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Leptonic $g-2$ moments, CP phases and the Higgs boson mass constraint

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Abstract

Higgs boson mass measurement at ~ 125 GeV points to a high scale for SUSY specifically the scalar masses. If all the scalars are heavy, supersymmetric contribution to the leptonic $g-2$ moments will be significantly reduced. On the other hand the Brookhaven experiment indicates a $\sim 3\sigma$ deviation from the standard model prediction. Here we analyze the leptonic $g-2$ moments in an extended MSSM model with inclusion of a vector like leptonic generation which brings in new sources of CP violation. In this work we consider the contributions to the leptonic $g-2$ moments arising from the exchange of charginos and neutralinos, sleptons and mirror sleptons, and from the exchange of W and Z bosons and of leptons and mirror leptons. We focus specifically on the $g-2$ moments for the muon and the electron where sensitive measurements exist. Here it is shown that one can get consistency with the current data on $g-2$ under the Higgs boson mass constraint. Dependence of the moments on CP phases from the extended sector are analyzed and it is shown that they are sensitively dependent on the phases from the new sector. It is shown that the corrections to the leptonic moments arising from the extended MSSM sector will be non-vanishing even if the SUSY scale extends into the PeV region.

Keywords: Leptonic moments, CP phases, Higgs mass, PeV scale.

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I. 1. INTRODUCTION

The observation by ATLAS [1] and by CMS [2] of the Higgs boson with a mass of $\sim$ 125 GeV has put very stringent constraints on low scale supersymmetry. Since the tree level mass of the Higgs boson lies below M_Z, a large loop correction from the supersymmetric sector is needed which in turn implies a high scale for the weak scale supersymmetry and specifically for the scalar masses. A large SUSY scale also has direct implications for the $g_\mu - 2$ of the muon. Thus the current experimental result gives for the muon $g-2$ [3]

$$\Delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = (26.2 \pm 8.5) \times 10^{-10}, \tag{1}$$

which is about a three sigma deviation from the standard model prediction. Similarly for the electron the experimental determination of $g_e - 2$ is very accurate and the uncertainty is rather small, i.e., one has [4]

$$\Delta a_e = a_e^{\rm exp} - a_e^{\rm SM} = -10.5(8.1) \times 10^{-13}. \tag{2}$$

This result relies on a QED calculation up to four loops. Thus along with Eq. (1), Eq. (2) also acts as a constraint on the standard model extensions. Supersymmetric theories with low weak scale mass can make corrections to $g_\mu - 2$ which could be as large as the standard model electroweak corrections and even larger and have strong CP phase dependence [5–7] (for early work see [8]). These arise largely from the chargino and sneutrino exchange diagram with the neutralino and smuon exchange diagram making a relatively small contribution. However, if the scalar masses are large, the supersymmetric exchange contributions will be small due to the largeness of the sneutrino and the smuon masses.

In this work we give an analysis of the $g-2$ for the muon and for the electron in an extended MSSM model with a vector like leptonic generation. We note that vector like multiplets are anomaly free and they appear in a variety of settings which include grand unified models, strings and D brane models [9–13]. Further, it is known that $g-2$ has a sharp dependence on CP phases [5–7]. For this reason we investigate also the dependence of the muon and the electron $g-2$ on the CP phases in the extended MSSM model. Here we are particularly interested in the dependence on the CP phases that arise from the new sector involving vector like leptons. We note that the CP phases are constrained in this case by the electric dipole moment of the electron which currently has the value $|d_e| < 8.7 \times 10^{-29}$ $e$cm [34] while the upper limit on the muon EDM is $|d_\mu| < 1.9 \times 10^{-19} e$cm [3] and is rather weak. As discussed in several works even with large phases the EDMs can be suppressed either by mass suppression [14, 15] or via the cancellation mechanism [16, 16–19, 22]. Several analyses of the vector like extensions of MSSM already exist in the literature [20, 21, 23–32].

The outline of the rest of the paper is as follows: In section 2 we give an analytical computation for the contribution of the vectorlike lepton generation to $g-2$ of the muon and of the electron. In section (3) we give a numerical analysis of the contributions arising from MSSM and from the extended MSSM with a vector like leptonic generation. Conclusions are given in section 4. Details of the extended MSSM model with a vector like leptonic generation are given in the Appendix. The explanation of the muon anomaly with vector like leptons was considered previously in [30] within a non-supersymmetric framework. Our analysis is within a supersymmetric framework where we carry out a simultaneous fit to both the muon as well as the electron anomaly. Further, we explore the implications of the CP phases arising from the new sector.

II. 2. ANALYSIS OF $g_\mu - 2$ AND $g_e - 2$ WITH EXCHANGE OF VECTOR LIKE LEPTONS

The extended MSSM with a vector like leptonic generation is discussed in detail in the Appendix. Using the formalism described there we compute the contribution to the anomalous magnetic moment of a charged lepton ℓ_α. We discuss now in detail the various contributions. The contribution arising from the exchange of the charginos, sneutrinos and mirror sneutrinos as shown in the left diagram in Fig. 1 is given by

$$a_\alpha^{\chi^+} = -\sum_{i=1}^{2}\sum_{j=1}^{10} \frac{m_{\tau_\alpha}}{16\pi^2 m_{\chi_i^-}} \text{Re}(C^L_{\alpha ij} C^{R*}_{\alpha ij}) F_3\left(\frac{m^2_{\tilde{\nu}_j}}{m^2_{\chi_i^-}}\right) + \sum_{i=1}^{2}\sum_{j=1}^{10} \frac{m^2_{\tau_\alpha}}{96\pi^2 m^2_{\chi_i^-}} \left[|C^L_{\alpha ij}|^2 + |C^R_{\alpha ij}|^2\right] F_4\left(\frac{m^2_{\tilde{\nu}_j}}{m^2_{\chi_i^-}}\right), \tag{3}$$

where $m_{\chi_i^-}$ is the mass of chargino χ_i^- and $m_{\tilde{\nu}_j}$ is the mass of sneutrino $\tilde{\nu}_j$ and where the form factors F_3 and F_4 are given by

$$F_3(x) = \frac{1}{(x-1)^3}\left[3x^2 - 4x + 1 - 2x^2 \ln x\right], \tag{4}$$

and

$$F_4(x) = \frac{1}{(x-1)^4}\left[2x^3 + 3x^2 - 6x + 1 - 6x^2 \ln x\right]. \tag{5}$$

The couplings appearing in Eq. (3) are given by

$$C^L_{\alpha ij} = g(-\kappa_\tau U^*_{i2} D^{\tau *}_{R1\alpha} \tilde{D}^\nu_{1j} - \kappa_\mu U^*_{i2} D^{\tau *}_{R3\alpha} \tilde{D}^\nu_{5j} - \kappa_e U^*_{i2} D^{\tau *}_{R4\alpha} \tilde{D}^\nu_{7j} - \kappa_{4\ell} U^*_{i2} D^{\tau *}_{R5\alpha} \tilde{D}^\nu_{9j} + U^*_{i1} D^{\tau *}_{R2\alpha} \tilde{D}^\nu_{4j} - \kappa_N U^*_{i2} D^{\tau *}_{R2\alpha} \tilde{D}^\nu_{2j}), \tag{6}$$

$$C^R_{\alpha ij} = g(-\kappa_{\nu_\tau} V_{i2} D^{\tau *}_{L1\alpha} \tilde{D}^\nu_{3j} - \kappa_{\nu_\mu} V_{i2} D^{\tau *}_{L3\alpha} \tilde{D}^\nu_{6j} - \kappa_{\nu_e} V_{i2} D^{\tau *}_{L4\alpha} \tilde{D}^\nu_{8j} + V_{i1} D^{\tau *}_{L1\alpha} \tilde{D}^\nu_{1j} + V_{i1} D^{\tau *}_{L3\alpha} \tilde{D}^\nu_{5j} - \kappa_{\nu_4} V_{i2} D^{\tau *}_{L5\alpha} \tilde{D}^\nu_{10j} + V_{i1} D^{\tau *}_{L4\alpha} \tilde{D}^\nu_{7j} - \kappa_E V_{i2} D^{\tau *}_{L2\alpha} \tilde{D}^\nu_{4j}), \tag{7}$$

where $D^\tau_{L,R}$ and $\tilde{D}^\nu$ are the charged lepton and sneutrino diagonalizing matrices and are defined by Eq. (47) and Eq.(57) and U and V are the matrices that diagonalize the chargino mass matrix M_C so that [33]

$$U^* M_C V^{-1} = diag(m_{\chi_1^\pm} m_{\chi_2^\pm}). \tag{8}$$

Further,

$$(\kappa_N, \kappa_\tau, \kappa_\mu, \kappa_e, \kappa_{4\ell}) = \frac{(m_N, m_\tau, m_\mu, m_e, m_{4\ell})}{\sqrt{2} m_W \cos\beta}, \tag{9}$$

$$(\kappa_E, \kappa_{\nu_\tau}, \kappa_{\nu_\mu}, \kappa_{\nu_e}, \kappa_{\nu_4}) = \frac{(m_E, m_{\nu_\tau}, m_{\nu_\mu}, m_{\nu_e}, m_{\nu_4})}{\sqrt{2} m_W \sin\beta}, \tag{10}$$

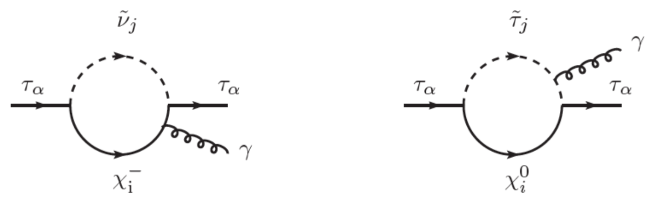

FIG. 1: The diagrams that contribute to the leptonic (τ_α) magnetic dipole moment via exchange of charginos (χ_i^-), sneutrinos and mirror sneutrinos ($\tilde{\nu}_j$) (left diagram) inside the loop and from the exchange of neutralinos (χ_i^0), sleptons and mirror sleptons ($\tilde{\tau}_j$) (right diagram) inside the loop.

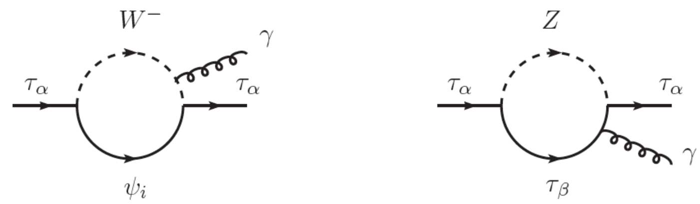

FIG. 2: The W loop (the left diagram) involving the exchange of sequential and vectorlike neutrinos ψ_i and the Z loop (the right diagram) involving the exchange of sequential and vectorlike charged leptons τ_β that contribute to the magnetic dipole moment of the charged lepton τ_α.

where m_W is the mass of the W boson and $\tan\beta = < H_2^2 > / < H_1^1 >$ where H_1, H_2 are the two Higgs doublets of MSSM.

