Quantum entanglement in three accelerating qubits coupled to scalar fields

Yue Dai, Zhejun Shen, and Yu Shi

Life and death of quantum entanglement in three accelerating qubits coupled with scalar fields

Yue Dai,[1] Zhejun Shen,[1] and Yu Shi[1,2,*]

[1]*Center for Field Theory and Particle Physics,*

Department of Physics & State Key Laboratory of Surface Physics, Fudan University,

Shanghai 200433, China

[2]*Collaborative Innovation Center of Advanced Microstructures, Fudan University,*

Shanghai 200433, China

Abstract

We consider quantum entanglement of three accelerating qubits, each of which is locally coupled with a real scalar field, without causal influence among the qubits or among the fields. The initial states are assumed to be the GHZ and the W states, which are the two representative three-partite entangled states. For each initial state, we study how various kinds of entanglement depend on the accelerations of the three qubits. All kinds of entanglement eventually suddenly die if at least two of three qubits have large enough accelerations. This result implies eventual sudden death of all kinds of entanglement among three particles coupled with scalar fields when they are sufficiently close to the horizon of a black hole.

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[*] yushi@fudan.edu.cn

I. INTRODUCTION

How quantum states are affected by gravity or acceleration is a subject of longstanding interest [1]. In presence of a black hole, the physical vacuum is that in the Kruskal-Szekers coordinates, which are nonsingular and cover the whole Schwarzschild spacetime, while a remote observer can observe particles. Known as Hawking radiation, it underlies the paradox of information loss [2]. Analogously, known as Unruh effect, an accelerating object coupled with a field detects a thermal bath of particles of this field, even though this field is in the Minkowski vacuum [3–6].

For a composite quantum system, the characterisitc quantum feature is quantum entanglement. An interesting question is how quantum entanglement among objects coupled with fields is affected by the Unruh effect. Various investigations were made on two entangled detectors, one or both of which accelerate [7–12].

Naturally one may wonder about the situation of three entangled field-coupled qubits. This is interesting and nontrivial because there are various different types of entanglement among three qubits A, B and C. There is bipartite entanglement between two qubits, and there is bipartite entanglement between one qubit and the remaining two qubits as one party. Most interestingly, there is tripartite entanglement among all the three qubits, which cannot be reduced to any combination of all kinds of bipartite entanglement [13]. This is profound and important in understanding many-body correlations. In quantum information theory, recent years witnessed much development in quantifying entanglement in terms of some measures. A convenient one is the so-called negativity, which is the sum of the negative eigenvalues of the partial transpose of the density matrix, ranging from 0 to 1/2 [14]. It can be used to quantify various kinds of bipartite entanglement in a three-qubit state. Twice of the negativity that quantifies the bipartite entanglement between a qubit and the remaining two qubits is called a one-tangle, ranging from 0 to 1. Twice of the negativity quantifies the bipartite entanglement between two qubits is called a two-tangle, ranging from 0 to 1. Using all the one-tangles and two-tangles, one can define a measure of tripartite entanglement called three-tangle, ranging from 0 to 1 [15].

In the present paper, by using these negativity-based entanglement measures, we make detailed investigations on how various types of entanglement in three field-coupled qubits vary with their accelerations, which have implications on particles near the horizon of a black

hole. It is well known that there are two inequivalent types of tripartite entanglement [16], typified respectively by the GHZ state

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle).$$ (1)

and the W state

$$\frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle).$$ (2)

So both GHZ and W states are considered in this paper.

II. METHOD

We consider three qubits A, B and C far away from each other. For simplicity, it is assumed that each qubit q ($q = A, B, C$) is coupled only locally with the field Φ_q around it, as described by the Unruh-Wald model [17]. The Hamiltionian of each qubit q itself is $H_q = \Omega_q Q_q^\dagger Q_q$, where the creation operator $Q_q^\dagger$ and annihilation operator Q_q are defined by $Q_q|0\rangle_q = Q_q^\dagger|1\rangle_q = 0$, $Q_q^\dagger|0\rangle_q = |1\rangle_q$ and $Q_q|1\rangle_q = |0\rangle_q$, Ω_q is the energy difference between the two eigenstates. Its coupling with the field is described by $H_{I_q}(t_q) = \epsilon_q(t_q) \int_{\Sigma_q} \Phi_q(\mathbf{x})[\psi_q(\mathbf{x})Q_q + \psi_q^*(\mathbf{x})Q_q^\dagger]\sqrt{-g}d^3x$, where $\mathbf{x}$ and t_q are spacetime coordinates in the comoving frame of the qubit, the integral is over the spacelike Cauchy surface Σ_q at the given time t_q, $\epsilon_q(t_q)$ is the coupling constant with a finite duration, $\psi_q(\mathbf{x})$ is a smooth function nonvanishing within a small region around the qubit. The total Hamiltonian of the system is thus

$$\sum_{q=A,B,C} (H_q + H_{\Phi_q} + H_{I_q}),$$ (3)

where H_{Φ_q} is the Klein-Gordon Hamiltonian for Φ_q.

It is assumed that the qubits are far away from each other such that during the interaction times, there is no physical coupling or influence between the fields around different qubits or between a qubit and the field around another qubit. We make this assumption to avoid the issue of global time slice and those complications caused by the different accelerations of the qubits. Consequently, one can describe the quantum state of the qubits by considering each qubit in its comoving reference frame. The interesting case that all qubits are coupled with a same field will be explored as future work.

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Therefore, in our consideration, after a time duration longer than the interacting times $T_q \gg 1/\Omega_q$, the state of the whole system in the interaction picture is transformed by

$$U_A \otimes U_B \otimes U_C,$$

where U_q is the unitary transformation acting on qubit q and the field Φ_q in its neighboring region. It can be obtained that [12]

$$U_q \approx 1 + iQ_q a^\dagger(\Gamma_q^*) - iQ_q^\dagger a(\Gamma_q^*), \tag{4}$$

where $a(\Gamma_q^*)$ and $a^\dagger(\Gamma_q^*)$ are the annihilation and creation operators of Γ_q^*, with

$$\Gamma_q(x) \equiv -2i \int [G_{Rq}(x; x') - G_{Aq}(x; x')] \epsilon_q(t') e^{i\Omega_q t} \psi_q^*(\mathbf{x}') \sqrt{-g'} d^4 x', \tag{5}$$

where G_{Rq} and G_{Aq} are the retarded and advanced Green functions of the field Φ_q [17].

For each field Φ_q, it has been argued that approximately the qubit q is only coupled with the field mode Γ_q^*, with frequency Ω_q, with the other modes decoupled [12, 17]. Consider the Fock state $|n\rangle_{\Gamma_q^*}$ containing n particles in the mode Γ_q^*, as observed in the Rindler wedge confining qubit q. We have [12]

$$U_q|0\rangle_q|n\rangle_{\Gamma_q^*} = |0\rangle_q|n\rangle_{\Gamma_q^*} - i\sqrt{n}\mu_q|1\rangle_q|n-1\rangle_{\Gamma_q^*}, \tag{6}$$

$$U_q|1\rangle_q|n\rangle_{\Gamma_q^*} = |1\rangle_q|n\rangle_{\Gamma_q^*} + i\sqrt{n+1}\mu_q^*|0\rangle_q|n+1\rangle_{\Gamma_q^*}, \tag{7}$$

where $\mu_q \equiv \langle \Gamma_q^*, \Gamma_q^* \rangle$. For an arbitrary mode χ, $\langle \Gamma_q^*, \chi \rangle = \int \epsilon_q(t) e^{i\Omega_q t} \psi_q^*(\mathbf{x}) \chi(t, \mathbf{x}) \sqrt{-g} d^4 x$ [17].

Suppose the initial state of the three qubits to be $|\Psi_i\rangle$. Without causal connection either between the qubits or between the fields, each qubit independently detects a thermal bath of the Unruh particles determined by its own acceleration. With each qubit in its own Rindler wedge, the initial state of the whole system, as observed by the observers comoving with the qubits respectively, is described by the density matrix

$$\rho_i = \rho_{\Gamma_A^*} \otimes \rho_{\Gamma_B^*} \otimes \rho_{\Gamma_C^*} \otimes |\Psi_i\rangle\langle\Psi_i|, \tag{8}$$

where $\rho_{\Gamma_q^*}$ is the density matrix of the mode $\chi_{\Gamma_q^*}$ of the field around qubit q, the decoupled modes are neglected.

For a uniformly moving qubit q,

$$\rho_{\Gamma_q^*} = |0\rangle_{\Gamma_q^*}\langle 0|, \tag{9}$$

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because the uniformly moving qubit sees a Minkowski vacuum.

For an accelerating qubit,

$$\rho_{\Gamma_q^*} = \eta_q \sum_{n_q} e^{-2\pi n_q \Omega_q / a_q} |n_q\rangle_{\Gamma_q^*} \langle n_q|, \tag{10}$$

where a_q is the acceleration of qubit q, n_q denotes the particle number of mode Ω_q, $\eta_q \equiv \sqrt{1 - e^{-2\pi\Omega_q / a_q}}$.

The final state of the system with respect to the comoving observers is

$$\rho_f = U_C^\dagger U_B^\dagger U_A^\dagger \rho_i U_A U_B U_C, \tag{11}$$

from which we obtain the reduced density matrix of the three qubits

$$\rho_{ABC} = Tr_{\Gamma_A^*, \Gamma_B^*, \Gamma_C^*}(\rho_f). \tag{12}$$

Then we study the dependence of various types of entanglement on the accelerations.

The one-tangle between qubit α and the remaining qubits β and γ is

$$\mathcal{N}_{\alpha(\beta\gamma)} \equiv \|\rho_{ABC}^{T_\alpha}\| - 1, \tag{13}$$

where T_α represents partial transpose with respect to α, $\|\rho\| \equiv \mathrm{Tr}\sqrt{\rho\rho^\dagger}$ represents the trace norm of ρ. The two-tangle between qubits α and β is

$$\mathcal{N}_{\alpha\beta} \equiv \|\rho_{\alpha\beta}^{T_A}\| - 1, \tag{14}$$

where $\rho_{\alpha\beta} = \mathrm{Tr}_\gamma \rho_{ABC}$ is the reduced density matrix of α and β. The three-tangle is

$$\pi \equiv \frac{1}{3} \sum_{\alpha=A,B,C} \pi_\alpha, \tag{15}$$

where

$$\pi_\alpha \equiv \mathcal{N}_{\alpha(\beta\gamma)}^2 - \mathcal{N}_{\alpha\beta}^2 - \mathcal{N}_{\alpha\gamma}^2. \tag{16}$$

Note that a monogamy relation

$$\mathcal{N}_{\alpha(\beta\gamma)}^2 \geq \mathcal{N}_{\alpha\beta}^2 + \mathcal{N}_{\alpha\gamma}^2 \tag{17}$$

is always valid, and is the basis for the definition (16).

In the following, for the GHZ and for the W states, we study various cases of the accelerations of the three qubits. Note the permutation symmetry of each of these two states.

