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Parton Transverse Momentum and Orbital Angular Momentum Distributions

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The quark orbital angular momentum component of proton spin, L_q, can be defined in QCD as the integral of a Wigner phase space distribution weighting the cross product of the quark's transverse position and momentum. It can also be independently defined from the operator product expansion for the off-forward Compton amplitude in terms of a twist-three generalized parton distribution. We provide an explicit link between the two definitions, connecting them through their dependence on partonic intrinsic transverse momentum. Connecting the definitions provides the key for correlating direct experimental determinations of L_q, and evaluations through Lattice QCD calculations. The direct observation of quark orbital angular momentum does not require transverse spin polarization, but can occur using longitudinally polarized targets.

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Orbital Angular Momentum (OAM), $L_{q,g}$, is generated inside the proton as a consequence of the quark and gluon transverse motion about the system's center of momentum. It has been identified as a critical component in the resolution of the proton spin puzzle [1], which has constituted a central focus of hadron physics since the seminal EMC experiments demonstrated that quark spin alone cannot account for the proton spin [2, 3]. Understanding OAM in the proton was the original motivation for introducing Generalized Parton Distributions (GPDs) in Refs. [4, 5], in that they provided a novel way of accessing angular momentum through a class of exclusive reactions including Deeply Virtual Compton Scattering (DVCS), Deeply Virtual Meson Production (DVMP), and related experiments. Through Ji's sum rule [5], one can, in fact, relate the components of the Energy Momentum Tensor (EMT) known as the gravitomagnetic form factors, $A_{q,g}$ and $B_{q,g}$, to the quark and gluon total angular momenta, $J_{q,g}$. The pivotal observation made in [5] is that $A_{q,g}$ and $B_{q,g}$ correspond to $n = 2$ Mellin moments of GPDs which, in turn, define the matrix elements for DVCS. These important developments rendered total angular momentum a measurable quantity. Although the decomposition of J_g into its spin and orbital components has proven difficult to define gauge invariantly, the orbital angular momentum of quarks is well defined through $J_q = L_q + S_q$. Even so, the direct observability of L_q remains a challenging question: the framework defined so far does not tell us how to access the dynamics of quark orbital motion since L_q is only obtained through the difference of the total angular momentum and spin components.

L_q has more recently been associated with precise operators and structure functions, given within two alternative approaches. On one side, a dynamical picture of quark orbital motion was given in terms of a Generalized Transverse Momentum Distribution (GTMD), *i.e.*, an unintegrated over transverse momentum GPD, in Refs. [6, 7]. The GTMD-based definition of quark OAM is

$$L_q^{\mathcal{U}}(x) = \int d^2k_T \int d^2b_T \, (b_T \times k_T)_3 \mathcal{W}^{\mathcal{U}}(x, k_T, b_T) \quad (1)$$

where $\mathcal{W}^{\mathcal{U}}$ is a Wigner distribution derived from the quark-quark off-forward correlator in a longitudinally polarized nucleon moving in the 3-direction[1]

$$\Phi^{\Gamma}_{\Lambda'\Lambda}(p', p; z', z) = \langle p', \Lambda' \mid \overline{\psi}(z')\Gamma\,\mathcal{U}\,\psi(z) \mid p, \Lambda\rangle \quad (2)$$

where Γ denotes an arbitrary γ-matrix structure. $\mathcal{W}^{\mathcal{U}}$ is obtained by Fourier-transforming (2) for $\Gamma = \gamma^+$ from $z - z'$ to the quark intrinsic momentum k (the average of the initial and final quark momenta in the symmetric system of variables [8]), projecting onto $(z - z')^+ = 0$, as well as from the (transverse) momentum transfer Δ_T (the difference between the initial and final quark momenta) to the transverse position b_T. If one foregoes the transformation to b_T, one can relate $L_q^{\mathcal{U}}$ to the k_T^2 moment of

[1] Throughout this paper we consider zero skewness, i.e., the plus component of the momentum transfer vanishes, $\Delta^+ = 0$. Moreover, we omit writing explicitly the Q^2 dependence which is, however, present in all expressions.

