Progressive gauge U(1) family symmetry for quarks and leptons

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Abstract

The pattern of quark and lepton mass matrices is unexplained in the standard model of particle interactions. I propose the novel idea of a progressive gauge $U(1)$ symmetry where it is a reflection of the regressive electroweak symmetry breaking pattern, caused by an extended Higgs scalar sector. Phenomenological implications of this new hypothesis are discussed.

The standard model (SM) of particle interactions is based on the gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$, with its associated vector gauge bosons, i.e. eight gluons, the weak $W^\pm$ and Z^0 bosons, and the photon. It consists of three families of quarks and leptons in left-handed doublets and right-handed singlets. It also has the all-important Higgs scalar doublet which provides mass directly to all particles with the possible exception of only the neutrinos. The resulting quark and lepton masses and their mixing patterns are unexplained in the SM. They are merely tunable parameters. To gain an understanding of these patterns, I propose that there is a family gauge $U(1)$ symmetry, which requires an extended scalar sector, which breaks this $U(1)$ as well as $SU(2)_L \times U(1)_Y$ in a regressive manner [1] so that the observed patterns of quark and lepton masses and mixing are qualitatively explained.

The fermion content of the SM is extended to include three singlet right-handed neutrinos ν_R. The new family gauge $U(1)_F$ symmetry is assumed coupled only to right-handed fermions, as shown in Table 1. The $[SU(3)_C]^2 U(1)_F$ anomaly is cancelled between u_R and d_R

Table 1: Fermion assignments under $U(1)_F$.

Particle	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$U(1)_F$
$Q_{Li} = (u, d)_{Li}$	3	2	1/6	$(0, 0, 0)$
u_{Ri}	3	1	2/3	(n_1, n_2, n_3)
d_{Ri}	3	1	-1/3	$(-n_1, -n_2, -n_3)$
$L_{Li} = (\nu, l)_{Li}$	1	2	-1/2	$(0, 0, 0)$
l_{Ri}	1	1	-1	$(-n_1, -n_2, -n_3)$
ν_{Ri}	1	1	0	(n_1, n_2, n_3)

for each family. The $[SU(2)_L]^2 U(1)_F$ anomaly is zero because left-handed fermions do not couple to $U(1)_F$. The $[U(1)_Y]^2 U(1)_F$ and $U(1)_Y [U(1)_F]^2$ anomalies are cancelled between u_R, d_R, and l_R for each family. The $[U(1)_F]^3$ anomaly is canceled between u_R and d_R, as well as l_R and ν_R for each family. This means that $U(1)_F$ is anomaly-free within each family (which is basically just $B - L - 2Y$), but it may have an overall different coupling for each,

as shown in Table 1.

To obtain quark and lepton masses, there should then be three Higgs doublets:

$$\Phi_i = (\phi^+, \phi^0)_i \sim (1, 2, 1/2; n_i), \tag{1}$$

coupling in turn to the three families. Let the scalar singlet $\sigma \sim [1, 1, 0; (n_2 - n_3)/2]$ be added, then the scalar potential has the relevant terms

$$V = m_2^2 \Phi_2^\dagger \Phi_2 + \kappa_2 \sigma^2 \Phi_2^\dagger \Phi_3 + ... \tag{2}$$

If m_2^2 is positive and large, then the vacuum expectation value of ϕ_2^0 is given by

$$v_2 \simeq \frac{-\kappa_2 u^2 v_3}{m_2^2}, \tag{3}$$

where $v_{2,3} = \langle \phi_{2,3}^0 \rangle$, $u = \langle \sigma \rangle$, and v_2 may be small because $\kappa_2 \to 0$ enlarges the symmetry of V. This mechanism based on Ref. [1] is easily generalized [2]. If $n_2 - n_3 = n_1 - n_2$ as well, then the term $\kappa_1 \sigma^2 \Phi_1^\dagger \Phi_2$ also exists, so that

$$v_1 \simeq \frac{-\kappa_1 u^2 v_2}{m_1^2}, \tag{4}$$

which yields $v_3 >> v_2 >> v_1$ and explains the hierarchy of quark and lepton masses. Thus the regressive pattern of electroweak symmetry breaking from $\Phi_{3,2,1}$ results in the progressive pattern of masses for the first, second, and third families.