III. GHZ STATE

First we consider the initial state to be the GHZ state,

$$|\Psi_i\rangle = |GHZ\rangle. \tag{18}$$

In the GHZ state, tracing over one qubit always yields a disentangled two-qubit state. On the other hand, the coupling between each qubit and the field around it does not increase the inter-qubit entanglement. Therefore, each two-tangle always remains zero,

$$\mathcal{N}_{AB} = \mathcal{N}_{BC} = \mathcal{N}_{AC} = 0. \tag{19}$$

A. C accelerating

Let us assume qubit C accelerates while A and B move uniformly. In this case, the density matrix of the three qubits is obtained as

$$
\begin{aligned}
\rho_{ABC} = & \eta_C^2 \sum_{n_C} \frac{e^{-2\pi n_C \Omega_C / a_C}}{Z_{n_C}} \Big[\left(1 + (n_C + 1)|\mu_A|^2 |\mu_B|^2 |\mu_C|^2\right) |000\rangle \langle 000| \\
& + |000\rangle \langle 111| + |111\rangle \langle 000| + |111\rangle \langle 111| + (n_C + 1)|\mu_B|^2 |\mu_C|^2 |100\rangle \langle 100| \\
& + (n_C + 1)|\mu_A|^2 |\mu_C|^2 |010\rangle \langle 010| + \left(n_C |\mu_C|^2 + |\mu_A|^2 |\mu_B|^2\right) |001\rangle \langle 001| \\
& + (n_C + 1)|\mu_C|^2 |110\rangle \langle 110| + |\mu_B|^2 |101\rangle \langle 101| + |\mu_A|^2 |011\rangle \langle 011| \Big],
\end{aligned}
\tag{20}
$$

where

$$
\begin{aligned}
Z_{n_C} = & 2 + |\mu_A|^2 + |\mu_B|^2 + (2n_C + 1)|\mu_C|^2 + |\mu_A|^2 |\mu_B|^2 \\
& + (n_C + 1)|\mu_A|^2 |\mu_C|^2 + (n_C + 1)|\mu_B|^2 |\mu_C|^2 + (n_C + 1)|\mu_A|^2 |\mu_B|^2 |\mu_C|^2.
\end{aligned}
\tag{21}
$$

In the GHZ state, any qubit is maximally entangled with the other two qubits as a single party. Hence the one-tangles are

$$\mathcal{N}_{A(BC)} = \mathcal{N}_{B(AC)} = \mathcal{N}_{C(AB)} = 1. \tag{22}$$

When $a_C \neq 0$, the entanglement decreases. $\mathcal{N}_{A(BC)}$ decreases with the increase of acceleration-frequency-ration (AFR) a_C / Ω_C till its sudden death, as shown in Fig. 1(a). This is a phenomenon of entanglement sudden death [18]. The result here on one-tangles extends the previous result of bipartite entanglement [12] from pure states to mixed states.
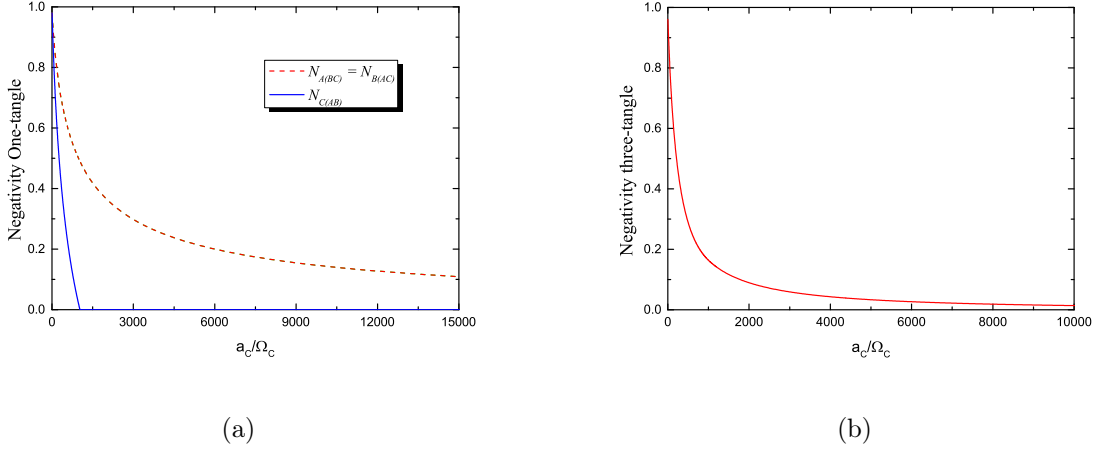
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FIG. 1. Dependence of (a) the one-tangles and (b) the three-tangle on the AFR of qubit C. Qubits A and B move uniformly. The qubits are in the GHZ state.

However, $\mathcal{N}_{A(BC)} = \mathcal{N}_{B(AC)}$ approaches zero asymptotically, presumably because for these two one-tangles, C is only one of the two qubits constituting a party, with the other qubit moving uniformly.

In the present case, since the two-tangles remain zero, the three-tangle turns out to be the average of the sum of the three one-tangles, as depicted in Fig. 1(b). Note that due to the sudden death of $\mathcal{N}_{A(BC)}$, there is a sudden change in the three-tangle, after which the three-tangle is just $2\mathcal{N}_{A(BC)}/3 = 2\mathcal{N}_{B(AC)}/3$.

B. B and C accelerating

In the case of B and C accelerating while A moving uniformly, we obtain

$$
\begin{aligned}
\rho_{ABC} = \eta_B^2 \eta_C^2 \sum_{n_B, n_C} & \frac{e^{-2\pi(n_B\Omega_B/a_B + n_C\Omega_C/a_C)}}{Z_{n_B,n_C}} \left[|000\rangle \langle 111| + |111\rangle \langle 000| \right. \\
& + |111\rangle \langle 111| + \left(1 + (n_B+1)(n_C+1)|\mu_A|^2|\mu_B|^2|\mu_C|^2\right)|000\rangle \langle 000| \\
& + (n_B+1)(n_C+1)|\mu_B|^2|\mu_C|^2 |100\rangle \langle 100| + (n_B+1)|\mu_B|^2 |101\rangle \langle 101| \\
& + (n_C+1)|\mu_C|^2 |110\rangle \langle 110| + \left(n_B n_C |\mu_B|^2|\mu_C|^2 + |\mu_A|^2\right)|011\rangle \langle 011| \\
& + \left(n_B|\mu_B|^2 + (n_C+1)|\mu_A|^2|\mu_C|^2\right)|010\rangle \langle 010| \\
& \left. + \left(n_C|\mu_C|^2 + (n_B+1)|\mu_A|^2|\mu_B|^2\right)|001\rangle \langle 001| \right],
\end{aligned}
\tag{23}
$$

where

$$Z_{n_B,n_C} = 2 + |\mu_A|^2 + (2n_B + 1)|\mu_B|^2 + (\frac{3}{2}n_C + 1)|\mu_C|^2$$

$$+ (n_B + 1)|\mu_A|^2|\mu_B|^2 + [n_B n_C + (n_B + 1)(n_C + 1)]|\mu_B|^2|\mu_C|^2 \qquad (24)$$

$$+ (n_C + 1)|\mu_A|^2|\mu_C|^2 + (n_B + 1)(n_C + 1)|\mu_A|^2|\mu_B|^2|\mu_C|^2.$$

The dependence of $\mathcal{N}_{A(BC)}$ on a_B/Ω_B and a_C/Ω_C is symmetric, as shown in Fig. 2(a). When one of the accelerations is zero, $\mathcal{N}_{A(BC)}$ decreases towards zero asymptotically, as discussed in the preceding subsection. When both are nonzero, $\mathcal{N}_{A(BC)}$ decreases quickly towards zero, reaching sudden death, at finite values of the two AFRs.

As shown in Fig. 2(b), $\mathcal{N}_{B(AC)}$ strongly depends on a_B/Ω_B and suddenly dies at a finite value of a_B/Ω_B, while it depends on a_C/Ω_C weakly, especially when a_B/Ω_B is so large that $\mathcal{N}_{B(AC)}$ is close to zero. This is because B is one party by itself, while C is only one of the two qubits constituting the other party. $\mathcal{N}_{C(AB)}$ can be obtained from $\mathcal{N}_{B(CA)}$ by exchanging B and C, as shown in Fig. 2(c).

We now look at some 2D cross sections of the 3D plots in Fig. 2. Fig. 3(a) is for $a_B/\Omega_B = a_C/\Omega_C$, hence $\mathcal{N}_{B(AC)} = \mathcal{N}_{C(AB)}$. Among the three one-tangles, $\mathcal{N}_{A(BC)}$ is the smallest, presumably because B and C constituting the party (BC) both accelerate. The three one-tangles die at a same value of $a_B/\Omega_B = a_C/\Omega_C$.

Fig. 3(b) is for $a_C/\Omega_C = 1.5a_B/\Omega_B$, while Fig. 3(c) is for $a_C/\Omega_C = 2a_B/\Omega_B$. The values of a_B/Ω_B for the sudden death of $\mathcal{N}_{B(AC)}$ in these two cases are close to that in Fig. 3(a), as $\mathcal{N}_{B(AC)}$ is mainly determined by a_B/Ω_B when it is close to zero. On the other hand, $\mathcal{N}_{C(AB)}$ dies first, as a_C/Ω_C is larger than a_B/Ω_B in these cross sections, while $a_A/\Omega_A = 0$. When a_B/Ω_B is less than the value at which $\mathcal{N}_{C(AB)}$ suddenly dies, $\mathcal{N}_{B(AC)} < \mathcal{N}_{A(BC)}$ as $a_B/\Omega_B > a_A/\Omega_a = 0$. When $\mathcal{N}_{C(AB)}$ suddenly dies, $\mathcal{N}_{B(AC)} = \mathcal{N}_{A(BC)}$. When a_B/Ω_B is larger than the value for the death of $\mathcal{N}_{C(AB)}$, $\mathcal{N}_{A(BC)} < \mathcal{N}_{B(AC)}$, till $\mathcal{N}_{B(AC)}$ dies at a larger value of a_B/Ω_B. The above-mentioned feature in the case of $a_B/\Omega_B = a_C/\Omega_C$ that the three one-tangles die at the same value of a_B/Ω_B is a special case, because when it is constrained that $\mathcal{N}_{C(AB)} = \mathcal{N}_{B(AC)}$, the sudden death of $\mathcal{N}_{C(AB)}$ implies that of $\mathcal{N}_{B(AC)}$, as the two are equal. On the other hand, when $\mathcal{N}_{C(AB)}$ suddenly dies, there must be $\mathcal{N}_{A(BC)} = \mathcal{N}_{B(AC)}$, hence the three have to suddenly die altogether, in other words, the only option for $\mathcal{N}_{A(BC)}$ to be between these two is that it dies also.