the GTMD F_{14} [7, 9, 10] for $\Delta_T \to 0$,

$$L_q^{\mathcal{U}}(x) = -F_{14}^{(1)} \equiv - \int d^2k_T \, \frac{k_T^2}{M^2} F_{14}(x, k_T^2, k_T \cdot \Delta_T, \Delta_T^2)\,. \tag{3}$$

F_{14} is a GTMD describing an unpolarized quark inside a longitudinally polarized proton [10]. Finally, $\mathcal{U}$ in Eq. (2) denotes the gauge link, *i.e.*, the Wilson path-ordered exponential connecting the coordinates z and z'. We will restrict the discussion in the present Letter to the case of a straight gauge link, corresponding to what is known as Ji's decomposition of angular momentum [11–13], and defer the analogous treatment of other relevant gauge link structures to an expanded exposition.

In another approach [14–16], it was observed that OAM is associated with a twist-three GPD, G_2. Similar to the treatment of the forward case [17–19], one can write the Mellin moments of G_2, which appears in the parametrization of the off-forward amplitude, in terms of both twist-two operators and (genuine) twist-three operators. For the second moment, the genuine twist-three contribution vanishes and one obtains, for $\Delta_T \to 0$,

$$\begin{aligned}\int_0^1 dx\, x G_2 &= -\frac{1}{2}\int_0^1 dx\, x(H+E) + \frac{1}{2}\int_0^1 dx\, \tilde{H} \\ &= -J_q + S_q = -L_q^{\rm Ji}\end{aligned} \tag{4}$$

where only a straight gauge link structure applies in such a relation involving only GPDs. This result can be viewed as an extension of the Efremov-Leader-Teryaev (ELT) sum rule [20], written for the polarized structure functions, to off-forward kinematics.

Notwithstanding these developments, two main problems remain to be solved: 1) relating the two distinct structures, one (F_{14}) appearing at twist two, and one (G_2) at twist three, both describing OAM within the same gauge invariant framework; 2) singling out an experimental measurement to access directly OAM, possibly through the newly defined structures. In this Letter, we provide a direct link between the k_T^2 moment of the GTMD and the twist-three GPD describing OAM, elucidating the underlying dependence on partonic intrinsic transverse momentum and off-shellness. The GTMD-based definition is calculable in Lattice QCD using the techniques of Ref. [21]. On the other side, the twist-three GPD-based definition can be measured directly in DVCS-type experiments, through the azimuthal angle modulations which are sensitive to twist-three GPDs in DVCS off a longitudinally polarized target [8]; this is at variance with the notion that transverse polarization, or proton spin-flip processes are necessary to obtain information on quark OAM.

Our central result is the following integral relations (6),(7) connecting F_{14}, G_2, $\widetilde{E}_{2T}$, H, E and $\widetilde{H}$ in the limit $\Delta_T \to 0$, where $\widetilde{E}_{2T}$ is a twist-three GPD in the classification of [10] related to the GPD G_2 in the classification of [15] by

$$\int dx\, x\widetilde{E}_{2T} = -\int dx\, x(H+E+G_2) \tag{5}$$

(note that, in [8], $\widetilde{E}_{2T}$ and G_2 were identified as encoding similar twist-three structures, without, however, providing the precise relationship given in (5)).

$$\text{(LIR)} \qquad F_{14}^{(1)} = -\int_x^1 dy\, \left(\widetilde{E}_{2T} + H + E\right) \quad \Rightarrow \quad -L_q^{\rm Ji} = \int_0^1 dx F_{14}^{(1)} = \int_0^1 dx\, x\, G_2 \tag{6}$$

$$\text{(EoM)} \quad x(\widetilde{E}_{2T} + H + E) = x\left[(H+E) - \int_x^1 \frac{dy}{y}(H+E) - \frac{1}{x}\widetilde{H} + \int_x^1 \frac{dy}{y^2}\widetilde{H}\right] + G^{(3)} = x(\widetilde{E}_{2T} + H + E)^{WW} + G^{(3)} \tag{7}$$

