# Higgs boson mass constraint and the CP even-CP odd Higgs boson mixing in an MSSM extension

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# The Higgs boson mass constraint and the CP even-CP odd Higgs boson mixing in an MSSM extension

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**Abstract**

One loop contributions to the CP even-CP odd Higgs boson mixings arising from contributions due to exchange of a vectorlike multiplet are computed under the Higgs boson mass constraint. The vectorlike multiplet consists of a fourth generation of quarks and a mirror generation. This sector brings in new CP phases which can be large consistent with EDM constraints. In this work we compute the contributions from the exchange of quarks and mirror quarks $t_{4L}, t_{4R}, T_L, T_R$, and their scalar partners, the squarks and the mirror squarks. The effect of their contributions to the Higgs boson masses and mixings are computed and analyzed. The possibility of measuring the effects of mixing of CP even and CP odd Higgs in experiment is discussed. It is shown that the branching ratios of the Higgs bosons into fermion pairs are sensitive to new physics and specifically to CP phases.

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# 1 Introduction

One of the important phenomenon in MSSM is the observation that the CP even-CP odd Higgs bosons can mix in the presence of an explicit CP violation [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. Such mixings give rise to effects which are observable at colliders. All of the early analyses, however, were done in the era before the experimental observation of the light Higgs boson at 125 GeV by ATLAS [14] and by CMS [15]. It turns out that the Higgs boson mass constraint is rather stringent and severely limits the parameter space of supersymmetry models. In this work we consider the effects of including a vectorlike multiplet in an MSSM extension. In this case the loop correction to the Higgs boson arises from two contributions: one from the MSSM sector and the other from the vectorlike multiplet. It is shown that such an inclusion leads to a significant enhancement of the CP even-CP odd mixing. The explicit CP violation in the Higgs sector can be in conformity with the current limits on the EDM of quarks and the leptons due to either mass suppression [16, 17] in the sfermion sector or via the cancellation mechanism [18, 19, 20, 18, 21, 22]. The neutral Higgs boson mixing is of great import since the observation of such a mixing would be a direct indication of the existence of a new source of CP violation beyond what is observed in the Kaon and the B-meson system (for a review see [23]).

The outline of the rest of the paper is as follows: In section 2 we describe the model and define notation. Inclusion of a vectorlike generation allowing for mixings between the vectorlike and the regular generations increases the dimensionality of the quark mass matrices from three to five and increases the dimensionality of the squark mass squared matrices from six to ten. In section 3 the effect of the vectorlike generation on the induced CP violation in the Higgs sector as a consequence of CP violation in the matter sector including the vectorlike matter is discussed. In section 4 a detailed computation of the corrections to the Higgs boson mass matrices is given. A numerical analysis of the mixing of the CP even-CP odd sector is discussed in section 5. A discussion of the constraints arising from the EDM of the quarks is also given in this section. Conclusions are given in section 6. Further details of the squark mass squared matrices including the vectorlike squarks are given in the Appendix.

# 2 The Model and Notation

Here we briefly describe the model and further details are given in the appendix. The model we consider is an extension of MSSM with an additional vectorlike multiplet. Like MSSM the vectorlike extension is free of anomalies and vectorlike multiplets appear in a variety of settings which include grand unified models, string and D brane models [24, 25, 26, 27]. Several analyses have recently appeared which utilize vectorlike multiplets [28, 29, 30, 31, 32, 33, 34, 35, 36]

Here we focus on the quark sector where the vectorlike multiplet consists of a fourth generation of quarks and their mirror quarks. Thus the quark sector of the extended MSSM model is given by

$$q_{iL} \equiv \begin{pmatrix} t_{iL} \\ b_{iL} \end{pmatrix} \sim \left(3, 2, \frac{1}{6}\right) \; ; \quad t^c_{iL} \sim \left(3^*, 1, -\frac{2}{3}\right) \; ; \quad b^c_{iL} \sim \left(3^*, 1, \frac{1}{3}\right) \; ; \quad i = 1, 2, 3, 4. \tag{1}$$

$$Q^c \equiv \begin{pmatrix} B^c_L \\ T^c_L \end{pmatrix} \sim \left(3^*, 2, -\frac{1}{6}\right) \; ; \; T_L \sim \left(3, 1, \frac{2}{3}\right) \; ; \; B_L \sim \left(3^*, 1, -\frac{1}{3}\right). \tag{2}$$

The numbers in the braces show the properties under $SU(3)_C \times SU(2)_L \times U(1)_Y$ where the first two entries label the representations for $SU(3)_C$ and $SU(2)_L$ and the last one gives the value of the hypercharge normalized so that $Q = T_3 + Y$. We allow the mixing of the vectorlike generation with the first three generations. Specifically we will focus on the mixings of the mirrors in the vectorlike generation with the first three generations. Here we display some relevant features. In the up quark sector we choose a basis as follows

$$\bar{\xi}^T_R = \begin{pmatrix} \bar{t}_R & \bar{T}_R & \bar{c}_R & \bar{u}_R & \bar{t}_{4R} \end{pmatrix}, \;\; \xi^T_L = \begin{pmatrix} t_L & T_L & c_L & u_L & \bar{t}_{4L} \end{pmatrix} . \tag{3}$$

and we write the mass term so that

$$-\mathcal{L}^u_m = \bar{\xi}^T_R (M_u) \xi_L + \text{h.c.}, \tag{4}$$

The superpotential of the theory (as shown in the appendix) leads to the up-quark mass matrix $M_u$ which is given by

$$M_u = \begin{pmatrix} y'_1 v_2/\sqrt{2} & h_5 & 0 & 0 & 0 \\ -h_3 & y_2 v_1/\sqrt{2} & -h'_3 & -h''_3 & -h_6 \\ 0 & h'_5 & y'_3 v_2/\sqrt{2} & 0 & 0 \\ 0 & h''_5 & 0 & y'_4 v_2/\sqrt{2} & 0 \\ 0 & h_8 & 0 & 0 & y'_5 v_2/\sqrt{2} \end{pmatrix} \tag{5}$$

This mass matrix is not hermitian and a bi-unitary transformation is needed to diagonalize it. Thus one has

$$D_R^{u\dagger}(M_u)D_L^u = \text{diag}(m_{u_1}, m_{u_2}, m_{u_3}, m_{u_4}, m_{u_5}). \tag{6}$$

Under the bi-unitary transformations the basis vectors transform so that

$$\begin{pmatrix} t_R \\ T_R \\ c_R \\ u_R \\ t_{4R} \end{pmatrix} = D_R^u \begin{pmatrix} u_{1_R} \\ u_{2_R} \\ u_{3_R} \\ u_{4_R} \\ u_{5_R} \end{pmatrix}, \quad \begin{pmatrix} t_L \\ T_L \\ c_L \\ u_L \\ t_{4L} \end{pmatrix} = D_L^u \begin{pmatrix} u_{1_L} \\ u_{2_L} \\ u_{3_L} \\ u_{4_L} \\ u_{5_L} \end{pmatrix}. \tag{7}$$

A similar analysis can be carried out for the down quarks. Here we choose the basis set as

$$\bar{\eta}_R^T = \begin{pmatrix} \bar{b}_R & \bar{B}_R & \bar{s}_R & \bar{d}_R & \bar{b}_{4R} \end{pmatrix}, \quad \eta_L^T = \begin{pmatrix} b_L & B_L & s_L & d_L & b_{4L} \end{pmatrix}. \tag{8}$$

In this basis the down quark mass terms are given by

$$-\mathcal{L}_m^d = \bar{\eta}_R^T(M_d)\eta_L + \text{h.c.}, \tag{9}$$

where the superpotential of the theory leads to the down-quark mass matrix $M_d$ of the following form

$$M_d = \begin{pmatrix} y_1 v_1/\sqrt{2} & h_4 & 0 & 0 & 0 \\ h_3 & y_2' v_2/\sqrt{2} & h_3' & h_3'' & h_6 \\ 0 & h_4' & y_3 v_1/\sqrt{2} & 0 & 0 \\ 0 & h_4'' & 0 & y_4 v_1/\sqrt{2} & 0 \\ 0 & h_7 & 0 & 0 & y_5 v_1/\sqrt{2} \end{pmatrix}. \tag{10}$$

In general $h_3, h_4, h_5, h_3', h_4', h_5', h_3'', h_4'', h_5'', h_6, h_7, h_8$ can be complex and we define their phases so that

$$h_k = |h_k|e^{i\chi_k}, \quad h_k' = |h_k'|e^{i\chi_k'}, \quad h_k'' = |h_k''|e^{i\chi_k''} \tag{11}$$

The squark sector of the model contains a variety of terms including F -type, D-type, soft as well as mixings terms involving squarks and mirror squarks. The details of these contributions to squark mass square matrices are discussed in the appendix.

## 3 Computation of correction to the Higgs boson mass

In MSSM the Higgs sector at the one loop level is described by the scalar potential

$$V(H_1, H_2) = V_0 + \Delta V$$

In our analysis we use the renormalization group improved effective potential where

$$V_0 = m_1^2|H_1|^2 + m_2^2|H_2|^2 + (m_3^2 H_1.H_2 + H.C.)$$
$$+\frac{(g_2^2+g_1^2)}{8}|H_1|^4 + \frac{(g_2^2+g_1^2)}{8}|H_2|^4 - \frac{g_2^2}{2}|H_1.H_2|^2 + \frac{(g_2^2-g_1^2)}{4}|H_1|^2|H_2|^2 \tag{12}$$

where $m_1^2 = m_{H_1}^2 + |\mu|^2$, $m_2^2 = m_{H_2}^2 + |\mu|^2$, $m_3^2 = |\mu B|$ and $m_{H_{1,2}}$ and $B$ are the soft SUSY breaking parameters, and $\Delta V$ is the one loop correction to the effective potential and is given by

$$\Delta V = \frac{1}{64\pi^2} Str(M^4(H_1, H_2)(log\frac{M^2(H_1, H_2)}{Q^2} - \frac{3}{2})) \tag{13}$$

where $Str = \sum_i C_i(2J_i + 1)(-1)^{2J_i}$ where the sum runs over all particles with spin $J_i$ and $C_i(2J_i + 1)$ counts the degrees of freedom of the particle i, and Q is the running scale. In the evaluation of $\Delta V$ one should include the contributions of all of the fields that enter in MSSM. This includes the Standard Model fields and their superpartners, the sfermions, the higgsinos and the gauginos. The one loop corrections to the effective potential make significant contributions to the minimization conditions.

It is well known that the presence of CP violating effect in the one loop effective potential induce CP violating phase in the Higgs VEV through the minimization of the effective potential. One can parametrize this effect by the CP phase $\theta_H$ where

$$(H_1) = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(v_1 + \phi_1 + i\psi_1) \\ H_1^- \end{pmatrix} \tag{14}$$

$$(H_2) = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} = e^{i\theta_H} \begin{pmatrix} H_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + \phi_2 + i\psi_2) \end{pmatrix} \tag{15}$$

The non-vanishing of the phase $\theta_H$ can be seen by looking at the minimization of the effective potential. For the present case with the inclusion of CP violating effects the variations with respect to the fields $\phi_1, \phi_2, \psi_1, \psi_2$ give the following

$$-\frac{1}{v_1}(\frac{\partial \Delta V}{\partial \phi_1})_0 = m_1^2 + \frac{g_2^2+g_1^2}{8}(v_1^2-v_2^2) + m_3^2 \tan\beta\cos\theta_H \tag{16}$$

$$-\frac{1}{v_2}(\frac{\partial \Delta V}{\partial \phi_2})_0 = m_2^2 - \frac{g_2^2+g_1^2}{8}(v_1^2-v_2^2) + m_3^2 cot\beta\cos\theta_H \tag{17}$$

$$\frac{1}{v_1}(\frac{\partial \Delta V}{\partial \psi_2})_0 = m_3^2\sin\theta_H = \frac{1}{v_2}(\frac{\partial \Delta V}{\partial \psi_1})_0 \tag{18}$$

where the subscript 0 means that the quantities are evaluated at the point $\phi_1 = \phi_2 = \psi_1 = \psi_2 = 0$.

The masses $M$ to be included in the $\Delta V$ analysis are the masses of three MSSM quark and their squark partners along with the masses of the generations in the vectorlike sector of the theory. In this case the phase $\theta_H$ is determined by

$$\begin{aligned} m_3^2\sin\theta_H =& \frac{1}{2}\beta_{h_t}|\mu||A_t|\sin\gamma_t f_1(M^2_{\tilde{u}_1}, M^2_{\tilde{u}_3}) + \frac{1}{2}\beta_{h_u}|\mu||A_u|\sin\gamma_u f_1(M^2_{\tilde{u}_7}, M^2_{\tilde{u}_8}) \\ &+\frac{1}{2}\beta_{h_c}|\mu||A_c|\sin\gamma_c f_1(M^2_{\tilde{u}_5}, M^2_{\tilde{u}_6}) + \frac{1}{2}\beta_{h_{4t}}|\mu||A_{4t}|\sin\gamma_{4t} f_1(M^2_{\tilde{u}_9}, M^2_{\tilde{u}_{10}}) \\ &+\frac{1}{2}\beta_{h_T}|\mu||A_T|\sin\gamma_T f_1(M^2_{\tilde{u}_2}, M^2_{\tilde{u}_4}) + \frac{1}{2}\beta_{h_b}|\mu||A_b|\sin\gamma_b f_1(M^2_{\tilde{d}_1}, M^2_{\tilde{d}_3}) \\ &+\frac{1}{2}\beta_{h_d}|\mu||A_d|\sin\gamma_d f_1(M^2_{\tilde{d}_7}, M^2_{\tilde{d}_8}) + \frac{1}{2}\beta_{h_s}|\mu||A_s|\sin\gamma_s f_1(M^2_{\tilde{d}_5}, M^2_{\tilde{d}_6}) \\ &+\frac{1}{2}\beta_{h_{4b}}|\mu||A_{4b}|\sin\gamma_{4b} f_1(M^2_{\tilde{d}_9}, M^2_{\tilde{d}_{10}}) + \frac{1}{2}\beta_{h_B}|\mu||A_B|\sin\gamma_B f_1(M^2_{\tilde{d}_2}, M^2_{\tilde{d}_4}) \end{aligned} \tag{19}$$

where

$$\begin{aligned} f_1(x,y) &= -2 + \log(\frac{xy}{Q^2}) + \frac{y+x}{y-x}\log\frac{y}{x} \\ \beta_{h_q} &= \frac{3h_q^2}{16\pi^2}, \quad \gamma_q = \theta_\mu + \alpha_{A_q} \end{aligned} \tag{20}$$