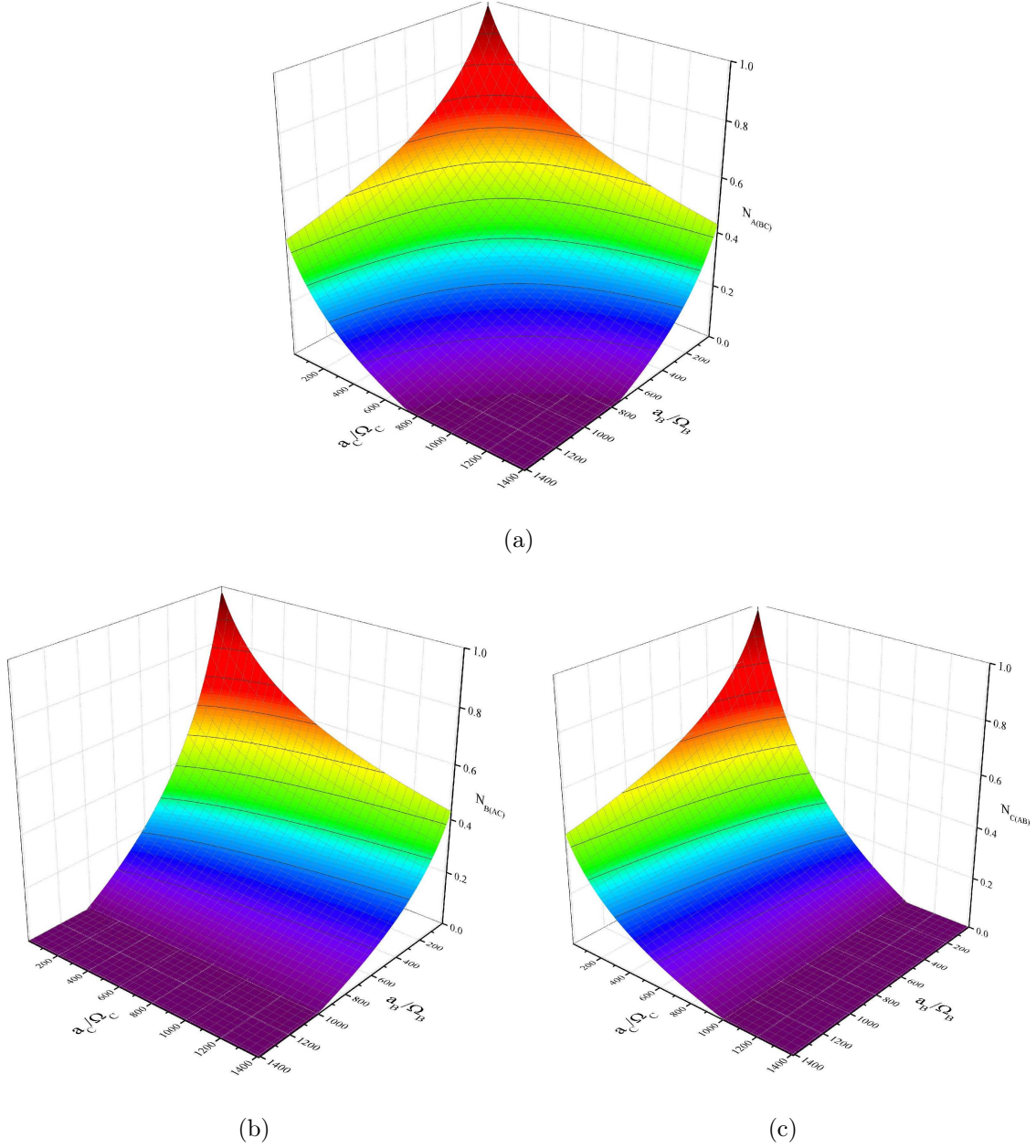We have also examined several cases of given values of a_B/Ω_B, as shown in Figs. 4(a),

(a)

(b)

(c)

FIG. 2. Dependence of the one-tangle (a) $\mathcal{N}_{A(BC)}$, (b) $\mathcal{N}_{B(AC)}$, (c) $\mathcal{N}_{C(AB)}$ on the AFRs of qubit B and C. Qubit A moves uniformly. The qubits are in the GHZ state.

4(b), and 4(c), with $a_B/\Omega_B = 300$, 750 and 1200 respectively. For $a_C/\Omega_C < a_B/\Omega_B$, $\mathcal{N}_{B(AC)} < \mathcal{N}_{C(AB)} < \mathcal{N}_{A(BC)}$. For $a_C/\Omega_C > a_B/\Omega_B$ up to the death of $\mathcal{N}_{C(AB)}$, $\mathcal{N}_{C(AB)} < \mathcal{N}_{B(AC)} < \mathcal{N}_{A(BC)}$. After the death of $\mathcal{N}_{C(AB)}$, $\mathcal{N}_{A(BC)} < \mathcal{N}_{B(AC)}$.

The three-tangle is shown in the 3D plot in Fig. 5. The condition of the three-tangle sudden death is that each of the two nonzero AFRs should be large enough. The reason is that the three-tangle is now the average of the squares of one-tangles. Hence it suddenly dies
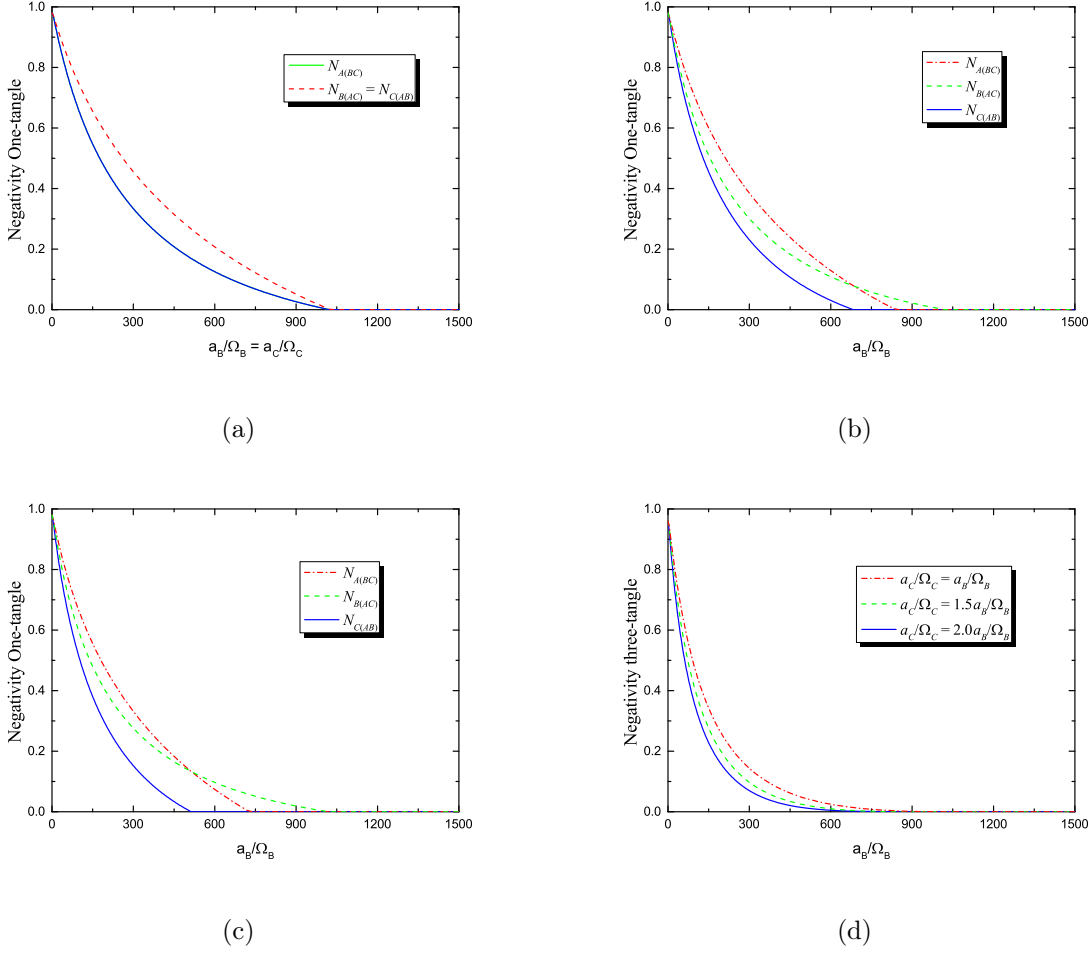
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FIG. 3. Dependence of the one-tangles on the AFRs of qubits B and C in the case that (a) $a_C/\Omega_C = a_B/\Omega_B$, (b) $a_C/\Omega_C = 1.5a_B/\Omega_B$, (c) $a_C/\Omega_C = 2a_B/\Omega_B$. (d) The three-tangle in these cases. Qubit A moves uniformly. The qubits are in the GHZ state.

when all one-tangles suddenly die. Some 2D cross sections of Fig. 5 are shown in Fig. 3(d) and Fig. 4(d). Note in the cases that $a_B/\Omega_B = 300$ and $a_B/\Omega_B = 750$ while $a_A = 0$, as shown in Fig. 4(d), the three-tangle only approaches zero asymptotically with the increase of a_C/Ω_C, since the values of a_B/Ω_B are not large enough.

C. A, B, and C all accelerating

In the case that all qubits accelerate, we have

$$\rho_{ABC} = \frac{1}{2}\eta_A^2\eta_B^2\eta_C^2 \sum_{n_A,n_B,n_C} \frac{e^{-2\pi(n_A\Omega_A/a_A+n_B\Omega_B/a_B+n_C\Omega_C/a_C)}}{Z_{n_A,n_B,n_C}}$$
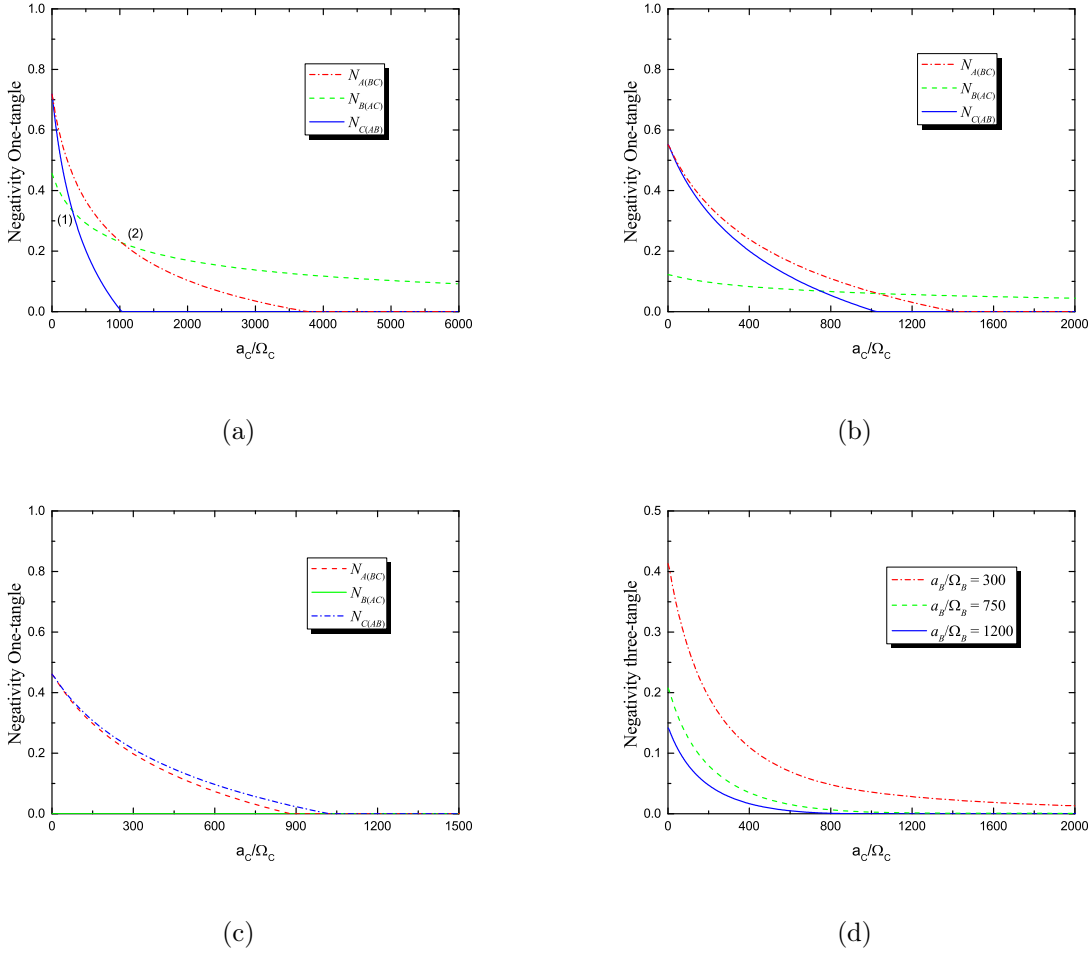
10

FIG. 4. Dependence of the one-tangles on the AFR of qubit C in the case that (a) $a_B/\Omega_B = 300$, (b) $a_B/\Omega_B = 750$, (c) $a_B/\Omega_B = 1200$. (d) The three-tangle in these cases. Qubit A moves uniformly. The qubits are in the GHZ state.