As an explicit example, let $n_{1,2,3} = (2, 1, 0)$ with $\sigma \sim (1, 1, 0; 1/2)$, then the most general scalar potential consisting of $\Phi_{1,2,3}$ and σ is given by

$$
\begin{aligned}
V &= m_1^2 \Phi_1^\dagger \Phi_1 + m_2^2 \Phi_2^\dagger \Phi_2 + m_3^2 \Phi_3^\dagger \Phi_3 + m_4^2 \sigma^* \sigma \\
&+ \frac{1}{2}\lambda_1(\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2}\lambda_2(\Phi_2^\dagger \Phi_2)^2 + \frac{1}{2}\lambda_3(\Phi_3^\dagger \Phi_3)^2 + \frac{1}{2}\lambda_4(\sigma^* \sigma)^2 \\
&+ \lambda_{12}(\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_{13}(\Phi_1^\dagger \Phi_1)(\Phi_3^\dagger \Phi_3) + \lambda_{23}(\Phi_2^\dagger \Phi_2)(\Phi_3^\dagger \Phi_3) \\
&+ \lambda'_{12}(\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \lambda'_{13}(\Phi_1^\dagger \Phi_3)(\Phi_3^\dagger \Phi_1) + \lambda'_{23}(\Phi_2^\dagger \Phi_3)(\Phi_3^\dagger \Phi_2) \\
&+ \lambda_{14}(\Phi_1^\dagger \Phi_1)(\sigma^* \sigma) + \lambda_{24}(\Phi_2^\dagger \Phi_2)(\sigma^* \sigma) + \lambda_{34}(\Phi_3^\dagger \Phi_3)(\sigma^* \sigma) \\
&+ [\lambda_{123}(\Phi_1^\dagger \Phi_2)(\Phi_3^\dagger \Phi_2) + \kappa_1 \sigma^2 \Phi_1^\dagger \Phi_2 + \kappa_2 \sigma^2 \Phi_2^\dagger \Phi_3 + H.c.]
\end{aligned} \tag{5}
$$

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For large positive $m_{1,2}^2$ and negative $m_{3,4}^2$, the minimum of V satisfies the conditions:

$$m_4^2 + \lambda_4|u|^2 + \lambda_{34}|v_3|^2 \simeq 0, \tag{6}$$

$$m_3^2 + \lambda_3|v_3|^2 + \lambda_{34}|u|^2 \simeq 0, \tag{7}$$

$$v_2 \simeq \frac{-\kappa_2 u^2 v_3}{m_2^2 + (\lambda_{23} + \lambda'_{23})|v_3|^2 + \lambda_{24}|u|^2}, \tag{8}$$

$$v_1 \simeq \frac{-\kappa_1 u^2 v_2}{m_1^2 + (\lambda_{13} + \lambda'_{13})|v_3|^2 + \lambda_{14}|u|^2}. \tag{9}$$

This regressive pattern of electroweak symmetry breaking allows a qualitative understanding of why $m_{u,d} << m_{c,s} << m_{t,b}$. The quark mass matrix linking $(\bar{d}, \bar{s}, \bar{b})_L$ to $(d, s, b)_R$ is of the form

$$\mathcal{M}_d = \begin{pmatrix} U_{1d} & U_{1s} & U_{1b} \\ U_{2d} & U_{2s} & U_{2b} \\ U_{3d} & U_{3s} & U_{3b} \end{pmatrix} \begin{pmatrix} m'_d & 0 & 0 \\ 0 & m'_s & 0 \\ 0 & 0 & m'_b \end{pmatrix}, \tag{10}$$

where $m'_{d,s,b} \propto v_{1,2,3}$, and $\sum_i |U_{id}|^2 = \sum_i |U_{is}|^2 = \sum_i |U_{ib}|^2 = 1$. However, $\sum_i U_{id}^* U_{is}$, etc. are not necessarily zero, so $m'_{d,s,b}$ are not necessarily the mass eigenvalues. If $U_{1d} = U_{2s} = U_{3b} = 1$, $\mathcal{M}_d$ is diagonal. Similarly if $U_{1u} = U_{2c} = U_{3t} = 1$, $\mathcal{M}_u$ is also diagonal. This corresponds to the alignment limit where there is a separate global U(1) symmetry for each family. Hence it is technically natural to expect the mixing between families to be small, i.e. $|U_{2d,3d}| << |U_{1d}|$, etc. Note that this argument does not work in the SM, because there is no mechanism there to enforce the hierarchy of quark masses, so that an off-diagonal term in $\mathcal{M}_d$ for example may be bigger than m_d itself. Here the mass scale for each column of $\mathcal{M}_d$ is dictated by a specific hierarchical v_i.