Eq. (6) is a Lorentz Invariance Relation (LIR), obtained from the analysis of the most general Lorentz decomposition of the quark-quark correlation function. Eq. (7) is obtained applying the QCD EoM for the quark fields to the unintegrated correlation function. Through Eq. (7), in analogy to the derivation for the polarized structure functions g_1 and g_2 [22], one separates the Wandzura-Wilczek (WW) part from the quark-gluon-quark correlation, $G^{(3)}$, which is given by

$$G^{(3)} = -\widetilde{\mathcal{M}} + x\int_x^1 \frac{dy}{y^2}\widetilde{\mathcal{M}} \tag{8}$$

with $\widetilde{\mathcal{M}}$ given in Eq. (19) below. The contribution of $G^{(3)}$ to angular momentum vanishes in the case of a straight gauge link, since the complete momentum integral of $\widetilde{\mathcal{M}}^i_{\Lambda'\Lambda}$, cf. Eq. (15), vanishes, as can be seen by explicit evaluation of the gauge link structure. Therefore, Eq. (7) reduces to Eq. (4) upon integration over x. Because of the validity of both relations (6),(7), we find a remarkable equivalence between the OAM densities defined through a Wigner distribution, Eq. (3), and through a twist-three GPD, $\widetilde{E}_{2T}$, as well as their connection to the OAM density defined through Ji's sum rule, cf. the right-hand side of Eq. (4). This constitutes our central result.

We now sketch the derivation of Eqs. (6),(7), highlighting the role of quark k_T and, thus, the off-shellness of partons in generating proton spin. The completely unintegrated off-forward quark-quark correlation function $W^\Gamma_{\Lambda'\Lambda}$, *i.e.*, (half) the four-dimensional Fourier transform of (2) from $z-z'$ to k [10, 17–19, 23], can be parametrized [10] in terms of invariant functions A_i. On the other hand, its k^- integral $\widetilde{W}^\Gamma_{\Lambda',\Lambda} = \int dk^- W^\Gamma_{\Lambda',\Lambda}$ is parametrized by the GTMDs. This implies the following twist-two relations already given in [10], taking into account that functions A_i associated with a staple link direction N included in [10] are discarded in the straight link case, and also specializing to zero skewness,

$$\frac{k_T\cdot\Delta_T}{\Delta_T^2}F_{12}+F_{13} = 2P^+\int dk^-\left(\frac{k_T\cdot\Delta_T}{\Delta_T^2}A_5+A_6 - \frac{xP^2-k\cdot P}{M^2}(A_8+xA_9)\right) \quad (9)$$

$$F_{14} = 2P^+\int dk^-\,(A_8+xA_9)\ . \quad (10)$$

We supplement these by the twist-three relation

$$\frac{k_T\cdot\Delta_T}{\Delta_T^2}F_{27}+F_{28} = 2P^+\int dk^-\left(\frac{k_T\cdot\Delta_T}{\Delta_T^2}A_5+A_6 + \frac{1}{M^2}\left(\frac{(k_T\cdot\Delta_T)^2}{\Delta_T^2}-k_T^2\right)A_9\right)\ . \quad (11)$$

Combining integrals over transverse k_T of these relations, one arrives at the LIR

$$\frac{d}{dx}\int d^2k_T\,\frac{k_T^2}{M^2}\,F_{14} = \widetilde{E}_{2T}+H+E \quad (12)$$

in the limit $\Delta_T\to 0$, having identified the GPD combinations $H+E$ and $\widetilde{E}_{2T}$ resulting after k_T integration of the GTMD combinations appearing in (9) and (11) [10]. Finally, integrating over x, one arrives at Eq. (6).