The contribution arising from the exchange of neutralinos, charged sleptons and charged mirror sleptons as shown in the right diagram in Fig. 1 is given by

$$a_\alpha^{\chi^0} = \sum_{i=1}^{4}\sum_{j=1}^{10} \frac{m_{\tau_\alpha}}{16\pi^2 m_{\chi_i^0}} \mathrm{Re}(C'^{L}_{\alpha ij} C'^{R*}_{\alpha ij}) F_1\left(\frac{m^2_{\tilde{\tau}_j}}{m^2_{\chi_i^0}}\right) + \sum_{i=1}^{4}\sum_{j=1}^{10} \frac{m^2_{\tau_\alpha}}{96\pi^2 m^2_{\chi_i^0}} \left[|C'^{L}_{\alpha ij}|^2 + |C'^{R}_{\alpha ij}|^2\right] F_2\left(\frac{m^2_{\tilde{\tau}_j}}{m^2_{\chi_i^0}}\right), \tag{11}$$

where the form factors are

$$F_1(x) = \frac{1}{(x-1)^3}\left[1 - x^2 + 2x\ln x\right], \tag{12}$$

and

$$F_2(x) = \frac{1}{(x-1)^4}\left[-x^3 + 6x^2 - 3x - 2 - 6x\ln x\right]. \tag{13}$$

The couplings that enter in Eq. 11 are given by

$$C'^{L}_{\alpha ij} = \sqrt{2}(\alpha_{\tau i} D^{\tau *}_{R1\alpha}\tilde{D}^{\tau}_{1j} - \delta_{Ei} D^{\tau *}_{R2\alpha}\tilde{D}^{\tau}_{2j} - \gamma_{\tau i} D^{\tau *}_{R1\alpha}\tilde{D}^{\tau}_{3j} + \beta_{Ei} D^{\tau *}_{R2\alpha}\tilde{D}^{\tau}_{4j} + \alpha_{\mu i} D^{\tau *}_{R3\alpha}\tilde{D}^{\tau}_{5j} - \gamma_{\mu i} D^{\tau *}_{R3\alpha}\tilde{D}^{\tau}_{6j} + \alpha_{ei} D^{\tau *}_{R4\alpha}\tilde{D}^{\tau}_{7j} - \gamma_{ei} D^{\tau *}_{R4\alpha}\tilde{D}^{\tau}_{8j} + \alpha_{4\ell i} D^{\tau *}_{R5\alpha}\tilde{D}^{\tau}_{9j} - \gamma_{4\ell i} D^{\tau *}_{R5\alpha}\tilde{D}^{\tau}_{10j}), \tag{14}$$

$$C'^{R}_{\alpha ij} =\sqrt{2}(\beta_{\tau i}D^{\tau*}_{L1\alpha}\tilde{D}^{\tau}_{1j} - \gamma_{Ei}D^{\tau*}_{L2\alpha}\tilde{D}^{\tau}_{2j} - \delta_{\tau i}D^{\tau*}_{L1\alpha}\tilde{D}^{\tau}_{3j} + \alpha_{Ei}D^{\tau*}_{L2\alpha}\tilde{D}^{\tau}_{4j} + \beta_{\mu i}D^{\tau*}_{L3\alpha}\tilde{D}^{\tau}_{5j} - \delta_{\mu i}D^{\tau*}_{L3\alpha}\tilde{D}^{\tau}_{6j}$$
$$+ \beta_{ei}D^{\tau*}_{L4\alpha}\tilde{D}^{\tau}_{7j} - \delta_{ei}D^{\tau*}_{L4\alpha}\tilde{D}^{\tau}_{8j} + \beta_{4\ell i}D^{\tau*}_{L5\alpha}\tilde{D}^{\tau}_{9j} - \delta_{4\ell i}D^{\tau*}_{L5\alpha}\tilde{D}^{\tau}_{10j}), \tag{15}$$

where

$$\alpha_{Ei} = \frac{gm_E X^*_{4i}}{2m_W \sin\beta} \ ; \qquad \beta_{Ei} = eX'_{1i} + \frac{g}{\cos\theta_W}X'_{2i}\left(\frac{1}{2} - \sin^2\theta_W\right) \tag{16}$$
$$\gamma_{Ei} = eX'^*_{1i} - \frac{g\sin^2\theta_W}{\cos\theta_W}X'^*_{2i} \ ; \qquad \delta_{Ei} = -\frac{gm_E X_{4i}}{2m_W \sin\beta}, \tag{17}$$

and

$$\alpha_{\tau i} = \frac{gm_\tau X_{3i}}{2m_W\cos\beta} \ ; \qquad \alpha_{\mu i} = \frac{gm_\mu X_{3i}}{2m_W\cos\beta} \ ; \qquad \alpha_{ei} = \frac{gm_e X_{3i}}{2m_W\cos\beta}; \qquad \alpha_{4\ell i} = \frac{gm_{4\ell} X_{3i}}{2m_W\cos\beta} \tag{18}$$
$$\delta_{\tau i} = -\frac{gm_\tau X^*_{3i}}{2m_W\cos\beta} \ ; \qquad \delta_{\mu i} = -\frac{gm_\mu X^*_{3i}}{2m_W\cos\beta} \ ; \qquad \delta_{ei} = -\frac{gm_e X^*_{3i}}{2m_W\cos\beta}; \qquad \delta_{4\ell i} = -\frac{gm_{4\ell} X^*_{3i}}{2m_W\cos\beta}, \tag{19}$$

and where

$$\beta_{\tau i} = \beta_{\mu i} = \beta_{ei} = \beta_{4\ell i} = -eX'^*_{1i} + \frac{g}{\cos\theta_W}X'^*_{2i}\left(-\frac{1}{2} + \sin^2\theta_W\right), \tag{20}$$
$$\gamma_{\tau i} = \gamma_{\mu i} = \gamma_{ei} = \gamma_{4\ell i} = -eX'_{1i} + \frac{g\sin^2\theta_W}{\cos\theta_W}X'_{2i}. \tag{21}$$

Here X' are defined by

$$X'_{1i} = X_{1i}\cos\theta_W + X_{2i}\sin\theta_W, \tag{22}$$
$$X'_{2i} = -X_{1i}\sin\theta_W + X_{2i}\cos\theta_W, \tag{23}$$

where X diagonalizes the neutralino mass matrix, i.e.,

$$X^T M_{\chi^0} X = \text{diag}(m_{\chi^0_1}, m_{\chi^0_2}, m_{\chi^0_3}, m_{\chi^0_4}). \tag{24}$$

Further, $\tilde{D}^\tau$ that enter in Eqs. (14) and (15) is a matrix which diagonalizes the charged slepton mass squared matrix and is defined in Eq. (53).

Next we compute the contribution from the exchange of the W and Z bosons. Thus the exchange of the W and the exchange of neutrinos and mirror neutrinos as shown in the left diagram of Fig. 2 gives

$$a^W_{\tau_\alpha} = \frac{m^2_{\tau_\alpha}}{16\pi^2 m^2_W}\sum_{i=1}^{5}[|C^W_{Li\alpha}|^2 + |C^W_{Ri\alpha}|^2]F_W\left(\frac{m^2_{\psi_i}}{m^2_W}\right) + \frac{m_{\psi_i}}{m_{\tau_\alpha}}\text{Re}(C^W_{Li\alpha}C^{W*}_{Ri\alpha})G_W\left(\frac{m^2_{\psi_i}}{m^2_W}\right), \tag{25}$$

where the form factors are given by

$$F_W(x) = \frac{1}{6(x-1)^4}\left[4x^4 - 49x^3 + 18x^3\ln x + 78x^2 - 43x + 10\right], \tag{26}$$

and

$$G_W(x) = \frac{1}{(x-1)^3}\left[4 - 15x + 12x^2 - x^3 - 6x^2\ln x\right]. \tag{27}$$

The couplings that enter in Eq. (25) are given by

$$C^W_{L_{i\alpha}} = \frac{g}{\sqrt{2}}[D^{\nu *}_{L1i}D^{\tau}_{L1\alpha} + D^{\nu *}_{L3i}D^{\tau}_{L3\alpha} + D^{\nu *}_{L4i}D^{\tau}_{L4\alpha} + D^{\nu *}_{L5i}D^{\tau}_{L5\alpha}], \tag{28}$$

$$C^W_{R_{i\alpha}} = \frac{g}{\sqrt{2}}[D^{\nu *}_{R2i}D^{\tau}_{R2\alpha}]. \tag{29}$$

Here $D^{\nu}_{L,R}$ are matrices of a bi-unitary transformation that diagonalizes the neutrino mass matrix and are defined in Eq. (43).

Finally the exchange of the Z and the exchange of leptons and mirror leptons as shown in the right diagram of Fig. 2 gives

$$a^Z_{\tau_\alpha} = \frac{m^2_{\tau_\alpha}}{32\pi^2 m^2_Z}\sum_{\beta=1}^{5}[[|C^Z_{L\beta\alpha}|^2 + |C^Z_{R\beta\alpha}|^2]F_Z\left(\frac{m^2_{\tau_\beta}}{m^2_Z}\right) + \frac{m_{\tau_\beta}}{m_{\tau_\alpha}}\mathrm{Re}(C^Z_{L\beta\alpha}C^{Z*}_{R\beta\alpha})G_Z\left(\frac{m^2_{\tau_\beta}}{m^2_Z}\right), \tag{30}$$

where

$$F_Z(x) = \frac{1}{3(x-1)^4}\left[-5x^4 + 14x^3 - 39x^2 + 18x^2\ln x + 38x - 8\right], \tag{31}$$

and

$$G_Z(x) = \frac{2}{(x-1)^3}\left[x^3 + 3x - 6x\ln x - 4\right], \tag{32}$$

and m_Z is the Z boson mass. The couplings that enter in Eq. (30) are given by

$$C^Z_{L_{\alpha\beta}} = \frac{g}{\cos\theta_W}[x(D^{\tau\dagger}_{L\alpha 1}D^{\tau}_{L1\beta} + D^{\tau\dagger}_{L\alpha 2}D^{\tau}_{L2\beta} + D^{\tau\dagger}_{L\alpha 3}D^{\tau}_{L3\beta} + D^{\tau\dagger}_{L\alpha 4}D^{\tau}_{L4\beta} + D^{\tau\dagger}_{L\alpha 5}D^{\tau}_{L5\beta}) -\frac{1}{2}(D^{\tau\dagger}_{L\alpha 1}D^{\tau}_{L1\beta} + D^{\tau\dagger}_{L\alpha 3}D^{\tau}_{L3\beta} + D^{\tau\dagger}_{L\alpha 4}D^{\tau}_{L4\beta} + D^{\tau\dagger}_{L\alpha 5}D^{\tau}_{L5\beta})], \tag{33}$$

and

$$C^Z_{R_{\alpha\beta}} = \frac{g}{\cos\theta_W}[x(D^{\tau\dagger}_{R\alpha 1}D^{\tau}_{R1\beta} + D^{\tau\dagger}_{R\alpha 2}D^{\tau}_{R2\beta} + D^{\tau\dagger}_{R\alpha 3}D^{\tau}_{R3\beta} + D^{\tau\dagger}_{R\alpha 4}D^{\tau}_{R4\beta} + D^{\tau\dagger}_{R\alpha 5}D^{\tau}_{R5\beta}) -\frac{1}{2}(D^{\tau\dagger}_{R\alpha 2}D^{\tau}_{R2\beta})]\,. \tag{34}$$

III. 3. ESTIMATES OF Δa_μ AND Δa_e

We begin by discussing the prediction for Δa_μ and Δa_e for MSSM when the scalar masses are large lying in the several TeV region. In Tables I and II we exhibit the results for two benchmark points where we assume universality and take the scalar masses and the trilinear couplings to be all equal. Table I exhibits the result of the computation for Δa_μ where individual contributions arising from the chargino exchange, neutralino exchange, W exchange and Z exchange are listed. The entries exhibit the contributions over and above what one expects from the standard model and so the entries for the W and Z exchanges show a null value. Thus the entire contribution in this case arises from the chargino and the neutralino exchange and their sum gives a value $\mathcal{O}(10^{-11})$ which is two orders of magnitude smaller than the experimental result of Eq. (1). A very similar analysis is given in Table II for Δa_e where again the contribution to Δa_e arises from the exchange of charginos and neutralinos and their sum is $\mathcal{O}(10^{-16})$ which is three orders of magnitude smaller than the result of Eq. (2). Thus with a high scale of the scalar masses one cannot explain the results of Eq. (1) and Eq. (2).