To construct the mass squared matrix of the Higgs scalars we need to compute the quantities

$$M^2_{ab} = (\frac{\partial^2 V}{\partial \Phi_a \partial \Phi_b})_0 \tag{21}$$

where $\Phi_a$ (a=1-4) are defined by

$$\{\Phi_a\} = \{\phi_1, \phi_2, \psi_1, \psi_2\} \tag{22}$$

and as already specified the subscript 0 means that we set $\phi_1 = \phi_2 = \psi_1 = \psi_2 = 0$ after the evaluation of the mass matrix. The tree and loop contributions to $M^2_{ab}$ are given by

$$M^2_{ab} = M^{2(0)}_{ab} + \Delta M^2_{ab} \tag{23}$$

where $M^{2(0)}_{ab}$ are the contributions at the tree level and $\Delta M^2_{ab}$ are the loop contributions where

$$\Delta M^2_{ab} = \frac{1}{32\pi^2} Str(\frac{\partial M^2}{\partial \Phi_a}\frac{\partial M^2}{\partial \Phi_b} log\frac{M^2}{Q^2} + M^2 \frac{\partial^2 M^2}{\partial \Phi_a \partial \Phi_b} log\frac{M^2}{eQ^2})_0 \tag{24}$$

where e=2.718. Computation of the $4\times 4$ Higgs mass matrix in the basis of Eq.(22) gives

$$\begin{pmatrix} M_Z^2 c_\beta^2 + M_A^2 s_\beta^2 + \Delta_{11} & -(M_Z^2 + M_A^2) s_\beta c_\beta + \Delta_{12} & \Delta_{13} s_\beta & \Delta_{13} c_\beta \\ -(M_Z^2 + M_A^2) s_\beta c_\beta + \Delta_{12} & M_Z^2 s_\beta^2 + M_A^2 c_\beta^2 + \Delta_{22} & \Delta_{23} s_\beta & \Delta_{23} c_\beta \\ \Delta 13 s_\beta & \Delta_{23} s_\beta & (M_A^2 + \Delta_{33}) s_\beta^2 & (M_A^2 + \Delta_{33}) s_\beta c_\beta \\ \Delta_{13} c_\beta & \Delta_{23} c_\beta & (M_A^2 + \Delta_{33}) s_\beta c_\beta & (M_A^2 + \Delta_{33}) c_\beta^2 \end{pmatrix} \tag{25}$$

where $(c_\beta, s_\beta) = (\cos\beta, \sin\beta)$. In the above the explicit Q dependence has been absorbed in $m_A^2$ which is given by

$$\begin{aligned} m_A^2 = \frac{1}{\sin\beta\cos\beta}[-m_3^2\cos\theta_H + \frac{1}{2}\beta_{h_t}|\mu||A_t|\cos\gamma_t f_1(M^2_{\tilde{u}_1}, M^2_{\tilde{u}_3}) + \frac{1}{2}\beta_{h_u}|\mu||A_u|\cos\gamma_u f_1(M^2_{\tilde{u}_7}, M^2_{\tilde{u}_8}) \\ +\frac{1}{2}\beta_{h_c}|\mu||A_c|\cos\gamma_c f_1(M^2_{\tilde{u}_5}, M^2_{\tilde{u}_6}) + \frac{1}{2}\beta_{h_{4t}}|\mu||A_{4t}|\cos\gamma_{4t} f_1(M^2_{\tilde{u}_9}, M^2_{\tilde{u}_{10}}) \\ +\frac{1}{2}\beta_{h_T}|\mu||A_T|\cos\gamma_T f_1(M^2_{\tilde{u}_2}, M^2_{\tilde{u}_4}) + \frac{1}{2}\beta_{h_b}|\mu||A_b|\cos\gamma_b f_1(M^2_{\tilde{d}_1}, M^2_{\tilde{d}_3}) \\ +\frac{1}{2}\beta_{h_d}|\mu||A_d|\cos\gamma_d f_1(M^2_{\tilde{d}_7}, M^2_{\tilde{d}_8}) + \frac{1}{2}\beta_{h_s}|\mu||A_s|\cos\gamma_s f_1(M^2_{\tilde{d}_5}, M^2_{\tilde{d}_6}) \\ +\frac{1}{2}\beta_{h_{4b}}|\mu||A_{4b}|\cos\gamma_{4b} f_1(M^2_{\tilde{d}_9}, M^2_{\tilde{d}_{10}}) + \frac{1}{2}\beta_{h_B}|\mu||A_B|\cos\gamma_B f_1(M^2_{\tilde{d}_2}, M^2_{\tilde{d}_4})] \end{aligned} \tag{26}$$

The first term in the second brace on the right hand side of the above equation is the tree term, while the rest ten terms are coming from the three generations of MSSM (six terms) and four terms from the vectorlike multiplet. One may reduce the $4 \times 4$ matrix of the Higgs matrix by introducing a new basis $\{\phi_1, \phi_2, \psi_{1D}, \psi_{2D}\}$ where

$$\begin{aligned} \psi_{1D} &= \sin\beta\psi_1 + \cos\beta\psi_2 \\ \psi_{2D} &= -\cos\beta\psi_1 + \sin\beta\psi_2 \end{aligned} \tag{27}$$

In this basis the field $\psi_{2D}$ decouples from the other three fields as a Goldstone field with a zero mass eigen value. The Higgs mass squared matrix of the remaining three fields are given by

$$M^2_{Higgs} = \begin{pmatrix} M_Z^2 c_\beta^2 + M_A^2 s_\beta^2 + \Delta_{11} & -(M_Z^2 + M_A^2) s_\beta c_\beta + \Delta_{12} & \Delta_{13} \\ -(M_Z^2 + M_A^2) s_\beta c_\beta + \Delta_{12} & M_Z^2 s_\beta^2 + M_A^2 c_\beta^2 + \Delta_{22} & \Delta_{23} \\ \Delta_{13} & \Delta_{23} & (M_A^2 + \Delta_{33}) \end{pmatrix} \tag{28}$$

# 4 Computation of Corrections $\Delta_{ij}$ to the Higgs boson mass squared matrix

We consider the exchange contribution from the quarks/mirror quarks and from the squarks/mirror squarks in the susy standard model enriched with the vectorlike generation.

$$\Delta V(u, \tilde{u}, d, \tilde{d}) = \frac{1}{64\pi^2} \left( \sum_{a=1}^{10} 6 M^4_{\tilde{u}_a} (\log \frac{M^2_{\tilde{u}_a}}{Q^2} - \frac{3}{2}) - 12 \sum_{q=u,c,t,t_4,T} m_q^4 (\log \frac{m_q^2}{Q^2} - \frac{3}{2}) \right)$$
$$+ \frac{1}{64\pi^2} \left( \sum_{a=1}^{10} 6 M^4_{\tilde{d}_a} (\log \frac{M^2_{\tilde{d}_a}}{Q^2} - \frac{3}{2}) - 12 \sum_{q=d,s,b,b_4,B} m_q^4 (\log \frac{m_q^2}{Q^2} - \frac{3}{2}) \right) \tag{29}$$

Note that in the supersymmetric limit, quark masses would be equal to the squark masses and the loop corrections vanish.

Using the above loop corrections we can calculate the corrections to the different Higgs mass squared elements as

$$\Delta_{ij} = \Delta_{ij\tilde{q}_u} + \Delta_{ij\tilde{q}_d} \tag{30}$$

where

$$\Delta_{ij\tilde{q}_u} = \Delta_{ij\tilde{t}} + \Delta_{ij\tilde{c}} + \Delta_{ij\tilde{u}} + \Delta_{ij\tilde{t}_4} + \Delta_{ij\tilde{T}}$$
$$\Delta_{ij\tilde{q}_d} = \Delta_{ij\tilde{b}} + \Delta_{ij\tilde{s}} + \Delta_{ij\tilde{d}} + \Delta_{ij\tilde{b}_4} + \Delta_{ij\tilde{B}} \tag{31}$$

For the up quarks/squarks we have the contributions

$$\Delta_{11\tilde{q}} = -2\beta_{hq} m_q^2 |\mu|^2 \frac{(|A_q|\cos\gamma_q - |\mu|\cot\beta)^2}{(M^2_{\tilde{u}_i} - M^2_{\tilde{u}_j})^2} f_2(M^2_{\tilde{u}_i}, M^2_{\tilde{u}_j})$$

$$\Delta_{22\tilde{q}} = -2\beta_{hq} m_q^2 |A_q|^2 \frac{(|A_q| - |\mu|\cot\beta\cos\gamma_q)^2}{(M^2_{\tilde{u}_i} - M^2_{\tilde{u}_j})^2} f_2(M^2_{\tilde{u}_i}, M^2_{\tilde{u}_j}) +$$

$$2\beta_{hq} m_q^2 \log(\frac{M^2_{\tilde{u}_i} M^2_{\tilde{u}_j}}{m_q^4}) + 4\beta_{hq} m_q^2 |A_q| \frac{(|A_q| - |\mu|\cot\beta\cos\gamma_q)}{(M^2_{\tilde{u}_i} - M^2_{\tilde{u}_j})} \log(\frac{M^2_{\tilde{u}_i}}{M^2_{\tilde{u}_j}})$$

$$\Delta_{12\tilde{q}} = -2\beta_{hq} m_q^2 |\mu| \frac{(|A_q|\cos\gamma_q - |\mu|\cot\beta)}{(M^2_{\tilde{u}_i} - M^2_{\tilde{u}_j})} \log(\frac{M^2_{\tilde{u}_i}}{M^2_{\tilde{u}_j}}) +$$

$$2\beta_{hq} m_q^2 |\mu||A_q| \frac{(|A_q|\cos\gamma_q - |\mu|\cot\beta)(|A_q| - |\mu|\cot\beta\cos\gamma_q)}{(M^2_{\tilde{u}_i} - M^2_{\tilde{u}_j})^2} f_2(M^2_{\tilde{u}_i}, M^2_{\tilde{u}_j})$$

$$\Delta_{13\tilde{q}} = -2\beta_{hq} m_q^2 |\mu|^2 |A_q| \sin\gamma_q \frac{(|\mu|\cot\beta - |A_q|\cos\gamma_q)}{\sin\beta(M^2_{\tilde{u}_i} - M^2_{\tilde{u}_j})^2} f_2(M^2_{\tilde{u}_i}, M^2_{\tilde{u}_j})$$

$$\Delta_{23\tilde{q}} = -2\beta_{hq} m_q^2 |\mu||A_q|^2 \sin\gamma_q \frac{(|A_q| - |\mu|\cot\beta\cos\gamma_q)}{\sin\beta(M^2_{\tilde{u}_i} - M^2_{\tilde{u}_j})^2} f_2(M^2_{\tilde{u}_i}, M^2_{\tilde{u}_j})$$

$$+2\beta_{hq} \frac{m_q^2 |\mu||A_q| \sin\gamma_q}{\sin\beta(M^2_{\tilde{u}_i} - M^2_{\tilde{u}_j})} \log(\frac{M^2_{\tilde{u}_i}}{M^2_{\tilde{u}_j}})$$

$$\Delta_{33\tilde{q}} = -2\beta_{hq} \frac{m_q^2 |\mu|^2 |A_q|^2 \sin^2\gamma_q}{\sin^2\beta(M^2_{\tilde{u}_i} - M^2_{\tilde{u}_j})^2} f_2(M^2_{\tilde{u}_i}, M^2_{\tilde{u}_j}) \tag{32}$$

where $(i, j) = (1, 3)$ for $q = t$, $(i, j) = (7, 8)$ for $q = u$, $(i, j) = (5, 6)$ for $q = c$, $(i, j) = (9, 10)$ for $q = t_4$ and

$$f_2(x, y) = -2 + \frac{y + x}{y - x} \log\frac{y}{x} \tag{33}$$

For the mirror $q = T$ the contribution is given by

$$\Delta_{11\tilde{T}} = -2\beta_{hT} m_T^2 |A_T|^2 \frac{(|A_T| - |\mu| \tan\beta \cos\gamma_T)^2}{(M_{\tilde{u}_2}^2 - M_{\tilde{u}_4}^2)^2} f_2(M_{\tilde{u}_2}^2, M_{\tilde{u}_4}^2) +$$
$$2\beta_{hT} m_T^2 \log(\frac{M_{\tilde{u}_2}^2 M_{\tilde{u}_4}^2}{m_T^4}) + 4\beta_{hT} m_T^2 |A_T| \frac{(|A_T| - |\mu| \tan\beta \cos\gamma_T)}{(M_{\tilde{u}_2}^2 - M_{\tilde{u}_4}^2)} \log(\frac{M_{\tilde{u}_2}^2}{M_{\tilde{u}_4}^2})$$
$$\Delta_{22\tilde{T}} = -2\beta_{hT} m_T^2 |\mu|^2 \frac{(|A_T| \cos\gamma_T - |\mu| \tan\beta)^2}{(M_{\tilde{u}_2}^2 - M_{\tilde{u}_4}^2)^2} f_2(M_{\tilde{u}_2}^2, M_{\tilde{u}_4}^2)$$
$$\Delta_{12\tilde{T}} = -2\beta_{hT} m_T^2 |\mu| \frac{(|A_T| \cos\gamma_T - |\mu| \tan\beta)}{(M_{\tilde{u}_2}^2 - M_{\tilde{u}_4}^2)} \log(\frac{M_{\tilde{u}_2}^2}{M_{\tilde{u}_4}^2}) +$$
$$2\beta_{hT} m_T^2 |\mu||A_T| \frac{(|A_T| \cos\gamma_T - |\mu| \tan\beta)(|A_T| - |\mu| \tan\beta \cos\gamma_T)}{(M_{\tilde{u}_2}^2 - M_{\tilde{u}_4}^2)^2} f_2(M_{\tilde{u}_2}^2, M_{\tilde{u}_4}^2)$$
$$\Delta_{13\tilde{T}} = -2\beta_{hT} m_T^2 |\mu||A_T|^2 \sin\gamma_T \frac{(|A_T| - |\mu| \tan\beta \cos\gamma_T)}{\cos\beta (M_{\tilde{u}_2}^2 - M_{\tilde{u}_4}^2)^2} f_2(M_{\tilde{u}_2}^2, M_{\tilde{u}_4}^2)$$
$$+2\beta_{hT} \frac{m_T^2 |\mu||A_T| \sin\gamma_T}{\cos\beta (M_{\tilde{u}_2}^2 - M_{\tilde{u}_4}^2)} \log(\frac{M_{\tilde{u}_2}^2}{M_{\tilde{u}_4}^2})$$
$$\Delta_{23\tilde{T}} = -2\beta_{hT} m_T^2 |\mu|^2 |A_T| \sin\gamma_T \frac{(|\mu| \tan\beta - |A_T| \cos\gamma_T)}{\cos\beta (M_{\tilde{u}_2}^2 - M_{\tilde{u}_4}^2)^2} f_2(M_{\tilde{u}_2}^2, M_{\tilde{u}_4}^2)$$
$$\Delta_{33\tilde{T}} = -2\beta_{hT} \frac{m_T^2 |\mu|^2 |A_T|^2 \sin^2\gamma_T}{\cos^2\beta (M_{\tilde{u}_2}^2 - M_{\tilde{u}_4}^2)^2} f_2(M_{\tilde{u}_2}^2, M_{\tilde{u}_4}^2) \quad (34)$$