The EoM relation in Eq. (7) was obtained by considering (2) for $\Gamma = i\sigma^{i+}\gamma_5$, $(i=1,2)$, and inserting the equation of motion for the quark operator (the symmetrized form serving to cancel the mass terms),

$$0 = \int\frac{dz^-d^2z_T}{(2\pi)^3}e^{ixP^+z^--ik_T\cdot z_T}\times \langle p',\Lambda'\mid\overline{\psi}(-z/2)(\Gamma\,\mathcal{U}\,i\overrightarrow{\not{D}}+i\overleftarrow{\not{D}}\,\Gamma\,\mathcal{U})\psi(z/2)\mid p,\Lambda\rangle_{z^+=0}, \quad (13)$$

where the differentiations act on the arguments of the quark fields before evaluation at the specified positions. This yields the following relation between the k^- integrated correlators $\widetilde{W}^\Gamma_{\Lambda',\Lambda}$ already referenced in introducing Eqs. (9)-(11) above,

$$-xP^+i\epsilon_T^{ij}\widetilde{W}^{\gamma^j}_{\Lambda'\Lambda} = \frac{\Delta_T^i}{2}\widetilde{W}^{\gamma^+\gamma_5}_{\Lambda'\Lambda} - k_T^j i\epsilon_T^{ij}\widetilde{W}^{\gamma^+}_{\Lambda'\Lambda}+\mathcal{M}^i_{\Lambda'\Lambda}\ , \quad (14)$$

with the genuine twist-three quark-gluon-quark correlator (still denoting $\Gamma = i\sigma^{i+}\gamma_5$),

$$\mathcal{M}^i_{\Lambda'\Lambda} = \frac{1}{4}\int\frac{dz^-d^2z_T}{(2\pi)^3}e^{ixP^+z^--ik_T\cdot z_T}\langle p',\Lambda'\mid\overline{\psi}(-z/2)\left[(\overleftarrow{\not{\partial}}-ig\not{A})\mathcal{U}\Gamma\Big|_{-z/2}+\Gamma\mathcal{U}(\overrightarrow{\not{\partial}}+ig\not{A})\Big|_{z/2}\right]\psi(z/2)\mid p,\Lambda\rangle_{z^+=0} \quad (15)$$

The form (15) is valid for any form of gauge link $\mathcal{U}$; to obtain more explicit expressions, specific choices for $\mathcal{U}$ must be made. In particular, varying the endpoints of $\mathcal{U}$ in general implies varying the entire path along which $\mathcal{U}$ is calculated; the derivatives evaluate that variation. For the straight gauge link case considered here, if one parametrizes

$$\mathcal{U}(a,b) = \mathcal{P}\exp\left(-ig\int_a^b dt(y'-y)^\mu A_\mu(y+t(y'-y))\right) \quad (16)$$

such that, in (15), $\mathcal{U}\equiv\mathcal{U}(0,1)$ with the endpoints to be identified as $y = -z/2$, $y' = z/2$, then one has

$$(\partial/\partial y^\nu - igA_\nu(y))\,\mathcal{U} = ig(y'-y)^\mu \times \int_0^1 ds(1-s)\mathcal{U}(0,s)F_{\mu\nu}(y+s(y'-y))\mathcal{U}(s,1) \tag{17}$$

and an analogous expression for the adjoint term in (15). This vanishes as $(y'-y) \to 0$.

By taking the proton non-flip spin components, $(\Lambda',\Lambda) = (+,+)-(-,-)$, that identify OAM [8] in (14), using the GTMD parametrizations [10] of the $\widetilde{W}^{\Gamma}_{\Lambda',\Lambda}$, and integrating over k_T, one has, in the $\Delta_T \to 0$ limit,

$$-x\widetilde{E}_{2T} = \widetilde{H} - \int d^2k_T \, \frac{k_T^2}{M^2} F_{14} + \widetilde{\mathcal{M}} \tag{18}$$

having again identified the GPD $\widetilde{E}_{2T}$ as in the LIR derivation above, as well as the GPD $\widetilde{H} = \int d^2k_T \, G_{14}$. The genuine twist-three term $\widetilde{\mathcal{M}}$ is given by

$$\widetilde{\mathcal{M}} = 2M \frac{\Delta_T^i}{\Delta_T^2} \int d^2k_T \left[\mathcal{M}^i_{++} - \mathcal{M}^i_{--}\right] . \tag{19}$$

Note that, since in the identity (18) the functions $\widetilde{E}_{2T}$, $\widetilde{H}$ and F_{14} are regular as $\Delta_T \to 0$, also the genuine twist-three term is. In other words, the k_T-integral in (19) must vanish as $\Delta_T \to 0$. The final expression defining the EoM relation in Eq. (7) is obtained by taking the derivative in x of (18), inserting (12), dividing by x and integrating as indicated by (7).