$$
\begin{aligned}
\times\,\Big[& \left(1 + (n_A + 1)(n_B + 1)(n_C + 1)|\mu_A|^2|\mu_B|^2|\mu_C|^2\right)|000\rangle\langle 000| \\
& + \left(1 + n_A n_B n_C |\mu_A|^2|\mu_B|^2|\mu_C|^2\right)|111\rangle\langle 111| + |111\rangle\langle 000| \\
& + |000\rangle\langle 111| + \left(n_A|\mu_A|^2 + (n_B + 1)(n_C + 1)|\mu_B|^2|\mu_C|^2\right)|100\rangle\langle 100| \\
& + \left(n_B|\mu_B|^2 + (n_A + 1)(n_C + 1)|\mu_A|^2|\mu_C|^2\right)|010\rangle\langle 010| \\
& + \left(n_C|\mu_C|^2 + (n_A + 1)(n_B + 1)|\mu_A|^2|\mu_B|^2\right)|001\rangle\langle 001| \\
& + \left((n_A + 1)|\mu_A|^2 + n_B n_C|\mu_B|^2|\mu_C|^2\right)|110\rangle\langle 110| \\
& + \left((n_B + 1)|\mu_B|^2 + n_A n_C|\mu_A|^2|\mu_C|^2\right)|101\rangle\langle 101|
\end{aligned}
$$

11
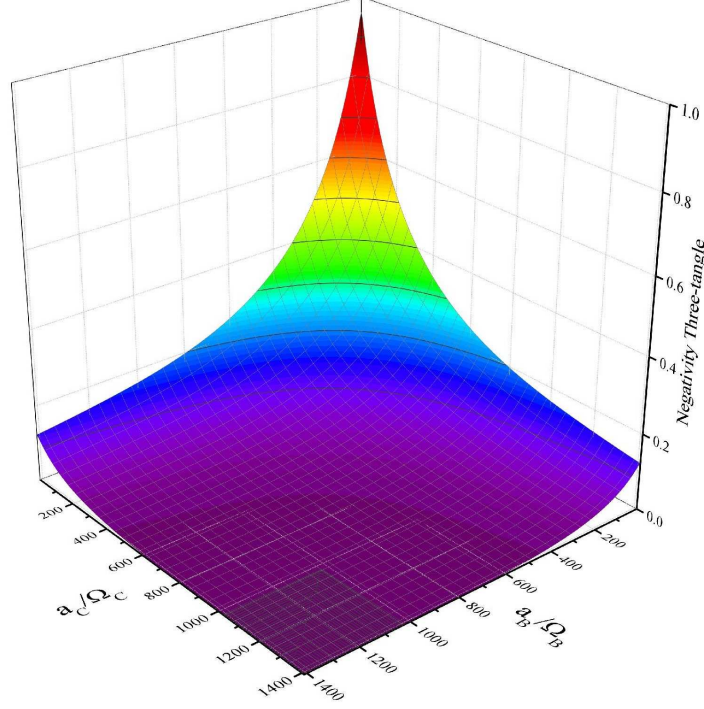
FIG. 5. Dependence of the three-tangle on the AFRs of qubits B and C. Qubit A moves uniformly. The qubits are in the GHZ state.

$$+ \left((n_C + 1)\,|\mu_C|^2 + n_A n_B |\mu_A|^2 |\mu_B|^2 \right) |011\rangle \langle 011| \Big], \tag{25}$$

where

$$
\begin{aligned}
Z_{n_A,n_B,n_C} =\; & 2 + [n_A n_B n_C + (n_A + 1)(n_B + 1)(n_C + 1)]\,|\mu_A|^2 |\mu_B|^2 |\mu_C|^2 \\
& + (2n_A + 1)|\mu_A|^2 + [n_B n_C + (n_B + 1)(n_C + 1)]\,|\mu_B|^2 |\mu_C|^2 \\
& + (2n_B + 1)|\mu_B|^2 + [n_A n_C + (n_A + 1)(n_C + 1)]\,|\mu_A|^2 |\mu_C|^2 \\
& + (2n_C + 1)|\mu_C|^2 + [n_A n_B + (n_A + 1)(n_B + 1)]\,|\mu_A|^2 |\mu_B|^2,
\end{aligned}
\tag{26}
$$

$\eta_q = \sqrt{1 - e^{-2\pi\Omega_q/a_q}}$, $(q = A, B, C)$.

Fig. 6 is for $a_A/\Omega_A = 100$ and $a_C/\Omega_C = 2a_B/\Omega_B$, where we show the three one-tangles. The plots have three intersections. Intersection (1) is at $a_A/\Omega_A = a_C/\Omega_C$ and thus $\mathcal{N}_{A(BC)} = \mathcal{N}_{C(AB)}$. Intersection (2) is at $a_A/\Omega_A = a_B/\Omega_B$ and thus $\mathcal{N}_{A(BC)} = \mathcal{N}_{B(AC)}$. Intersection (3) is another point where $\mathcal{N}_{A(BC)} = \mathcal{N}_{B(AC)}$. In the cases that qubit A moves uniformly, intersection (3) is at the value of a_B/Ω_B where $\mathcal{N}_{C(AB)}$ suddenly dies, as shown in Figs. 3(b), 3(c), 4(a) and 4(b). Now the nonzero a_A delays this intersection. The three-tangle suddenly
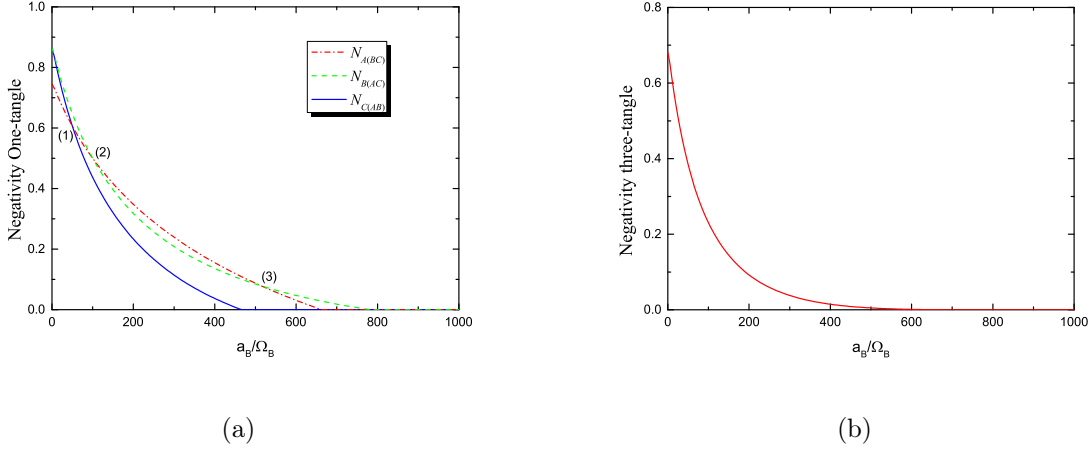
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FIG. 6. Dependence of (a) the one-tangles, (b) the three-tangle on the AFR of qubit B in the case that $a_A/\Omega_A = 100$, $a_C/\Omega_C = 2a_B/\Omega_B$. The qubits are in the GHZ state.
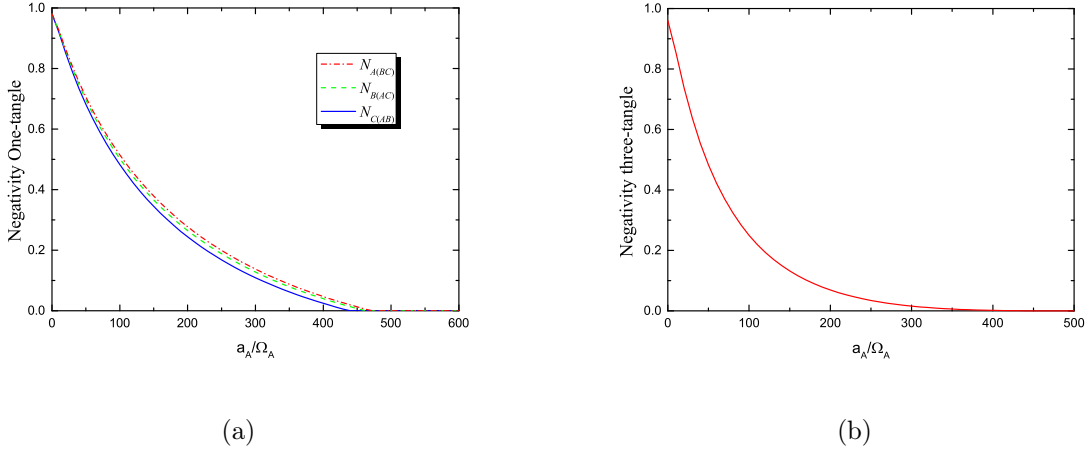

FIG. 7. Dependence of (a) the one-tangles, (b) the three-tangle on the AFR of qubit A in the case that $a_B/\Omega_B = 1.2a_A/\Omega_A$, $a_C/\Omega_C = 1.5a_A/\Omega_A$. The qubits are in the GHZ state.

dies after all three one-tangles become zero, as shown in Fig. 6(b).

In Fig. 7 we show the one-tangles and the three-tangle for $a_B/\Omega_B = 1.2a_A/\Omega_A$, $a_C/\Omega_C = 1.5a_A/\Omega_A$. The three one-tangles suddenly die successively, afterwards the three-tangle becomes zero. After $\mathcal{N}_{C(AB)}$ suddenly dies, $\mathcal{N}_{A(BC)}$ and $\mathcal{N}_{B(AC)}$ do not intersect. This is because a_A/Ω_A is so large that the would-be intersection is shifted to some value of a_A/Ω_A larger than the value of sudden death of each of them.

IV. W STATE

Now we consider the initial state to be the W state,

$$|\Psi_i\rangle = |W\rangle. \tag{27}$$

In the W state, the one-tangles are $\mathcal{N}_{A(BC)} = \mathcal{N}_{B(AC)} = \mathcal{N}_{C(AB)} = 2\sqrt{2}/3$. The two-tangles are nonzero now, because for the W state, tracing out one qubit does not yield a separable state. One obtains $\mathcal{N}_{AB} = \mathcal{N}_{BA} = \mathcal{N}_{AC} = \mathcal{N}_{CA} = \mathcal{N}_{BC} = \mathcal{N}_{CB} = (\sqrt{5} - 1)/3 \approx 0.412$. The three-tangle is less than 1, because the two-tangles are nonzero. One obtains $\pi = 4(\sqrt{5} - 1)/9 \approx 0.549$.

A. C accelerating

First we consider the case that only qubit C accelerates, while A and B move uniformly. We obtain

$$
\begin{aligned}
\rho_{ABC} =\eta_C^2 \sum_{n_C} \frac{e^{-2\pi n_C \Omega_C/a_C}}{Z_{n_C}} & \left[|001\rangle\langle010| + |001\rangle\langle100| + |100\rangle\langle001| + |100\rangle\langle010| \right. \\
& + |100\rangle\langle100| + |010\rangle\langle010| + \left(|\mu_A|^2 + |\mu_B|^2 + (n_C + 1)|\mu_C|^2 \right)|000\rangle\langle000| \\
& + |010\rangle\langle001| + n_C|\mu_C|^2 \left(|101\rangle\langle101| + |011\rangle\langle101| + |101\rangle\langle011| + |011\rangle\langle011| \right) \\
& \left. + |010\rangle\langle100| + \left(1 + n_C|\mu_A|^2|\mu_C|^2 + n_C|\mu_B|^2|\mu_C|^2 \right)|001\rangle\langle001| \right],
\end{aligned}
\tag{28}
$$

where

$$Z_{n_C} = 3 + |\mu_A|^2 + |\mu_B|^2 + (3n_C + 1)|\mu_C|^2 + n_C|\mu_A|^2|\mu_C|^2 + n_C|\mu_B|^2|\mu_C|^2. \tag{29}$$

The entanglement among the qubits decreases when C accelerates. As shown in Fig. 8(a), the one-tangle $\mathcal{N}_{C(AB)}$ suddenly dies at a certain value of a_C/Ω_C. This is similar to the GHZ state. However, differing from the GHZ state, with $a_A/\Omega_A = a_B/\Omega_B = 0$, $\mathcal{N}_{A(BC)} = \mathcal{N}_{B(CA)}$ has a minimum at a certain value of a_C/Ω_C, but then increase towards an nonzero asymptotical value.
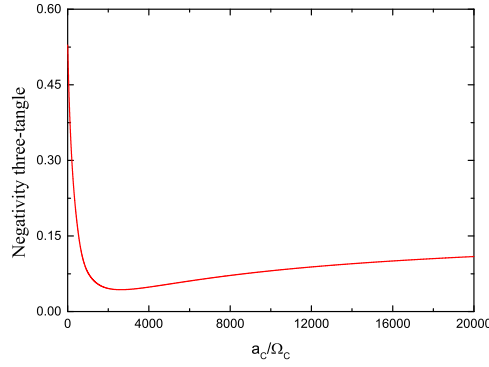
As shown in Fig. 8(b), with the increase of a_C/Ω_C, $\mathcal{N}_{AC} = \mathcal{N}_{BC}$ decreases and suddenly dies at a certain value, while $\mathcal{N}_{AB}$ remains constant since it has nothing to do with C.

