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Generalized mass ordering degeneracy in neutrino oscillation experiments

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We consider the impact of neutral-current (NC) non-standard neutrino interactions (NSI) on the determination of the neutrino mass ordering. We show that in presence of NSI there is an exact degeneracy which makes it impossible to determine the neutrino mass ordering and the octant of the solar mixing angle θ_{12} at oscillation experiments. The degeneracy holds at the probability level and for arbitrary matter density profiles, and hence, solar, atmospheric, reactor, and accelerator neutrino experiments are affected simultaneously. The degeneracy requires order-one corrections from NSI to the NC electron neutrino–quark interaction and can be tested in electron neutrino NC scattering experiments.

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I. INTRODUCTION

Neutrino oscillation physics has entered the precision era. Present data determines all three leptonic mixing angles and the absolute value of the two mass-squared differences with few percent precision [1]. Crucial goals of future oscillation experiments are (a) the determination of the neutrino mass ordering and the CP-violating phase δ , and (b) establishing the robustness of three-flavour oscillations with respect to physics beyond the Standard Model (SM). In the present work we show that those two items are intimately related. We consider the hypothesis that additional interactions affect the neutrino sector, beyond the SM weak interaction [2-4], see [5, 6] for recent reviews. We will show that, for a certain choice of these non-standard interactions (NSI), the determination of the neutrino mass ordering—one of the main goals of upcoming oscillation experiments [7–11]—becomes impossible, due to an exact degeneracy in the evolution equation governing neutrino oscillations in matter.

The paper is structured as follows. In Sec. II we introduce the NSI framework and the notation used in the rest of the paper. Section III shows the origin of the degeneracy and how it can be realized in both vacuum and matter regimes. In Sec. IV we explain how the degeneracy affects neutrino oscillation data, while in Sec. V we explore the possible combination with neutrino scattering data to try to remove the degeneracy. Finally, our conclusions are summarized in Sec. VI.

II. NON-STANDARD INTERACTIONS IN NEUTRINO PROPAGATION

Three-flavour neutrino evolution in an arbitrary matter potential is described by the Schroedinger equation

$$i\frac{d}{dx}\Psi = H(x)\Psi, \qquad (1)$$

where Ψ is a vector of the flavour amplitudes, $\Psi = (a_e, a_\mu, a_\tau)^T$, and $H(x) = H_{\text{vac}} + H_{\text{mat}}(x)$. The Hamiltonian describing evolution in vacuum is

$$H_{\rm vac} = U {\rm diag}(0, \Delta_{21}, \Delta_{31}) U^{\dagger} , \qquad (2)$$

with $\Delta_{ij} = \Delta m_{ij}^2/(2E_{\nu})$, where $\Delta m_{ij}^2 \equiv m_i^2 - m_j^2$ stands for the neutrino mass-squared difference, and E_{ν} is the neutrino energy. From neutrino oscillation data, we know that $|\Delta m_{31}^2| \approx |\Delta m_{32}^2| \approx 30 \Delta m_{21}^2$. The neutrino mass ordering is parametrized by the sign of the larger masssquared difference, with normal ordering (NO) corresponding to $\Delta m_{31}^2 > 0$ and inverted ordering (IO) to $\Delta m_{31}^2 < 0$. The sign of Δm_{21}^2 by convention is chosen positive. The standard parametrization for the leptonic mixing matrix is $U = O_{23}U_{13}O_{12}$, where O_{ij} (U_{ij}) denotes a real (complex) rotation in the ij sector, with mixing angle θ_{ij} . Here we find it convenient to use an equivalent parametrization, where we put the complex phase δ in the 12 rotation, such that $U = O_{23}O_{13}U_{12}$. After subtracting a term proportional to the unit matrix, the vacuum Hamiltonian becomes

$$H_{\rm vac} = O_{23}O_{13} \begin{pmatrix} H^{(2)} & 0\\ 0 & \Delta_{31} - \frac{\Delta_{21}}{2} \end{pmatrix} O_{13}^T O_{23}^T, \quad (3)$$

with the 12 block given by $% \left({{{\rm{block}}} \right)$

$$H^{(2)} = \frac{\Delta_{21}}{2} \begin{pmatrix} -\cos 2\theta_{12} & \sin 2\theta_{12}e^{i\delta} \\ \sin 2\theta_{12}e^{-i\delta} & \cos 2\theta_{12} \end{pmatrix} .$$
(4)