The argument of technical naturalness for an inequality is only as good as the symmetry which maintains it. Here it is separate global $U(1)$ for each family. Hence the fact of a not-so-small Cabibbo angle is a reflection that this symmetry is not very strong, but it does not invalidate the argument. On the other hand, this does not work at all in the lepton sector, because of the additional arbitrary contribution of the Majorana mass matrix of the heavy right-handed neutrinos, so the angles are not expected to be small in general.

Now $\mathcal{M}_d$ is diagonalized in general by

$$\mathcal{M}_d = U_{dL} \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} U_{dR}^\dagger, \tag{11}$$

where both U_{dL} and U_{dR} are unitary matrices and assumed here to be close to the identity matrix. To minimize the appearance of flavor-changing neutral currents (FCNC) in the $U(1)_F$ sector, it will be assumed [3] that $U_{dR} = U_{uR} = 1$. As usual the charged-current mixing matrix in the electroweak sector is

$$V_{CKM} = U_{uL}^\dagger U_{dL}, \tag{12}$$

and the neutral-current interaction through the Z boson is diagonal and universal as in the SM. Thus the gauge sector here is absent of tree-level flavor-changing neutral currents. The mass-squared matrix spanning the (Z, Z_F) gauge bosons is given by

$$\mathcal{M}_{Z,Z_F}^2 = \begin{pmatrix} (1/2)g_Z^2(v_1^2 + v_2^2 + v_3^2) & g_Z g_F(2v_1^2 + v_2^2) \\ g_Z g_F(2v_1^2 + v_2^2) & (1/2)g_F^2 u^2 \end{pmatrix}. \tag{13}$$

The mixing between Z and Z_F is of order $(2g_Z/g_F)(2v_1^2 + v_2^2)/u^2$ which is very small, say at most 10^{-5} in this model, and may be safely neglected. Using Table 1, the branching fraction of Z_F to $e^- e^+ + \mu^- \mu^+$ is about $1/8$. The $c_{u,d}$ coefficients used in the experimental search [4, 5] of Z_F are then

$$c_u = c_d = 4g_F^2(1/8). \tag{14}$$

For $g_F = 0.1$, a lower bound of about 3.0 TeV on m_{Z_F} is obtained from the Large Hadron Collider (LHC) based on data from the 7 and 8 TeV runs. In that case, the lower limit on u is about 42.4 TeV. If Z_F is discovered, then it may be distinguished from other Z' models by the ratio

$$\frac{\Gamma(Z_F \to e^- e^+)}{\Gamma(Z_F \to \mu^- \mu^+)} = \frac{n_1^2}{n_2^2} = 4. \tag{15}$$

The particle spectrum of this model consists of the heavy vector gauge boson Z_F as well as the heavy scalar $\Phi_{1,2}$ doublets and the heavy scalar σ singlet. The rest are just the SM

particles, with the important difference that the SM Higgs boson is now replaced by a linear combination $h = \sum_i a_i h_i$, where $h_{1,2,3} = \sqrt{2} Re(\phi_{1,2,3}^0)$. [There may also be a σ component which is assumed negligible in this study. If it is included, then since σ does not couple to the SM fermions, its effect is to reduce all h couplings by an overall factor.] This h should of course be identified as the 125 GeV particle [6, 7] discovered at the LHC. If $a_i = v_i (\sum_i v_i^2)^{-1/2}$, then h is the SM Higgs boson. If not, there could be significant deviations in the production and decay of h, as discussed below. The couplings of $h_{1,2,3}$ to quarks and leptons are given by

$$
\begin{aligned}
\mathcal{L}_Y &= \frac{1}{\sqrt{2}} (\bar{u}, \bar{c}, \bar{t})_L U_{uL}^\dagger \begin{pmatrix} m_u h_1/v_1 & 0 & 0 \\ 0 & m_c h_2/v_2 & 0 \\ 0 & 0 & m_t h_3/v_3 \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}_R \\
&+ \frac{1}{\sqrt{2}} (\bar{d}, \bar{s}, \bar{b})_L U_{dL}^\dagger \begin{pmatrix} m_d h_1/v_1 & 0 & 0 \\ 0 & m_s h_2/v_2 & 0 \\ 0 & 0 & m_b h_3/v_3 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_R \\
&+ \frac{1}{\sqrt{2}} (\bar{e}, \bar{\mu}, \bar{\tau})_L U_{lL}^\dagger \begin{pmatrix} m_e h_1/v_1 & 0 & 0 \\ 0 & m_\mu h_2/v_2 & 0 \\ 0 & 0 & m_\tau h_3/v_3 \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}_R + H.c. \quad (16)
\end{aligned}
$$

