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Erratum: Binary Neutron Stars with Arbitrary Spins in Numerical Relativity

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The code used in [1] erroneously computed the enthalpy at the center of the neutron stars. Upon correcting this error, density oscillations in evolutions of rotating neutron stars are significantly reduced (from ~ 20 percent to ~ 0.5 percent). Furthermore, it is possible to construct neutron stars with faster rotation rates.

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Ref. [1] presents a computational code for the construction and evolution of binary neutron stars with arbitrary spin vectors. Following Tichy [2], the 3-velocity of the neutron star fluid in an inspiralling binary is written as the sum of an irrotational part and a rotational part,

$$U^i = \frac{\Psi^{-4}\tilde{\gamma}^{ij}}{h\gamma_n}\left(\nabla_j\phi + W_j\right). \tag{1}$$

Here Ψ denotesthe conformal factor, $\tilde{\gamma}_{ij}$ the conformal spatial metric, h the specific enthalpy, γ_n the Lorentz term $\gamma_n = \left(1-\gamma_{ij}U^iU^j\right)^{-1/2}$, and ϕ the irrotational velocity potential. The vector W_i represents a rotation term designed to endow a uniform roation to the star,

$$W_i = \epsilon_{ijk}\omega^j r^k, \tag{2}$$

where ω^j is the rotation vector chosen by hand, and r^k is the distance to the center of the star. In this construction, the solution of the Euler equation is

$$h = \sqrt{L^2 - \left(\nabla_i\phi + W_i\right)\left(\nabla^i\phi + W^i\right)}, \tag{3}$$

where

$$L^2 = \frac{(x+y) + \sqrt{x^2+2xy}}{2\alpha^2}, \tag{4}$$

$$x = \left(\beta^i\nabla_i\phi + C\right)^2, \tag{5}$$

and

$$y = 2\alpha^2\left(\nabla_i\phi + W_i\right)W^i. \tag{6}$$

Here C denotes the Euler constant, α the lapse function and β^i the shift-vector.

The code reported in Ref. [1] has a mistake in the computation of h. Instead of Eq. 3, we computed the following quantity.

$$h' = \sqrt{L^2 - \left(\nabla_i\phi\right)\left(\nabla^i\phi\right)}, \tag{7}$$

and instead of Eq. (6), we computed

$$y' = \left(\nabla_i\phi + W_i\right)W^i. \tag{8}$$

This error causes h' to deviate from the correct h by

$$\begin{aligned} h'^2 - h^2 =& \frac{(y'-y)}{2\alpha^2} + \frac{\sqrt{x^2+2xy'} - \sqrt{x^2+2xy}}{2\alpha^2} \\ &+ W_iW^i + 2W^i\nabla_i\phi. \end{aligned} \tag{9}$$

For non-rotating stars, $W^i = 0$, the error dissapears: $h' = h$. In the limit of fast rotation, i.e. large W, we expect this difference to be dominated by the terms quadratic in W,

$$h'^2 - h^2 \approx \frac{W^2}{2\alpha^2}. \tag{10}$$

This implies the constructed BNS had an enthalpy lower than the correct equilibrium configurations. This picture is consistent with Fig. 20 of [1] (and Fig. 1 below): for high NS spin, the central density $\rho(t)$ immediately *increases* in an evolution, and oscillates around values *larger* than the initial density.

A. Updated Results

We construct initial data with the same input parameters as for the case `S0.4z - Ecc3` in [1], and evolve it with the same evolution code. For this evolution, we find:

1. Convergence of the Hamiltonian and Momentum constraints, and of the ADM energy and ADM angular momentum do not appreciably differ. Convergence of the neutron star spin is somewhat improved.

2. As noted in [3], the absolute difference between the Komar mass M_K and the ADM energy $M_{\rm ADM}$

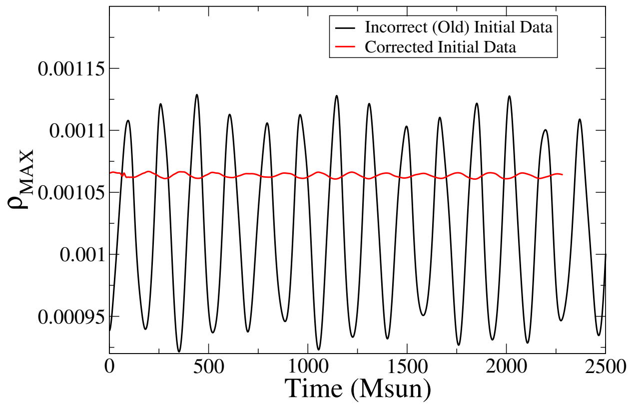

FIG. 1. Density oscillations for the `S0.4z` run from [1] and a new evolution of the corrected initial data.