It should be noted that the relations discussed here are perturbatively divergent and require consistent regularization/renormalization at each step. An interesting aspect, e.g., of the LIR (6) is that it connects a GTMD which does not have a GPD limit, F_{14}, to GPDs. In particular, taking k_T-moments of GTMDs in general calls for additional regularization of the integral at large k_T. To treat both sides on an equal footing implies utilizing such a transverse momentum-dependent regularization and renormalization scheme, and thus interpreting the GPDs in terms of the underlying GTMDs of which they are the GPD limit. On the other hand, it seems tempting to speculate that relations of the type (6) may ultimately be useful to connect the renormalization of quantities which are intrinsically defined as transverse momentum-dependent, such as F_{14}, to the more standard schemes employed for GPDs.

As an application of the relations between the different ways to access angular momentum, we compile and correlate in Fig. 1 determinations of J_q, L_q and S_q from several sources, including experiment, lattice QCD, and models. The value of $J_{u-d} = J_u - J_d$ is plotted versus $L_{u-d} = L_u - L_d$. The horizontal bands represent measurements/calculations of J_{u-d} using DVCS data [25]/GPD evaluations; the slanted band is given by

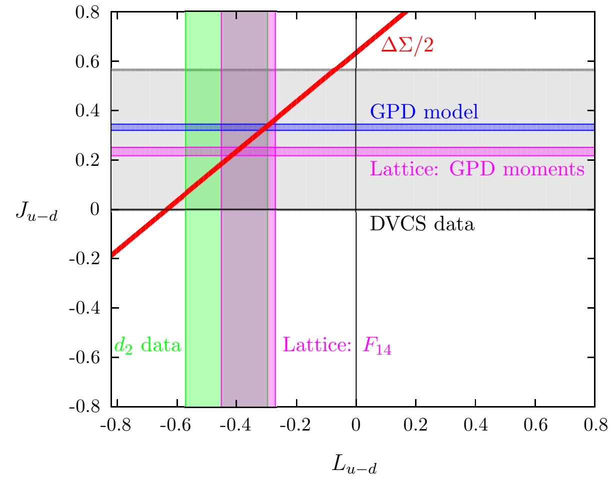

FIG. 1: J_{u-d} plotted vs. L_{u-d}. The (red) slanted band represents $J_{u-d} = L_{u-d} + (1/2)\Delta\Sigma_{u-d}$ using $\Delta\Sigma$ from Ref. [24]. The horizontal bands represent J_{u-d} from experiment (gray) [25], from the GPD model extraction (blue) [26, 27], and from lattice QCD (magenta) [28]. The vertical bands are the preliminary lattice QCD evaluation of L_{u-d} using the definition in Eq. (1) (magenta) [29], and the GPD model normalized according to Eq. (20) (green).

the relation $J_q = L_q + \Delta\Sigma_q/2$, where the experimental value for $\Delta\Sigma_{u-d}$ was taken from Ref. [24]. The vertical bands correspond to preliminary data for L_q obtained in a lattice QCD calculation at an artificially high pion mass of $m_\pi = 518\,\text{MeV}$ using an approach related to the GTMD F_{14} from Eq. (3) [29], and to a phenomenological extraction using inclusive scattering data. It should be noted that a comprehensive analysis of the systematic uncertainties and corrections affecting the lattice result given here is still pending; among them is an expected enhancement by roughly 30% as one goes to the physical pion mass. The extraction from data uses a calculation of $\widetilde{E}_{2T}$ in the reggeized diquark model [26, 27], the detailed presentation of which is deferred to a separate publication. Briefly, the extension of this model to twist three GPDs was carried out by extending to off-forward kinematics the approach first presented in Ref. [30]. This model produces a parametrization of the GPDs H_q and E_q, $q = u, d$, which is fitted to both the nucleon unpolarized PDFs for the u and d quarks, and to the flavor-separated nucleon electromagnetic form factors [31]. The latter determine the normalization constants of the flavor-dependent GPDs so that they reproduce the nucleon charges and measured magnetic moments. An independent experimental constraint is necessary to determine the normalization of the genuine twist-three part of $\widetilde{E}_{2T}$. This is obtained by using its third Mellin mo-