In the above, the left-handed fermions are not mass eigenstates. They are rotated by the unitary U_L matrices. This is easily seen by replacing $h_{1,2,3}$ with $\sqrt{2} v_{1,2,3}$ in Eq. (16), which reduces the coupling matrices to mass matrices. The mismatch between the *up* and *down* sectors generates thus Eq. (12). Nevertheless, each h_i couples diagonally to all fermions in their mass-eigenvalue bases, i.e. h_1 couples to $\bar{u}_L u_R$, $\bar{d}_L d_R$, $\bar{e}_L e_R$; h_2 couples to $\bar{c}_L c_R$, $\bar{s}_L s_R$, $\bar{\mu}_L \mu_R$; and h_3 couples to $\bar{t}_L t_R$, $\bar{b}_L b_R$, $\bar{\tau}_L \tau_R$. This is a remarkable result, because flavor-changing neutral-current couplings are supposed to be unavoidable in models with several Higgs doublets. Its origin is the assumption $U_{uR} = U_{dR} = U_{lR} = 1$, which coresoponds to a symmetry limit, where all the right-handed fermions are already mass eigenstates, whereas the left-handed fermions are rotated by U_L to form mass eigenstates. Because of the $U(1)_F$ symmetry, h_1 couples to u_R, d_R, e_R only, h_2 couples to c_R, s_R, μ_R only, and h_3 couples to t_R, b_R, τ_R only. The observed Higgs boson h is a linear combination of $h_{1,2,3}$, so its Yukawa

couplings are diagonal. An immediate prediction is that there is no $h \to \tau\mu$ coupling here in the mass-eigenvalue basis of charged leptons. If the preliminary indication [8] of a nonzero branching fraction for this process is confirmed, this assumption must be relaxed.

In this model, the important thing to realize is that the new physics couples only to right-handed fermions. If this new physics scale is very high, the standard model is recovered. If this new physics is diagonal in family space, i.e $U_{uR} = U_{dR} = U_{lR} = 1$, then there is no tree-level flavor-changing neutral currents. This means that any FCNC effect is suppressed by the mass of the new physics scale, as well as the nondiagonal entries of the right-handed unitary matrices. Stringent limits on these parameters are then applicable. A very crude estimate on $|U_{ds}^R|$ and $|U_{sd}^R|$ from the $K_L - K_S$ mass difference is that both should be less than about 2×10^{-3} for $g_F = 0.1$ and $m_{Z_F} = 3$ TeV. This bound will be proportionally weaker as m_{Z_F} increases. Detailed study of these effects will be presented elsewhere.

Let $a_i = x_i v_i (\sum_i v_i^2)^{-1/2}$ with $\sum_i a_i^2 = 1$. Then for $x_i \neq 1$, there are possible observable deviations from the SM in Higgs interactions. For example, for $x_3 \neq 1$, the production of h through the t and b quark loops in gluon fusion is changed by the factor x_3^2. This is probably a small effect, because v_3 dominates over $v_{2,1}$, so v_3 is still very close to the SM value of $v = \sqrt{v_3^2 + v_2^2 + v_1^2}$. However, h decay to the second and first families may be strongly affected. For example, for $v = 174$ GeV, let $v_1 = 0.5$ GeV, $v_2 = 10$ GeV, then $v_3 = 173.7$ GeV, and $v_3^2/v^2 = 0.9967$ and $v_2^2/v^2 = 0.0033$. Now

$$x_3^2 = 1.0033(1 - 0.0033x_2^2), \tag{17}$$

where $v_1^2/v^2 = 8.3 \times 10^{-6}$ has been neglected. The SM limit is $x_2 = x_3 = 1$, but x_2 may easily be much larger, e.g. $x_2 = 2$ and $x_3 = 0.995$. Whereas the effect of the small deviation of x_3 from unity is very hard to observe, the consequence of a large x_2 is potentially observable in $h \to \mu^-\mu^+$ which would be enhanced by a factor of x_2^2. At present the LHC bounds [9, 10] are about 7 times the SM value at 95% CL, hence $x_2 < 2.6$ is allowed. If $h \to \mu^-\mu^+$ is

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indeed observed at a rate much above the SM prediction, then in this model, the same must be true for $h \to c\bar{c}$. The SM prediction for $h \to c\bar{c}$ is about 2.5%, but it is obscured by a large background from the strong production of charm quarks. If it is enhanced by x_2^2, it may then be marginally observable [11].