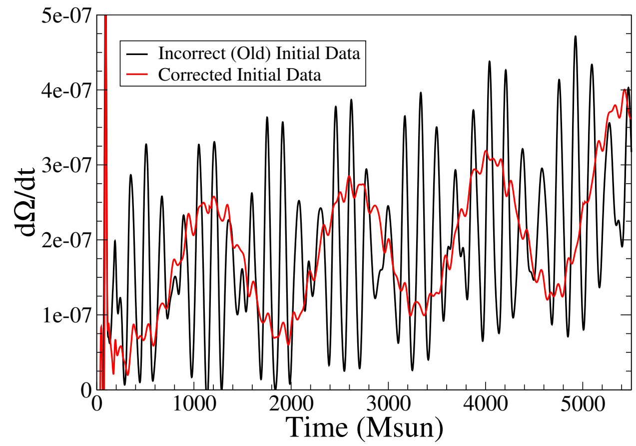

FIG. 2. Derivative of the orbital angular frequency from the S0.4z run from [1] and from a new run with the same parameters.

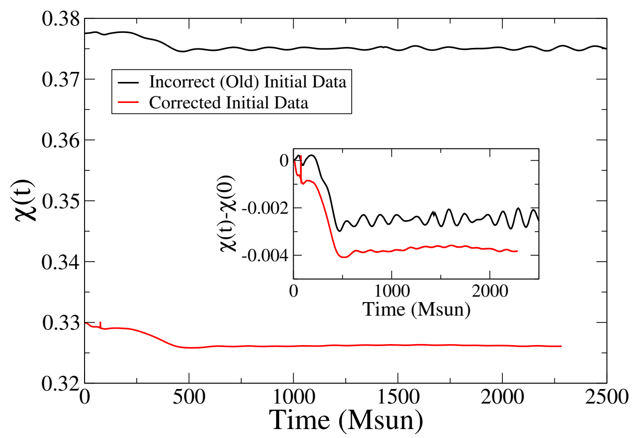

FIG. 3. Dimensionless spin, $\chi = J/M_{\rm ADM}^2$, measured during the evolution of the S0.4z-Ecc3 run, computed from old and corrected initial data. The inset subtracts the value of the spin at $t = 0$ from both curves.

is an indicator of deviations from equilibrium, as $M_K = M_{\rm ADM}$ for equilibrium systems in circular orbits. The difference between the Komar mass M_K and ADM energy $E_{\rm ADM}$ is reduced by an order of magnitude, from $|M'_K - M'_{\rm ADM}| = 2.6 \times 10^{-3}$ to $M_K - M_{\rm ADM} = 2.1 \times 10^{-4}$. This supports the idea that the neutron stars themselves are closer to being in equilibrium.

3. Evolution of the corrected initial data yields substantially smaller density oscillations. Figure 1 shows the density oscillations for the evolution reported in [1] and for the evolution of the corrected initial data. Peak-to-peak density oscillations are reduced from $\sim 20\%$ to about 0.5%. Density oscillations of $\sim 0.5\%$ also occur in our simulations of non-spinning binary neutron stars. The frequency of density-oscillation is unchanged, consistent with our interpretation that it represents a quasi-normal mode. We note that the phase of oscillation has changed by approximately half of a period.

4. The orbital frequency $\Omega(t)$ has significantly smaller oscillations at periods $\approx 200M_\odot$. Figure 2 compares $\dot{\Omega}(t)$ between evolutions of the old (erroneous) and new (corrected) initial data. High-frequency oscillations are strongly suppressed with the corrected initial data, allowing a clearer view of the lower-frequency sinusoidal features which are due to the overall trajectory of the binary.

5. The corrected code yields higher central density and therefore more compact stars (at same mass). At the same rotational frequency parameter ω (as defined in Eq. 2) we therefore expect the corrected code to yield stars with smaller angular momentum. This is indeed the case as is shown in Fig. 3. The subsequent evolution of the spin magnitude is comparable for both incorrect and corrected initial data (cf. inset of Fig. 3) although the oscillations present in the data are reduced.