For the down quarks/squarks we have the contributions

$$\Delta_{11\tilde{q}} = -2\beta_{hq} m_q^2 |A_q|^2 \frac{(|A_q| - |\mu| \tan\beta \cos\gamma_q)^2}{(M_{\tilde{d}_i}^2 - M_{\tilde{d}_j}^2)^2} f_2(M_{\tilde{d}_i}^2, M_{\tilde{d}_j}^2) +$$

$$2\beta_{hq} m_q^2 \log(\frac{M_{\tilde{d}_i}^2 M_{\tilde{d}_j}^2}{m_q^4}) + 4\beta_{hq} m_q^2 |A_q| \frac{(|A_q| - |\mu| \tan\beta \cos\gamma_q)}{(M_{\tilde{d}_i}^2 - M_{\tilde{d}_j}^2)} \log(\frac{M_{\tilde{d}_i}^2}{M_{\tilde{d}_j}^2})$$

$$\Delta_{22\tilde{q}} = -2\beta_{hq} m_q^2 |\mu|^2 \frac{(|A_q| \cos\gamma_q - |\mu| \tan\beta)^2}{(M_{\tilde{d}_i}^2 - M_{\tilde{d}_j}^2)^2} f_2(M_{\tilde{d}_i}^2, M_{\tilde{d}_j}^2)$$

$$\Delta_{12\tilde{q}} = -2\beta_{hq} m_q^2 |\mu| \frac{(|A_q| \cos\gamma_q - |\mu| \tan\beta)}{(M_{\tilde{d}_i}^2 - M_{\tilde{d}_j}^2)} \log(\frac{M_{\tilde{d}_i}^2}{M_{\tilde{d}_j}^2}) +$$

$$2\beta_{hq} m_q^2 |\mu||A_q| \frac{(|A_q| \cos\gamma_q - |\mu| \tan\beta)(|A_q| - |\mu| \tan\beta \cos\gamma_q)}{(M_{\tilde{d}_i}^2 - M_{\tilde{d}_j}^2)^2} f_2(M_{\tilde{d}_i}^2, M_{\tilde{d}_j}^2)$$

$$\Delta_{13\tilde{q}} = -2\beta_{hq} m_q^2 |\mu||A_q|^2 \sin\gamma_q \frac{(|A_q| - |\mu| \tan\beta \cos\gamma_q)}{\cos\beta (M_{\tilde{d}_i}^2 - M_{\tilde{d}_j}^2)^2} f_2(M_{\tilde{d}_i}^2, M_{\tilde{d}_j}^2)$$

$$+2\beta_{hq} \frac{m_q^2 |\mu||A_q| \sin\gamma_q}{\cos\beta (M_{\tilde{d}_i}^2 - M_{\tilde{d}_j}^2)} \log(\frac{M_{\tilde{d}_i}^2}{M_{\tilde{d}_j}^2})$$

$$\Delta_{23\tilde{q}} = -2\beta_{hq} m_q^2 |\mu|^2 |A_q| \sin\gamma_q \frac{(|\mu| \tan\beta - |A_q| \cos\gamma_q)}{\cos\beta (M_{\tilde{d}_i}^2 - M_{\tilde{d}_j}^2)^2} f_2(M_{\tilde{d}_i}^2, M_{\tilde{d}_j}^2)$$

$$\Delta_{33\tilde{q}} = -2\beta_{hq} \frac{m_q^2 |\mu|^2 |A_q|^2 \sin^2\gamma_q}{\cos^2\beta (M_{\tilde{d}_i}^2 - M_{\tilde{d}_j}^2)^2} f_2(M_{\tilde{d}_i}^2, M_{\tilde{d}_j}^2) \quad (35)$$

where $(i,j) = (1,3)$ for $q = b$, $(i,j) = (7,8)$ for $q = d$, $(i,j) = (5,6)$ for $q = s$ and $(i,j) = (9,10)$ for $q = b_4$.

Finally the contribution of the mirror $B$ is given by

$$\Delta_{11\tilde{B}} = -2\beta_{hB} m_B^2 |\mu|^2 \frac{(|A_B|\cos\gamma_B - |\mu|\cot\beta)^2}{(M^2_{\tilde{d}_2} - M^2_{\tilde{d}_4})^2} f_2(M^2_{\tilde{d}_2}, M^2_{\tilde{d}_4})$$

$$\Delta_{22\tilde{B}} = -2\beta_{hB} m_B^2 |A_B|^2 \frac{(|A_B| - |\mu|\cot\beta\cos\gamma_q)^2}{(M^2_{\tilde{d}_2} - M^2_{\tilde{d}_4})^2} f_2(M^2_{\tilde{d}_2}, M^2_{\tilde{d}_4}) +$$

$$2\beta_{hB} m_B^2 \log(\frac{M^2_{\tilde{d}_2} M^2_{\tilde{d}_4}}{m_B^4}) + 4\beta_{hB} m_B^2 |A_B| \frac{(|A_B| - |\mu|\cot\beta\cos\gamma_B)}{(M^2_{\tilde{d}_2} - M^2_{\tilde{d}_4})} \log(\frac{M^2_{\tilde{d}_2}}{M^2_{\tilde{d}_4}})$$

$$\Delta_{12\tilde{B}} = -2\beta_{hB} m_B^2 |\mu| \frac{(|A_B|\cos\gamma_B - |\mu|\cot\beta)}{(M^2_{\tilde{d}_2} - M^2_{\tilde{d}_4})} \log(\frac{M^2_{\tilde{d}_2}}{M^2_{\tilde{d}_4}}) +$$

$$2\beta_{hB} m_B^2 |\mu||A_B| \frac{(|A_B|\cos\gamma_B - |\mu|\cot\beta)(|A_B| - |\mu|\cot\beta\cos\gamma_B)}{(M^2_{\tilde{d}_2} - M^2_{\tilde{d}_4})^2} f_2(M^2_{\tilde{d}_2}, M^2_{\tilde{d}_4})$$

$$\Delta_{13\tilde{B}} = -2\beta_{hB} m_B^2 |\mu|^2 |A_B| \sin\gamma_B \frac{(|\mu|\cot\beta - |A_q|\cos\gamma_B)}{\sin\beta(M^2_{\tilde{d}_2} - M^2_{\tilde{d}_4})^2} f_2(M^2_{\tilde{d}_2}, M^2_{\tilde{d}_4})$$

$$\Delta_{23\tilde{B}} = -2\beta_{hB} m_B^2 |\mu||A_B|^2 \sin\gamma_B \frac{(|A_B| - |\mu|\cot\beta\cos\gamma_B)}{\sin\beta(M^2_{\tilde{d}_2} - M^2_{\tilde{d}_4})^2} f_2(M^2_{\tilde{d}_2}, M^2_{\tilde{d}_4})$$

$$+2\beta_{hB} \frac{m_B^2 |\mu||A_B| \sin\gamma_B}{\sin\beta(M^2_{\tilde{d}_2} - M^2_{\tilde{d}_4})} \log(\frac{M^2_{\tilde{d}_2}}{M^2_{\tilde{d}_4}})$$

$$\Delta_{33\tilde{B}} = -2\beta_{hB} \frac{m_B^2 |\mu|^2 |A_B|^2 \sin^2\gamma_B}{\sin^2\beta(M^2_{\tilde{d}_2} - M^2_{\tilde{d}_4})^2} f_2(M^2_{\tilde{d}_2}, M^2_{\tilde{d}_4}) \quad (36)$$

The Yukawa couplings and quark masses in the $\Delta_{ij}$ elements are defined as follows

$$h_{t_4} = y'_5,\ h_t = y'_1,\ h_c = y'_3,\ h_u = y'_4, h_T = y_2$$

$$h_{b_4} = y_5,\ h_b = y_1,\ h_s = y_3,\ h_d = y_4, h_B = y'_2$$

$$m_T^2 = \frac{v_1^2|y_2|^2}{2},\ m_{t_4}^2 = \frac{v_2^2|y'_5|^2}{2},\ m_u^2 = \frac{v_2^2|y'_4|^2}{2}$$

$$m_c^2 = \frac{v_2^2|y'_3|^2}{2},\ m_t^2 = \frac{v_2^2|y'_1|^2}{2},\ m_B^2 = \frac{v_2^2|y'_2|^2}{2}$$

$$m_{b_4}^2 = \frac{v_1^2|y_5|^2}{2},\ m_d^2 = \frac{v_1^2|y_4|^2}{2},\ m_s^2 = \frac{v_1^2|y_3|^2}{2},\ m_b^2 = \frac{v_1^2|y_1|^2}{2} \quad (37)$$

The mass eigen values of the squark mass squared matrices $M^2_{\tilde{q}_i}$ are defined in the appendix.

# 5 Numerical Analysis

We present now a numerical analysis of the CP even-CP odd mixings of the Higgs bosons. The mixings arise from the Higgs boson mass squared matrix which as discussed above will be $3 \times 3$. In the preceding section this mass squared matrix has been computed in the basis $\phi_1, \phi_2, \psi_{1D}$ as explained in the text of the previous section. The Higgs mass squared matrix computed in section 4 is a real symmetric $3 \times 3$ matrix and can be diagonalized by an orthogonal transformations so that

$$DM^2D^T = diag(M_{H1}^2, M_{H2}^2, M_{H3}^2) \tag{38}$$

Here the $H_1$ is the lightest field and the remaining two fields $H_2, H_3$ are typically significantly heavier than $H_1$. We can investigate the CP structure of the two heavy fields through the estimate of the eigen vectors of the Higgs mass squared matrix.

$$\begin{aligned} H_2 &= D_{21}\phi_1 + D_{22}\phi_2 + D_{23}\psi_{1D} \\ H_3 &= D_{31}\phi_1 + D_{32}\phi_2 + D_{33}\psi_{1D} \end{aligned} \tag{39}$$

The percentage of CP odd part of $H_2$ is defined to be $|D_{23}|^2 \times 100$ and its CP even part is defined to be $(|D_{22}|^2 + |D_{21}|^2) \times 100$. The same definitions apply to the other neutral heavy Higgs $H_3$. The CP even-CP odd Higgs mixing depends directly on CP phases. On the other hand CP phases also generate EDM for the quarks and for the neutron. The current experimental limit on the EDM of the neutron is [37] $|d_n| < 2.9 \times 10^{-26} e$cm(90%CL). We note that the combinations of the phases that enter in the EDM of the quarks are not the same that enter in the CP even-CP odd Higgs mixings. Thus significant CP even-CP odd Higgs mixings can occur while at the same time the EDM constraint can be satisfied.

We give now an analysis of the $CP$ structure of the two heavy physical fields $H_2$ and $H_3$. We order the eigen values so that in the limit of no mixing between the $CP$ even and the $CP$ odd states one has that ($M_{H1}$, $M_{H2}$, $M_{H3}$) tend to ($m_h$, $m_H$, $m_A$) where $m_h$ is the mass of the light $CP$ even state, $m_H$ the mass of the heavy $CP$ even and $m_A$ is the mass of the $CP$ odd Higgs in MSSM when all $CP$ phases are set to zero. In the squark sector we assume $m_0^{u^2} = M_{\tilde{T}}^2 = M_{\tilde{t}_1}^2 = M_{\tilde{t}_2}^2 = M_{\tilde{t}_3}^2$ and $m_0^{d^2} = M_{\tilde{1}L}^2 = M_{\tilde{B}}^2 = M_{\tilde{b}_1}^2 = M_{\tilde{Q}}^2 = M_{\tilde{2}L}^2 = M_{\tilde{b}_2}^2 = M_{\tilde{3}L}^2 = M_{\tilde{b}_3}^2$. and $m_0^u = m_0^d = m_0$. Additionally the trilinear couplings are chosen so that: $A_0^u = A_t = A_T = A_c = A_u = A_{4t}$ and $A_0^d = A_b = A_B = A_s = A_d = A_{4b}$.

One expects the CP even-CP odd mixing to be a very sensitive function of the CP phases. We study this sensitivity for the case of MSSM first. In Fig. 1 we exhibit this dependence as a function of $\theta_\mu$. The left panel exhibits the CP even and CP odd components of the Higgs boson $H_2$ while the right panel exhibits the CP even and CP odd components of the Higgs boson $H_3$. In figure 2 we exhibit this dependence for the case of $\alpha_{A_0}$ where ($\alpha_{A_0} = \alpha_{A_0^u} = \alpha_{A_0^d}$). Next let us suppose that not all the loop correction to the light Higgs boson mass arises from the MSSM sector. Rather there are two components to this correction, one that arises from MSSM while the other arises from exchange of a vectorlike quark multiplet. In this case the vectorlike multiplet brings in new sources of CP violation which can contribute to the CP even-CP odd Higgs mixings. We give an illustration of this in table 1 and table 2. Table 1 gives the contribution to the Higgs mass from the MSSM sector alone which is a few GeV smaller than the desired value. The deficit is made up by exchange of a vectorlike multiplet. The contributions of the MSSM and of the vectorlike multiplet together are exhibited in table 2 which gives the Higgs mass consistent with the experimental value within a small error corridor of $\pm 2$ GeV. A comparison of tables 1 and 2, especially of the last three lines, shows that the CP even-CP odd mixing for the case of table 2 is very different from the case of table 1. Thus for $H_2(H_3)$, the CP odd (even) component is as much as 10% for the case when the vector multiplet is included whereas without the inclusion of the vector multiplet the even-odd mixing was vanishing. Thus inclusion of the vectorlike multiplet in the analysis has a strong effect on the CP even-CP odd mixing.

| | $M_{H1}$ | $CP$even | $CP$odd | $M_{H2}$ | $CP$even | $CP$odd | $M_{H3}$ | $CP$even | $CP$odd |
|---|---|---|---|---|---|---|---|---|---|
| (1) | 118.02 | 99.99 | 0.01 | 501.57 | 93.96 | 6.04 | 499.56 | 6.05 | 93.95 |
| (2) | 116.76 | 100 | 0.00 | 500.44 | 95.46 | 4.54 | 499.88 | 4.54 | 95.46 |
| (3) | 117.21 | 100 | 0.00 | 500.22 | 97.57 | 2.43 | 499.95 | 2.43 | 97.57 |
| (4) | 117.36 | 100 | 0.00 | 500.14 | 100 | 0.00 | 500 | 0.00 | 100 |
| (5) | 119.53 | 100 | 0.00 | 500.10 | 100 | 0.00 | 500 | 0.00 | 100 |
| (6) | 119.82 | 100 | 0.00 | 500.07 | 100 | 0.00 | 500 | 0.00 | 100 |