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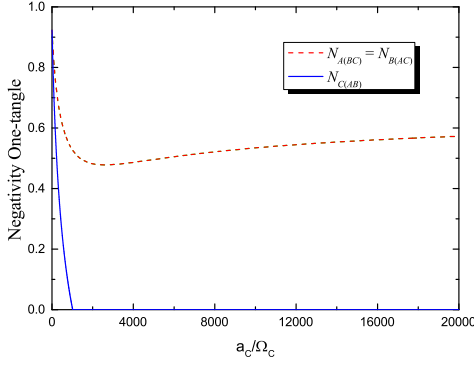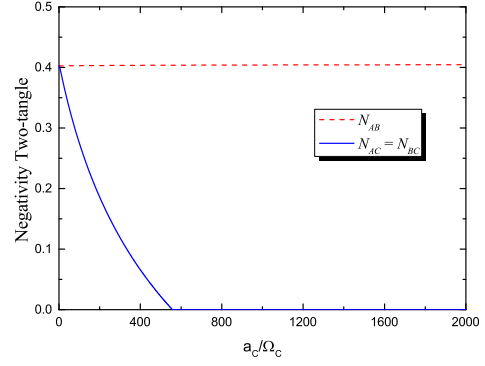
FIG. 8. Dependence of (a) the one-tangles, (b) the two-tangles, (c) the three-tangle on the AFR of qubit C. Qubits A and B move uniformly. The qubits are in the W state.

As shown in Fig. 8(c), with the increase of a_C/Ω_C, the three-tangle first decreases towards a minimum, and then increases towards an nonzero asymptotic value. This can be inferred from the features of the one-tangles and two-tangles, according to the definition of the three-tangle.

In the limit of $a_C/\Omega_C \to \infty$,

$$\rho_{ABC} \to \frac{1}{3}\left[|000\rangle\langle 000| + |011\rangle\langle 011| + |101\rangle\langle 101| + |101\rangle\langle 011| + |011\rangle\langle 101|\right], \qquad (30)$$

so the asymptotic values of the one-tangles are

$$\lim_{a_C \to \infty} \mathcal{N}_{A(BC)} = \lim_{a_C \to \infty} \mathcal{N}_{B(AC)} = \frac{2}{3}, \qquad (31)$$

$$\lim_{a_C \to \infty} \mathcal{N}_{C(AB)} = 0. \qquad (32)$$

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The two tangles are the following. $\mathcal{N}_{AB} = (\sqrt{5} - 1)/3$, which is a constant independent of a_C/Ω_C. $\mathcal{N}_{AC} = \mathcal{N}_{BC} = 0$ after its sudden death. Consequently, the three-tangle approaches asymptotically $4(\sqrt{5} - 1)/27 \approx 0.183$.

B. B and C accelerating

In the case that B and C accelerate while A moves uniformly, we have

$$
\begin{aligned}
\rho_{ABC} = \eta_B^2 \eta_C^2 \sum_{n_B,n_C} \frac{e^{-2\pi(n_B\Omega_B/a_B + n_C\Omega_C/a_C)}}{Z_{n_B,n_C}} & [|001\rangle\langle 010| + |001\rangle\langle 100| + |100\rangle\langle 001| \\
& + |100\rangle\langle 010| + |010\rangle\langle 001| + |010\rangle\langle 100| + |100\rangle\langle 100| + n_B n_C |\mu_B|^2 |\mu_C|^2 |111\rangle\langle 111| \\
& + n_B |\mu_B|^2 |011\rangle\langle 110| + n_C |\mu_C|^2 |101\rangle\langle 101| + n_B |\mu_B|^2 |110\rangle\langle 110| \\
& + n_C |\mu_C|^2 |011\rangle\langle 101| + \left(|\mu_A|^2 + (n_B + 1)|\mu_B|^2 + (n_C + 1)|\mu_C|^2\right) |000\rangle\langle 000| \\
& + \left(1 + n_C |\mu_A|^2 |\mu_C|^2 + (n_B + 1) n_C |\mu_B|^2 |\mu_C|^2\right) |001\rangle\langle 001| \\
& + n_C |\mu_C|^2 |101\rangle\langle 011| + \left(1 + n_B |\mu_A|^2 |\mu_B|^2 + n_B (n_C + 1)|\mu_B|^2 |\mu_C|^2\right) |010\rangle\langle 010| \\
& + n_B |\mu_B|^2 |110\rangle\langle 011| + \left(n_B n_C |\mu_A|^2 |\mu_B|^2 |\mu_C|^2 + n_B |\mu_B|^2 + n_C |\mu_C|^2\right) |011\rangle\langle 011| \Big],
\end{aligned}
\tag{33}
$$

where

$$
\begin{aligned}
Z_{n_B,n_C} = & 1 + \frac{1}{3}|\mu_A|^2 + \left(n_B + \frac{1}{3}\right)|\mu_B|^2 + \left(n_C + \frac{1}{3}\right)|\mu_C|^2 \\
& + \frac{1}{3} n_B |\mu_A|^2 |\mu_B|^2 + \frac{1}{3} n_C |\mu_A|^2 |\mu_C|^2 + \left(n_B n_C + \frac{1}{3} n_B + \frac{1}{3} n_C\right)|\mu_B|^2 |\mu_C|^2 \\
& + \frac{1}{3}(n_A + 1) n_B n_C |\mu_A|^2 |\mu_B|^2 |\mu_C|^2.
\end{aligned}
\tag{34}
$$

The one-tangle $\mathcal{N}_{A(BC)}$ is symmetric with respect to a_B/Ω_B and a_C/Ω_C, as shown in Fig. 9(a). $\mathcal{N}_{B(AC)}$ is shown in Fig. 9(b), and $\mathcal{N}_{C(AB)}$ can be obtained from $\mathcal{N}_{B(AC)}$ by exchanging B and C, as shown in Fig. 9(c). These symmetries are common with the GHZ state.

The two-tangle $\mathcal{N}_{AB}$ is shown in Fig. 9(d), and $\mathcal{N}_{AC}$ can be obtained from $\mathcal{N}_{AB}$ by replacing B as C, as shown in Fig. 9(e). And $\mathcal{N}_{BC}$ is shown in Fig. 9(f). All exhibit sudden death.

To see more clearly the dependence on a_B/Ω_B, we examined the two-dimensional cross sections of the 3D plots with $a_C/\Omega_C = a_B/\Omega_B$ (Fig. 10), $a_C/\Omega_C = 1.5 a_B/\Omega_B$ (Fig. 11) and $a_C/\Omega_C = 2 a_B/\Omega_B$ (Fig. 12).
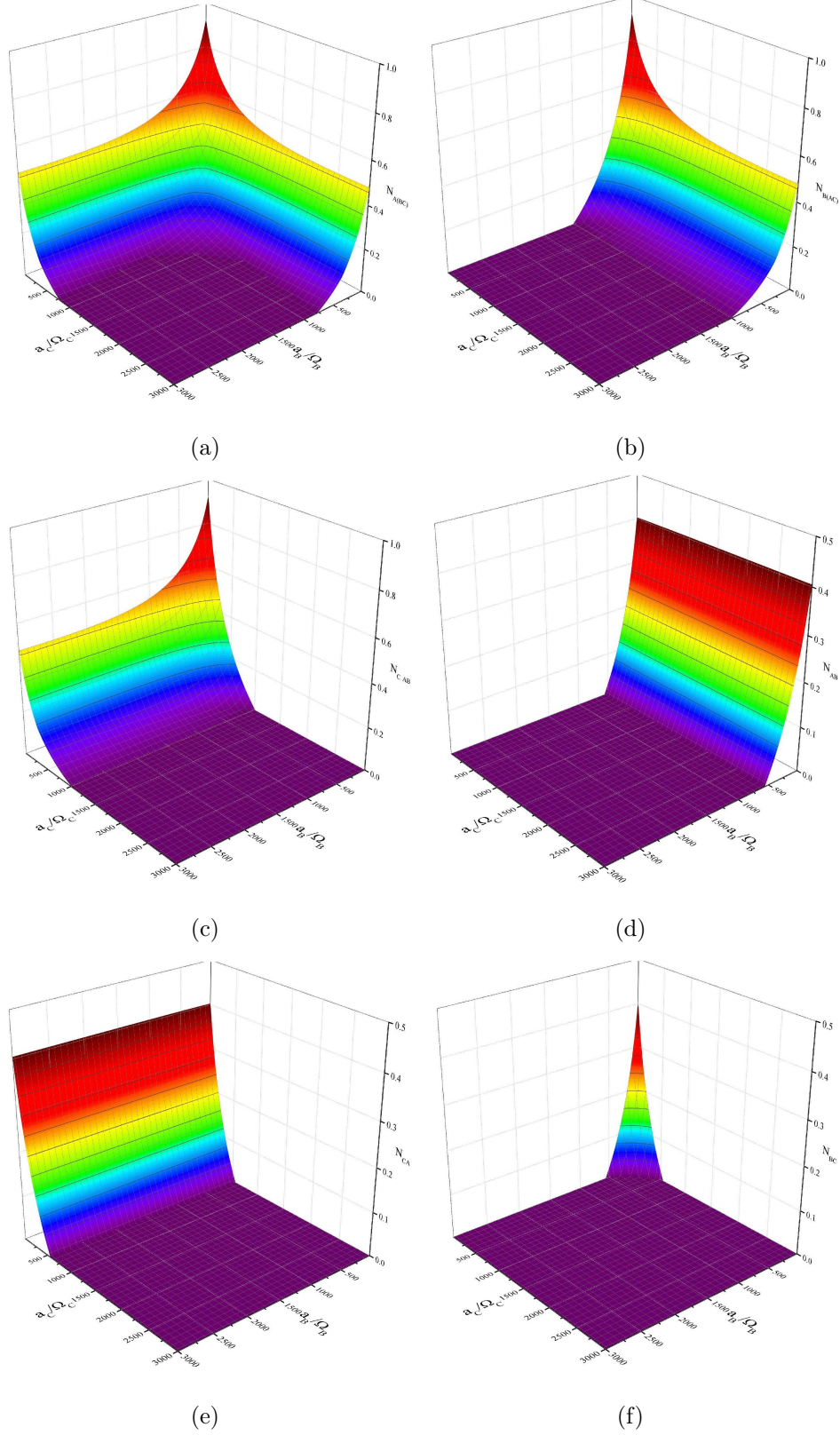
FIG. 9. Dependence of the one-tangles (a) $\mathcal{N}_{A(BC)}$, (b) $\mathcal{N}_{B(AC)}$, (c) $\mathcal{N}_{C(AB)}$ and the two-tangles (d) $\mathcal{N}_{AB}$, (e) $\mathcal{N}_{AC}$, (f) $\mathcal{N}_{BC}$ on the AFRs of qubits B and C. Qubit A moves uniformly. The qubits are in the W state.
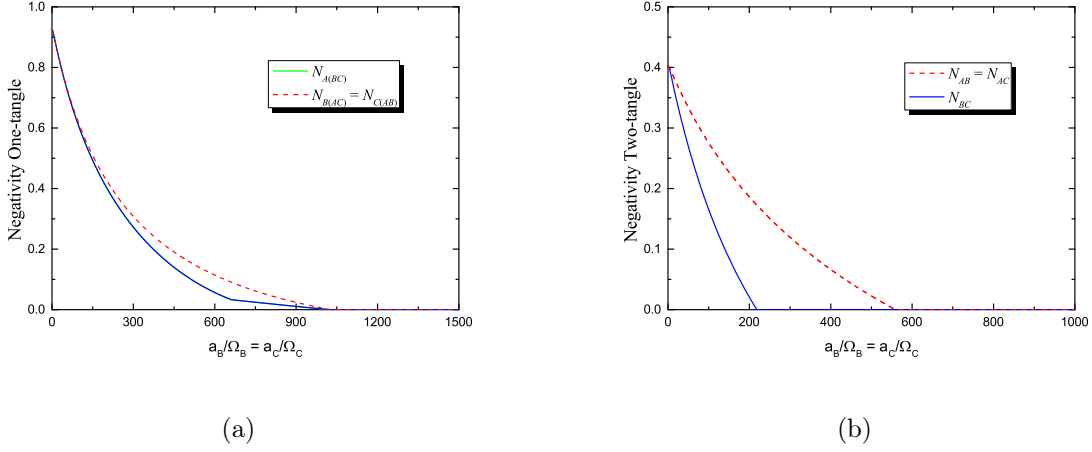
(a)　　　　　　　　　　　　　　(b)