We turn now to the analysis within the extended MSSM with a vector like leptonic generation. As in the analysis within MSSM here also we assume the universality of the soft parameters so that we set $m_0^2 = \tilde{M}^2_{\tau L} = \tilde{M}^2_E = \tilde{M}^2_\tau = \tilde{M}^2_\chi = \tilde{M}^2_{\mu L} = \tilde{M}^2_\mu = \tilde{M}^2_{eL} = \tilde{M}^2_e = \tilde{M}^2_{4L} = \tilde{M}^2_4$ and $A_0 = A_\tau = A_E = A_\mu = A_e = A_{4\ell}$ in the computation of the charged slepton mass squared matrix. Similarly we assume $m_0^{\tilde{\nu}^2} = \tilde{M}^2_N = \tilde{M}^2_{\nu_\tau} = \tilde{M}^2_{\nu_\mu} = \tilde{M}^2_{\nu_e} = \tilde{M}^2_{4L} = \tilde{M}^2_{\nu 4}$ and $A_{\nu_\tau} = A_{\nu_\mu} = A_{\nu_e} = A_N = A_{4\nu} = A_0^{\tilde{\nu}}$ for the computation of the sneutrino mass squared matrix (see Appendix). The contributions from the chargino exchange, the neutralino exchange, and the W and Z exchange are listed in Table I and Table II for two benchmark points. In this case the W boson and the Z boson exchange contributions are non-vanishing and the contributions listed are those over and above what one expects in the standard model. As in the MSSM case here also one finds that the contributions from the chargino exchange and from the neutralino exchange fall significantly below the experimental results of Eq. (1) and Eq. (2). However, in this case including the contributions from the W exchange and from the Z boson exchange one finds that consistency with Eq. (1) and Eq. (2) is achieved. At the same time one has the Higgs boson mass in the model for both benchmarks (a) and (b) at ~ 125 GeV consistent with the experimental measurements by ATLAS [1] and by CMS [2]. Here the loop correction that gives mass to the Higgs boson comes from the MSSM sector while the extra vector like leptonic generation makes a negligible contribution.

In the analysis of Δa_μ and Δa_e the exchange of both the sequential leptons and the mirrors play a role with the mirror exchange being the more dominant. The analysis requires diagonalization of a 5×5 mass matrix in the charged lepton-charged mirror lepton sector and diagonalization of a 5×5 mass matrix in the neutrino-mirror neutrino sector. Parameter choices are made to ensure that the eigenvalues in the charged lepton sector give the desired experimental values for e, μ and τ along with two additional masses, one for the sequential fourth generation lepton and the other for the mirror charged lepton. Their values are listed in Table III for the case of two benchmark points (a) and (b). A similar analysis holds for the neutrino-mirror neutrino sector where we get two additional eigenvalues, one for the fourth generation neutrino and the other for the mirror neutrino. Their values are also listed in Table III for two benchmark points. The analysis also requires diagonalization of a 10×10 matrix in the charged slepton and charged mirror slepton sector, as well as diagonalization of a 10×10 matrix in the sneutrino and the mirror sneutrino sector.

Contribution		(a) MSSM	(a) Vectorlike	(b) MSSM	(b) Vectorlike
Chargino	$a_\mu^{\chi^\pm}$	$+1.68\times10^{-11}$	$+1.07\times10^{-11}$	$+1.68\times10^{-11}$	-8.54×10^{-11}
Neutralino	$a_\mu^{\chi^0}$	-3.09×10^{-13}	-1.50×10^{-12}	-3.09×10^{-13}	-6.58×10^{-13}
W Boson	a_μ^{W}	0	$+1.53\times10^{-9}$	0	$+2.56\times10^{-9}$
Z Boson	a_μ^{Z}	0	$+5.12\times10^{-10}$	0	$+8.76\times10^{-10}$
Total	Δa_μ	$+1.65\times10^{-11}$	$+2.05\times10^{-9}$	$+1.65\times10^{-11}$	$+3.35\times10^{-9}$

TABLE I: The contribution of the vectorlike multiplet vs the contribution from the MSSM sector to the anomalous magnetic moments of the muon for two illustrative benchmark points (a) and (b). They are: (a) $m_N=5$, $m_{4\ell}=450$, $|f_3'|=0.62$, $|f_3''|=6.62\times10^{-3}$, $|f_4'|=20$, $|h_6|=230$, $|h_8|=730$ and (b) $m_N=200$, $m_{4\ell}=250$, $|f_3'|=0.73$, $|f_3''|=5.23\times10^{-3}$, $|f_4'|=30$, $|h_6|=66$, $|h_8|=180$. Other parameters have the values $\tan\beta=15$, $m_0=m_0^{\tilde\nu}=5000$, $|A_0^{\tilde\nu}|=|A_0|=6000$, $|m_1|=224$, $|m_2|=407$, $|\mu|=2124$, $m_E=320$, $m_{\nu4}=350$, $m_{h^0}=124.66$, $|f_3|=1\times10^{-4}$, $|f_4|=1\times10^{-5}$, $|f_4''|=38$, $|f_5|=1\times10^{-4}$, $|f_5'|=5.0\times10^{-4}$, $|f_5''|=3.0\times10^{-3}$, $|h_7|=34$, $\alpha_{A_0}=\pi$, $\alpha_{A_0^{\tilde\nu}}=\pi$, $\xi_1=\xi_2=\theta_\mu=\chi_3=\chi_3'=\chi_3''=\chi_4=\chi_4'=\chi_4''=\chi_5=\chi_5'=\chi_5''=\chi_6=\chi_7=\chi_8=0$. For the MSSM analysis the following parameters were used for both cases (a) and (b): The scalar masses are taken to be universal with $m_0=5000$ and the trilinear coupling is taken to be universal $A_0=-6000$. Other inputs are: $\chi_1^\pm=439$, $\chi_1^0=223$, $\chi_2^\pm=2144$, $\chi_2^0=439$, $-\chi_3^0=2142$, $\chi_4^0=2143$, $\mu=2124$. All masses are in GeV and phases in rad.

Contribution		(a) MSSM	(a) Vectorlike	(b) MSSM	(b) Vectorlike
Chargino	$a_e^{\chi^\pm}$	$+3.92\times10^{-16}$	-2.88×10^{-16}	$+3.92\times10^{-16}$	-6.31×10^{-15}
Neutralino	$a_e^{\chi^0}$	-7.25×10^{-18}	-1.69×10^{-16}	-7.25×10^{-18}	-3.12×10^{-17}
W Boson	a_e^{W}	0	$+1.99\times10^{-13}$	0	$+1.71\times10^{-13}$
Z Boson	a_e^{Z}	0	$+5.89\times10^{-14}$	0	$+5.11\times10^{-14}$
Total	Δa_e	$+3.85\times10^{-16}$	$+2.58\times10^{-13}$	$+3.85\times10^{-16}$	$+2.16\times10^{-13}$

TABLE II: The contribution of the vectorlike multiplet vs the contribution from the MSSM sector to the anomalous magnetic moments of the electron for two illustrative benchmark points (a) and (b) as given in table I.

Particles	Mass Spectrum (GeV) (a)	(b)
Mirror Neutrino	208	207
Fourth Sequential Neutrino	816	395
Mirror Lepton	253	349
Fourth Sequential Lepton	545	226

TABLE III: The mass of the heavy particles obtained after diagonalizing the lepton and neutrino mass matrices for benchmark points (a) and (b) of Table I.

We discuss now some further features of the analysis which includes the vector like leptonic generation. In Figure 3 we show the variation of Δa_μ as a function of m_E the mass of the mirror lepton as given by Eq. (45), for four $\tan\beta$ values. A remarkable feature of this graph is the dependence on $\tan\beta$ it exhibits. Notice that for a fixed m_E, Δa_μ decreases for increasing values of $\tan\beta$ as $\tan\beta$ varies from 20 to 35. Now we recall that the Yukawa coupling of a charged lepton has a $1/\cos\beta$ dependence and as a consequence the contribution of the charged lepton to Δa_μ becomes larger for larger $\tan\beta$ which is a well known result. However, the Yukawa coupling of the mirror lepton goes like $1/\sin\beta$ [9] and so Δa_μ decreases for larger values of $\tan\beta$. This feature explains the $\tan\beta$ dependence in Figure 3. It also shows that the W and Z exchange contributions in this case are being controlled by exchange of the mirror particles. A very similar dependence on $\tan\beta$ is exhibited by Δa_e.

The anomalous magnetic moments are quite sensitive to CP phases as first demonstrated in the analysis of [5–7] for the case of CP phases that arise in $N = 1$ supergravity [5, 7] and more generally for the case of MSSM [6]. In those analyses it was also found that large CP phases could be made consistent with the experimental constraints on the EDMs by the cancellation mechanism [16–19, 22]. In the present analysis the contribution from the MSSM sector is suppressed and the dominant contribution arises from the W and Z exchanges. For the case of three generations this sector does not have any CP phases in the leptonic sector. However, the extended MSSM with a vector like leptonic multiplet allows for CP phases which cannot be removed by field redefinitions. It is of interest then to discuss the dependence of Δa_μ and Δa_e on the CP phases that arise in the extended MSSM. We discuss now the dependence of Δa_μ and Δa_e on such phases. In Fig. (4) we exhibit the dependence of Δa_μ and Δa_e on χ'_3, which is the phase of f'_3 (see Appendix). A sharp dependence on χ'_3 is seen for both Δa_μ and for Δa_e. A very similar sensitivity to the CP phase χ_6 which is the phase of h_6 (see Appendix) is exhibited in Fig. (5). To explore further the sensitivity of Δa_μ and of Δa_e to parameters in the vector like sector we exhibit in Fig. (6) the dependence of Δa_μ and Δa_e on h_6 which is the co-efficient of the term $\epsilon_{ij}\hat{\chi}^{ci}\hat{\psi}^j_{4L}$ in the superpotential (see Eq. (37)). One can see in Fig. (6) the strong dependence of Δa_μ and Δa_e on h_6. In the analyses given so far both Δa_μ and Δa_e have very significant dependence on the parameters arising from inclusion of the vector like sector. However, there are parameters which affect Δa_e and Δa_μ differently. This is the case for $|f''_3|$. Here as seen in the left panel of Fig. (7) Δa_e is a sensitive function of $|f''_3|$ but not so for the case for Δa_μ (not exhibited) because of its much larger size. Finally we note that even if the SUSY scale lies in the PeV region, the contributions from the W and Z exchange arising from Fig. (2) survive while the diagrams of Fig. (1) give a vanishingly small contribution. This is illustrated in the right panel of Fig. (7).