Table 1: An exhibition of the $CP$ structure of the $H_1$, $H_2$ and $H_3$ fields for the case without the contributions of the vectorlike generation. The analysis is for six benchmark points (1), (2), (3), (4), (5) and (6). Benchmark (1): $\tan\beta = 5$, $m_0 = m_0^u = m_0^d = 2300$, $|\mu| = 800$, $|A_0^u| = 8500$, $|A_0^d| = 9500$, $\theta_\mu = 0.9$, $\alpha_{A_0^u} = 0.5$, $\alpha_{A_0^d} = 1.5$. Benchmark (2): $\tan\beta = 10$, $m_0 = m_0^u = m_0^d = 2000$, $|\mu| = 380$, $|A_0^u| = 7400$, $|A_0^d| = 8300$, $\theta_\mu = 0.4$, $\alpha_{A_0^u} = 1.2$, $\alpha_{A_0^d} = 1.3$. Benchmark (3): $\tan\beta = 15$, $m_0 = m_0^u = m_0^d = 2300$, $|\mu| = 300$, $|A_0^u| = 8600$, $|A_0^d| = 8000$, $\theta_\mu = 0.9$, $\alpha_{A_0^u} = 3.5$, $\alpha_{A_0^d} = 2.2$. Benchmark (4): $\tan\beta = 20$, $m_0 = m_0^u = m_0^d = 2100$, $|\mu| = 200$, $|A_0^u| = 7800$, $|A_0^d| = 7000$, $\theta_\mu = 1.7$, $\alpha_{A_0^u} = 1.4$, $\alpha_{A_0^d} = 1$. Benchmark (5): $\tan\beta = 25$, $m_0 = m_0^u = m_0^d = 2500$, $|\mu| = 260$, $|A_0^u| = 9350$, $|A_0^d| = 3500$, $\theta_\mu = 2.2$, $\alpha_{A_0^u} = 1$, $\alpha_{A_0^d} = 3.2$. Benchmark (6): $\tan\beta = 30$, $m_0 = m_0^u = m_0^d = 2400$, $|\mu| = 200$, $|A_0^u| = 8950$, $|A_0^d| = 1000$, $\theta_\mu = 2.37$, $\alpha_{A_0^u} = 0.9$, $\alpha_{A_0^d} = 2.8$. The common parameters are: $m_A = 500$, $|h_3| = 1.58$, $|h_3'| = 6.34 \times 10^{-2}$, $|h_3''| = 1.97 \times 10^{-2}$, $|h_4| = 4.42$, $|h_4'| = 5.07$, $|h_4''| = 12.87$, $|h_5| = 6.6$, $|h_5'| = 2.67$, $|h_5''| = 1.86 \times 10^{-1}$, $|h_6| = 1000$, $|h_7| = 1000$, $|h_8| = 1000$, $\chi_3 = 2 \times 10^{-2}$, $\chi_3' = 1 \times 10^{-3}$, $\chi_3'' = 4 \times 10^{-3}$, $\chi_4 = 7 \times 10^{-3}$, $\chi_4' = \chi_4'' = 1 \times 10^{-3}$, $\chi_5 = 9 \times 10^{-3}$, $\chi_5' = 5 \times 10^{-3}$, $\chi_5'' = 2 \times 10^{-3}$, $\chi_6 = \chi_7 = \chi_8 = 5 \times 10^{-3}$. All masses are in GeV and all phases in rad.

| | $M_{H1}$ | $CP$even | $CP$odd | $M_{H2}$ | $CP$even | $CP$odd | $M_{H3}$ | $CP$even | $CP$odd |
|---|---|---|---|---|---|---|---|---|---|
| (1) | 124.08 | 99.98 | 0.02 | 504.80 | 91.68 | 8.32 | 497.46 | 8.33 | 91.67 |
| (2) | 124.54 | 99.98 | 0.02 | 523.51 | 90.71 | 9.29 | 486.87 | 9.30 | 90.70 |
| (3) | 124.17 | 99.99 | 0.01 | 533.10 | 92.69 | 7.31 | 486.07 | 7.32 | 92.68 |
| (4) | 124.06 | 100 | 0.00 | 539.99 | 94.79 | 5.21 | 494.94 | 5.21 | 94.79 |
| (5) | 123.99 | 100 | 0.00 | 514.14 | 89.28 | 10.72 | 492.61 | 10.72 | 89.28 |
| (6) | 124.71 | 100 | 0.00 | 539.94 | 94.35 | 5.65 | 495.41 | 5.66 | 94.34 |

Table 2: An exhibition of the $CP$ structure of the $H_1$, $H_2$ and $H_3$ fields for the case with the contributions of the vectorlike generation. The analysis is for six benchmark points corresponding to the parameter set of table 1. The Yukawa couplings are: (1): $h_T = 1.5$, $h_B = 0.4$, $h_{t_4} = 0.6$, $h_{b_4} = 1.5$; (2): $h_T = 2.9$, $h_B = 0.4$, $h_{t_4} = 0.5$, $h_{b_4} = 2.9$; (3): $h_T = 4.3$, $h_B = 0.4$, $h_{t_4} = 0.5$, $h_{b_4} = 4.3$; (4): $h_T = 5.8$, $h_B = 0.4$, $h_{t_4} = 0.5$, $h_{b_4} = 5.8$; (5): $h_T = 7.2$, $h_B = 0.4$, $h_{t_4} = 0.5$, $h_{b_4} = 7.2$; (6): $h_T = 8.6$, $h_B = 0.4$, $h_{t_4} = 0.5$, $h_{b_4} = 8.6$. Masses for the vectorlike quarks are gotten by diagonalization of the matrices of Eqs. (5) and (10) and are given as follows: mirror up quark $m_{t'} = 980.14$, mirror down quark mass $m_{b'} = 1062.63$, fourth generation up quark mass $m_4^{\rm up} = 1025.14$, fourth generation down quark mass $m_4^{\rm down}$ = 937.64. All masses are in GeV. The inputs from the MSSM sector are listed in table 3.

The MSSM sector inputs of the six benchmark points in table 1 and table 2.

| (case) | $\tan\beta$ | $\|\mu\|$ | $\theta_\mu$ | $m_0$ | $\|A_0^u\|$ | $\|A_0^d\|$ | $\alpha_{A_0^u}$ | $\alpha_{A_0^d}$ |
|---|---|---|---|---|---|---|---|---|
| (1) | 5 | 800 | 0.9 | 2300 | 8500 | 9500 | 0.5 | 1.5 |
| (2) | 10 | 380 | 0.4 | 2000 | 7400 | 8300 | 1.2 | 1.3 |
| (3) | 15 | 300 | 0.9 | 2300 | 8600 | 8000 | 3.5 | 2.2 |
| (4) | 20 | 200 | 1.7 | 2100 | 7800 | 7000 | 1.4 | 1 |
| (5) | 25 | 260 | 2.2 | 2500 | 9350 | 3500 | 1 | 3.2 |
| (6) | 30 | 200 | 2.37 | 2400 | 8950 | 1000 | 0.9 | 2.8 |

Table 3: The inputs of the six benchmark points of table 1.

We give now a more detailed analysis of CP even-CP odd mixing for the case with inclusion of the vectorlike multiplet. Specifically we discuss three illustrative benchmark points of table 2. In figure 3 we exhibit this dependence as a function of $\theta_\mu$. The left panel exhibits the CP even and CP odd components of the Higgs boson $H_2$ while the right panel exhibits the CP even and CP odd components of the Higgs boson $H_3$. One finds that the mixing can be very substantial for a significant parameter range of $\theta_\mu$. A similar analysis is presented in figure 4 for the case of $\alpha_{A_0^u}$ dependence. The $\alpha_{A_0^d}$ dependence is very similar to that for $\alpha_{A_0^u}$ and is not exhibited. Figure 5 exhibits the dependence of the CP even-CP mixing for $H_2$ and $H_3$ as a function of $m_0$. In Fig. 6 we give an analysis of the sensitivity of the masses for the boson $H_1, H_2, H_3$ as a function of $\theta_\mu$ and a similar analysis as a function of $\alpha_{A_0^u}$ is given in Fig. 7. One finds only a mild sensitivity of the light Higgs $H_1$ mass but much larger sensitivity of the masses of $H_2$ and $H_3$ on the CP phases. This is consistent with the significant CP even -CP odd mixing among the two heavy neutral Higgs.

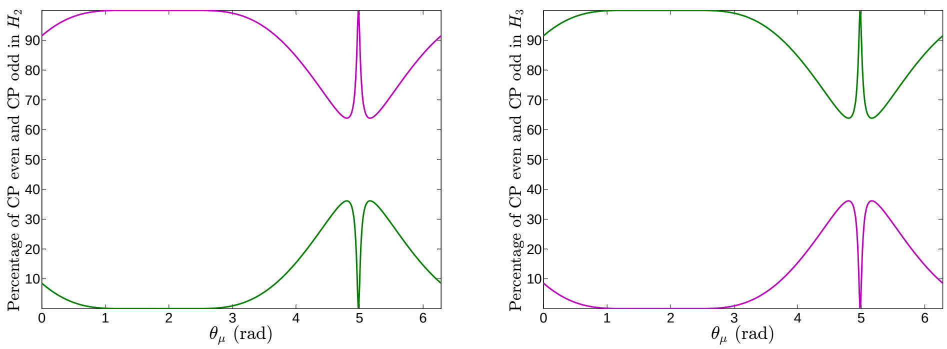


Figure 1: Left panel: Variation of the $CP$ even component of $H_2$ (upper curve) and the $CP$ odd component of $H_2$ (lower curve) without including the contributions of the vectorlike generation versus $\theta_\mu$. The input parameters are: $\tan\beta = 20$,$m_A = 500$, $m_0 = m_0^u = m_0^d = 2400$, $|\mu| = 300$, $|A_0^u| = |A_0^d| = 8750$, $\alpha_{A_0^u} = \alpha_{A_0^d} = 1.3$, $|h_3| = 1.58$, $|h_3'| = 6.34 \times 10^{-2}$, $|h_3''| = 1.97 \times 10^{-2}$, $|h_4| = 4.42$, $|h_4'| = 5.07$, $|h_4''| = 12.87$, $|h_5| = 6.6$, $|h_5'| = 2.67$, $|h_5''| = 1.86 \times 10^{-1}$, $|h_6| = 1000$, $|h_7| = 1000$, $|h_8| = 1000$, $\chi_3 = 2 \times 10^{-2}$, $\chi_3' = 1 \times 10^{-3}$, $\chi_3'' = 4 \times 10^{-3}$, $\chi_4 = 7 \times 10^{-3}$, $\chi_4' = \chi_4'' = 1 \times 10^{-3}$, $\chi_5 = 9 \times 10^{-3}$, $\chi_5' = 5 \times 10^{-3}$, $\chi_5'' = 2 \times 10^{-3}$, $\chi_6 = \chi_7 = \chi_8 = 5 \times 10^{-3}$. Right panel: Variation of the $CP$ even component of $H_3$ (lower curve) and the $CP$ odd component of $H_3$ (upper curve) without including the contributions of the vectorlike generation versus $\theta_\mu$ for the same inputs as left panel. All masses are in GeV and all phases in rad.

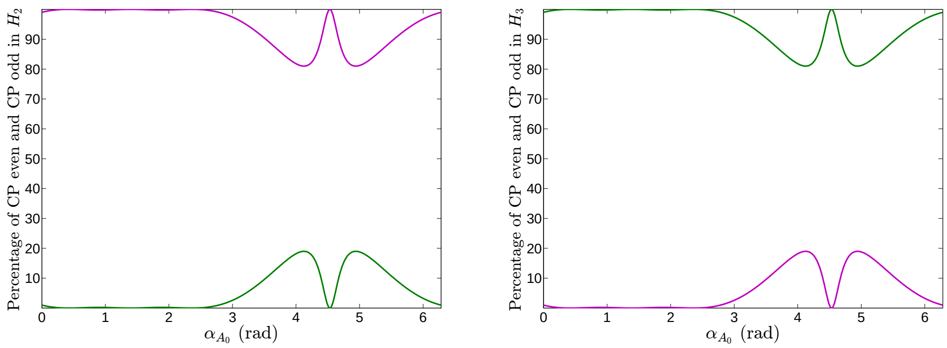


Figure 2: Left panel: Variation of the $CP$ even component of $H_2$ (upper curve) and the $CP$ odd component of $H_2$ (lower curve) without including the contributions of the vectorlike generation versus $\alpha_{A_0}$ ($\alpha_{A_0} = \alpha_{A_0^u} = \alpha_{A_0^d}$).The input parameters are: $\tan\beta = 30$,$m_A = 500$, $m_0 = m_0^u = m_0^d = 2200$, $|\mu| = 180$, $|A_0^u| = |A_0^d| = 8000$, $\theta_\mu = 1.75$, $|h_3| = 1.58$, $|h_3'| = 6.34 \times 10^{-2}$, $|h_3''| = 1.97 \times 10^{-2}$, $|h_4| = 4.42$, $|h_4'| = 5.07$, $|h_4''| = 12.87$, $|h_5| = 6.6$, $|h_5'| = 2.67$, $|h_5''| = 1.86 \times 10^{-1}$, $|h_6| = 1000$, $|h_7| = 1000$, $|h_8| = 1000$, $\chi_3 = 2 \times 10^{-2}$, $\chi_3' = 1 \times 10^{-3}$, $\chi_3'' = 4 \times 10^{-3}$, $\chi_4 = 7 \times 10^{-3}$, $\chi_4' = \chi_4'' = 1 \times 10^{-3}$, $\chi_5 = 9 \times 10^{-3}$, $\chi_5' = 5 \times 10^{-3}$, $\chi_5'' = 2 \times 10^{-3}$, $\chi_6 = \chi_7 = \chi_8 = 5 \times 10^{-3}$. Right panel: Variation of the $CP$ even component of $H_3$ (lower curve) and the $CP$ odd component of $H_3$ (upper curve) without including the contributions of the vectorlike generation versus $\alpha_{A_0}$ for the same inputs as left panel.