FIG. 10. Dependence of (a) the one-tangles, (b) the two-tangles on the AFRs of qubits B and C in the case that they are equal. Qubit A moves uniformly. The qubits are in the W state.
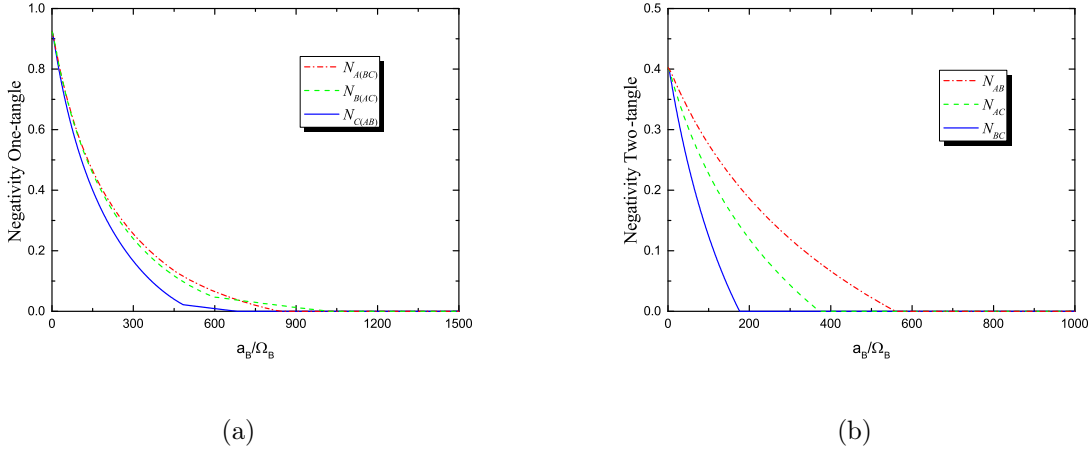

(a)　　　　　　　　　　　　　　(b)

FIG. 11. Dependence of (a) the one-tangles, (b) the two-tangles on the AFR of qubit B in the case that $a_C/\Omega_C = 1.5 a_B/\Omega_B$. Qubit A moves uniformly. The qubits are in the W state.

In these cases, in which $a_c/\Omega_C \propto a_B/\Omega_B$, the behavior of the one-tangles is similar to that in the GHZ state: each one-tangle suddenly dies at certain values of a_B/Ω_B. For $0 = a_A/\Omega_A < a_B/\Omega_B < a_C/\Omega_C$, $\mathcal{N}_{A(BC)} > \mathcal{N}_{B(AC)} > \mathcal{N}_{C(AB)}$ till the sudden death the smallest one $\mathcal{N}_{C(AB)}$, afterwards $\mathcal{N}_{B(AC)} > \mathcal{N}_{C(AB)}$. See Fig. 10(a), Fig. 11(a) and Fig. 12(a). A feature different from GHZ is that in addition to the sudden death, there appears sudden change.

We also look at one-tangles for some given values of a_B/Ω_B, as shown in Fig. 13(a) for $a_B/\Omega_B = 300$, and in Fig. 14(a) for $a_B/\Omega_B = 750$. In both figures, a feature common with
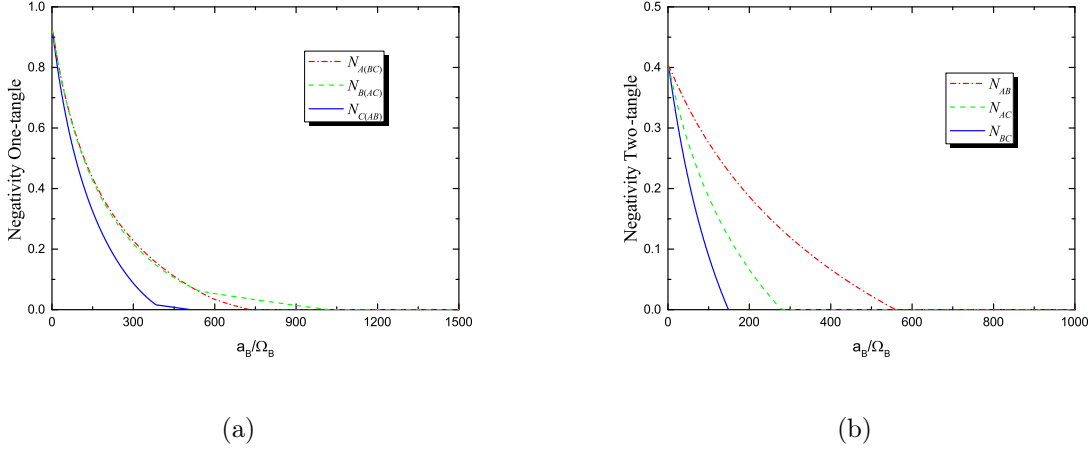
18

FIG. 12. Dependence of (a) the one-tangles, (b) the two-tangles on the AFR of qubit B in the case that $a_C/\Omega_C = 2a_B/\Omega_B$. Qubit A moves uniformly. The qubits are in the W state.

the GHZ state is that when the smallest one-tangle, namely $\mathcal{N}_{C(AB)}$, suddenly dies, the other two one-tangles exchange the order of magnitude. Compared with GHZ state, a new feature is that $\mathcal{N}_{A(BC)}$ and $\mathcal{N}_{B(AC)}$ are not monotonic with respect to a_C/Ω_C. As a_B/Ω_B is larger in Fig. 14(a) than in Fig. 13(a), the one-tangles decrease faster. In Fig. 13(a), only $\mathcal{N}_{C(AB)}$ has sudden death, while the minima of the other two one-tangles are nonzero. In Fig. 14(a), both $\mathcal{N}_{C(AB)}$ and $\mathcal{N}_{A(BC)}$ have sudden death, but $\mathcal{N}_{A(BC)}$ revives and increases at larger values of a_C/Ω_C, because B and C constitute one party, and B is fixed to be not large enough. In each of the two Figures, $\mathcal{N}_{B(AC)}$ has a nonzero minimum, as a_B/Ω_B is now fixed, while C is only one of the two qubits constituting the other party. Note that C is the qubit on which the acceleration-frequency can always be large enough.

Now we look at the two-tangles. The 2D cross sections at $a_C/\Omega_C = a_B/\Omega_B$, $a_C/\Omega_C = 1.5a_B/\Omega_B$ and $a_C/\Omega_C = 2a_B/\Omega_B$ are shown in Figs. 10(b), 11(b) and 12(b), respectively. $\mathcal{N}_{AB}$ decreases with a_B/Ω_B, $\mathcal{N}_{AC}$ decreases with a_C/Ω_C, while $\mathcal{N}_{BC}$ decreases with both. For a given value of a_B/Ω_B, $\mathcal{N}_{BC}$ is the smallest two-tangle in each of these cross sections. For a same value of a_B/Ω_B, $\mathcal{N}_{AC}$ in Fig. 10(b) is larger than in Fig. 11(b), which is larger than in Fig. 12(b), because a_C/Ω_C in Fig. 10(b) is smaller than in Fig. 11(b), which is smaller than in Fig. 12(b). Each two-tangle suddenly dies when the AFR of one or both of the parties are large enough. This is consistent with the behavior of entanglement of a two-qubit system [12].

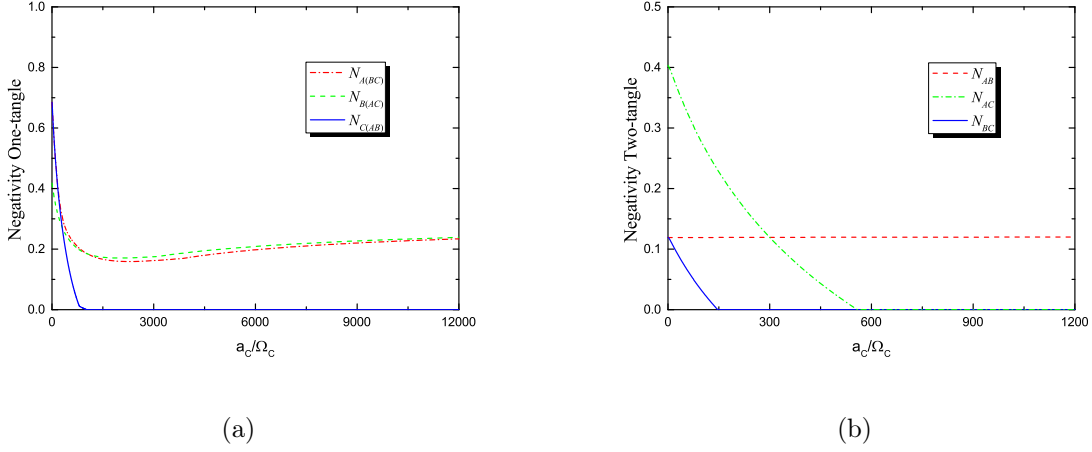In Fig. 13(b) and Fig. 14(b) we show the cases of $a_B/\Omega_B = 300$ and $a_B/\Omega_B = 750$,

(a)

(b)

FIG. 13. Dependence of (a) the one-tangles, (b) the two-tangles on the AFR of qubit C in the case that $a_B/\Omega_B = 300$. Qubit A moves uniformly. The qubits are in the W state.
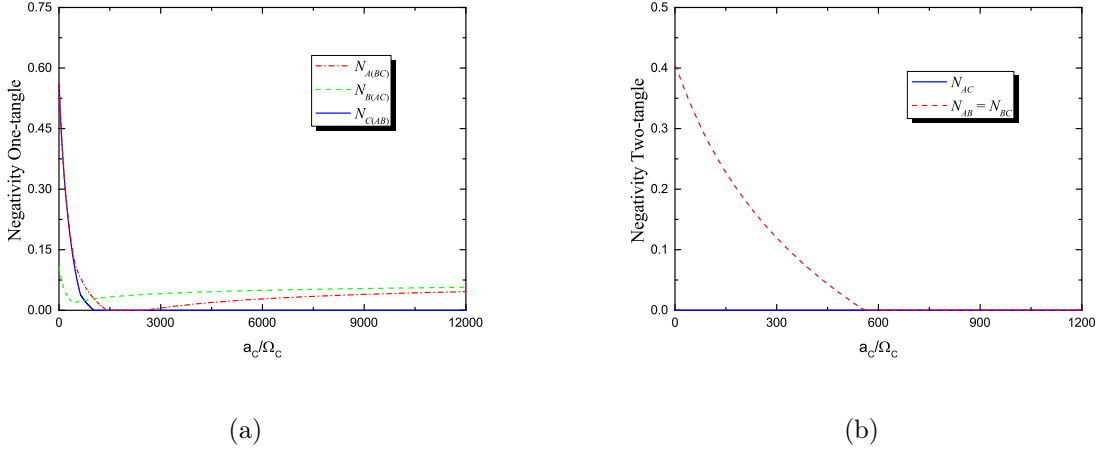

(a)

(b)

FIG. 14. Dependence of (a) the one-tangles, (b) the two-tangles on the AFR of qubit C in the case that $a_B/\Omega_B = 750$. Qubit A moves uniformly. The qubits are in the W state.

respectively. In Fig. 13(b), $\mathcal{N}_{AB}$, measuring the entanglement between A and B, is a constant independent of a_C/Ω_C. In Fig. 14(b), because a_B/Ω_B is large enough, $\mathcal{N}_{AB}$ and $\mathcal{N}_{BC}$ both keep zero, independent of a_C/Ω_C.