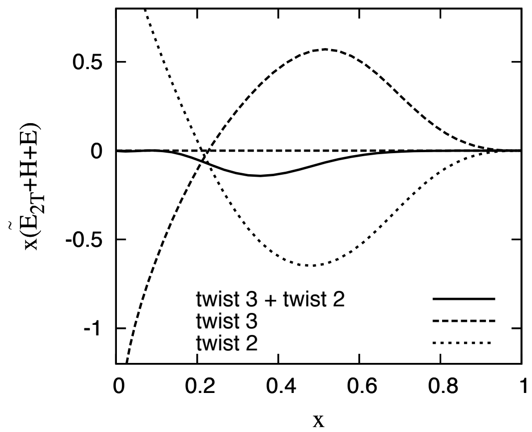

FIG. 2: Contributions in Eq. (7) calculated in the reggeized diquark model [27]. The dashed line is the genuine twist-three contribution, the dotted line is the twist-two term, $x(\widetilde{E}_{2T} + H + E)^{WW}$, and the full line is their sum. All quantities are evaluated for $\Delta_T = 0$ at the initial scale of the model.

ment, which can be related to

$$d_2 = 3 \int_0^1 dx x^2 g_2^{tw3}(x) \;, \tag{20}$$

where g_2, the transverse spin-dependent structure function, is obtained in double-spin asymmetry measurements of longitudinally polarized electrons scattering from longitudinally and transversely polarized nucleons. We used, in particular, the SLAC data for the u and d quark values of d_2 at a common Q^2 value of 5 GeV^2 [32]. With the normalization of the twist-three part of $\widetilde{E}_{2T}$ obtained from Eq. (20) we then evaluated L_q. The result is the vertical green band. This is consistent, although with a large error, with the values extracted from the lattice. No experimental determinations of L_q to corroborate our analysis can be placed on the graph at this point, although future extractions will be possible from analyses of the $\sin 2\phi$ modulation of DVCS data [8].

In Fig. 2 we exhibit in more detail the contributions in Eq. (7) as a function of x, i.e., the behavior of $x(\widetilde{E}_{2T}+H+E)^{WW}$, the genuine twist-three term, and their sum at the initial scale of the model. As for g_2, the genuine twist-three part is predicted to be large. Due to the Regge behavior of the functions, we expect measurements at low x, *i.e.*, in a regime which would be best accessible at an Electron Ion Collider to be important.

Finally, future developments will include the extension of our study to the Jaffe-Manohar [1] decomposition of angular momentum, which, as shown in Ref. [13], involves a final state interaction (encoded in a staple-shaped gauge link), and is related to the Ji decomposition by

$$L_q^{\rm JM} = L_q^{\rm Ji} + \langle \tau_3 \rangle \tag{21}$$

where $\langle \tau_3 \rangle$ is an off-forward extension of a Qiu-Sterman term [33]. $\langle \tau_3 \rangle$ has been interpreted physically as a change in OAM due to a torque - a final state interaction - exerted on the outgoing quark by the color-magnetic field produced by the spectators [13].

In conclusion, understanding quark OAM entails cross-correlating phenomenology, theory and lattice QCD efforts to bring them to bear simultaneously on the subject. We provided relations that are key for realizing such a coordinated approach, utilizing directly non-local, k_T-unintegrated quark-quark correlation functions. This approach opens up an avenue to explore the role of partonic transverse momentum and off-shellness for OAM, while providing a formalism which connects to lattice QCD calculations on one side and to experiment on the other. A first, exploratory direct calculation of quark OAM in lattice QCD using an approach related to the GTMD F_{14} was incorporated into the analysis, and confronted with independent determinations, e.g., via Ji's sum rule, and through d_2 measurements. Our relations bring to the fore the intricacies of connecting a twist-two GTMD moment and a twist-three GPD, before the backdrop of a field theoretic rendition of OAM.

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