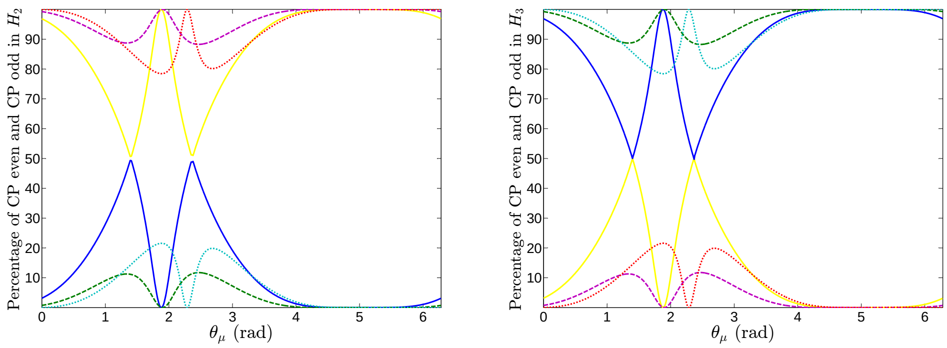


Figure 3: Left panel: Variation of the $CP$ even component of $H_2$ (upper curves) and the $CP$ odd component of $H_2$ (lower curves) including the contributions of the vectorlike generation versus $\theta_\mu$. The input for the solid curves is $\tan\beta = 10$, $m_0 = m_0^u = m_0^d = 2000$, $|\mu| = 380$, $|A_0^u| = 7400$, $|A_0^d| = 8300$, $\alpha_{A_0^u} = 1.2$, $\alpha_{A_0^d} = 1.3$, $h_T = 2.9$, $h_B = 0.4$, $h_{t_4} = 0.5$, $h_{b_4} = 2.9$ (Point 2). The input for the dashed curves is $\tan\beta = 20$, $m_0 = m_0^u = m_0^d = 2100$, $|\mu| = 200$, $|A_0^u| = 7800$, $|A_0^d| = 7000$, $\alpha_{A_0^u} = 1.4$, $\alpha_{A_0^d} = 1$, $h_T = 5.8$, $h_B = 0.4$, $h_{t_4} = 0.5$, $h_{b_4} = 5.8$ (Point 4). The input for the dotted curves is $\tan\beta = 30$, $m_0 = m_0^u = m_0^d = 2400$, $|\mu| = 200$, $|A_0^u| = 8950$, $|A_0^d| = 1000$, $\alpha_{A_0^u} = 0.9$, $\alpha_{A_0^d} = 2.8$, $h_T = 8.6$, $h_B = 0.4$, $h_{t_4} = 0.5$, $h_{b_4} = 8.6$ (Point 6). The common parameters are: $m_A = 500$, $|h_3| = 1.58$, $|h'_3| = 6.34 \times 10^{-2}$, $|h''_3| = 1.97 \times 10^{-2}$, $|h_4| = 4.42$, $|h'_4| = 5.07$, $|h''_4| = 12.87$, $|h_5| = 6.6$, $|h'_5| = 2.67$, $|h''_5| = 1.86 \times 10^{-1}$, $|h_6| = 1000$, $|h_7| = 1000$, $|h_8| = 1000$, $\chi_3 = 2 \times 10^{-2}$, $\chi'_3 = 1 \times 10^{-3}$, $\chi''_3 = 4 \times 10^{-3}$, $\chi_4 = 7 \times 10^{-3}$, $\chi'_4 = \chi''_4 = 1 \times 10^{-3}$, $\chi_5 = 9 \times 10^{-3}$, $\chi'_5 = 5 \times 10^{-3}$, $\chi''_5 = 2 \times 10^{-3}$, $\chi_6 = \chi_7 = \chi_8 = 5 \times 10^{-3}$. Right panel: Variation of the $CP$ even component of $H_3$ (lower curves) and the $CP$ odd component of $H_3$ (upper curves) including the contributions of the vectorlike generation versus $\theta_\mu$ for the same inputs as left panel. All masses are in GeV and all phases in rad.

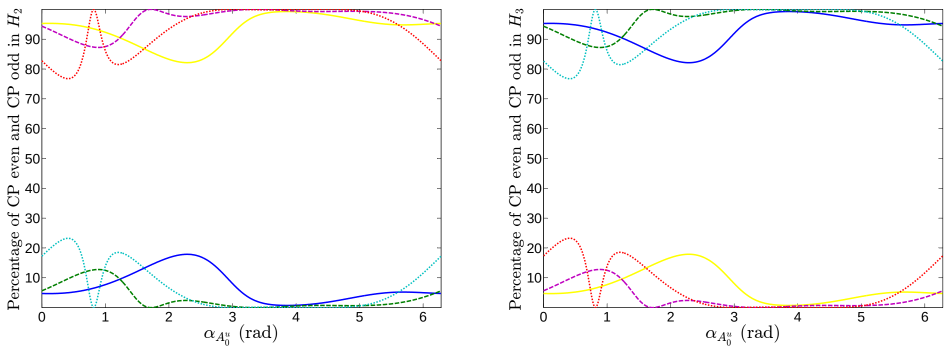


Figure 4: Left panel: Variation of the $CP$ even component of $H_2$ (upper curves) and the $CP$ odd component of $H_2$ (lower curves) including the contributions of the vectorlike generation versus $\alpha_{A_0^u}$. Right panel: Variation of the $CP$ even component of $H_3$ (lower curves) and the $CP$ odd component of $H_3$ (upper curves) including the contributions of the vectorlike generation versus $\alpha_{A_0^u}$ for the same inputs as figure 3.

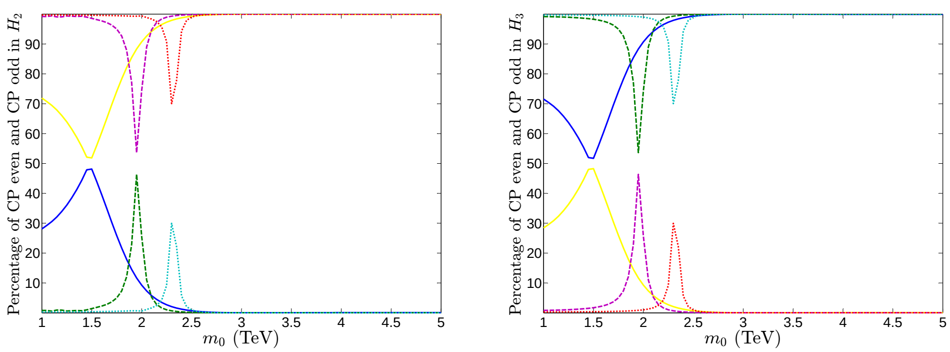


Figure 5: Left panel: Variation of the $CP$ even component of $H_2$ (upper curves) and the $CP$ odd component of $H_2$ (lower curves) including the contributions of the vectorlike generation versus $m_0$. Right panel: Variation of the $CP$ even component of $H_3$ (lower curves) and the $CP$ odd component of $H_3$ (upper curves) including the contributions of the vectorlike generation versus $m_0$ for the same inputs as figure 3.

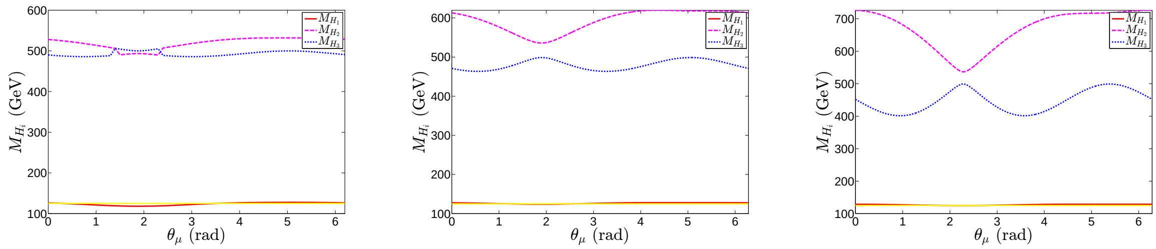


Figure 6: Left panel: Variation of the $M_{H_1}$ (solid curve), $M_{H_2}$ (dashed curve) and $M_{H_3}$ (dotted curve) versus $\theta_\mu$ for $\tan\beta = 10$, $m_0 = m_0^u = m_0^d = 2000$, $|\mu| = 380$, $|A_0^u| = 7400$, $|A_0^d| = 8300$, $\alpha_{A_0^u} = 1.2$, $\alpha_{A_0^d} = 1.3$, $h_T = 2.9$, $h_B = 0.4$, $h_{t_4} = 0.5$, $h_{b_4} = 2.9$. Middle panel: Variation of the $M_{H_1}$ (solid curve), $M_{H_2}$ (dashed curve) and $M_{H_3}$ (dotted curve) versus $\theta_\mu$ for $\tan\beta = 20$, $m_0 = m_0^u = m_0^d = 2100$, $|\mu| = 200$, $|A_0^u| = 7800$, $|A_0^d| = 7000$, $\alpha_{A_0^u} = 1.4$, $\alpha_{A_0^d} = 1$, $h_T = 5.8$, $h_B = 0.4$, $h_{t_4} = 0.5$, $h_{b_4} = 5.8$. Right panel: Variation of the $M_{H_1}$ (solid curve), $M_{H_2}$ (dashed curve) and $M_{H_3}$ (dotted curve) versus $\theta_\mu$ for $\tan\beta = 30$, $m_0 = m_0^u = m_0^d = 2400$, $|\mu| = 200$, $|A_0^u| = 8950$, $|A_0^d| = 1000$, $\alpha_{A_0^u} = 0.9$, $\alpha_{A_0^d} = 2.8$, $h_T = 8.6$, $h_B = 0.4$, $h_{t_4} = 0.5$, $h_{b_4} = 8.6$. The common parameters are: $m_A = 500$, $|h_3| = 1.58$, $|h_3'| = 6.34 \times 10^{-2}$, $|h_3''| = 1.97 \times 10^{-2}$, $|h_4| = 4.42$, $|h_4'| = 5.07$, $|h_4''| = 12.87$, $|h_5| = 6.6$, $|h_5'| = 2.67$, $|h_5''| = 1.86 \times 10^{-1}$, $|h_6| = 1000$, $|h_7| = 1000$, $|h_8| = 1000$, $\chi_3 = 2 \times 10^{-2}$, $\chi_3' = 1 \times 10^{-3}$, $\chi_3'' = 4 \times 10^{-3}$, $\chi_4 = 7 \times 10^{-3}$, $\chi_4' = \chi_4'' = 1 \times 10^{-3}$, $\chi_5 = 9 \times 10^{-3}$, $\chi_5' = 5 \times 10^{-3}$, $\chi_5'' = 2 \times 10^{-3}$, $\chi_6 = \chi_7 = \chi_8 = 5 \times 10^{-3}$. All masses are in GeV and all phases in rad.

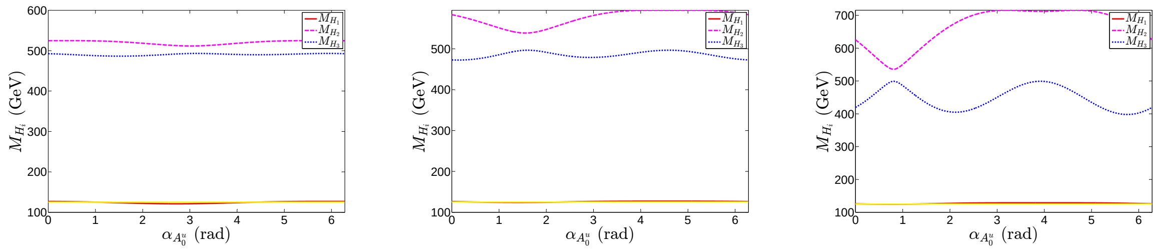


Figure 7: Left panel: Variation of the $M_{H_1}$ (solid curve), $M_{H_2}$ (dashed curve) and $M_{H_3}$ (dotted curve) versus $\alpha_{A_0^u}$ for $\tan\beta = 10$, $m_0 = m_0^u = m_0^d = 2000$, $|\mu| = 380$, $|A_0^u| = 7400$, $|A_0^d| = 8300$, $\theta_\mu = 0.4$, $\alpha_{A_0^d} = 1.3$, $h_T = 2.9$, $h_B = 0.4$, $h_{t_4} = 0.5$, $h_{b_4} = 2.9$. Middle panel: Variation of the $M_{H_1}$ (solid curve), $M_{H_2}$ (dashed curve) and $M_{H_3}$ (dotted curve) versus $\alpha_{A_0^u}$ for $\tan\beta = 20$, $m_0 = m_0^u = m_0^d = 2100$, $|\mu| = 200$, $|A_0^u| = 7800$, $|A_0^d| = 7000$, $\theta_\mu = 1.7$, $\alpha_{A_0^d} = 1$, $h_T = 5.8$, $h_B = 0.4$, $h_{t_4} = 0.5$, $h_{b_4} = 5.8$. Right panel: Variation of the $M_{H_1}$ (solid curve), $M_{H_2}$ (dashed curve) and $M_{H_3}$ (dotted curve) versus $\alpha_{A_0^u}$ for $\tan\beta = 30$, $m_0 = m_0^u = m_0^d = 2400$, $|\mu| = 200$, $|A_0^u| = 8950$, $|A_0^d| = 1000$, $\theta_\mu = 2.37$, $\alpha_{A_0^d} = 2.8$, $h_T = 8.6$, $h_B = 0.4$, $h_{t_4} = 0.5$, $h_{b_4} = 8.6$. The common parameters are: $m_A = 500$, $|h_3| = 1.58$, $|h_3'| = 6.34 \times 10^{-2}$, $|h_3''| = 1.97 \times 10^{-2}$, $|h_4| = 4.42$, $|h_4'| = 5.07$, $|h_4''| = 12.87$, $|h_5| = 6.6$, $|h_5'| = 2.67$, $|h_5''| = 1.86 \times 10^{-1}$, $|h_6| = 1000$, $|h_7| = 1000$, $|h_8| = 1000$, $\chi_3 = 2 \times 10^{-2}$, $\chi_3' = 1 \times 10^{-3}$, $\chi_3'' = 4 \times 10^{-3}$, $\chi_4 = 7 \times 10^{-3}$, $\chi_4' = \chi_4'' = 1 \times 10^{-3}$, $\chi_5 = 9 \times 10^{-3}$, $\chi_5' = 5 \times 10^{-3}$, $\chi_5'' = 2 \times 10^{-3}$, $\chi_6 = \chi_7 = \chi_8 = 5 \times 10^{-3}$. All masses are in GeV and all phases in rad.