Finally, let us look at the three-tangle, whose 3D plot is shown in Fig. 15. To clearly see its difference with the three-tangle of GHZ state (Fig. 5), we examine some 2D cross sections. In Fig. 16(a), in which it is set that $a_C/\Omega_C \propto a_B/\Omega_B$, three-tangle dies when a_B/Ω_B are large enough, and the larger a_C/Ω_C, the quicker the decrease of three-tangle with a_B/Ω_B. This feature is similar to that of the GHZ state (Fig. 3(d)). But on the other
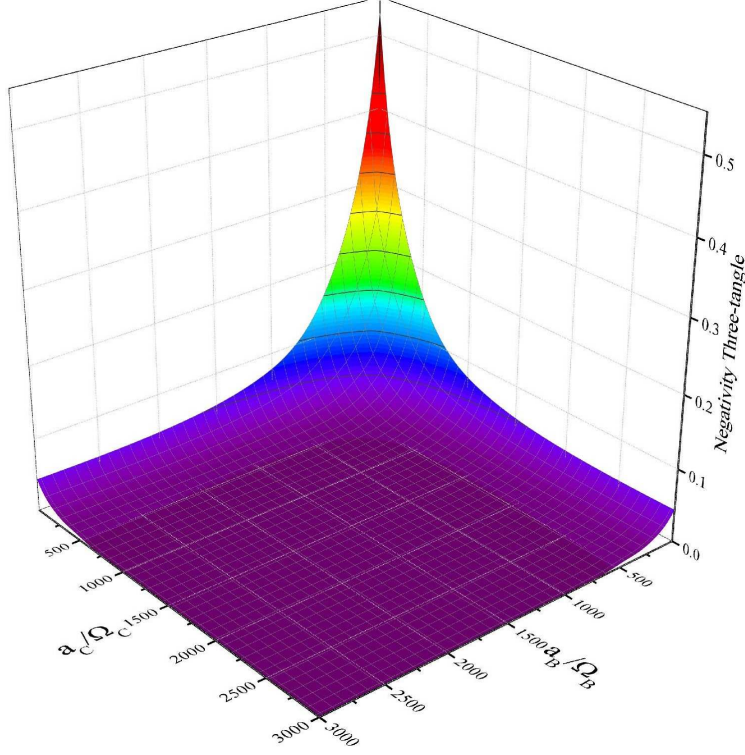
FIG. 15. Dependence of the three-tangle on the AFR of qubits B and C, while qubit A moves uniformly. The qubits are in the W state.

hand, when two of the three qubits' accelerations are not large enough, for example, when it is fixed that $a_A/\Omega_A = 0$ while $a_B/\Omega_B = 300$ or 750 (Fig. 16(b)), with the increase of a_C/Ω_C, the three-tangle decreases to a nonzero minimum, and then increases towards an asymptotic value. Also refer to Fig. 8(c) for the case of $a_A/\Omega_A = a_B/\Omega_B = 0$. Therefore it is implied that the AFRs of at least two qubits should be large enough in order that the three-tangle has sudden death.

Recall that in the GHZ state, when the AFRs of two qubits are not large enough, the three-tangle approaches zero asymptotically with the AFR of the remaining qubit tends to infinity (Figs. 1(b) and 4(d)). This is a difference between the two states.

C. A, B, and C all accelerating

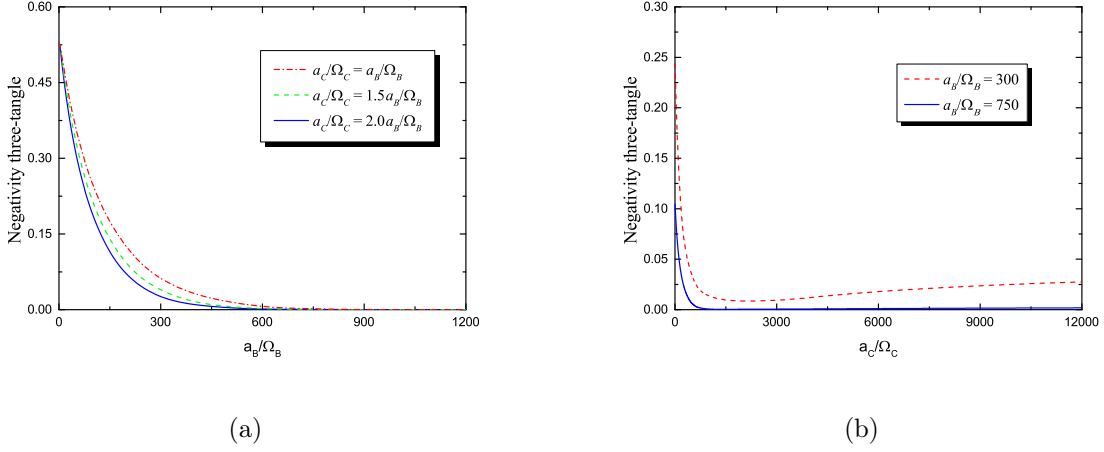Now we consider the case that all three qubits accelerate, obtaining

FIG. 16. (a) Dependence of the three-tangle on the AFR of qubit B in the case that $a_C/\Omega_C = a_B/\Omega_B$, $1.5a_B/\Omega_B$ and $2a_B/\Omega_B$. (b) Dependence of the three-tangle on the AFR of qubit C in the case that $a_B/\Omega_B = 300$ and $a_B/\Omega_B = 750$. Qubit A moves uniformly. The qubits are in the W state.

$$
\begin{aligned}
\rho_{ABC} = \eta_A^2\eta_B^2\eta_C^2 \sum_{n_A,n_B,n_C} & \frac{e^{-2\pi(n_A\Omega_A/a_A+n_B\Omega_B/a_B+n_C\Omega_C/a_C)}}{Z_{n_A,n_B,n_C}} \big[|001\rangle\langle010| \\
& +|001\rangle\langle100| + |100\rangle\langle001| + |100\rangle\langle010| + |010\rangle\langle001| + |010\rangle\langle100| \\
& +n_B|\mu_B|^2|011\rangle\langle110| + n_A|\mu_A|^2|101\rangle\langle110| + n_C|\mu_C|^2|011\rangle\langle101| \\
& +n_A|\mu_A|^2|110\rangle\langle101| + n_C|\mu_C|^2|101\rangle\langle011| + n_B|\mu_B|^2|110\rangle\langle011| \\
& + \big((n_A + 1)\,|\mu_A|^2 + (n_B + 1)\,|\mu_B|^2 + (n_C + 1)\,|\mu_C|^2\big)\,|000\rangle\langle000| \\
& + \big(1 + (n_A + 1)\,n_C|\mu_A|^2|\mu_C|^2 + (n_B + 1)\,n_C|\mu_B|^2|\mu_C|^2\big)\,|001\rangle\langle001| \\
& + \big(1 + (n_A + 1)\,n_B|\mu_A|^2|\mu_B|^2 + n_B\,(n_C + 1)\,|\mu_B|^2|\mu_C|^2\big)\,|010\rangle\langle010| \\
& + \big((n_A + 1)\,n_Bn_C|\mu_A|^2|\mu_B|^2|\mu_C|^2 + n_B|\mu_B|^2 + n_C|\mu_C|^2\big)\,|011\rangle\langle011| \\
& + \big(1 + n_A\,(n_B + 1)\,|\mu_A|^2|\mu_B|^2 + n_A\,(n_C + 1)\,|\mu_A|^2|\mu_C|^2\big)\,|100\rangle\langle100| \\
& + \big(n_A\,(n_B + 1)\,n_C|\mu_A|^2|\mu_B|^2|\mu_C|^2 + n_A|\mu_A|^2 + n_C|\mu_C|^2\big)\,|101\rangle\langle101| \\
& + \big(n_A|\mu_A|^2 + n_B|\mu_B|^2 + n_An_B\,(n_C + 1)\,|\mu_A|^2|\mu_B|^2|\mu_C|^2\big)\,|110\rangle\langle110| \\
& + \big(n_An_B|\mu_A|^2|\mu_B|^2 + n_An_C|\mu_A|^2|\mu_C|^2 + n_Bn_C|\mu_B|^2|\mu_C|^2\big)\,|111\rangle\langle111|\big],
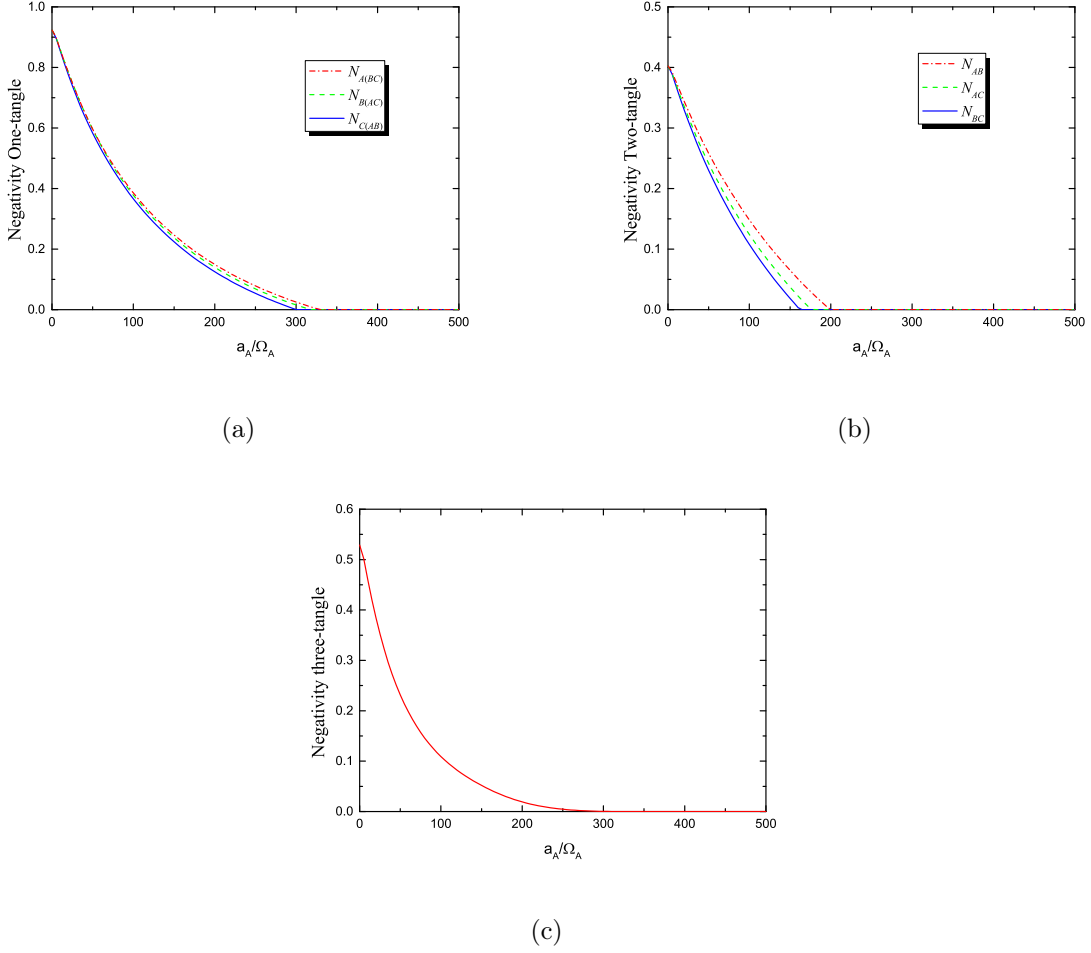\end{aligned}
$$