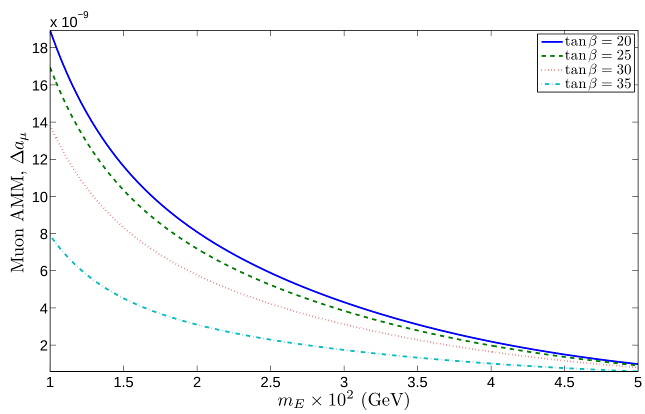

FIG. 3: Δa_μ as a function of m_E when $\tan\beta$ = 20, 25, 30, 35. Other parameters are $m_0 = m_0^{\tilde{\nu}} = 5000$, $|A_0^{\tilde{\nu}}| = |A_0| = 6000$, $|m_1| = 224$, $|m_2| = 407$, $|\mu| = 2124$, $m_{4\ell} = 250$, $m_N = 300$, $m_{\nu 4} = 350$, $m_{h^0} = 124.66$, $|f_3| = 1 \times 10^{-5}$, $|f'_3| = 8.18$, $|f''_3| = 4.32 \times 10^{-2}$, $|f_4| = 1 \times 10^{-3}$, $|f'_4| = 3.61$, $|f''_4| = 3.85$, $|f_5| = 1 \times 10^{-4}$, $|f'_5| = 5.0 \times 10^{-4}$, $|f''_5| = 3.0 \times 10^{-6}$, $|h_6| = 10$, $|h_7| = 19$, $|h_8| = 10$, $\alpha_{A_0} = \pi$, $\alpha_{A_0^{\tilde{\nu}}} = \pi$, $\xi_1 = \xi_2 = \theta_\mu = \chi_3 = \chi'_3 = \chi''_3 = \chi_4 = \chi'_4 = \chi''_4 = \chi_5 = \chi'_5 = \chi''_5 = \chi_6 = \chi_7 = \chi_8 = 0$. All masses are in GeV and phases in rad.

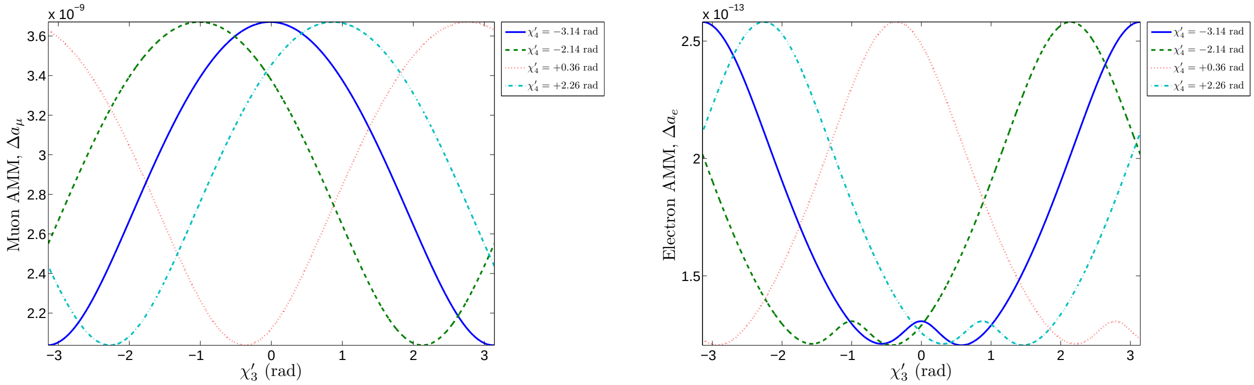

FIG. 4: Δa_μ (left panel) and Δa_e (right panel) as a function of χ'_3 in the range $[-\pi, +\pi]$ when $\chi'_4 = -3.14, -2.14, +0.36, +2.26$. Other parameters are $\tan\beta = 15$, $m_0 = m_0^{\tilde{\nu}} = 5000$, $|A_0^{\tilde{\nu}}| = |A_0| = 6000$, $|m_1| = 224$, $|m_2| = 407$, $|\mu| = 2124$, $m_N = 5$, $m_{\nu 4} = 350$, $m_E = 320$, $m_{4\ell} = 450$, $m_{h^0} = 124.66$, $|f_3| = 1 \times 10^{-4}$, $|f'_3| = 0.62$, $|f''_3| = 6.62 \times 10^{-3}$, $|f_4| = 1 \times 10^{-5}$, $|f'_4| = 20$, $|f''_4| = 38$, $|f_5| = 1 \times 10^{-4}$, $|f'_5| = 5.0 \times 10^{-4}$, $|f''_5| = 3.0 \times 10^{-6}$, $|h_6| = 230$, $|h_7| = 34$, $|h_8| = 730$, $\alpha_{A_0} = \pi$, $\alpha_{A_0^{\tilde{\nu}}} = \pi$, $\xi_1 = \xi_2 = \theta_\mu = \chi_3 = \chi''_3 = \chi_4 = \chi''_4 = \chi_5 = \chi'_5 = \chi''_5 = \chi_6 = \chi_7 = \chi_8 = 0$. All masses are in GeV and phases in rad.

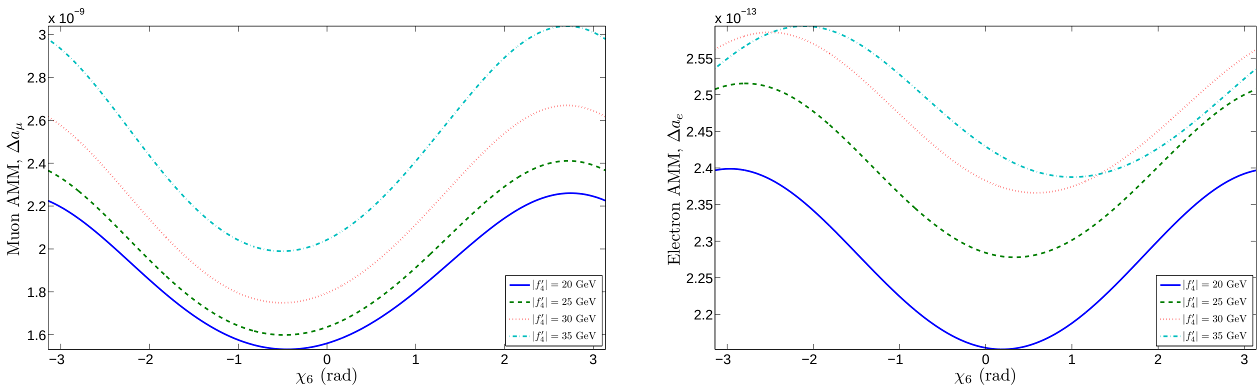

FIG. 5: Δa_μ (left panel) and Δa_e (right panel) as a function of χ_6 in the range $[-\pi, +\pi]$ when $|f'_4| = 20, 25, 30, 35$. Other parameters are $\tan\beta = 15$, $m_0 = m_0^{\tilde{\nu}} = 5000$, $|A_0^{\tilde{\nu}}| = |A_0| = 6000$, $|m_1| = 224$, $|m_2| = 407$, $|\mu| = 2124$, $m_N = 5$, $m_{\nu 4} = 350$, $m_E = 320$, $m_{4\ell} = 450$, $m_{h^0} = 124.66$, $|f_3| = 1 \times 10^{-4}$, $|f'_3| = 0.627$, $|f''_3| = 6.605 \times 10^{-3}$, $|f_4| = 1 \times 10^{-3}$, $|f''_4| = 38$, $|f_5| = 1 \times 10^{-4}$, $|f'_5| = 5.0 \times 10^{-4}$, $|f''_5| = 3.0 \times 10^{-6}$, $|h_6| = 230$, $|h_7| = 34$, $|h_8| = 730$, $\alpha_{A_0} = \pi$, $\alpha_{A_0^{\tilde{\nu}}} = \pi$, $\xi_1 = \xi_2 = \theta_\mu = \chi_3 = \chi_4 = \chi_5 = \chi'_5 = \chi''_5 = 0$, $\chi'_3 = 2.96$, $\chi''_3 = -1.54$, $\chi'_4 = 2.86$, $\chi''_4 = 1.46$, $\chi_7 = -2.94$, $\chi_8 = 0.6$. All masses are in GeV and phases in rad.

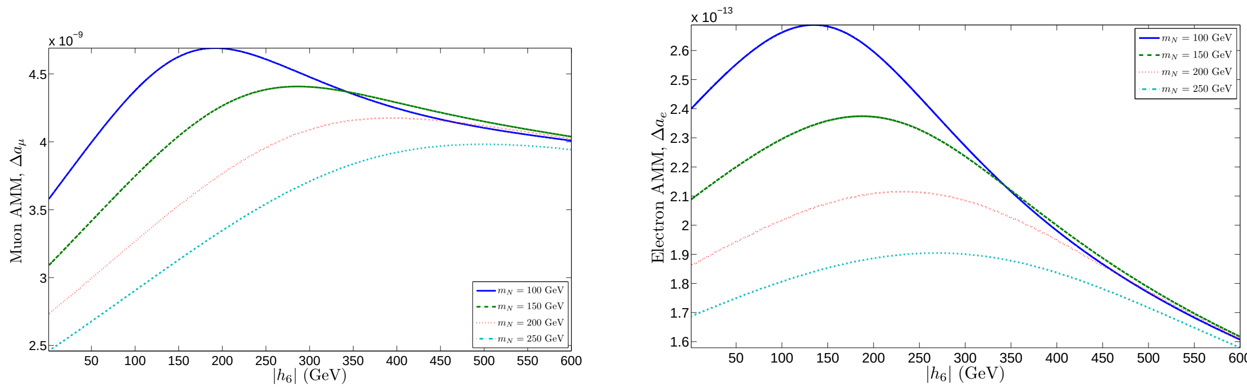

FIG. 6: Δa_μ (left panel) and Δa_e (right panel) as a function of $|h_6|$ when $m_N = 100, 150, 200, 250$. Other parameters are $\tan\beta = 15$, $m_0 = m_0^{\tilde{\nu}} = 5000$, $|A_0^{\tilde{\nu}}| = |A_0| = 6000$, $|m_1| = 224$, $|m_2| = 407$, $|\mu| = 2124$, $m_{\nu 4} = 350$, $m_E = 320$, $m_{4\ell} = 250$, $m_{h^0} = 124.66$, $|f_3| = 1 \times 10^{-5}$, $|f_3'| = 0.73$, $|f_3''| = 5.23 \times 10^{-3}$, $|f_4| = 1 \times 10^{-3}$, $|f_4'| = 20$, $|f_4''| = 38$, $|f_5| = 1 \times 10^{-4}$, $|f_5'| = 5.0 \times 10^{-4}$, $|f_5''| = 3.0 \times 10^{-6}$, $|h_7| = 34$, $|h_8| = 180$, $\alpha_{A_0} = \pi$, $\alpha_{A_0^{\tilde{\nu}}} = \pi$, $\xi_1 = \xi_2 = \theta_\mu = \chi_3 = \chi_4 = \chi_5 = \chi_5' = \chi_5'' = 0$, $\chi_3' = 2.96$, $\chi_3'' = -1.54$, $\chi_4' = 2.86$, $\chi_4'' = 1.46$, $\chi_6 = 3.06$, $\chi_7 = -2.94$, $\chi_8 = 0.6$. All masses are in GeV and phases in rad.