## 5.1 Decays of the Higgs bosons to fermion pairs

Decays of the Higgs bosons are important channels for tests of new physics beyond the standard model. A convenient ratio for this purpose is $R_{if}$ defined by [4]

$$R_{if} = \frac{\Gamma(H_i \to \bar{f}f)}{\Gamma(H_i \to \bar{f}f)_0} = \frac{(D_{ik})^2(1 - x_f^2)^{3/2} + f^2(D_{i3})^2(1 - x_f^2)^{1/2}}{(D_{ik}(0))^2(1 - x_{f0}^2)^{3/2} + f^2(D_{i3}(0))^2(1 - x_{f0}^2)^{1/2}} \tag{40}$$

where $x_f^2 = 4m_f^2/M_{H_i}^2$, $x_{f0}^2 = 4m_f^2/M_{H_i}(0)^2$, where $k = 2(1)$ and $f = \cos\beta(\sin\beta)$ for $u$-type quarks ($d$-type quarks and charged leptons). The argument 0 in D and in the subscript of $x_f$ in the denominator indicates that $\theta_\mu + \alpha_{A_0} = 0$. For the case when there is no contribution from the vectorlike multiplet the ratio between the decay widths of the higgs into quark pairs is exhibited in table 4 for the model point 3 in table 1. As a comparison we exhibit the same ratios for the case when a vectorlike multiplet is included again for model point 3 of table 5. One finds significant differences between the two tables for certain decay width ratios which points to the significant contribution from the vectorlike multiplet to the ratio. We now study the CP phase dependence for the case with contributions from the vectorlike multiplet are included. In Fig. 8 we give the dependence of $R_{1b}$ and $R_{1c}$ on $\theta_\mu$ and on $\alpha_{A_0} = \alpha_{A_0^u} = \alpha_{A_0^d}$. One finds a large sensitivity of the ratio to the CP phases. A similar analysis for $R_{2b}, R_{2c}$ is given in Fig. 9 and for $R_{3b}, R_{3c}$ in Fig. 10.

| $R_{if}$ | $b$ | $s$ | $d$ | $t$ | $c$ | $u$ |
|---|---|---|---|---|---|---|
| $i = 1$ | 1.125 | 1.125 | 1.125 | ... | 0.999 | 0.999 |
| $i = 2$ | 0.999 | 0.999 | 0.999 | 1.154 | 1.133 | 1.133 |
| $i = 3$ | 1 | 1 | 1 | 0.984 | 0.992 | 0.992 |

Table 4: An exhibition of the ratio between the decay widths of the higgs scalars into quark pairs for the case without the contributions of vectorlike multiplet. The parameter space corresponding to point 3 in table 1.

| $R_{if}$ | $b$ | $s$ | $d$ | $t$ | $c$ | $u$ |
|---|---|---|---|---|---|---|
| $i = 1$ | 2.069 | 2.07 | 2.07 | ... | 0.997 | 0.997 |
| $i = 2$ | 0.998 | 0.998 | 0.998 | 1.701 | 1.861 | 1.861 |
| $i = 3$ | 0.999 | 0.999 | 0.999 | 1.001 | 1.134 | 1.134 |

Table 5: An exhibition of the ratio between the decay widths of the higgs scalars into quark pairs for the case with the contributions of vectorlike multiplet. The parameter space corresponding to point 3 in table 1.

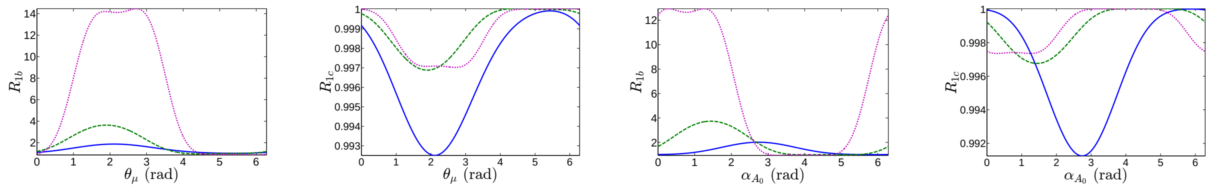


Figure 8: Left panel: Variation of the $R_{1b}$ versus $\theta_\mu$ for the case with the contributions of the vectorlike generation. Second left panel: Variation of the $R_{1c}$ versus $\theta_\mu$ for the case with the contributions of the vectorlike generation. The inputs correspond to point 2 (solid curve), point 4 (dashed curve) and point 6 (dotted curve) in table 3. Second right panel: Variation of the $R_{1b}$ versus $\alpha_{A_0}$ ($\alpha_{A_0} = \alpha_{A_0^u} = \alpha_{A_0^d}$) for the case with the contributions of the vectorlike generation. Right panel: Variation of the $R_{1c}$ versus $\alpha_{A_0}$ for the case with the contributions of the vectorlike generation. The inputs correspond to point 2 (solid curve), point 4 (dashed curve) and point 6 (dotted curve) in table 3.

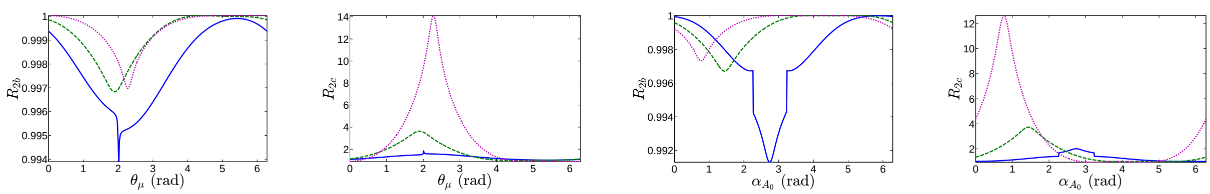


Figure 9: Left panel: Variation of the $R_{2b}$ versus $\theta_\mu$ for the case with the contributions of the vectorlike generation. Second left panel: Variation of the $R_{2c}$ versus $\theta_\mu$ for the case with the contributions of the vectorlike generation. The inputs correspond to point 2 (solid curve), point 4 (dashed curve) and point 6 (dotted curve) in table 3. Second right panel: Variation of the $R_{2b}$ versus $\alpha_{A_0}$ ($\alpha_{A_0} = \alpha_{A_0^u} = \alpha_{A_0^d}$) for the case with the contributions of the vectorlike generation. Right panel: Variation of the $R_{2c}$ versus $\alpha_{A_0}$ for the case with the contributions of the vectorlike generation. The inputs correspond to point 2 (solid curve), point 4 (dashed curve) and point 6 (dotted curve) in table 3.

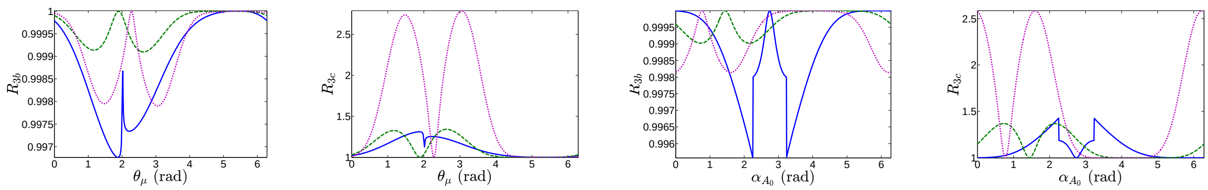


Figure 10: Left panel: Variation of the $R_{3b}$ versus $\theta_\mu$ for the case with the contributions of the vectorlike generation. Second left panel: Variation of the $R_{3c}$ versus $\theta_\mu$ for the case with the contributions of the vectorlike generation. The inputs correspond to point 2 (solid curve), point 4 (dashed curve) and point 6 (dotted curve) in table 3. Second right panel: Variation of the $R_{3b}$ versus $\alpha_{A_0}$ ($\alpha_{A_0} = \alpha_{A_0^u} = \alpha_{A_0^d}$) for the case with the contributions of the vectorlike generation. Right panel: Variation of the $R_{3c}$ versus $\alpha_{A_0}$ for the case with the contributions of the vectorlike generation. The inputs correspond to point 2 (solid curve), point 4 (dashed curve) and point 6 (dotted curve) in table 3.

# 6 Conclusion

An important phenomenon in supersymmetric models with inclusion of explicit CP violation relates to the mixing of CP even and CP odd Higgs bosons. In this work we have investigated the implication of a vectorlike quark multiplet on the CP even-CP odd mixing within an extended MSSM model. The sector brings with it new sources of CP violation and our analysis shows that the vectorlike multiplet can generate substantial CP even-CP odd Higgs mixing even in regions where the mixing from the MSSM sector is small. We have investigated the dependence of the mixings on the phases and find that large mixings can occur in certain regions of the parameter space of CP phases. The decays of the Higgs bosons into fermions are sensitive to new physics. We have investigated these decays for the case of MSSM and for the case when one has in addition a vectorlike multiplet. Further, for the latter case we have investigated the dependence of the Higgs decays widths into fermions as a function of CP phases. These decays show a sharp dependence on the phase of $\mu$ and on the phase of the trilinear coupling. These results are of interest regarding the new data expected from the LHC and the search for the heavy Higgs bosons.

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# 7 Appendix: Squark mass matrices

In this Appendix we give further details of the model discussed in section 2. As discussed in section 2 we allow for mixing between the vector generation and specifically the mirrors and the standard three generations of quarks. The superpotential allowing such mixings is given by

$$
\begin{aligned}
W = \epsilon_{ij}[& y_1 \hat{H}_1^i \hat{q}_{1L}^j \hat{b}_{1L}^c + y_1' \hat{H}_2^j \hat{q}_{1L}^i \hat{t}_{1L}^c + y_2 \hat{H}_1^i \hat{Q}^{cj} \hat{T}_L + y_2' \hat{H}_2^j \hat{Q}^{ci} \hat{B}_L \\
& + y_3 \hat{H}_1^i \hat{q}_{2L}^j \hat{b}_{2L}^c + y_3' \hat{H}_2^j \hat{q}_{2L}^i \hat{t}_{2L}^c + y_4 \hat{H}_1^i \hat{q}_{3L}^j \hat{b}_{3L}^c + y_4' \hat{H}_2^j \hat{q}_{3L}^i \hat{t}_{3L}^c + y_5 \hat{H}_1^i \hat{q}_{4L}^j \hat{b}_{4L}^c + y_5' \hat{H}_2^j \hat{q}_{4L}^i \hat{t}_{4L}^c] \\
& + h_3 \epsilon_{ij} \hat{Q}^{ci} \hat{q}_{1L}^j + h_3' \epsilon_{ij} \hat{Q}^{ci} \hat{q}_{2L}^j + h_3'' \epsilon_{ij} \hat{Q}^{ci} \hat{q}_{3L}^j + h_6 \epsilon_{ij} \hat{Q}^{ci} \hat{q}_{4L}^j + h_4 \hat{b}_{1L}^c \hat{B}_L + h_5 \hat{t}_{1L}^c \hat{T}_L \\
& + h_4' \hat{b}_{2L}^c \hat{B}_L + h_5' \hat{t}_{2L}^c \hat{T}_L + h_4'' \hat{b}_{3L}^c \hat{B}_L + h_5'' \hat{t}_{3L}^c \hat{T}_L + h_7 \hat{b}_{4L}^c \hat{B}_L + h_8 \hat{t}_{4L}^c \hat{T}_L - \mu \epsilon_{ij} \hat{H}_1^i \hat{H}_2^j \ , \qquad (41)
\end{aligned}
$$

Here the couplings are in general complex. Thus, for example, $\mu$ is the complex Higgs mixing parameter so that $\mu = |\mu| e^{i\theta_\mu}$. The mass terms for the ups, mirror ups, downs and mirror downs arise from the term

$$
\mathcal{L} = -\frac{1}{2} \frac{\partial^2 W}{\partial A_i \partial A_j} \psi_i \psi_j + \text{h.c.}, \qquad (42)
$$

where $\psi$ and $A$ stand for generic two-component fermion and scalar fields. After spontaneous breaking of the electroweak symmetry, ($\langle H_1^1 \rangle = v_1/\sqrt{2}$ and $\langle H_2^2 \rangle = v_2/\sqrt{2}$), we have the following set of mass terms written in the four-component spinor notation so that

$$
-\mathcal{L}_m = \bar{\xi}_R^T (M_u) \xi_L + \bar{\eta}_R^T (M_d) \eta_L + \text{h.c.}, \qquad (43)
$$

where the basis vectors are defined in Eq. 3 and Eq. 8.

Next we consider the mixing of the down squarks and the charged mirror sdowns. The mass squared matrix of the sdown - mirror sdown comes from three sources: the F term, the D term of the potential and the soft SUSY breaking terms. Using the superpotential of the mass terms arising from it after the breaking of the electroweak symmetry are given by the Lagrangian

$$
\mathcal{L} = \mathcal{L}_F + \mathcal{L}_D + \mathcal{L}_{\text{soft}} \ , \qquad (44)
$$

where $\mathcal{L}_F$ is deduced from $F_i = \partial W/\partial A_i$, and $-\mathcal{L}_F = V_F = F_i F_i^*$ while the $\mathcal{L}_D$ is given by

$$
\begin{aligned}
-\mathcal{L}_D = &\frac{1}{2}m_Z^2\cos^2\theta_W\cos 2\beta\{\tilde{t}_L\tilde{t}_L^* - \tilde{b}_L\tilde{b}_L^* + \tilde{c}_L\tilde{c}_L^* - \tilde{s}_L\tilde{s}_L^* + \tilde{u}_L\tilde{u}_L^* - \tilde{d}_L\tilde{d}_L^* + \tilde{t}_{4L}\tilde{t}_{4L}^* - \tilde{b}_{4L}\tilde{b}_{4L}^* \\
&+ \tilde{B}_R\tilde{B}_R^* - \tilde{T}_R\tilde{T}_R^*\} + \frac{1}{2}m_Z^2\sin^2\theta_W\cos 2\beta\{-\frac{1}{3}\tilde{t}_L\tilde{t}_L^* + \frac{4}{3}\tilde{t}_R\tilde{t}_R^* - \frac{1}{3}\tilde{c}_L\tilde{c}_L^* + \frac{4}{3}\tilde{c}_R\tilde{c}_R^* \\
&- \frac{1}{3}\tilde{u}_L\tilde{u}_L^* + \frac{4}{3}\tilde{u}_R\tilde{u}_R^* + \frac{1}{3}\tilde{T}_R\tilde{T}_R^* - \frac{4}{3}\tilde{T}_L\tilde{T}_L^* - \frac{1}{3}\tilde{b}_L\tilde{b}_L^* - \frac{2}{3}\tilde{b}_R\tilde{b}_R^* \\
&- \frac{1}{3}\tilde{s}_L\tilde{s}_L^* - \frac{2}{3}\tilde{s}_R\tilde{s}_R^* - \frac{1}{3}\tilde{d}_L\tilde{d}_L^* - \frac{2}{3}\tilde{d}_R\tilde{d}_R^* + \frac{1}{3}\tilde{B}_R\tilde{B}_R^* \\
&+ \frac{2}{3}\tilde{B}_L\tilde{B}_L^* - \frac{1}{3}\tilde{t}_{4L}\tilde{t}_{4L}^* + \frac{4}{3}\tilde{t}_{4R}\tilde{t}_{4R}^* - \frac{1}{3}\tilde{b}_{4L}\tilde{b}_{4L}^* - \frac{2}{3}\tilde{b}_{4R}\tilde{b}_{4R}^*\}.
\end{aligned} \tag{45}
$$