In summary, the h of this model couples diagonally to all fermions as in the standard model, but differs from it by having an additional factor $x_i \neq 1$ for each family.

In the neutrino sector, the 3×3 Yukawa coupling matrix linking $\bar{\nu}_{iL}$ to ν_{jR}, again with the assumption $U_{\nu R} = 1$, is given by

$$\mathcal{L}_\nu = \frac{1}{\sqrt{2}} (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)_L U_{\nu L}^\dagger \begin{pmatrix} m_{D1} h_1 / v_1 & 0 & 0 \\ 0 & m_{D2} h_2 / v_2 & 0 \\ 0 & 0 & m_{D3} h_3 / v_3 \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}_R + H.c. \qquad (18)$$

Adding a scalar singlet $\sigma' \sim (1, 1, 0; 3)$ to break $U(1)_F$, the 3×3 Majorana mass matrix spanning $(\nu_{e,\mu,\tau})_R$ is of the form

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & M_3 & 0 \\ M_3 & 0 & 0 \\ 0 & 0 & M_0 \end{pmatrix}, \qquad (19)$$

where M_0 is an allowed mass term, and M_3 comes from $\langle \sigma' \rangle$. Hence the mismatch between U_{lL} and $U_{\nu L}$ generates the neutrino mixing matrix, whereas the neutrino mass eigenvalues are $\pm m_{D1} m_{D2} / M_3$ and $-m_{D3}^2 / M_0$. This approximates the realistic case of two almost degenerate neutrinos for solar oscillations and one other for atmospheric oscillations. The splitting of the two degenerate masses may be achieved with a slight relaxation of the $U_{\nu R} = 1$ assumption for example. The addition of σ' means that the mass of Z_F gets another contribution. It is now given by

$$m_{Z_F}^2 = \frac{1}{2} g_F^2 (u^2 + 36 u'^2). \qquad (20)$$

This allows a smaller value for u, say 1 TeV, in Eqs. (3) and (4). Setting $m_{Z_F} > 3$ TeV for $g_F = 0.1$, a lower limit $u' > 7$ TeV is obtained. To connect σ' to V of Eq. (5), another scalar $\sigma'' \sim (1, 1, 0; -3/2)$ may be added to allow the terms $\sigma'(\sigma'')^2$ and $\sigma'' \sigma^3$.

In conclusion, a progressive gauge $U(1)_F$ family symmetry is proposed for quarks and leptons. It is anomaly-free within each family, but it has a different overall coupling for each, as shown in Table 1. Three scalar doublets $\Phi_{1,2,3}$ are required, coupling each to a different family. The regressive pattern of electroweak symmetry breaking, i.e. $v_3 >> v_2 >> v_1$, as shown in Eqs. (6) to (9), offers an understanding to the observed hierarchy of fermion masses, i.e. $m_u << m_c << m_t$, and $m_d << m_s << m_b$, and $m_e << m_\mu << m_\tau$. Since each family has its own mass scale, the limit of no mixing is technically natural because it corresponds to an extra global U(1) symmetry. This is a possible explanation of the observed small mixing in the quark sector. In the lepton sector, because of the additional Majorana mass matrix of the right-handed neutrinos, this limit of no mixing is spoiled. Hence small mixing is not expected. The two main predictions of this new proposal are:

- There exists a Z_F gauge boson which couples right-handedly to the three families of quarks and leptons with different overall couplings. In particular, the ratio $\Gamma(Z_F \to e^-e^+)/\Gamma(Z_F \to \mu^-\mu^+)$ is n_1^2/n_2^2 which is in general not equal to one. In the example studied in this paper, it is 4. For $g_F = 0.1$, the mass of Z_F is greater than 3 TeV from current data [4, 5].

- The 125 GeV particle observed at the LHC is identified as h which does not exactly correspond to the one Higgs boson of the SM. In the simplest scenario studied in this paper, it should have only diagonal couplings to quarks and leptons as in the SM, but with an additional overall factor x_i for the different families. Hence $h \to \mu^-\mu^+$ as well as $h \to c\bar{c}$ are allowed to be several times those of the SM, which will be probed with more data in the future; whereas $h \to \tau\mu$ is forbidden, despite a hint [8] from the CMS Collaboration that it may be nonzero. This absence of flavor-changing neutral-current couplings corresponds to a symmetry limit where $U_{uR} = U_{dR} = U_{lR} = 1$.

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