Let us consider now the presence of neutral-current (NC) NSI in the form of dimension-6 four-fermion operators, which may contribute to the effective potential in matter in H_{mat} . We follow the notation of [12], for a recent review see *e.g.* [6]. NSI are described by the Lagrangian

$$\mathcal{L}_{\rm NSI} = -2\sqrt{2}G_F \epsilon^f_{\alpha\beta} (\overline{\nu}_{\alpha L} \gamma^\mu \nu_{\beta L}) (\overline{f} \gamma_\mu f) , \qquad (5)$$

where, $\alpha, \beta = e, \mu, \tau$, and f denotes a fermion present in the background medium. The parameter $\epsilon_{\alpha\beta}^{f}$ parametrizes the strength of the new interaction with respect to the Fermi constant G_F . Hermiticity requires that $\epsilon_{\alpha\beta}^{f} = (\epsilon_{\beta\alpha}^{f})^{*}$. Note that we restrict to vector interactions, since we are interested in the contribution to the effective matter potential. In generic models of new physics NSI parameters are expected to be small. However, examples of viable gauge models leading to $\epsilon_{\alpha\beta}^{u,d} \sim \mathcal{O}(1)$ can be found in [13, 14] (see also [6] for a discussion of NSI models).

The matter part of the Hamiltonian is then obtained as

$$H_{\rm mat} = \sqrt{2}G_F N_e(x) \begin{pmatrix} 1 + \epsilon_{ee} & \epsilon_{e\mu} & \epsilon_{e\tau} \\ \epsilon^*_{e\mu} & \epsilon_{\mu\mu} & \epsilon_{\mu\tau} \\ \epsilon^*_{e\tau} & \epsilon^*_{\mu\tau} & \epsilon_{\tau\tau} \end{pmatrix}, \quad (6)$$

$$\epsilon_{\alpha\beta} = \sum_{f=e,u,d} Y_f(x) \epsilon^f_{\alpha\beta} , \qquad (7)$$

with $Y_f(x) \equiv N_f(x)/N_e(x)$, $N_f(x)$ being the density of fermion f along the neutrino path. This implies that the effective NSI parameters $\epsilon_{\alpha\beta}$ may depend on x. The "1" in the *ee* entry in eq. (6) corresponds to the standard matter potential [2, 15]. For neutral matter, the densities of electrons and protons are equal. Thus, the relative densities of up and down quarks are

$$Y_u(x) = 2 + Y_n(x), \quad Y_d(x) = 1 + 2Y_n(x), \quad (8)$$

where $Y_n(x)$ is the relative neutron density along the neutrino path. Below we will use the notation $\epsilon_{\alpha\beta}^{\oplus}$ and $\epsilon_{\alpha\beta}^{\odot}$ to indicate when the $\epsilon_{\alpha\beta}$ refer to the specific matter composition of the Earth or the Sun, respectively.

III. THE GENERALIZED MASS ORDERING DEGENERACY

Let us consider first the vacuum part of the Hamiltonian, $H_{\rm vac}$ defined in eqs. (3) and (4). It is easy to show that the transformation

$$\Delta m_{31}^2 \to -\Delta m_{31}^2 + \Delta m_{21}^2 = -\Delta m_{32}^2, \sin \theta_{12} \leftrightarrow \cos \theta_{12}, \delta \to \pi - \delta$$
(9)

implies that $H_{\rm vac} \rightarrow -H_{\rm vac}^*$. Inserting this into eq. (1) and taking the complex conjugate we recover exactly the same evolution equation, when we take into account that

complex conjugation of the amplitudes $(\Psi \to \Psi^*)$ is irrelevant, as only moduli of flavour amplitudes are observable.^a This proves that the transformation (9) leaves the three-flavour evolution in vacuum invariant.

