA alone. It should be said, however, that this is a lower bound on the improved precision by which the parameters can be estimated since it is based only on SNR considerations. As pointed out by McWilliams in an unpublished document [28], the Doppler frequency shift together with the larger diurnal amplitude modulations experienced by the GW signal in the gLISA TDI measurements will further improve the precision of the reconstructed parameters beyond that due to only the enhanced SNR. We will analyze and quantify this point in a follow-up article as this is beyond the scope of this letter.

From the expressions of the SNR and of the Fourier amplitude of the GW signal, $\tilde{h}(f)$ (Eqs. 1, 3), and by fixing the SNR to a specific value for each operational configuration (stand-alone vs. network), it is possible to infer the corresponding average luminosity distance to a BHB in terms of its chirp mass parameter [27]. In Fig. 2 we plot the results of this analysis by assuming a SNR of 10. As expected, the three derived luminosity distances are monotonically increasing functions of the chirp-mass. For chirp-masses in the range $[10 - 36)$ $M_\odot$ gLISA can see signals further away than LISA because of its better sensitivity at higher frequencies. Systems with a chirp-mass larger that 36 $M_\odot$ instead can be seen by LISA at a larger luminosity distance than that achievable by gLISA alone. Note that the LISA-gLISA network out-performs the stand-alone configurations by as much as 40 percent for BHBs with chirp-masses in the interval $(30 - 40)$ $M_\odot$. This results into a number of observable events that is about 3 times larger than that detectable by each interferometer alone. Finally, a GW150914-like signal characterized by a chirp mass of 28.1 $M_\odot$ can be see by LISA at an average luminosity distance of about 400 Mpc, by gLISA out to 600 Mpc, and by the LISA-gLISA system out to about 800 Mpc.

In addition to derive the luminosity distance as a function of the chirp-mass (with a chosen value of the SNR), we have also calculated the SNR as a function of the chirp-mass
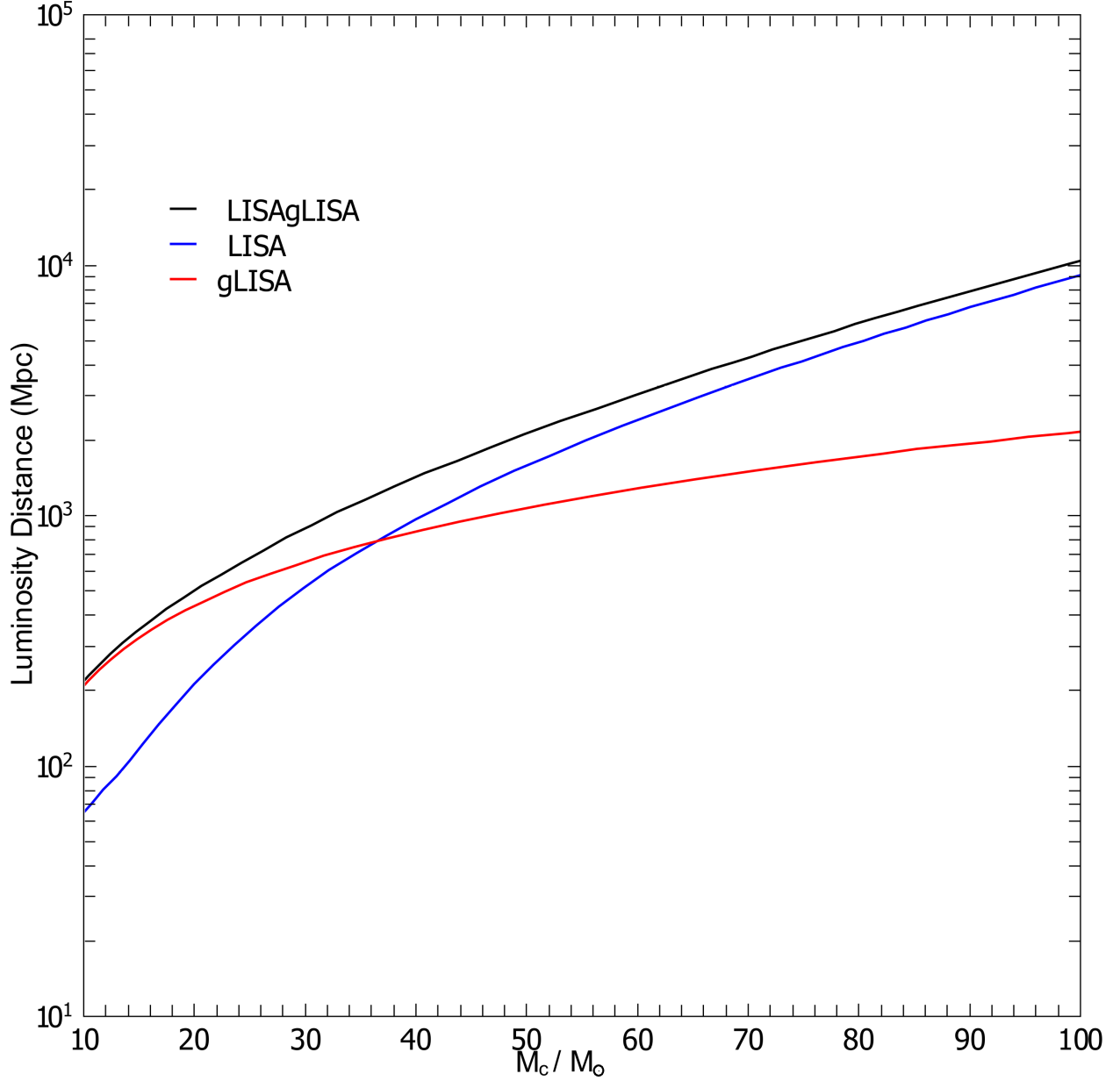
FIG. 2. Average luminosity distance achievable by LISA, gLISA and the LISA-gLISA network as function of the chirp mass. The SNR of each configuration has been assumed to be equal to 10 with coalescence times equal to five years. The initial frequency was derived by using Eq. 4.