(35)

(a)

(b)

(c)

FIG. 17. Dependence of (a) the one-tangles, (b) the two-tangles, (c) the three-tangle on the AFR of qubit A in the case that $a_B/\Omega_B = 1.2a_A/\Omega_A$, $a_C/\Omega_C = 1.5a_A/\Omega_A$. The qubits are in the W state.

where

$$
\begin{aligned}
Z_{n_A,n_B,n_C} =&3 + (3n_A n_B n_C + n_A n_B + n_A n_C + n_B n_C)|\mu_A|^2|\mu_B|^2|\mu_C|^2 \\
&+ (3n_A + 1)|\mu_A|^2 + (3n_B n_C + n_B + n_C)|\mu_B|^2|\mu_C|^2 \\
&+ (3n_B + 1)|\mu_B|^2 + (3n_A n_C + n_A + n_C)|\mu_A|^2|\mu_C|^2 \\
&+ (3n_C + 1)|\mu_C|^2 + (3n_A n_B + n_A + n_B)|\mu_A|^2|\mu_B|^2.
\end{aligned}
\tag{36}
$$

As an example, we specify $a_B/\Omega_B = 1.2a_A/\Omega_A$, $a_C/\Omega_C = 1.5a_A/\Omega_A$. One-tangles, two-tangles and three-tangle are shown in Figs. 17(a), 17(b) and Fig. 17(c), respectively. The behavior of one-tangles and three-tangle of the W state is similar to that of the GHZ state as shown in Fig. 7, but under the same condition, the values of these tangles are smaller in

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W state. Like in the GHZ case, $\mathcal{N}_{C(AB)}$ suddenly dies first, because of the larger value of a_C/Ω_C. Afterwards, $\mathcal{N}_{A(BC)}$ and $\mathcal{N}_{B(AC)}$ do not intersect because a_A/Ω_A is too large. The three-tangle suddenly dies when all one-tangles become zero. Note that when one-tangle dies, the relevant two two-tangles have already died because of the relation (17). Also note that a sudden change occurs in the three-tangle when the one-tangles and two-tangles suddenly die.

The cases that one or more AFRs are not large enough can be inferred from the cases of only one or two qubits accelerate discussed above.

V. SUMMARY

In this paper, we have studied how Unruh effect influences various kinds of entanglement of three qubits, each of which is coupled with an ambient scalar field. The initial states have been considered to be two typical three-qubit entangled states, the GHZ and the W states, which represent two types of tripartite entanglement. We have studied the cases of one qubit accelerating, two qubits accelerating and three qubits accelerating. The details are summarized in Table I.

A two-tangle measures bipartite entanglement between two qubits, with the other qubit traced out. For the GHZ state as the initial state, the two-tangles always remain zero. For the W state as the initial state, the two-tangle between two qubits remains a nonzero constant if these two qubits move uniformly. If at least one of the two qubits accelerates, their two-tangle suddenly dies when the AFR of one or both of these two qubits are large enough.

When two qubits move uniformly while the other accelerates, the one-tangle between the accelerating qubit and the party consisting of the two uniformly moving qubits suddenly dies if the AFR of the accelerating qubit is large enough. However, each of the other two one-tangles, between one uniformly moving qubit and the party consisting of the other uniformly moving qubit and the accelerating qubit, approaches an asymptotic value as the AFR tends to infinity. For the GHZ state, the asymptotic value is zero. For the W state, the asymptotic value is nonzero and is larger than the local minima existing at finite values of the AFR.

When two qubits accelerate while the other moves uniformly, the one-tangle between the uniformly moving qubit and the party consisting of the two accelerating qubits suddenly dies

if the AFRs of both of the two accelerating qubits are large enough. Each of the other two one-tangles, between one accelerating qubit and the party consisting of the other accelerating qubit and the uniformly moving qubit, suddenly dies when the AFR of the qubit which is by itself one party is large enough. It weakly depends on the AFR of the other accelerating qubit. These features are common in the GHZ and the W states.

		GHZ	**W**
$a_A = 0,$ $a_B = 0,$ $a_C \neq 0.$	1-tangles	$\mathcal{N}_{C(AB)}$ SD at a finite a_C/Ω_C. $\mathcal{N}_{A(BC)} = \mathcal{N}_{B(AC)} \to 0$ as $a_C/\Omega_C \to \infty$.	$\mathcal{N}_{C(AB)}$ SD at a finite a_C/Ω_C. $\mathcal{N}_{A(BC)} = \mathcal{N}_{B(AC)}$ is non-monotonic, with nonzero local minima, $\to \frac{2}{3}$ as $a_C/\Omega_C \to \infty$.
	2-tangles	0.	$\mathcal{N}_{AB}$ remains constant. $\mathcal{N}_{AC} = \mathcal{N}_{BC}$ SD at a finite a_C/Ω_C.
	3-tangle	$\to 0$ as $a_C/\Omega_C \to \infty$. There is a sudden change where $\mathcal{N}_{C(AB)}$ SD.	Nonmonotonic, with a nonzero local minimum at a finite a_C/Ω_C, $\to (\sqrt{5} - 1)/27$ as $a_C/\Omega_C \to \infty$.
$a_A = 0,$ $a_B \neq 0,$ $a_C \neq 0.$	1-tangles	$\mathcal{N}_{A(BC)}$ SD at finite values of both a_B/Ω_B and a_C/Ω_C. $\mathcal{N}_{\beta(A\gamma)}$ SD at a finite value of a_β/Ω_β, while weakly varies with a_γ/Ω_γ.	$\mathcal{N}_{A(BC)}$ eventually SD if both a_B/Ω_B and a_C/Ω_C are large enough. It is nonmonotonic, with nonzero local minima, and can SD and revive with the increase of one AFR while the other is not large enough. $\mathcal{N}_{\beta(A\gamma)}$ SD when a_β/Ω_β is large enough. It has nonzero local minima and has sudden changes if a_β/Ω_β is not large enough no matter how large is a_γ/Ω_γ.

		GHZ	**W**
$a_A \neq 0,$ $a_B \neq 0,$ $a_C \neq 0.$	2-tangles	0.	SD when one or both of the two relevant qubits have large enough AFRs.
	3-tangle	SD if a_B/Ω_B and a_C/Ω_C are both large enough.	SD if a_B/Ω_B and a_C/Ω_C are both large enough. It has nonmonotonicity and local nonzero minima, $\rightarrow$ a nonzero asymptotic value when only one AFR $\rightarrow \infty$ while the other nonzero AFR is not large enough.
	1-tangles	$\mathcal{N}_{\alpha(\beta\gamma)}$ SD if the AFR of α or AFRs of both β and γ are large enough.	$\mathcal{N}_{\alpha(\beta\gamma)}$ SD if the AFR of α or AFRs of both β and γ are large enough. There exists nonmonotonicity.
	2-tangles	0.	SD when one or both of the two relevant qubits have large enough AFRs.
	3-tangle	SD after all 1-tangles SD, that is, if at least two of three qubits have large enough AFRs.	SD after all 1-tangles SD, that is, if at least two of three qubits have large enough AFRs.

TABLE I: Comparison between the GHZ and the W states of
the entanglement behavior caused by the Unruh fields. SD is
the acronym for "sudden death" or "suddenly die". By non-
monoticity, it is with respect to one AFR.

When all qubits accelerate, for both the GHZ and the W states, the one-tangle $\mathcal{N}_{\alpha(\beta\gamma)}$
suddenly dies if a_α/Ω_α is large enough, or both a_β/Ω_β and a_γ/Ω_γ are large enough.

Therefore, overall speaking, all the one-tangles eventually suddenly die if at least two of three qubits have large enough AFRs. When all the one-tangles suddenly die, all the two-tangles must have also died, as dictated by the monogamy relation (17), consequently the three-tangles also suddenly die. The main difference between W state and the GHZ state is that for the W state, there exists nonmonotocity with respect to the AFR of each qubit alone.

It is well known that near the horizon $r = 2m$ of a black hole, the Schwarzschild metric can be approximated as the Rindler metric with the acceleration $a = \frac{m}{r^2}(1 - \frac{2m}{r})^{-1/2}$, while the uniform movement corresponds to free falling into the black hole [19]. Therefore, the above result can be translated to be near the horizon of a black hole, with $r \approx 2m[1 - \frac{1}{(4ma)^2}]^{-1}$ corresponding to acceleration a.

The calculations in this paper imply that near the horizon of a black hole, for three qubits coupled with scalar fields, all the one-tangles and then all the two-tangles and the three-tangle eventually die if at least two of the three qubits are close enough to the horizon $2m$. That is, all kinds of entanglement of the field-coupled qubits are eventually killed by the black hole horizon. Finally, we conjecture that for N particles, each of which is coupled with a scalar field, all kinds of entanglement suddenly die if $N-1$ particles are close enough to the horizon of a black hole.

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[1] V. F. Mukhanov and S. Winitzki, *Introduction to Quantum Effects in Gravity*, Cambridge University Press (2007).

[2] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975).

[3] S. A. Fulling, Phys. Rev. D **7**, 2850 (1973).

[4] P. C. W. Davies, J. Phys. A **8**, 609 (1975).

[5] B. S. DeWitt, Phys. Rep. **19**, 295 (1975).

[6] W. G. the Unruh, Phys. Rev. D **14**, 870 (1976).

[7] S.-Y. Lin, C.-H. Chou and B. L. Hu, Phys. Rev. D. **78**, 125025 (2008).

[8] A. G. S. Landulfo and G. E. A. Matsas, Phys. Rev. A **80**, 032315 (2009).

[9] J. Doukas and L. C. L. Hollenberg, Phys. Rev. A **79**, 052109 (2009).

[10] J. Doukas and B. Carson, Phys. Rev. A 81, 062320 (2010).

[11] J. Hu and H. Yu, Phys. Rev. A **91**, 012327 (2015).

[12] Y. Dai, Z. Shen and Y. Shi, JHEP **09**, 071 (2015).

[13] C. H. Bennett, S. Popescu, D. Rohrlich, J. A. Smolin, A. V. Thapliyal, Phys. Rev. A **63**, 012307 (2000).

[14] G. Vedal and R. F. Werner, Phys. Rev. A **65**, 032314 (2002).

[15] Y. C. Ou and H. Fan, Phys. Rev. A **75**, 062308 (2007).

[16] W. Dür, G. Vidal and J. I. Cirac, Phys. Rev. A **62**, 062314 (2000).

[17] W. G. the Unruh and R. M. Wald, Phys. Rev. D **29**, 1047 (1984).

[18] T. Yu and J. H. Eberly, Science **23**, 598 (2009).

[19] W. Rindler, Am. J. Phys. **34**, 1174 (1966).