Note that this transformation corresponds to a complete inversion of the neutrino mass spectrum. The transformation $\Delta m_{31}^2 \leftrightarrow -\Delta m_{32}^2$ exchanges NO and IO, while changing the octant of θ_{12} exchanges the amount of ν_e present in ν_1 and ν_2 . We denote the effect of the transformation (9) as "flipping" the mass spectrum. The corresponding degeneracy is known in limiting cases, for instance, the so-called mass ordering degeneracy in the context of long-baseline experiments [17]. It is manifest also in the exact expression for the three-flavour ν_e survival-probability P_{ee} in vacuum, relevant for medium-baseline reactor experiments [18].

It is clear that for a non-zero standard matter effect, eq. (6) with $\epsilon_{\alpha\beta} = 0$, the transformation (9) no longer leaves the evolution invariant, since H_{mat} remains constant. The matter effect in the 13-sector is the basis of the mass ordering determination in long-baseline accelerator [7, 8] and atmospheric neutrino [10, 11] experiments. Moreover, the observation of the MSW [2, 15] matter resonance in the Sun requires that $\theta_{12} < 45^{\circ}$, which forbids the transformation in the second line of eq. (9). This allows, in principle, for the determination of the mass ordering via a precise measurement of P_{ee} in vacuum [19], as intended for instance by the JUNO collaboration [9].

However, if in addition to the transformation (9), it is also possible to transform $H_{\text{mat}} \rightarrow -H_{\text{mat}}^*$, then the full Hamiltonian including matter would transform as $H \rightarrow -H^*$, leaving the evolution equation invariant. This can be achieved in presence of NSI, supplementing the transformation (9) with [18]

$$\epsilon_{ee} \to -\epsilon_{ee} - 2, \\ \epsilon_{\alpha\beta} \to -\epsilon^*_{\alpha\beta} \quad (\alpha\beta \neq ee).$$
 (10)

The transformation of ϵ_{ee} is crucial to change the sign of the *ee* element of H_{mat} including the standard matter effect. Note that eq. (10) depends on the parametrization used for H_{vac} in eq. (3). If the standard parametrization with $U = O_{23}U_{13}O_{12}$ was used instead, then we would obtain $\epsilon_{e\mu} \to \epsilon^*_{e\mu}$, $\epsilon_{e\tau} \to \epsilon^*_{e\tau}$, and all other $\epsilon_{\alpha\beta}$ transforming as in eq. (10).

Since in general the $\epsilon_{\alpha\beta}$ are dependent on the neutron density, the degeneracy can be broken in principle by comparing experiments in matter with different neutron densities, or in configurations where the neutron density changes significantly along the neutrino path. However,

^a The invariance of the evolution under the transformation $H \rightarrow -H^*$ is a consequence of CPT invariance. It has been noted in the context of NSI in [16] and applied in some limiting cases, see also [12].

one can choose couplings such that NSI with neutrons are zero and take place only with protons and/or electrons, by choosing $\epsilon_{\alpha\beta}^q$ proportional to the quark electric charge, *i.e.*, $\epsilon_{\alpha\beta}^u = -2\epsilon_{\alpha\beta}^d$. In this situation the $\epsilon_{\alpha\beta}$ are always independent of x, the degeneracy is complete and cannot be broken by any combination of neutrino oscillation experiments.

Let us illustrate the degeneracy by the following example: assume that there are no NSI in Nature. Then we can fit data from any neutrino oscillation experiment either with standard oscillations and the correct spectrum, or equally well with a flipped spectrum and $\epsilon_{ee} = -2$. For

$$\epsilon_{ee}^{u} = -4/3, \quad \epsilon_{ee}^{d} = 2/3$$
 (11)

we obtain $\epsilon_{ee} = -2$ independent of the neutron density, and hence the degeneracy will be perfect, irrespective of the matter environment.