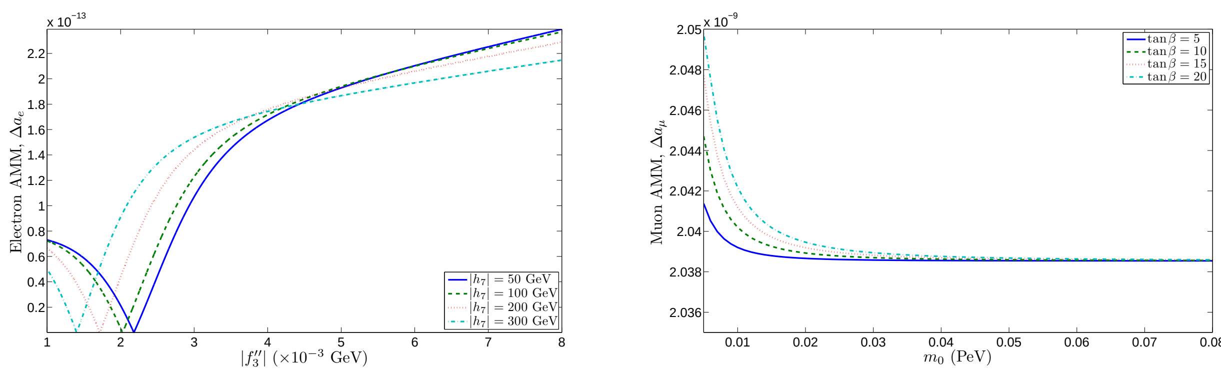

FIG. 7: Left panel: Variation of Δa_e as a function of $|f_3''|$ for four values of $|h_7|$ when $|h_7| = 50, 100, 200, 300$. Other parameters are $\tan\beta = 15$, $m_0 = m_0^{\tilde{\nu}} = 5000$, $|A_0^{\tilde{\nu}}| = |A_0| = 6000$, $|m_1| = 224$, $|m_2| = 407$, $|\mu| = 2124$, $m_N = 200$, $m_{\nu 4} = 350$, $m_E = 320$, $m_{4\ell} = 250$, $m_{h_0} = 124.66$, $|f_3| = 1 \times 10^{-5}$, $|f_3'| = 0.73$, $|f_4| = 1 \times 10^{-3}$, $|f_4'| = 20$, $|f_4''| = 38$, $|f_5| = 1 \times 10^{-4}$, $|f_5'| = 5.0 \times 10^{-4}$, $|f_5''| = 3.0 \times 10^{-6}$, $|h_6| = 66$, $|h_8| = 180$, $\alpha_{A_0} = \pi$, $\alpha_{A_0^{\tilde{\nu}}} = \pi$, $\xi_1 = \xi_2 = \theta_\mu = \chi_3 = \chi_4 = \chi_5 = \chi_5' = \chi_5'' = 0$, $\chi_3' = 2.96$, $\chi_3'' = -1.54$, $\chi_4' = 2.86$, $\chi_4'' = 1.46$, $\chi_6 = 3.06$, $\chi_7 = -2.94$, $\chi_8 = 0.6$. All masses are in GeV and phases in rad. Right panel: A plot of Δa_μ as a function of the common scalar mass m_0 exhibiting a residual correction Δa_μ even when m_0 lies in the PeV region. The parameters used in the plot are for benchmark (a) in Table (I).

IV. 4. CONCLUSION

The Higgs boson mass measurement at 126 GeV indicates a high SUSY scale, and specifically a high scale for the scalar masses. If the scalar masses are all heavy, the contribution to the leptonic moments and specifically to $\Delta a_\ell = a_\ell^{\text{exp}} - a_\ell^{\text{SM}}$ becomes negligible in this case. In this work we have investigated leptonic $g-2$ moments within an extended MSSM model with an extra vector like generation and CP phase dependent couplings. It is found that one can achieve consistency with the experimental measurements of Δa_μ and Δa_e under the constraint of the Higgs boson mass. The dependence of the moments on CP phases from the new sector are also investigated and shown to have a very sensitive dependence. Further, it is shown that Δa_μ and Δa_e will be non-vanishing even when the SUSY scale lies in the PeV region. The model presented here can be made UV complete by including a full generation of vector like matter including both quarks and leptons. Finally we note that the work presented here has some overlap with [35] which appeared after this work was finished. For another recent work on this topic see [36].

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V. APPENDIX ON THE EXTENDED MSSM WITH A VECTOR LIKE LEPTONIC GENERATION

In this Appendix we define the notation for the vector generation and their properties under $SU(3)_C \times SU(2)_L \times U(1)_Y$. For the four sequential families we use the notation

$$\psi_{iL} \equiv \begin{pmatrix} \nu_{iL} \\ \ell_{iL} \end{pmatrix} \sim (1,2,-\frac{1}{2}), \ell^c_{iL} \sim (1,1,1), \nu^c_{iL} \sim (1,1,0), \tag{35}$$

where the last entry on the right hand side of each $\sim$ is the value of the hypercharge Y defined so that $Q = T_3 + Y$ and we have included in our analysis the singlet field ν^c_i, where i runs from $1-4$. For the mirrors we use the notation

$$\chi^c \equiv \begin{pmatrix} E^c_{\mu L} \\ N^c_L \end{pmatrix} \sim (1,2,\frac{1}{2}), E_{\mu L} \sim (1,1,-1), N_L \sim (1,1,0). \tag{36}$$

The main difference between the leptons and the mirrors is that while the leptons have $V-A$ type interactions with $SU(2)_L \times U(1)_Y$ gauge bosons the mirrors have $V+A$ type interactions. In the analysis we assume R parity conservation. All of the neutral scalar fields in the new sector carry odd R parity and giving them a VEV will violate R parity conservation. For that reason only the Higgs fields are given VEVs. Further, in MSSM one can make field redefinitions to make the VEVs of both of the neutral Higgs fields to be real. One of these can become complex at the loop level leading to mixing of CP even-CP odd neutral Higgs. The induced phases are, however, small. An analysis including the CP even-CP odd Higgs mixing requires a separate treatment (see, e.g., [32]). We do not include loop induced CP phases in our analysis. Their effects of the analysis would in any case be negligible.

We assume that the mirrors of the vector like generation escape acquiring mass at the GUT scale and remain light down to the electroweak scale where the superpotential of the model for the lepton part may be written in the form

$$\begin{aligned}
W = &-\mu\epsilon_{ij}\hat{H}^i_1\hat{H}^j_2 + \epsilon_{ij}[f_1\hat{H}^i_1\hat{\psi}^j_L\hat{\tau}^c_L + f'_1\hat{H}^j_2\hat{\psi}^i_L\hat{\nu}^c_{\tau L} + f_2\hat{H}^i_1\hat{\chi}^{cj}\hat{N}_L + f'_2\hat{H}^j_2\hat{\chi}^{ci}\hat{E}_L \\
&+ h_1\hat{H}^i_1\hat{\psi}^j_{\mu L}\hat{\mu}^c_L + h'_1\hat{H}^j_2\hat{\psi}^i_{\mu L}\hat{\nu}^c_{\mu L} + h_2\hat{H}^i_1\hat{\psi}^j_{eL}\hat{e}^c_L + h'_2\hat{H}^j_2\hat{\psi}^i_{eL}\hat{\nu}^c_{eL} + y_5\hat{H}^i_1\hat{\psi}^j_{4L}\hat{\ell}^c_{4L} + y'_5\hat{H}^j_2\hat{\psi}^i_{4L}\hat{\nu}^c_{4L}] \\
&+ f_3\epsilon_{ij}\hat{\chi}^{ci}\hat{\psi}^j_L + f'_3\epsilon_{ij}\hat{\chi}^{ci}\hat{\psi}^j_{\mu L} + f_4\hat{\tau}^c_L\hat{E}_L + f_5\hat{\nu}^c_{\tau L}\hat{N}_L + f'_4\hat{\mu}^c_L\hat{E}_L + f'_5\hat{\nu}^c_{\mu L}\hat{N}_L \\
&+ f''_3\epsilon_{ij}\hat{\chi}^{ci}\hat{\psi}^j_{eL}x + f''_4\hat{e}^c_L\hat{E}_L + f''_5\hat{\nu}^c_{eL}\hat{N}_L \ + h_6\epsilon_{ij}\hat{\chi}^{ci}\hat{\psi}^j_{4L} + h_7\hat{\ell}^c_{4L}\hat{E}_L + h_8\hat{\nu}^c_{4L}\hat{N}_L,
\end{aligned} \tag{37}$$

where $\hat{}$ implies superfields, $\hat{\psi}_L$ stands for $\hat{\psi}_{3L}$, $\hat{\psi}_{\mu L}$ stands for $\hat{\psi}_{2L}$ and $\hat{\psi}_{eL}$ stands for $\hat{\psi}_{1L}$.