For $\mathcal{L}_{\text{soft}}$ we assume the following form

$$
\begin{aligned}
-\mathcal{L}_{\text{soft}} = &M_{\tilde{1}L}^2\tilde{q}_{1L}^{k*}\tilde{q}_{1L}^k + M_{\tilde{4}L}^2\tilde{q}_{4L}^{k*}\tilde{q}_{4L}^k + M_{\tilde{2}L}^2\tilde{q}_{2L}^{k*}\tilde{q}_{2L}^k + M_{\tilde{3}L}^2\tilde{q}_{3L}^{k*}\tilde{q}_{3L}^k + M_{\tilde{Q}}^2\tilde{Q}^{ck*}\tilde{Q}^{ck} + M_{\tilde{t}_1}^2\tilde{t}_{1L}^{c*}\tilde{t}_{1L}^c \\
&+ M_{\tilde{b}_1}^2\tilde{b}_{1L}^{c*}\tilde{b}_{1L}^c + M_{\tilde{t}_2}^2\tilde{t}_{2L}^{c*}\tilde{t}_{2L}^c + M_{\tilde{b}_4}^2\tilde{b}_{4L}^{c*}\tilde{b}_{4L}^c + M_{\tilde{t}_4}^2\tilde{t}_{4L}^{c*}\tilde{t}_{4L}^c \\
&+ M_{\tilde{t}_3}^2\tilde{t}_{3L}^{c*}\tilde{t}_{3L}^c + M_{\tilde{b}_2}^2\tilde{b}_{2L}^{c*}\tilde{b}_{2L}^c + M_{\tilde{b}_3}^2\tilde{b}_{3L}^{c*}\tilde{b}_{3L}^c + M_{\tilde{B}}^2\tilde{B}_L^*\tilde{B}_L + M_{\tilde{T}}^2\tilde{T}_L^*\tilde{T}_L \\
&+ \epsilon_{ij}\{y_1A_bH_1^i\tilde{q}_{1L}^j\tilde{b}_{1L}^c - y_1'A_tH_2^i\tilde{q}_{1L}^j\tilde{t}_{1L}^c + y_5A_{b_4}H_1^i\tilde{q}_{4L}^j\tilde{b}_{4L}^c - y_5'A_{t_4}H_2^i\tilde{q}_{4L}^j\tilde{t}_{4L}^c + y_3A_sH_1^i\tilde{q}_{2L}^j\tilde{b}_{2L}^c \\
&- y_3'A_cH_2^i\tilde{q}_{2L}^j\tilde{t}_{2L}^c + y_4A_dH_1^i\tilde{q}_{3L}^j\tilde{b}_{3L}^c - y_4'A_uH_2^i\tilde{q}_{3L}^j\tilde{t}_{3L}^c + y_2A_TH_1^i\tilde{Q}^{cj}\tilde{T}_L - y_2'A_BH_2^i\tilde{Q}^{cj}\tilde{B}_L + \text{h.c.}\} \ .
\end{aligned} \tag{46}
$$

Here $M_{\tilde{1}L}, M_{\tilde{T}}$, etc are the soft masses and $A_t, A_b$, etc are the trilinear couplings. The trilinear couplings are complex and we define their phases so that

$$
A_b = |A_b|e^{i\alpha_{A_b}} \ , \quad A_t = |A_t|e^{i\alpha_{A_t}} \ , \cdots \ . \tag{47}
$$

From these terms we construct the scalar mass squared matrices. Thus we define the scalar mass squared matrix $M_{\tilde{d}}^2$ in the basis $(\tilde{b}_L, \tilde{B}_L, \tilde{b}_R, \tilde{B}_R, \tilde{s}_L, \tilde{s}_R, \tilde{d}_L, \tilde{d}_R, \tilde{b}_{4L}, \tilde{b}_{4R})$. We label the matrix elements of these as $(M_{\tilde{d}}^2)_{ij} = M_{ij}^2$ which is a hermitian matrix. We can diagonalize this hermitian mass squared matrix by the unitary transformation

$$
\tilde{D}^{d\dagger}M_{\tilde{d}}^2\tilde{D}^d = diag(M_{\tilde{d}_1}^2, M_{\tilde{d}_2}^2, M_{\tilde{d}_3}^2, M_{\tilde{d}_4}^2, M_{\tilde{d}_5}^2, M_{\tilde{d}_6}^2, M_{\tilde{d}_7}^2, M_{\tilde{d}_8}^2 M_{\tilde{d}_9}^2, M_{\tilde{d}_{10}}^2) \ . \tag{48}
$$

Similarly we write the mass squared matrix in the up squark sector in the basis $(\tilde{t}_L, \tilde{T}_L, \tilde{t}_R, \tilde{T}_R, \tilde{c}_L, \tilde{c}_R, \tilde{u}_L, \tilde{u}_R, \tilde{t}_{4L}, \tilde{t}_{4R})$. Thus here we denote the up squark mass squared matrix in the form $(M_{\tilde{u}}^2)_{ij} = m_{ij}^2$ which is also a hermitian matrix. We can diagonalize this mass square matrix by the unitary transformation

$$
\tilde{D}^{u\dagger}M_{\tilde{u}}^2\tilde{D}^u = \text{diag}(M_{\tilde{u}_1}^2, M_{\tilde{u}_2}^2, M_{\tilde{u}_3}^2, M_{\tilde{u}_4}^2, M_{\tilde{u}_5}^2, M_{\tilde{u}_6}^2, M_{\tilde{u}_7}^2, M_{\tilde{u}_8}^2, M_{\tilde{u}_9}^2, M_{\tilde{u}_{10}}^2) \ . \tag{49}
$$

We label the matrix elements of these as $(M^2_{\tilde{d}})_{ij} = M^2_{ij}$ where the elements of the matrix are given by

$$
\begin{aligned}
M^2_{11} &= M^2_{\tilde{1}L} + \frac{v_1^2|y_1|^2}{2} + |h_3|^2 - m_Z^2\cos 2\beta\left(\frac{1}{2} - \frac{1}{3}\sin^2\theta_W\right),\\
M^2_{22} &= M^2_{\tilde{B}} + \frac{v_2^2|y_2'|^2}{2} + |h_4|^2 + |h_4'|^2 + |h_4''|^2 + |h_7|^2 + \frac{1}{3}m_Z^2\cos 2\beta\sin^2\theta_W,\\
M^2_{33} &= M^2_{\tilde{b}_1} + \frac{v_1^2|y_1|^2}{2} + |h_4|^2 - \frac{1}{3}m_Z^2\cos 2\beta\sin^2\theta_W,\\
M^2_{44} &= M^2_{\tilde{Q}} + \frac{v_2^2|y_2'|^2}{2} + |h_3|^2 + |h_3'|^2 + |h_3''|^2 + |h_6|^2 + m_Z^2\cos 2\beta\left(\frac{1}{2} - \frac{1}{3}\sin^2\theta_W\right),
\end{aligned}
$$

$$
\begin{aligned}
M^2_{55} &= M^2_{\tilde{2}L} + \frac{v_1^2|y_3|^2}{2} + |h_3'|^2 - m_Z^2\cos 2\beta\left(\frac{1}{2} - \frac{1}{3}\sin^2\theta_W\right),\\
M^2_{66} &= M^2_{\tilde{b}_2} + \frac{v_1^2|y_3|^2}{2} + |h_4'|^2 - \frac{1}{3}m_Z^2\cos 2\beta\sin^2\theta_W,\\
M^2_{77} &= M^2_{\tilde{3}L} + \frac{v_1^2|y_4|^2}{2} + |h_3''|^2 - m_Z^2\cos 2\beta\left(\frac{1}{2} - \frac{1}{3}\sin^2\theta_W\right),\\
M^2_{88} &= M^2_{\tilde{b}_3} + \frac{v_1^2|y_4|^2}{2} + |h_4''|^2 - \frac{1}{3}m_Z^2\cos 2\beta\sin^2\theta_W\ .\\
M^2_{99} &= M^2_{\tilde{4}L} + \frac{v_1^2|y_5|^2}{2} + |h_6|^2 - m_Z^2\cos 2\beta\left(\frac{1}{2} - \frac{1}{3}\sin^2\theta_W\right)\\
M^2_{1010} &= M^2_{\tilde{b}_4} + \frac{v_1^2|y_5|^2}{2} + |h_7|^2 - \frac{1}{3}m_Z^2\cos 2\beta\sin^2\theta_W\ .
\end{aligned}
\tag{50}
$$

$$M^2_{12}=M^{2*}_{21}=\frac{v_2y_2'h_3^*}{\sqrt{2}}+\frac{v_1h_4y_1^*}{\sqrt{2}}, M^2_{13}=M^{2*}_{31}=\frac{y_1^*}{\sqrt{2}}(v_1A_b^*-\mu v_2), M^2_{14}=M^{2*}_{41}=0,$$
$$M^2_{15}=M^{2*}_{51}=h_3'h_3^*, M^2_{16}=M^{2*}_{61}=0, M^2_{17}=M^{2*}_{71}=h_3''h_3^*, M^2_{18}=M^{2*}_{81}=0, M^2_{19}=M^{2*}_{91}=h_3^*h_6,$$
$$M^2_{110}=M^{2*}_{101}=0, M^2_{23}=M^{2*}_{32}=0, M^2_{24}=M^{2*}_{42}=\frac{y_2'^*}{\sqrt{2}}(v_2A_B^*-\mu v_1), M^2_{25}=M^{2*}_{52}=\frac{v_2h_3'y_2'^*}{\sqrt{2}}+\frac{v_1y_3h_4^*}{\sqrt{2}},$$
$$M^2_{26}=M^{2*}_{62}=0, M^2_{27}=M^{2*}_{72}=\frac{v_2h_3''y_2'^*}{\sqrt{2}}+\frac{v_1y_4h_4''^*}{\sqrt{2}}, M^2_{28}=M^{2*}_{82}=0,$$
$$M^2_{29}=M^{2*}_{92}=\frac{v_1h_7^*y_5}{\sqrt{2}}+\frac{v_2y_2'^*h_6}{\sqrt{2}}, M^2_{210}=M^{2*}_{102}=0,$$
$$M^2_{34}=M^{2*}_{43}=\frac{v_2h_4y_2'^*}{\sqrt{2}}+\frac{v_1y_1h_3^*}{\sqrt{2}}, M^2_{35}=M^{2*}_{53}=0, M^2_{36}=M^{2*}_{63}=h_4h_4'^*,$$
$$M^2_{37}=M^{2*}_{73}=0, M^2_{38}=M^{2*}_{83}=h_4h_4''^*,$$
$$M^2_{39}=M^{2*}_{93}=0, M^2_{310}=M^{2*}_{103}=h_4h_7^*,$$
$$M^2_{45}=M^{2*}_{54}=0, M^2_{46}=M^{2*}_{64}=\frac{v_2y_2'h_4'^*}{\sqrt{2}}+\frac{v_1h_3'y_3^*}{\sqrt{2}},$$
$$M^2_{47}=M^{2*}_{74}=0, M^2_{48}=M^{2*}_{84}=\frac{v_2y_2'h_4''^*}{\sqrt{2}}+\frac{v_1h_3''y_4^*}{\sqrt{2}},$$
$$M^2_{49}=M^{2*}_{94}=0, M^2_{410}=M^{2*}_{104}=\frac{v_2y_2'h_7^*}{\sqrt{2}}+\frac{v_1h_6y_5^*}{\sqrt{2}},$$
$$M^2_{56}=M^{2*}_{65}=\frac{y_3^*}{\sqrt{2}}(v_1A_s^*-\mu v_2), M^2_{57}=M^{2*}_{75}=h_3''h_3'^*,$$
$$M^2_{58}=M^{2*}_{85}=0, M^2_{59}=M^{2*}_{95}=h_3'^*h_6, M^2_{510}=M^{2*}_{105}=0, M^2_{67}=M^{2*}_{76}=0,$$
$$M^2_{68}=M^{2*}_{86}=h_4'h_4''^*, M^2_{69}=M^{2*}_{96}=0, M^2_{610}=M^{2*}_{106}=h_4'h_7^*, M^2_{78}=M^{2*}_{87}=\frac{y_4^*}{\sqrt{2}}(v_1A_d^*-\mu v_2)\ .$$
$$M^2_{79}=M^{2*}_{97}=h_3''^*h_6, M^2_{710}=M^{2*}_{107}=0$$
$$M^2_{89}=M^{2*}_{98}=0, M^2_{810}=M^{2*}_{108}=h_4''h_7^*, M^2_{910}=M^{2*}_{109}=\frac{y_5^*}{\sqrt{2}}(v_1A_{b_4}^*-\mu v_2)\ .$$

We can diagonalize this hermitian mass squared matrix of the scalar downs by the unitary transformation

$$\tilde{D}^{d\dagger}M^2_{\tilde{d}}\tilde{D}^d=diag(M^2_{\tilde{d}_1},M^2_{\tilde{d}_2},M^2_{\tilde{d}_3},M^2_{\tilde{d}_4},M^2_{\tilde{d}_5},M^2_{\tilde{d}_6},M^2_{\tilde{d}_7},M^2_{\tilde{d}_8},M^2_{\tilde{d}_9},M^2_{\tilde{d}_{10}})\ . \qquad (51)$$