IV. IMPACT OF THE DEGENERACY AT OSCILLATION EXPERIMENTS

A manifestations of this result is the so-called LMA-dark solution for solar neutrinos [20], which corresponds to a fit to solar neutrino data with $\theta_{12} > 45^{\circ}$ ("dark octant") and values of $\epsilon_{ee}^{u,d} \simeq -1$. In the Sun the neutron fraction Y_n drops from about 1/2 in the centre to about 1/6at the border of the solar core. From eqs. (8) and (7) follows, that for $\epsilon_{ee}^{u,d} \simeq -1$ we obtain $\epsilon_{ee}^{\odot} \simeq -2$, close to the value needed for the generalized mass ordering degeneracy. In [12] a recent analysis of solar neutrino data has been performed, assuming either NSI with up or down quarks. In this case Y_n does not drop out of $\epsilon_{\alpha\beta}$ defined in eq. (7), and hence the condition $\epsilon_{ee} = -2$ cannot be fulfilled along the whole neutrino path in the Sun. Therefore, the degeneracy is not perfect. In [12] the $\Delta \chi^2$ of the LMA-dark solution is nearly zero for NSI on up quarks and $\lesssim 2$ for down quarks. While the sign of Δm_{31}^2 is irrelevant for solar neutrino phenomenology, it has been realised in [18], that the $\sin \theta_{12} \leftrightarrow \cos \theta_{12}$ ambiguity introduced by the LMA-dark solution leads to a mass ordering ambiguity in the planned reactor experiment JUNO [9]. This is a manifestation of the generalized degeneracy discussed above.

As another example, we will now demonstrate the impact of the generalized degeneracy on the sensitivity of the long-baseline Deep Underground Neutrino Experiment (DUNE) [8] to the mass ordering. In absence of NSI, the DUNE experiment would be able to reject the wrong hypothesis for the mass ordering with a significance above ~ 5σ regardless of the true value of δ [8]. We calculate expected data for NO, $\delta = 40^{\circ}$, $\sin^2 \theta_{12} = 0.3$, and no NSI. The simulation is performed using GLoBES [21]; the simulation details are the same as in Ref. [22]. Then these artificial data are fitted by allowing for the simultaneous presence of ϵ_{ee} and $\epsilon_{e\tau}$, while all other NSI



FIG. 1: Results from a fit to simulated data for DUNE. We assume a true NO and no NSI, and perform a fit allowing for non-zero values of ϵ_{ee} and $\epsilon_{e\tau}$. In the upper panel we fit with the correct mass spectrum, while in the lower panel we adopt IO and exchange $\sin \theta_{12} \leftrightarrow \cos \theta_{12}$. The shaded regions correspond to DUNE alone, whereas the contour curves include the constraints from global oscillation data [12] and from the CHARM experiment [24] on the NC cross section. We marginalize over Δm_{31}^2 , δ , θ_{23} and the complex phase of $\epsilon_{e\tau}$.

parameters are set to zero, for simplicity. Results are shown by the shaded regions in Fig. 1. The lower panel confirms the perfect degeneracy of the flipped mass spectrum at $\epsilon_{ee}^{\oplus} = -2$ and $\epsilon_{e\tau} = 0$, with $\Delta \chi^2 = 0$ with respect to the true best fit point. In both panels of Fig. 1 we observe also a strong correlation between ϵ_{ee} and $\epsilon_{e\tau}$, see [22]. Therefore, while the degeneracy is exact for ($\epsilon_{ee}, \epsilon_{e\tau}$) = (-2,0), it is recovered to a good accuracy for nonzero values of $\epsilon_{e\tau}$ as long as $|\epsilon_{e\tau}| \simeq 0.2|\epsilon_{ee} + 2|$. The importance of ϵ_{ee} and $\epsilon_{e\tau}$ for the mass ordering determination in DUNE has been pointed out recently in [23].



FIG. 2: Allowed region at 1, 2, 3σ (2 d.o.f.) from CHARM [24] (assuming NSI from a heavy mediator) in the plane of ϵ_{ee}^{u} and ϵ_{ee}^{d} , for $\epsilon_{e\mu}^{u,d} = \epsilon_{e\tau}^{u,d} = 0$. The cross corresponds to the point of exact degeneracy, eq. (11). The diagonal lines indicate the parameters for which $\epsilon_{ee} = -2$ in Earth and solar matter.