The mass terms for the neutrinos, mirror neutrinos, leptons and mirror leptons arise from the term

$$\mathcal{L} = -\frac{1}{2}\frac{\partial^2 W}{\partial A_i \partial A_j}\psi_i\psi_j + \text{H.c.}, \tag{38}$$

where ψ and A stand for generic two-component fermion and scalar fields. After spontaneous breaking of the electroweak symmetry, ($\langle H^1_1\rangle = v_1/\sqrt{2}$ and $\langle H^2_2\rangle = v_2/\sqrt{2}$), we have the following set of mass terms written in the

4-component spinor notation so that

$$-\mathcal{L}_m = \bar{\xi}_R^T (M_f) \xi_L + \bar{\eta}_R^T (M_\ell) \eta_L + \text{H.c.}, \tag{39}$$

where the basis vectors in which the mass matrix is written is given by

$$\begin{aligned}
\bar{\xi}_R^T &= \begin{pmatrix} \bar{\nu}_{\tau R} & \bar{N}_R & \bar{\nu}_{\mu R} & \bar{\nu}_{eR} & \bar{\nu}_{4R} \end{pmatrix}, \\
\xi_L^T &= \begin{pmatrix} \nu_{\tau L} & N_L & \nu_{\mu L} & \nu_{eL} & \nu_{4L} \end{pmatrix}, \\
\bar{\eta}_R^T &= \begin{pmatrix} \bar{\tau_R} & \bar{E}_R & \bar{\mu_R} & \bar{e_R} & \bar{\ell}_{4R} \end{pmatrix}, \\
\eta_L^T &= \begin{pmatrix} \tau_L & E_L & \mu_L & e_L & \ell_{4L} \end{pmatrix},
\end{aligned} \tag{40}$$

and the mass matrix M_f of neutrinos is given by

$$M_f = \begin{pmatrix} f_1' v_2/\sqrt{2} & f_5 & 0 & 0 & 0 \\ -f_3 & f_2 v_1/\sqrt{2} & -f_3' & -f_3'' & -h_6 \\ 0 & f_5' & h_1' v_2/\sqrt{2} & 0 & 0 \\ 0 & f_5'' & 0 & h_2' v_2/\sqrt{2} & 0 \\ 0 & h_8 & 0 & 0 & y_5' v_2/\sqrt{2} \end{pmatrix}. \tag{41}$$

We define the matrix element (22) of the mass matrix as m_N so that

$$m_N = f_2 v_1/\sqrt{2}. \tag{42}$$

The mass matrix is not hermitian and thus one needs bi-unitary transformations to diagonalize it. We define the bi-unitary transformation so that

$$D_R^{\nu\dagger} (M_f) D_L^\nu = \text{diag}(m_{\psi_1}, m_{\psi_2}, m_{\psi_3}, m_{\psi_4}, m_{\psi_5}). \tag{43}$$

where $\psi_1, \psi_2, \psi_3, \psi_4, \psi_5$ are the mass eigenstates for the neutrinos. In the limit of no mixing we identify ψ_1 as the light tau neutrino, ψ_2 as the heavier mass mirror eigen state, ψ_3 as the muon neutrino, ψ_4 as the electron neutrino and ψ_5 as the other heavy 4-sequential generation neutrino. A similar analysis goes to the lepton mass matrix M_ℓ where

$$M_\ell = \begin{pmatrix} f_1 v_1/\sqrt{2} & f_4 & 0 & 0 & 0 \\ f_3 & f_2' v_2/\sqrt{2} & f_3' & f_3'' & h_6 \\ 0 & f_4' & h_1 v_1/\sqrt{2} & 0 & 0 \\ 0 & f_4'' & 0 & h_2 v_1/\sqrt{2} & 0 \\ 0 & h_7 & 0 & 0 & y_5 v_1/\sqrt{2} \end{pmatrix}. \tag{44}$$

We introduce now the mass parameter m_E for the (22) element of the mass matrix above so that

$$m_E = f_2' v_2/\sqrt{2}. \tag{45}$$

CP phases that arise from the new sector are defined so that

$$\begin{aligned}
&f_i = |f_i| e^{i\chi_i}, \;\; f_i' = |f_i'| e^{i\chi_i'}, \;\; f_i^{''} = |f_i^{''}| e^{i\chi_i^{''}} \quad (i = 3, 4, 5) \\
&h_k = |h_k| e^{i\chi_k}, \quad k = 6, 7, 8\,.
\end{aligned} \tag{46}$$

As in the neutrino mass matrix case, the charged slepton mass matrix is not hermitian and thus one needs again a bi-unitary transformations to diagonalize it. We define the bi-unitary transformation so that

$$D^{\tau\dagger}_R(M_\ell)D^\tau_L = \text{diag}(m_{\tau_1}, m_{\tau_2}, m_{\tau_3}, m_{\tau_4}, m_{\tau_5}), \tag{47}$$

where τ_α ($\alpha = 1-5$) are the mass eigenstates for the charged lepton matrix.

The mass squared matrices of the slepton-mirror slepton and sneutrino-mirror sneutrino sectors come from three sources: the F term, the D term of the potential and the soft SUSY breaking terms. After spontaneous breaking of the electroweak symmetry the Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_F + \mathcal{L}_D + \mathcal{L}_{\text{soft}}\ , \tag{48}$$

where $\mathcal{L}_F$ is deduced from $-\mathcal{L}_F = F_i F^*_i$, while the $\mathcal{L}_D$ is given by

$$\begin{aligned}
-\mathcal{L}_D = &\frac{1}{2}m^2_Z\cos^2\theta_W\cos 2\beta\{\tilde\nu_{\tau L}\tilde\nu^*_{\tau L} - \tilde\tau_L\tilde\tau^*_L + \tilde\nu_{\mu L}\tilde\nu^*_{\mu L} - \tilde\mu_L\tilde\mu^*_L + \tilde\nu_{eL}\tilde\nu^*_{eL} - \tilde e_L\tilde e^*_L \\
&+ \tilde E_R\tilde E^*_R - \tilde N_R\tilde N^*_R + \tilde\nu_{4L}\tilde\nu^*_{4L} - \tilde\ell_{4L}\tilde\ell^*_{4L}\} + \frac{1}{2}m^2_Z\sin^2\theta_W\cos 2\beta\{\tilde\nu_{\tau L}\tilde\nu^*_{\tau L} + \tilde\tau_L\tilde\tau^*_L + \tilde\nu_{\mu L}\tilde\nu^*_{\mu L} + \tilde\mu_L\tilde\mu^*_L \\
&+ \tilde\nu_{eL}\tilde\nu^*_{eL} + \tilde e_L\tilde e^*_L + \tilde\nu_{4L}\tilde\nu^*_{4L} + \tilde\ell_{4L}\tilde\ell^*_{4L} \\
&- \tilde E_R\tilde E^*_R - \tilde N_R\tilde N^*_R + 2\tilde E_L\tilde E^*_L - 2\tilde\tau_R\tilde\tau^*_R - 2\tilde\mu_R\tilde\mu^*_R - 2\tilde e_R\tilde e^*_R - 2\tilde\ell_{4R}\tilde\ell^*_{4R}\}.
\end{aligned} \tag{49}$$

For $\mathcal{L}_{\text{soft}}$ we assume the following form

$$\begin{aligned}
-\mathcal{L}_{\text{soft}} = &\tilde M^2_{\tau L}\tilde\psi^{i*}_{\tau L}\tilde\psi^i_{\tau L} + \tilde M^2_\chi\tilde\chi^{ci*}\tilde\chi^{ci} + \tilde M^2_{\mu L}\tilde\psi^{i*}_{\mu L}\tilde\psi^i_{\mu L} \\
&+ \tilde M^2_{eL}\tilde\psi^{i*}_{eL}\tilde\psi^i_{eL} + \tilde M^2_{\nu_\tau}\tilde\nu^{c*}_{\tau L}\tilde\nu^c_{\tau L} + \tilde M^2_{\nu_\mu}\tilde\nu^{c*}_{\mu L}\tilde\nu^c_{\mu L} \\
&+ \tilde M^2_{4L}\tilde\psi^{i*}_{4L}\tilde\psi^i_{4L} + \tilde M^2_{\nu_4}\tilde\nu^{c*}_{4L}\tilde\nu^c_{4L} + \tilde M^2_{\nu_e}\tilde\nu^{c*}_{eL}\tilde\nu^c_{eL} + \tilde M^2_\tau\tilde\tau^{c*}_L\tilde\tau^c_L + \tilde M^2_\mu\tilde\mu^{c*}_L\tilde\mu^c_L \\
&+ \tilde M^2_e\tilde e^{c*}_L\tilde e^c_L + \tilde M^2_E\tilde E^*_L\tilde E_L + \tilde M^2_N\tilde N^*_L\tilde N_L + \tilde M^2_4\tilde\ell^{c*}_{4L}\tilde\ell^c_{4L} \\
&+ \epsilon_{ij}\{f_1 A_\tau H^i_1\tilde\psi^j_{\tau L}\tilde\tau^c_L - f'_1 A_{\nu_\tau} H^i_2\tilde\psi^j_{\tau L}\tilde\nu^c_{\tau L} + h_1 A_\mu H^i_1\tilde\psi^j_{\mu L}\tilde\mu^c_L - h'_1 A_{\nu_\mu} H^i_2\tilde\psi^j_{\mu L}\tilde\nu^c_{\mu L} \\
&+ h_2 A_e H^i_1\tilde\psi^j_{eL}\tilde e^c_L - h'_2 A_{\nu_e} H^i_2\tilde\psi^j_{eL}\tilde\nu^c_{eL} + f_2 A_N H^i_1\tilde\chi^{cj}\tilde N_L - f'_2 A_E H^i_2\tilde\chi^{cj}\tilde E_L \\
&+ y_5 A_{4\ell} H^i_1\tilde\psi^j_{4L}\tilde\ell^c_{4L} - y'_5 A_{4\nu} H^i_2\tilde\psi^j_{4L}\tilde\nu^c_{4L} + \text{H.c.}\}\ .
\end{aligned} \tag{50}$$

We define the scalar mass squared matrix $M^2_{\tilde\tau}$ in the basis

$$(\tilde\tau_L, \tilde E_L, \tilde\tau_R, \tilde E_R, \tilde\mu_L, \tilde\mu_R, \tilde e_L, \tilde e_R, \tilde\ell_{4L}, \tilde\ell_{4R}). \tag{51}$$

We label the matrix elements of these as $(M^2_{\tilde{\tau}})_{ij} = M^2_{ij}$ where the elements of the matrix are given by

$$M^2_{11} = \tilde{M}^2_{\tau L} + \frac{v_1^2|f_1|^2}{2} + |f_3|^2 - m_Z^2 \cos 2\beta \left(\frac{1}{2} - \sin^2\theta_W\right),$$

$$M^2_{22} = \tilde{M}^2_{E} + \frac{v_2^2|f_2'|^2}{2} + |f_4|^2 + |f_4'|^2 + |f_4''|^2 + |h_7|^2 + m_Z^2 \cos 2\beta \sin^2\theta_W,$$

$$M^2_{33} = \tilde{M}^2_{\tau} + \frac{v_1^2|f_1|^2}{2} + |f_4|^2 - m_Z^2 \cos 2\beta \sin^2\theta_W,$$

$$M^2_{44} = \tilde{M}^2_{\chi} + \frac{v_2^2|f_2'|^2}{2} + |f_3|^2 + |f_3'|^2 + |f_3''|^2 + |h_6|^2 + m_Z^2 \cos 2\beta \left(\frac{1}{2} - \sin^2\theta_W\right),$$

$$M^2_{55} = \tilde{M}^2_{\mu L} + \frac{v_1^2|h_1|^2}{2} + |f_3'|^2 - m_Z^2 \cos 2\beta \left(\frac{1}{2} - \sin^2\theta_W\right),$$

$$M^2_{66} = \tilde{M}^2_{\mu} + \frac{v_1^2|h_1|^2}{2} + |f_4'|^2 - m_Z^2 \cos 2\beta \sin^2\theta_W,$$