We also find that the corrected code is capable of solving initial data sets for higher values of the NS rotation parameter ω. Figure 3 already showed that at the same rotation parameter ω, the corrected code yields smaller spin. Computing a sequence of initial data sets at different ω, we obtain Figure 4. For small ω, the $\chi(\omega)$ relation is unchangeindicating that the low-spin evolution reported in [1] is probably only mildly affected. For large ω, the initial data solver can create ID at spins up to $\chi \sim 0.63$, a factor ~ 1.4 larger than the erroneous code. This is, in fact, greater than the break-up spin of $\chi = 0.57$ found for $\Gamma = 2$ polytropes found in [4].

Being now able to construct ID at larger NS spins, we evolve an equal-mass, equal-spin ID set with $\omega = 0.019M_\odot^{-1}$, $\chi = 0.46$. In figure 5 we present a snapshot of the run, plotting the normalized density oscillations, spin, and the trajectories of the stars. The peak-to-peak density oscillations are now about 2%, higher than in the

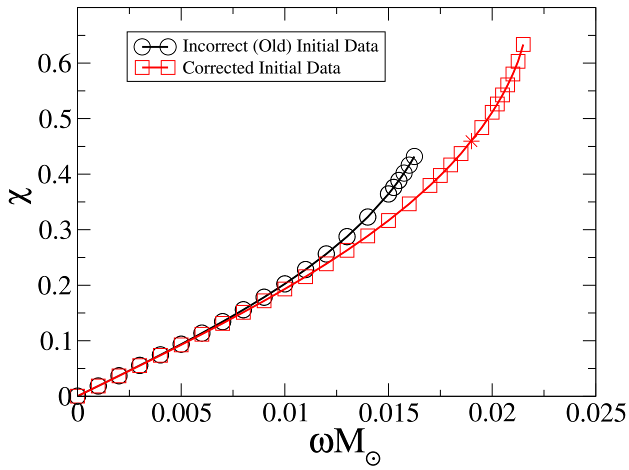

FIG. 4. χ as a function of ω in the initial data. The black curve is from Fig. 8 of [1], while the red curve is generated with the corrected initial data. The astericks indicates the configuration we evolve in Fig. 5.

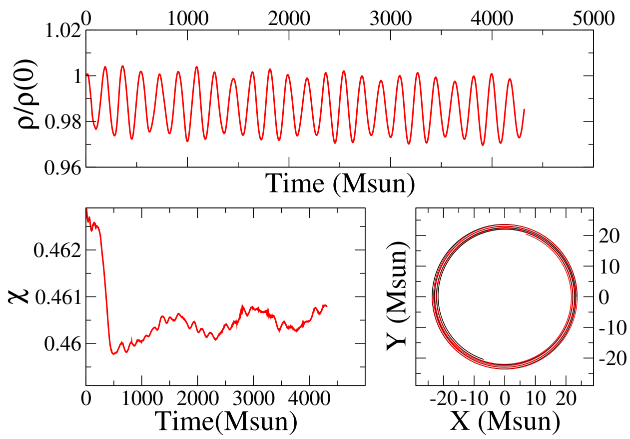

FIG. 5. A snapshot of an evolution with $\omega = 0.019$. The top panel shows the normalized density oscillations. The bottom-left panel shows the measured spin of a star. The bottom-right panel shows the orbtis of the stars as they inspiral.

$\chi \sim 0.33$ evolution, but still much smaller than for the erroneuous initial data despite the larger NS spin.

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[1] N. Tacik, F. Foucart, H. P. Pfeiffer, R. Haas, S. Ossokine, J. Kaplan, C. Muhlberger, M. D. Duez, L. E. Kidder, M. A. Scheel, and B. Szilágyi, Phys. Rev. D **92**, 124012 (2015), arXiv:1508.06986 [gr-qc].
[2] W. Tichy, Phys. Rev. D **86**, 064024 (2012), arXiv:1209.5336 [gr-qc].
[3] E. Gourgoulhon and S. Bonazzola, Classical and Quantum Gravity **11**, 443 (1994).
[4] M. Ansorg, A. Kleinwachter, and R. Meinel, Astron.Astrophys. **405**, 711 (2003), arXiv:astro-ph/0301173 [astro-ph].
[5] C. Loken, D. Gruner, L. Groer, R. Peltier, N. Bunn, M. Craig, T. Henriques, J. Dempsey, C.-H. Yu, J. Chen, L. J. Dursi, J. Chong, S. Northrup, J. Pinto, N. Knecht, and R. V. Zon, J. Phys.: Conf. Ser. **256**, 012026 (2010).