V. COMBINATION WITH NEUTRINO SCATTERING DATA

Since the generalized degeneracy is exact and holds for any oscillation experiment, the only way to break it are non-oscillation experiments. Indeed, operators of the type in eq. (5) contribute to the neutral current (NC) neutrino scattering cross section. Unfortunately, data on electron neutrino NC scattering is scarce. A relevant constraint on the parameters of interest to us comes from the historical CHARM experiment [24], which has measured the quantity $R_e = 0.406 \pm 0.140$, where R_e is ratio of the electron neutrino plus antineutrino NC cross sections to the corresponding charged current ones. In presence of NSI we have $R_e = \tilde{g}_L^2 + \tilde{g}_R^2$, where [25]

$$\tilde{g}_P^2 = \sum_{q=u,d} \left[\left(g_P^q + \frac{\epsilon_{ee}^q}{2} \right)^2 + \frac{|\epsilon_{e\mu}^q|^2 + |\epsilon_{e\tau}^q|^2}{4} \right], \quad (12)$$

with P = L, R, and g_P^q are the SM NC couplings. We have included only the vector-like NSI parameters. Note that the CHARM constraint is somewhat model dependent, since it would not apply if the mediator particle responsible for the NSI is much lighter than the momentum transfer in CHARM (typically of several tens of GeV) [13].

Assuming that the CHARM bound applies, we follow Ref. [25] and we show in Fig. 2 the allowed region for ϵ_{ee}^{u} and ϵ_{ee}^{d} from the CHARM data. The point from eq. (11), corresponding to perfect degeneracy for any matter profile, is indicated by the cross in the figure. We observe that it is excluded by CHARM data: for this point we

predict $R_e \approx 0.956$, which disagrees with the CHARM experimental value at 3.9σ . The diagonal lines in the figure indicate the parameters for which $\epsilon_{ee} = -2$ in Earth and solar matter. We use that in Earth matter, $Y_n \approx 1.05$, and for the Sun we show the spread induced by $Y_n = 1/2 \rightarrow 1/6$. For neutrino trajectories in the Earth, the generalized degeneracy holds along the line indicated in the plot. However, since the degeneracy for the Sun appears for slightly different values of $\epsilon_{ee}^{u,d}$ there is the potential to break it by the combination. Indeed, for the LMA-dark solution for NSI either on up or down quarks we have $\epsilon_{ee}^{u,d} \approx -1$ [12]. From Fig. 2 we see that ($\epsilon_{ee}^u, \epsilon_{ee}^d$) $\approx (0, -1)$ is in strong disagreement with CHARM, while ($\epsilon_{ee}^u, \epsilon_{ee}^d$) $\approx (-1, 0)$ is within the 1σ region.

Let us therefore adopt the hypothesis of NSI with up quarks only and check whether solar neutrino and CHARM data could break the mass ordering degeneracy for DUNE. The contour curves in Fig. 1 show the combined analysis, where we include global oscillation data (including the solar neutrino "SNO-POLY" analysis) from [12], assuming that ϵ_{ee}^{u} and $\epsilon_{e\tau}^{u}$ are approximately uncorrelated. If we restrict $\epsilon_{e\tau} = 0$, the degeneracy is broken, since $\epsilon_{ee}^u = -1$ (as required by solar data) implies $\epsilon_{ee}^{\oplus} \approx -3$, which can be excluded at high confidence level by DUNE for $\epsilon_{e\tau} = 0$. However, if we allow for non-zero $\epsilon_{e\tau}$, we see that a large region with the flipped mass spectrum remains below the 2σ level around $\epsilon_{ee}^{\oplus} \approx -3$ and $|\epsilon_{e\tau}^{\oplus}| \approx 0.2$. Hence, we conclude that including present constraints from oscillation and scattering data, the degeneracy will severely affect the mass ordering sensitivity of DUNE. Let us note that if more NSI parameters are allowed to vary, the fit with the flipped spectrum may even improve further.

VI. CONCLUSIONS

We have demonstrated that the so-called LMA-dark solution is a manifestation of an exact degeneracy at the level of the neutrino evolution equation. This degeneracy makes it impossible to determine the neutrino mass ordering by neutrino oscillation experiments. It requires $|\epsilon_{ee}^{u,d}| \simeq 1$, *i.e.*, NSI of electron neutrinos comparable in strength to weak interactions. The only way to break the degeneracy is via non-oscillation experiments. We have shown that taking into account current data on the ν_e NC cross section excludes NSI needed for the exact generalized degeneracy (subject to some model dependence); however, the degeneracy remains to be present at an approximate level, still destroying the mass ordering sensitivity of planned experiments. In order to break the degeneracy at high confidence level, improved data on ν_e NC interactions is mandatory. These may be provided, for instance, by coherent neutrino-nucleus interaction experiments [26–29].

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