by assuming the SNR of the LISA mission to be constant and equal to 10 (see Fig. 3). This of course implies that the luminosity distance of the system is uniquely determined by the chirp-mass through the assumed SNR value of 10 for LISA. Since we are focusing on chirp-masses defined in the interval $[10 - 100]\ M_{\odot}$, from Fig. 2 we infer that the values assumed by the luminosity distance will be greater than or equal to about 65 Mpc. This

minimum value of the luminosity distance defines an astrophysical interesting region as it is more than four times larger than the luminosity distance to the Virgo cluster of galaxies.

Fig. 3 shows the two SNRs of gLISA and LISA-gLISA to be decreasing functions of the chirp-mass, with their maximum values of 31 and 32.5 respectively at $M_c = 10$ $M_\odot$. At $M_c \simeq 36$ $M_\odot$ instead the gLISA SNR also becomes equal to 10 while, as expected, the LISA-gLISA SNR is $\sqrt{2}$ larger. This article has shown the scientific advantages of simultaneously
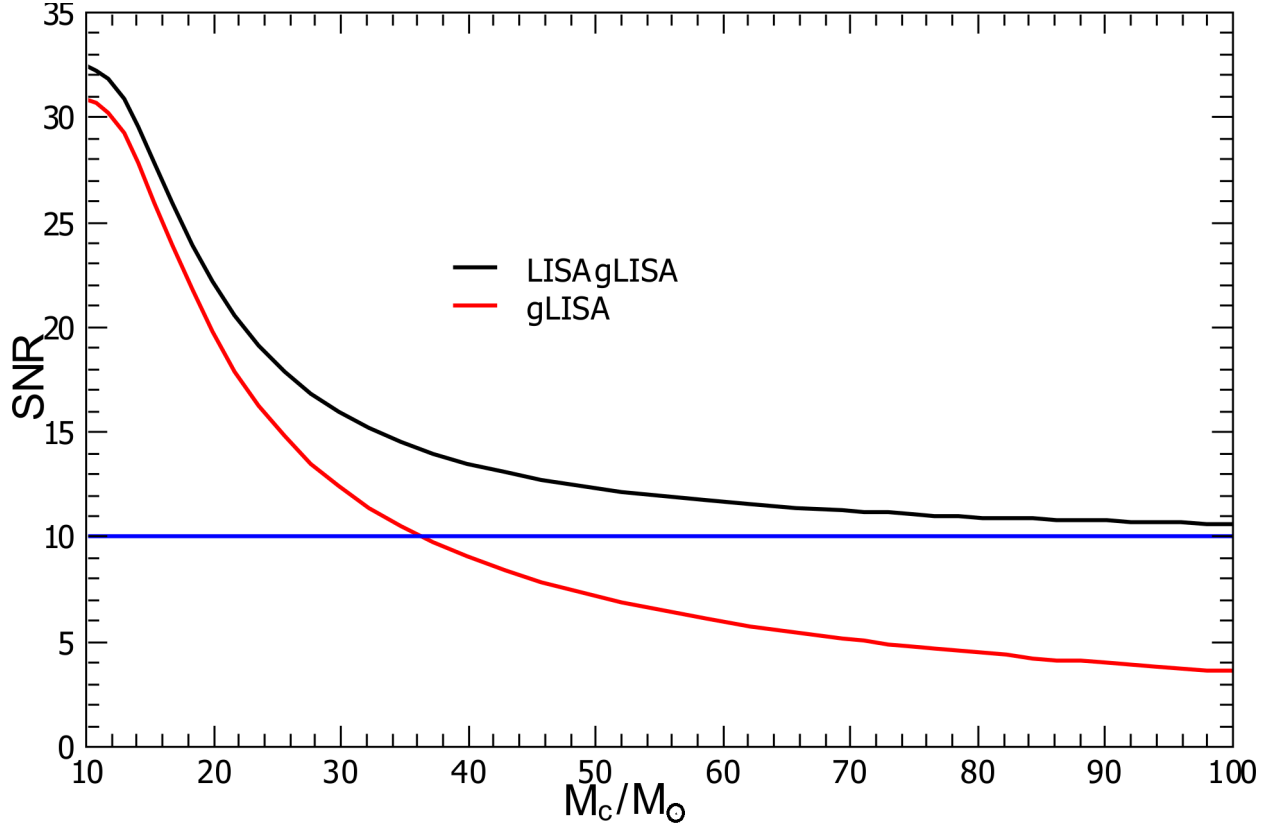
FIG. 3. SNR for gLISA and the LISA-gLISA network as a function of the chirp-mass of BHBs that are seen by LISA with an SNR = 10. A coalescence time of five years was assumed, and the initial frequency was derived by using Eq. 4.

flying with the LISA mission an additional, smaller size, space-based interferometer such as gLISA. Because of its smaller arm-length, gLISA will be able to survey the region of the GW frequency band that is in between those accessible by LISA and aLIGO. By covering the entire mHz and kHz GW frequency band, LISA, gLISA and aLIGO will detect all known sources emitting in this broad frequency region, and observe signals requiring multi-band detection for understanding the physical nature of their sources.

ACKNOWLEDGMENTS

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[27] To derive the redshift, z, we have assumed a fiducial ΛCDM flat cosmology with the matter density parameter $\Omega_M = 0.3$, the cosmological constant density parameter $\Omega_\Lambda = 0.7$, and the value of the Hubble parameter at the present time $H_0 = 70\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$. To obtain the luminosity distance, D_L, in terms of the chirp mass, M_c, we have rewritten the SNR as a

function of the redshift by expressing D_L in terms of z (see Eq. (13) in [13]). Then, for a given SNR, we have solved a transcendental equation to obtain z, and from it derived the luminosity distance.

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