$$M^2_{77} = \tilde{M}^2_{eL} + \frac{v_1^2|h_2|^2}{2} + |f_3''|^2 - m_Z^2\cos 2\beta\left(\frac{1}{2} - \sin^2\theta_W\right),$$
$$M^2_{88} = \tilde{M}^2_{e} + \frac{v_1^2|h_2|^2}{2} + |f_4''|^2 - m_Z^2\cos 2\beta\sin^2\theta_W,$$
$$M^2_{99} = \tilde{M}^2_{4L} + \frac{v_1^2|y_5|^2}{2} + |h_6|^2 - m_Z^2\cos 2\beta\left(\frac{1}{2} - \sin^2\theta_W\right),$$
$$M^2_{1010} = \tilde{M}^2_{4} + \frac{v_1^2|y_5|^2}{2} + |h_7|^2 - m_Z^2\cos 2\beta\sin^2\theta_W,$$
$$M^2_{12} = M^{2*}_{21} = \frac{v_2 f_2' f_3^*}{\sqrt{2}} + \frac{v_1 f_4 f_1^*}{\sqrt{2}},$$
$$M^2_{13} = M^{2*}_{31} = \frac{f_1^*}{\sqrt{2}}(v_1 A_\tau^* - \mu v_2),$$
$$M^2_{14} = M^{2*}_{41} = 0, M^2_{15} = M^{2*}_{51} = f_3' f_3^*,$$
$$M^2_{16} = M^{2*}_{61} = 0, M^2_{17} = M^{2*}_{71} = f_3'' f_3^*, M^2_{18} = M^{2*}_{81} = 0, M^2_{23} = M^{2*}_{32} = 0,$$
$$M^2_{24} = M^{2*}_{42} = \frac{f_2'^*}{\sqrt{2}}(v_2 A_E^* - \mu v_1), M^2_{25} = M^{2*}_{52} = \frac{v_2 f_3' f_2'^*}{\sqrt{2}} + \frac{v_1 h_1 f_4^*}{\sqrt{2}},$$
$$M^2_{26} = M^{2*}_{62} = 0, M^2_{27} = M^{2*}_{72} = \frac{v_2 f_3'' f_2'^*}{\sqrt{2}} + \frac{v_1 h_1 f_4'^*}{\sqrt{2}}, M^2_{28} = M^{2*}_{82} = 0,$$
$$M^2_{34} = M^{2*}_{43} = \frac{v_2 f_4 f_2'^*}{\sqrt{2}} + \frac{v_1 f_1 f_3^*}{\sqrt{2}}, M^2_{35} = M^{2*}_{53} = 0, M^2_{36} = M^{2*}_{63} = f_4 f_4'^*,$$
$$M^2_{37} = M^{2*}_{73} = 0, M^2_{38} = M^{2*}_{83} = f_4 f_4''^*, M^2_{45} = M^{2*}_{54} = 0, M^2_{46} = M^{2*}_{64} = \frac{v_2 f_2' f_4'^*}{\sqrt{2}} + \frac{v_1 f_3' h_1^*}{\sqrt{2}},$$
$$M^2_{47} = M^{2*}_{74} = 0, M^2_{48} = M^{2*}_{84} = \frac{v_2 f_2' f_4''^*}{\sqrt{2}} + \frac{v_1 f_3'' h_2^*}{\sqrt{2}},$$
$$M^2_{56} = M^{2*}_{65} = \frac{h_1^*}{\sqrt{2}}(v_1 A_\mu^* - \mu v_2), M^2_{57} = M^{2*}_{75} = f_3'' f_3'^*, M^2_{58} = M^{2*}_{85} = 0, M^2_{67} = M^{2*}_{76} = 0,$$
$$M^2_{68} = M^{2*}_{86} = f_4' f_4''^*, M^2_{78} = M^{2*}_{87} = \frac{h_2^*}{\sqrt{2}}(v_1 A_e^* - \mu v_2)$$
$$M^2_{19} = M^{2*}_{91} = f_3^* h_6, M^2_{110} = M^{2*}_{101} = 0,$$
$$M^2_{29} = M^{2*}_{92} = \frac{v_1 y_5 h_7^*}{\sqrt{2}} + \frac{v_2 h_6 f_2'^*}{\sqrt{2}}, M^2_{210} = M^{2*}_{102} = 0,$$
$$M^2_{39} = M^{2*}_{93} = 0, M^2_{310} = M^{2*}_{103} = f_4 h_7^*,$$
$$M^2_{49} = M^{2*}_{94} = 0, M^2_{410} = M^{2*}_{104} = \frac{v_2 f_2' h_7^*}{\sqrt{2}} + \frac{v_1 h_6 y_5^*}{\sqrt{2}},$$
$$M^2_{59} = M^{2*}_{95} = f_3'^* h_6, M^2_{510} = M^{2*}_{105} = 0,$$
$$M^2_{69} = M^{2*}_{96} = 0, M^2_{610} = M^{2*}_{106} = f_4' h_7^*,$$
$$M^2_{79} = M^{2*}_{97} = f_3''^* h_6, M^2_{710} = M^{2*}_{107} = 0,$$
$$M^2_{89} = M^{2*}_{98} = 0, M^2_{810} = M^{2*}_{108} = f_5'' h_7^*,$$
$$M^2_{910} = M^{2*}_{109} = \frac{y_5^*}{\sqrt{2}}(v_1 A_{4\ell}^* - \mu v_2). \tag{52}$$

We assume that the masses that enter the mass squared matrix for the scalars are all of electroweak size. This mass squared matrix is hermitian and can be diagonalized with a unitary transformation

$$\tilde{D}^{\tau\dagger} M^2_{\tilde{\tau}} \tilde{D}^{\tau} = \mathrm{diag}(\mathrm{M}^2_{\tilde{\tau}_1}, \mathrm{M}^2_{\tilde{\tau}_2}, \mathrm{M}^2_{\tilde{\tau}_3}, \mathrm{M}^2_{\tilde{\tau}_4}, \mathrm{M}^2_{\tilde{\tau}_5}, \mathrm{M}^2_{\tilde{\tau}_6}, \mathrm{M}^2_{\tilde{\tau}_7}, \mathrm{M}^2_{\tilde{\tau}_8} \mathrm{M}^2_{\tilde{\tau}_9}, \mathrm{M}^2_{\tilde{\tau}_{10}}). \tag{53}$$

The mass squared matrix in the sneutrino sector has a similar structure. In the basis

$$(\tilde{\nu}_{\tau L}, \tilde{N}_L, \tilde{\nu}_{\tau R}, \tilde{N}_R, \tilde{\nu}_{\mu L}, \tilde{\nu}_{\mu R}, \tilde{\nu}_{eL}, \tilde{\nu}_{eR}, \tilde{\nu}_{4L}, \tilde{\nu}_{4R}), \tag{54}$$

the sneutrino mass squared matrix $(M^2_{\tilde{\nu}})_{ij} = m^2_{ij}$ has elements given by

$$\begin{gathered}
m^2_{11} = \tilde{M}^2_{\tau L} + \frac{v_2^2}{2}|f'_1|^2 + |f_3|^2 + \frac{1}{2}m_Z^2\cos 2\beta, \\
m^2_{22} = \tilde{M}^2_N + \frac{v_1^2}{2}|f_2|^2 + |f_5|^2 + |f'_5|^2 + |f''_5|^2 + |h_8|^2, \\
m^2_{33} = \tilde{M}^2_{\nu_\tau} + \frac{v_2^2}{2}|f'_1|^2 + |f_5|^2, \\
m^2_{44} = \tilde{M}^2_\chi + \frac{v_1^2}{2}|f_2|^2 + |f_3|^2 + |f'_3|^2 + |f''_3|^2 + |h_6|^2 - \frac{1}{2}m_Z^2\cos 2\beta, \\
m^2_{55} = \tilde{M}^2_{\mu L} + \frac{v_2^2}{2}|h'_1|^2 + |f'_3|^2 + \frac{1}{2}m_Z^2\cos 2\beta, \\
m^2_{66} = \tilde{M}^2_{\nu_\mu} + \frac{v_2^2}{2}|h'_1|^2 + |f'_5|^2, \\
m^2_{77} = \tilde{M}^2_{eL} + \frac{v_2^2}{2}|h'_2|^2 + |f''_3|^2 + \frac{1}{2}m_Z^2\cos 2\beta, \\
m^2_{88} = \tilde{M}^2_{\nu_e} + \frac{v_2^2}{2}|h'_2|^2 + |f''_5|^2, \\
m^2_{99} = \tilde{M}^2_{4L} + \frac{v_2^2}{2}|y'_5|^2 + |h_6|^2 + \frac{1}{2}m_Z^2\cos 2\beta, \\
m^2_{1010} = \tilde{M}^2_{\nu 4} + |h_8|^2 + \frac{v_2^2}{2}|y'_5|^2, \\
m^2_{12} = m^{2*}_{21} = \frac{v_2 f_5 f'^*_1}{\sqrt{2}} - \frac{v_1 f_2 f^*_3}{\sqrt{2}}, \\
m^2_{13} = m^{2*}_{31} = \frac{f'^*_1}{\sqrt{2}}(v_2 A^*_{\nu_\tau} - \mu v_1), m^2_{14} = m^{2*}_{41} = 0, \\
m^2_{15} = m^{2*}_{51} = f'_3 f^*_3, m^2_{16} = m^{2*}_{61} = 0, \\
m^2_{17} = m^{2*}_{71} = f''_3 f^*_3, m^2_{18} = m^{2*}_{81} = 0, \\
m^2_{23} = m^{2*}_{32} = 0, m^2_{24} = m^{2*}_{42} = \frac{f^*_2}{\sqrt{2}}(v_1 A^*_N - \mu v_2), m^2_{25} = m^{2*}_{52} = -\frac{v_1 f^*_2 f'_3}{\sqrt{2}} + \frac{h'_1 v_2 f'^*_5}{\sqrt{2}}, \\
m^2_{26} = m^{2*}_{62} = 0, m^2_{27} = m^{2*}_{72} = -\frac{v_1 f^*_2 f''_3}{\sqrt{2}} + \frac{h'_2 v_2 f''^*_5}{\sqrt{2}},
\end{gathered} \tag{55}$$