Next we write the mass squared matrix in the sups sector the basis $(\tilde{t}_L,\tilde{T}_L,\tilde{t}_R,\tilde{T}_R,\tilde{c}_L,\tilde{c}_R,\tilde{u}_L,\tilde{u}_R,\tilde{t}_{4L},\tilde{t}_{4R})$. Thus here we denote the sups mass squared matrix in the form $(M^2_{\tilde{u}})_{ij}=m^2_{ij}$ where

$$m_{11}^2 = M_{\tilde{1}L}^2 + \frac{v_2^2|y_1'|^2}{2} + |h_3|^2 + m_Z^2 \cos 2\beta \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right),$$
$$m_{22}^2 = M_{\tilde{T}}^2 + \frac{v_1^2|y_2|^2}{2} + |h_5|^2 + |h_5'|^2 + |h_5''|^2 + |h_8|^2 - \frac{2}{3}m_Z^2 \cos 2\beta \sin^2\theta_W,$$
$$m_{33}^2 = M_{\tilde{t}_1}^2 + \frac{v_2^2|y_1'|^2}{2} + |h_5|^2 + \frac{2}{3}m_Z^2 \cos 2\beta \sin^2\theta_W,$$
$$m_{44}^2 = M_{\tilde{Q}}^2 + \frac{v_1^2|y_2|^2}{2} + |h_3|^2 + |h_3'|^2 + |h_3''|^2 + |h_6|^2 - m_Z^2 \cos 2\beta \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right),$$

$$m_{55}^2 = M_{\tilde{2}L}^2 + \frac{v_2^2|y_3'|^2}{2} + |h_3'|^2 + m_Z^2 \cos 2\beta \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right),$$
$$m_{66}^2 = M_{\tilde{t}_2}^2 + \frac{v_2^2|y_3'|^2}{2} + |h_5'|^2 + \frac{2}{3}m_Z^2 \cos 2\beta \sin^2\theta_W,$$
$$m_{77}^2 = M_{\tilde{3}L}^2 + \frac{v_2^2|y_4'|^2}{2} + |h_3''|^2 + m_Z^2 \cos 2\beta \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right),$$
$$m_{88}^2 = M_{\tilde{t}_3}^2 + \frac{v_2^2|y_4'|^2}{2} + |h_5''|^2 + \frac{2}{3}m_Z^2 \cos 2\beta \sin^2\theta_W,$$
$$m_{99}^2 = M_{\tilde{4}L}^2 + \frac{v_2^2|y_5'|^2}{2} + |h_6|^2 + m_Z^2 \cos 2\beta \left(\frac{1}{2} - \frac{2}{3}\sin^2\theta_W\right),$$
$$m_{1010}^2 = M_{\tilde{t}_4}^2 + \frac{v_2^2|y_5'|^2}{2} + |h_8|^2 + \frac{2}{3}m_Z^2 \cos 2\beta \sin^2\theta_W.$$

$$
\begin{aligned}
&m^2_{12} = m^{2*}_{21} = -\frac{v_1 y_2 h^*_3}{\sqrt{2}} + \frac{v_2 h_5 y'^*_1}{\sqrt{2}}, m^2_{13} = m^{2*}_{31} = \frac{y'^*_1}{\sqrt{2}}(v_2 A^*_t - \mu v_1), m^2_{14} = m^{2*}_{41} = 0,\\
&m^2_{15} = m^{2*}_{51} = h'_3 h^*_3, m^2_{16} = m^{2*}_{61} = 0, m^2_{17} = m^{2*}_{71} = h''_3 h^*_3, m^2_{18} = m^{2*}_{81} = 0,\\
&m^2_{23} = m^{2*}_{32} = 0, m^2_{24} = m^{2*}_{42} = \frac{y^*_2}{\sqrt{2}}(v_1 A^*_T - \mu v_2), m^2_{25} = m^{2*}_{52} = -\frac{v_1 h'_3 y^*_2}{\sqrt{2}} + \frac{v_2 y'_3 h'^*_5}{\sqrt{2}},\\
&m^2_{26} = m^{2*}_{62} = 0, m^2_{27} = m^{2*}_{72} = -\frac{v_1 h''_3 y^*_2}{\sqrt{2}} + \frac{v_2 y'_4 h''^*_5}{\sqrt{2}}, m^2_{28} = m^{2*}_{82} = 0,\\
&m^2_{34} = m^{2*}_{43} = \frac{v_1 h_5 y^*_2}{\sqrt{2}} - \frac{v_2 y'_1 h^*_3}{\sqrt{2}}, m^2_{35} = m^{2*}_{53} = 0, m^2_{36} = m^{2*}_{63} = h_5 h'^*_5,\\
&m^2_{37} = m^{2*}_{73} = 0, m^2_{38} = m^{2*}_{83} = h_5 h''^*_5,\\
&m^2_{45} = m^{2*}_{54} = 0, m^2_{46} = m^{2*}_{64} = -\frac{y'^*_3 v_2 h'_3}{\sqrt{2}} + \frac{v_1 y_2 h'^*_5}{\sqrt{2}},\\
&m^2_{47} = m^{2*}_{74} = 0, m^2_{48} = m^{2*}_{84} = \frac{v_1 y_2 h''^*_5}{\sqrt{2}} - \frac{v_2 y'^*_4 h''_3}{\sqrt{2}},\\
&m^2_{56} = m^{2*}_{65} = \frac{y'^*_3}{\sqrt{2}}(v_2 A^*_c - \mu v_1),\\
&m^2_{57} = m^{2*}_{75} = h''_3 h'^*_3, m^2_{58} = m^{2*}_{85} = 0,\\
&m^2_{67} = m^{2*}_{76} = 0, m^2_{68} = m^{2*}_{86} = h'_5 h''^*_5,\\
&m^2_{78} = m^{2*}_{87} = \frac{y'^*_4}{\sqrt{2}}(v_2 A^*_u - \mu v_1),\\
&m^2_{19} = m^{2*}_{91} = h_6 h^*_3, m^2_{110} = m^{2*}_{101} = 0,\\
&m^2_{29} = m^{2*}_{92} = -\frac{y^*_2 v_1 h_6}{\sqrt{2}} + \frac{v_2 y^*_5 h_8}{\sqrt{2}},\\
&m^2_{210} = m^{2*}_{102} = 0, m^2_{39} = m^{2*}_{93} = 0,\\
&m^2_{310} = m^{2*}_{103} = h_5 h^*_8,\\
&m^2_{49} = m^{2*}_{94} = 0, m^2_{410} = m^{2*}_{104} = -\frac{y'^*_5 v_2 h_6}{\sqrt{2}} + \frac{v_1 y_2 h^*_8}{\sqrt{2}},\\
&m^2_{59} = m^{2*}_{95} = h_6 h'^*_3, m^2_{510} = m^{2*}_{105} = 0\\
&m^2_{69} = m^{2*}_{96} = 0, m^2_{610} = m^{2*}_{106} = h'_5 h^*_8\\
&m^2_{79} = m^{2*}_{97} = h_6 h''^*_3, m^2_{710} = m^{2*}_{107} = 0,\\
&m^2_{89} = m^{2*}_{98} = 0, m^2_{810} = m^{2*}_{108} = h''_5 h^*_8,\\
&m^2_{910} = m^{2*}_{109} = \frac{y'^*_5}{\sqrt{2}}(v_2 A^*_{t_4} - \mu v_1)
\end{aligned}
\tag{52}
$$

We can diagonalize the scalar up mass squared matrix by the unitary transformation

$$\tilde{D}^{u\dagger} M^2_{\tilde{u}} \tilde{D}^u = \text{diag}(M^2_{\tilde{u}_1}, M^2_{\tilde{u}_2}, M^2_{\tilde{u}_3}, M^2_{\tilde{u}_4}, M^2_{\tilde{u}_5}, M^2_{\tilde{u}_6}, M^2_{\tilde{u}_7}, M^2_{\tilde{u}_8} M^2_{\tilde{u}_9}, M^2_{\tilde{u}_{10}}) \ . \tag{53}$$

# References


[1] A. Pilaftsis, Phys. Rev. D **58**, 096010 (1998) doi:10.1103/PhysRevD.58.096010 [hep-ph/9803297].

[2] A. Pilaftsis, Phys. Lett. B **435**, 88 (1998) doi:10.1016/S0370-2693(98)00771-0 [hep-ph/9805373].

[3] A. Pilaftsis and C. E. M. Wagner, Nucl. Phys. B **553**, 3 (1999) doi:10.1016/S0550-3213(99)00261-8 [hep-ph/9902371].

[4] D. A. Demir, Phys. Rev. D **60**, 055006 (1999) doi:10.1103/PhysRevD.60.055006 [hep-ph/9901389].

[5] S. Y. Choi, M. Drees and J. S. Lee, Phys. Lett. B **481**, 57 (2000) doi:10.1016/S0370-2693(00)00421-4 [hep-ph/0002287].

[6] M. Carena, J. R. Ellis, A. Pilaftsis and C. E. M. Wagner, Nucl. Phys. B **586**, 92 (2000) doi:10.1016/S0550-3213(00)00358-8 [hep-ph/0003180].

[7] T. Ibrahim and P. Nath, Phys. Rev. D **63**, 035009 (2001) doi:10.1103/PhysRevD.63.035009 [hep-ph/0008237].

[8] M. Carena, J. R. Ellis, A. Pilaftsis and C. E. M. Wagner, Nucl. Phys. B **625**, 345 (2002) doi:10.1016/S0550-3213(02)00014-7 [hep-ph/0111245].

[9] T. Ibrahim and P. Nath, Phys. Rev. D **66**, 015005 (2002) doi:10.1103/PhysRevD.66.015005 [hep-ph/0204092].

[10] J. R. Ellis, J. S. Lee and A. Pilaftsis, Phys. Rev. D **70**, 075010 (2004) doi:10.1103/PhysRevD.70.075010 [hep-ph/0404167].

[11] S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, "The Higgs sector of the complex MSSM at two-loop order: QCD contributions," Phys. Lett. B **652** (2007) 300 [arXiv:0705.0746 [hep-ph]];

[12] J. S. Lee, M. Carena, J. Ellis, A. Pilaftsis and C. E. M. Wagner, Comput. Phys. Commun. **184**, 1220 (2013) doi:10.1016/j.cpc.2012.11.006 [arXiv:1208.2212 [hep-ph]].

[13] M. Carena, J. Ellis, J. S. Lee, A. Pilaftsis and C. E. M. Wagner, JHEP **1602**, 123 (2016) doi:10.1007/JHEP02(2016)123 [arXiv:1512.00437 [hep-ph]].

[14] G. Aad *et al.* [ATLAS Collaboration], Phys. Lett. B **716**, 1 (2012) doi:10.1016/j.physletb.2012.08.020 [arXiv:1207.7214 [hep-ex]].

[15] S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B **716**, 30 (2012) doi:10.1016/j.physletb.2012.08.021 [arXiv:1207.7235 [hep-ex]].

[16] P. Nath, Phys. Rev. Lett. **66**, 2565 (1991). doi:10.1103/PhysRevLett.66.2565

[17] Y. Kizukuri and N. Oshimo, Phys. Rev. D **46**, 3025 (1992). doi:10.1103/PhysRevD.46.3025

[18] T. Ibrahim and P. Nath, Phys. Rev. D **58**, 111301 (1998) doi:10.1103/PhysRevD.60.099902. [hep-ph/9807501].

[19] T. Ibrahim and P. Nath, Phys. Rev. D **57**, 478 (1998) doi:10.1103/PhysRevD.58.019901 [hep-ph/9708456].

[20] T. Falk and K. A. Olive, Phys. Lett. B **439**, 71 (1998) doi:10.1016/S0370-2693(98)01022-3 [hep-ph/9806236].

[21] M. Brhlik, G. J. Good and G. L. Kane, Phys. Rev. D **59**, 115004 (1999) doi:10.1103/PhysRevD.59.115004 [hep-ph/9810457].

[22] T. Ibrahim and P. Nath, Phys. Rev. D **61**, 093004 (2000) doi:10.1103/PhysRevD.61.093004 [hep-ph/9910553].

[23] T. Ibrahim and P. Nath, Rev. Mod. Phys. **80**, 577 (2008) doi:10.1103/RevModPhys.80.577 [arXiv:0705.2008 [hep-ph]].

[24] H. Georgi, Nucl. Phys. B **156**, 126 (1979); F. Wilczek and A. Zee, Phys. Rev. D **25**, 553 (1982); J. Maalampi, J.T. Peltoniemi, and M. Roos, PLB 220, 441(1989); J. Maalampi and M. Roos, Phys. Rept. **186**, 53 (1990); K. S. Babu, I. Gogoladze, P. Nath and R. M. Syed, Phys. Rev. D **72**, 095011 (2005) [hep-ph/0506312]; Phys.

Rev. D **74**, 075004 (2006), [arXiv:hep-ph/0607244]; Phys. Rev. D **85**, 075002 (2012) [arXiv:1112.5387 [hep-ph]]; P. Nath and R. M. Syed, Phys. Rev. D **81**, 037701 (2010).

[25] K. S. Babu, I. Gogoladze, M. U. Rehman and Q. Shafi, Phys. Rev. D **78**, 055017 (2008) doi:10.1103/PhysRevD.78.055017 [arXiv:0807.3055 [hep-ph]].

[26] C. Liu, Phys. Rev. D **80**, 035004 (2009) doi:10.1103/PhysRevD.80.035004 [arXiv:0907.3011 [hep-ph]].

[27] S. P. Martin, Phys. Rev. D **81**, 035004 (2010) doi:10.1103/PhysRevD.81.035004 [arXiv:0910.2732 [hep-ph]].

[28] T. Ibrahim and P. Nath, Phys. Rev. D **78**, 075013 (2008) doi:10.1103/PhysRevD.78.075013 [arXiv:0806.3880 [hep-ph]].

[29] T. Ibrahim and P. Nath, Phys. Rev. D **81**, no. 3, 033007 (2010) doi:10.1103/PhysRevD.81.033007 [arXiv:1001.0231 [hep-ph]].

[30] T. Ibrahim and P. Nath, Phys. Rev. D **82**, 055001 (2010) doi:10.1103/PhysRevD.82.055001 [arXiv:1007.0432 [hep-ph]].

[31] T. Ibrahim and P. Nath, Phys. Rev. D **84**, 015003 (2011) doi:10.1103/PhysRevD.84.015003 [arXiv:1104.3851 [hep-ph]].

[32] T. Ibrahim and P. Nath, Phys. Rev. D **87**, no. 1, 015030 (2013) doi:10.1103/PhysRevD.87.015030 [arXiv:1211.0622 [hep-ph]].

[33] A. Aboubrahim, T. Ibrahim and P. Nath, Phys. Rev. D **88**, 013019 (2013) doi:10.1103/PhysRevD.88.013019 [arXiv:1306.2275 [hep-ph]]; Phys. Rev. D **91**, no. 9, 095017 (2015) doi:10.1103/PhysRevD.91.095017 [arXiv:1503.06850 [hep-ph]]; Phys. Rev. D **89**, no. 9, 093016 (2014) doi:10.1103/PhysRevD.89.093016 [arXiv:1403.6448 [hep-ph]]; Phys. Rev. D **89**, no. 5, 055009 (2014) doi:10.1103/PhysRevD.89.055009 [arXiv:1312.2505 [hep-ph]].

[34] A. Aboubrahim, T. Ibrahim, P. Nath and A. Zorik, Phys. Rev. D **92**, no. 3, 035013 (2015) doi:10.1103/PhysRevD.92.035013 [arXiv:1507.02668 [hep-ph]];

[35] T. Ibrahim, A. Itani and P. Nath, Phys. Rev. D **92**, no. 1, 015003 (2015) doi:10.1103/PhysRevD.92.015003 [arXiv:1503.01078 [hep-ph]].

[36] T. Ibrahim, A. Itani and P. Nath, Phys. Rev. D **90**, no. 5, 055006 (2014) doi:10.1103/PhysRevD.90.055006 [arXiv:1406.0083 [hep-ph]].

[37] C. A. Baker *et al.*, Phys. Rev. Lett. **97**, 131801 (2006) doi:10.1103/PhysRevLett.97.131801 [hep-ex/0602020].