$$
\begin{gathered}
m^2_{28}=m^{2*}_{82}=0, m^2_{34}=m^{2*}_{43}=\frac{v_1 f_2^* f_5}{\sqrt{2}}-\frac{v_2 f_1' f_3^*}{\sqrt{2}},\\
m^2_{35}=m^{2*}_{53}=0, m^2_{36}=m^{2*}_{63}=f_5 f_5'^*, m^2_{37}=m^{2*}_{73}=0, m^2_{38}=m^{2*}_{83}=f_5 f_5''^*, m^2_{45}=m^{2*}_{54}=0,\\
m^2_{46}=m^{2*}_{64}=-\frac{h_1'^* v_2 f_3'}{\sqrt{2}}+\frac{v_1 f_2 f_5'^*}{\sqrt{2}}, m^2_{47}=m^{2*}_{74}=0,\\
m^2_{48}=m^{2*}_{84}=\frac{v_1 f_2 f_5''^*}{\sqrt{2}}-\frac{v_2 h_2'^* f_3''}{\sqrt{2}}, m^2_{56}=m^{2*}_{65}=\frac{h_1'^*}{\sqrt{2}}(v_2 A^*_{\nu_\mu}-\mu v_1),\\
m^2_{57}=m^{2*}_{75}=f_3'' f_3'^*, m^2_{58}=m^{2*}_{85}=0, m^2_{67}=m^{2*}_{76}=0,\\
m^2_{68}=m^{2*}_{86}=f_5' f_5''^*, m^2_{78}=m^{2*}_{87}=\frac{h_2'^*}{\sqrt{2}}(v_2 A^*_{\nu_e}-\mu v_1),\\
m^2_{19}=m^{2*}_{91}=h_6 f_3^*, m^2_{110}=m^{2*}_{101}=0,\\
m^2_{29}=m^{2*}_{92}=-\frac{f_2 v_1 h_6}{\sqrt{2}}+\frac{v_2 h_8 y_5^*}{\sqrt{2}}, m^2_{210}=m^{2*}_{102}=0,\\
m^2_{39}=m^{2*}_{93}=0, m^2_{310}=m^{2*}_{103}=f_5 h_8^*,\\
m^2_{49}=m^{2*}_{94}=0, m^2_{410}=m^{2*}_{104}=-\frac{v_2 y_5' h_6}{\sqrt{2}}+\frac{v_1 h_8^* f_2}{\sqrt{2}},\\
m^2_{59}=m^{2*}_{95}=h_6 f_3'^*, m^2_{510}=m^{2*}_{105}=0,\\
m^2_{69}=m^{2*}_{96}=0, m^2_{610}=m^{2*}_{106}=f_5' h_8^*,\\
m^2_{79}=m^{2*}_{97}=h_6 f_3''^*, m^2_{710}=m^{2*}_{107}=0,\\
m^2_{89}=m^{2*}_{98}=0, m^2_{810}=m^{2*}_{108}=f_5'' h_8^*,\\
m^2_{910}=m^{2*}_{109}=\frac{y_5'}{\sqrt{2}}(v_2 A^*_{4\nu}-\mu v_1). \qquad (56)
\end{gathered}
$$

Again as in the charged lepton sector we assume that all the masses are of the electroweak size so all the terms enter in the mass squared matrix. This mass squared matrix can be diagonalized by the unitary transformation

$$
\tilde{D}^{\nu\dagger} M^2_{\tilde{\nu}} \tilde{D}^{\nu} = \text{diag}(M^2_{\tilde{\nu}_1}, M^2_{\tilde{\nu}_2}, M^2_{\tilde{\nu}_3}, M^2_{\tilde{\nu}_4}, M^2_{\tilde{\nu}_5}, M^2_{\tilde{\nu}_6}, M^2_{\tilde{\nu}_7}, M^2_{\tilde{\nu}_8}, M^2_{\tilde{\nu}_9}, M^2_{\tilde{\nu}_{10}}). \qquad (57)
$$

References

[1] G. Aad *et al.* [ATLAS Collaboration], Phys. Lett. B **716**, 1 (2012) doi:10.1016/j.physletb.2012.08.020 [arXiv:1207.7214 [hep-ex]].

[2] S. Chatrchyan *et al.* [CMS Collaboration], Phys. Lett. B **716**, 30 (2012) doi:10.1016/j.physletb.2012.08.021 [arXiv:1207.7235 [hep-ex]].

[3] J. Beringer *et al.* [Particle Data Group Collaboration], Phys. Rev. D **86**, 010001 (2012). doi:10.1103/PhysRevD.86.010001

[4] G. F. Giudice, P. Paradisi and M. Passera, JHEP **1211**, 113 (2012) doi:10.1007/JHEP11(2012)113 [arXiv:1208.6583 [hep-ph]].

[5] T. Ibrahim and P. Nath, Phys. Rev. D **61**, 095008 (2000) doi:10.1103/PhysRevD.61.095008 [hep-ph/9907555].

[6] T. Ibrahim and P. Nath, Phys. Rev. D **62**, 015004 (2000) doi:10.1103/PhysRevD.62.015004 [hep-ph/9908443].

[7] T. Ibrahim, U. Chattopadhyay and P. Nath, Phys. Rev. D **64**, 016010 (2001) doi:10.1103/PhysRevD.64.016010 [hep-ph/0102324].

[8] T. C. Yuan, R. L. Arnowitt, A. H. Chamseddine and P. Nath, Z. Phys. C **26**, 407 (1984). doi:10.1007/BF01452567; D. A. Kosower, L. M. Krauss and N. Sakai, Phys. Lett. B **133**, 305 (1983). doi:10.1016/0370-2693(83)90152-1; J. L. Lopez, D. V. Nanopoulos and X. Wang, Phys. Rev. D **49**, 366 (1994) doi:10.1103/PhysRevD.49.366 [hep-ph/9308336]; U. Chattopadhyay and P. Nath, Phys. Rev. D **53**, 1648 (1996) doi:10.1103/PhysRevD.53.1648 [hep-ph/9507386]; T. Moroi, Phys. Rev. D **53**, 6565 (1996) Erratum: [Phys. Rev. D **56**, 4424 (1997)] doi:10.1103/PhysRevD.53.6565, 10.1103/PhysRevD.56.4424 [hep-ph/9512396].

[9] T. Ibrahim and P. Nath, Phys. Rev. D **78**, 075013 (2008) doi:10.1103/PhysRevD.78.075013 [arXiv:0806.3880 [hep-ph]].

[10] H. Georgi, Nucl. Phys. B **156**, 126 (1979); F. Wilczek and A. Zee, Phys. Rev. D **25**, 553 (1982); G. Senjanovic, F. Wilczek and A. Zee, Phys. Lett. B **141**, 389 (1984), doi:10.1016/0370-2693(84)90269-7; J. Maalampi, J.T. Peltoniemi, and M. Roos, PLB 220, 441(1989); K. S. Babu, I. Gogoladze, P. Nath and R. M. Syed, Phys. Rev. D **74**, 075004 (2006): Phys. Rev. D **74**, 075004 (2006).

[11] K. S. Babu, I. Gogoladze, M. U. Rehman and Q. Shafi, Phys. Rev. D **78**, 055017 (2008) doi:10.1103/PhysRevD.78.055017 [arXiv:0807.3055 [hep-ph]].

[12] C. Liu, Phys. Rev. D **80**, 035004 (2009) doi:10.1103/PhysRevD.80.035004 [arXiv:0907.3011 [hep-ph]].

[13] S. P. Martin, Phys. Rev. D **81**, 035004 (2010) doi:10.1103/PhysRevD.81.035004 [arXiv:0910.2732 [hep-ph]].

[14] P. Nath, Phys. Rev. Lett. **66**, 2565 (1991). doi:10.1103/PhysRevLett.66.2565

[15] Y. Kizukuri and N. Oshimo, Phys. Rev. D **46**, 3025 (1992). doi:10.1103/PhysRevD.46.3025

[16] T. Ibrahim and P. Nath, Phys. Rev. D **58**, 111301 (1998) doi:10.1103/PhysRevD.60.099902, 10.1103/PhysRevD.58.111301 [hep-ph/9807501].

[17] T. Ibrahim and P. Nath, Phys. Rev. D **57**, 478 (1998); doi: 10.1103/PhysRevD.57.478 [hep-ph/9708456].

[18] T. Falk and K. A. Olive, Phys. Lett. B **439**, 71 (1998) doi:10.1016/S0370-2693(98)01022-3 [hep-ph/9806236].

[19] M. Brhlik, G. J. Good and G. L. Kane, Phys. Rev. D **59**, 115004 (1999) doi:10.1103/PhysRevD.59.115004 [hep-ph/9810457].

[20] L. Lavoura and J. P. Silva, Phys. Rev. D **47** (1993) 2046; N. Maekawa, Phys. Rev. D **52** (1995) 1684; . Maiani, G. Parisi and R. Petronzio, Nucl. Phys. B **136** (1978) 115 ; S. Theisen, N. D. Tracas and G. Zoupanos, Z. Phys. C **37** (1988) 597; D. Ghilencea, M. Lanzagorta and G. G. Ross, Phys. Lett. B **415** (1997) 253;

[21] T. Moroi and Y. Okada, Mod. Phys. Lett. A **7** (1992) 187; Phys. Lett. B **295** (1992) 73; K. S. Babu and J. C. Pati, hep-ph/0203029; R. Kitano and K. Yamamoto, Phys. Rev. D **62** (2000) 073007; L. J. Hall, K. Harigaya and Y. Nomura, arXiv:1605.03585 [hep-ph].

[22] T. Ibrahim and P. Nath, Phys. Rev. D **61**, 093004 (2000) doi:10.1103/PhysRevD.61.093004 [hep-ph/9910553].

[23] T. Ibrahim and P. Nath, Phys. Rev. D **81**, no. 3, 033007 (2010) Erratum: [Phys. Rev. D **89**, no. 11, 119902 (2014)] doi:10.1103/PhysRevD.81.033007, 10.1103/PhysRevD.89.119902 [arXiv:1001.0231 [hep-ph]].

[24] T. Ibrahim and P. Nath, Phys. Rev. D **82**, 055001 (2010) doi:10.1103/PhysRevD.82.055001 [arXiv:1007.0432 [hep-ph]].

[25] T. Ibrahim and P. Nath, Phys. Rev. D **84**, 015003 (2011) doi:10.1103/PhysRevD.84.015003 [arXiv:1104.3851 [hep-ph]].

[26] T. Ibrahim and P. Nath, Phys. Rev. D **87**, no. 1, 015030 (2013) doi:10.1103/PhysRevD.87.015030 [arXiv:1211.0622 [hep-ph]].

[27] A. Aboubrahim, T. Ibrahim and P. Nath, Phys. Rev. D **88**, 013019 (2013) doi:10.1103/PhysRevD.88.013019 [arXiv:1306.2275 [hep-ph]].

[28] A. Aboubrahim, T. Ibrahim, P. Nath and A. Zorik, Phys. Rev. D **92**, no. 3, 035013 (2015) doi:10.1103/PhysRevD.92.035013 [arXiv:1507.02668 [hep-ph]].

[29] T. Ibrahim, A. Itani and P. Nath, Phys. Rev. D **92**, no. 1, 015003 (2015) doi:10.1103/PhysRevD.92.015003 [arXiv:1503.01078 [hep-ph]]; Phys. Rev. D **90**, no. 5, 055006 (2014) doi:10.1103/PhysRevD.90.055006 [arXiv:1406.0083 [hep-ph]].

[30] R. Dermisek and A. Raval, Phys. Rev. D **88**, 013017 (2013) doi:10.1103/PhysRevD.88.013017 [arXiv:1305.3522 [hep-ph]].

[31] K. Nickel and F. Staub, JHEP **1507**, 139 (2015) doi:10.1007/JHEP07(2015)139 [arXiv:1505.06077 [hep-ph]].

[32] T. Ibrahim, P. Nath and A. Zorik, arXiv:1606.05799 [hep-ph].

[33] T. Ibrahim and P. Nath, Rev. Mod. Phys. **80**, 577 (2008) doi:10.1103/RevModPhys.80.577 [arXiv:0705.2008 [hep-ph]].

[34] J. Baron *et al.* [ACME Collaboration], Science **343**, 269 (2014) doi:10.1126/science.1248213 [arXiv:1310.7534 [physics.atom-ph]].

[35] M. Nishida and K. Yoshioka, arXiv:1605.06675 [hep-ph].

[36] N. Okada and H. M. Tran, arXiv:1606.05329 [hep-ph].