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Abstract

We present a new class of unified models based on $SO(10)$ symmetry which provides insights into the masses and mixings of quarks and leptons, including the neutrinos. The key feature of our proposal is the absence of Higgs boson 10_H belonging to the fundamental representation that is normally employed. Flavor mixing is induced via vector-like fermions in the $16 + \overline{16}$ representation. A variety of scenarios, both supersymmetric and otherwise, are analyzed involving a $\overline{126}_H$ along with either a 45_H or a 210_H of Higgs boson employed for symmetry breaking. It is shown that this framework, with only a limited number of parameters, provides an excellent fit to the full fermion spectrum, utilizing either type-I or type-II seesaw mechanism. These flavor models can be potentially tested and distinguished in their predictions for proton decay branching ratios, which are analyzed.

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1 Introduction

Grand unified theories [1–3] based on $SO(10)$ gauge symmetry [4] are attractive candidates for physics beyond the Standard Model (SM). These theories predict the existence of right-handed neutrinos needed for the seesaw mechanism, and unify all fermions of a given family into a single irreducible multiplet, the 16–dimensional spinor representation. Quarks and leptons are thus unified, as are the three gauge interactions of the SM. The unification of fermions into multiplets suggests that $SO(10)$ may serve as a fertile ground for understanding the flavor puzzle. There are challenges involved, since in particular, large neutrino mixing angles should emerge from the same underlying Yukawa structure that allows for small quark mixing angles. This indeed has been realized in a class of $SO(10)$ models with a minimal set of Yukawa coupling matrices [5–13], and we shall provide a new class of models that achieves this in this paper. Since $SO(10)$ admits an intermediate symmetry, the Pati-Salam symmetry $SU(4)_c \times SU(2)_L \times SU(2)_R$ or one of its subgroups, unification of gauge couplings can occur consistently even without low energy supersymmetry. Of course, $SO(10)$ may be realized in the supersymmetric context as well, in which case the intermediate symmetry breaking scale may be the same as the unification scale. As far as the Yukawa sector of the theory is concerned, the two scenarios (non-SUSY versus SUSY) are not all that different. In this paper we shall study a new class of $SO(10)$ models addressing the flavor puzzle both in the non-supersymmetric and in the SUSY contexts.

One of our motivations for the present study is the difficulty faced by a widely studied minimal renormalizable supersymmetric SO(10) [14–16] grand unified theory. This theory has attracted much attention in the past due to several attractive features which include:

- natural generation of neutrino masses and mixings through type I [17] and type II [18] seesaw mechanism;

- relation between neutrino and charged fermion mass matrices [5];

- good fit of fermion masses and mixings with an economic Yukawa sector with only two symmetric Yukawa matrices [5–13];

- automatic and exact low energy R-parity conservation leading to a compelling dark matter candidate [19–23];

- connection of the $b - \tau$ Yukawa unification and large atmospheric mixing angle in scenarios with dominant type II seesaw mechanism [24–26].

The Yukawa sector of this theory has only two symmetric matrices (in flavor space), involving a 10_H and a $\overline{126}_H$ of Higgs bosons. It is natural to include a 210_H for completing the symmetry breaking. In such a scenario, unfortunately, once the constraints from the Higgs sector are properly taken into account, the model can be ruled out [27–30], assuming that the low energy supersymmetric threshold corrections to the fermion masses are negligible. With

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the relatively large Higgs mass $m_H = 125$ GeV, the split supersymmetric scenario [31,32] of the minimal $SO(10)$ model [33] is also found to be inconsistent [34,35][4].

One should not abandon the whole elegant grand unified program simply because the simplest supersymmetric realization does not work perfectly. The usual way to rule in a theory that was ruled out is to increase the particle content and thus the number of model parameters. This was the approach of [36], where a new 120-dimensional Higgs representation has been added to the minimal model.[5] In this way the Yukawa sector increases by one antisymmetric matrix, which gives sufficient freedom to fit the data.

In this paper we will go, surprisingly, in the opposite direction, and ask ourselves, if it is possible to fit the data with less, not more, Yukawa matrices. This paradoxical question has obviously a hidden proviso, otherwise we would get no mixing at all. To account for the correct low energy mass spectrum, mixings, and CP violation we will thus make use of an extra vector-like generation $16 + \overline{16}$, similar to the one used in [37]. The difference with [37] is that we will assume the bilinear spinors 16_a to be coupled with $\overline{126}_H$ instead of 10_H. In this way we may hope to describe neutrino masses and mixings in a pattern similar to the charged fermions, which is one of the great achievements of the $SO(10)$ framework.

We shall see that this decreasing of the number of Yukawa matrices at the expense of an extra vector-like family can be successful and we will show several examples where it works. Although we will consider different possible Higgs sectors and take some of their constraints seriously, we will not consider a combined fit of the Higgs and Yukawa parameters, which can obviously pose extra restrictions. This more modest approach nevertheless shows that $SO(10)$ Yukawa sectors with a single Yukawa matrix can be realistic.

The rest of the paper is organized as follows. In Sec. II we present the key features of the new class of $SO(10)$ models. In Sec. III we set up the framework and the formalism. In Sec. IV we adopt a specific basis that removes redundancies, which is well suited for numerical analysis of the flavor observables. Sec. V discusses the constraints imposed on the SUSY models from the minimization of the Higgs potential. Sec. VI has our numerical fits to the fermion masses and mixings for the six models analyzed. Finally, in Sec. VII we conclude. In Appendices A and B some useful relations used for the fermion mass fits are given. Appendix C contains the numerical Yukawa matrices for various cases that result from the fits.

2   New class of $SO(10)$ models

The key feature of the new proposed models is the absence of 10_H. In its place we introduce a $16 + \overline{16}$ vector-like fermions. In addition to a $\overline{126}_H$, we employ either a 45_H or a 210_H for symmetry breaking. These fields have non-trivial couplings to the vector-like fermions,

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[4]BB thanks Ketan Patel for pointing this out.

[5]Another possibility, not yet fully explored, is to increase the Higgs sector parameters, for example with a 54_H, see Ref [7] for fermion fits.

which is needed to avoid certain unwanted relations among down-type quark and charged lepton masses. Additional Higgs fields (e.g. 54_H) are needed for consistent GUT symmetry breaking, but these fields do not enter into the Yukawa sector. The Yukawa Lagrangian of our models has a very simple form,

$$\mathcal{L}_{\text{Yuk}} = \overline{16} \left(m_a + \eta_a 45_H \right) 16_a + 16_a \, \mathcal{Y}_{ab} \, \overline{126}_H \, 16_b + \overline{16} \, \bar{y} \, 126_H \, \overline{16} + h.c. \qquad (2.1)$$

corresponding to the use of 45_H as the symmetry breaking field (in addition to the $\overline{126}_H$ field). Here a, $b = 1 - 4$ are the generation indices which include a 16 from the vector-like family. We thus see that the Yukawa sector has one 4×4 matrix $\mathcal{Y}_{ab}$, and two four-vectors m_a and η_a. Since $\mathcal{Y}_{ab}$ can be chosen to be diagonal and real, this amounts to $4 + 4 + 4$ flavor mixing parameters. The Yukawa coupling $\overline{16} \, \bar{y} \, 126_H$ does not have any effect on the light fermion masses and mixings. While in the diagonal and real basis for $\mathcal{Y}_{ab}$ the vectors m_a and η_a are in general complex, these being related to GUT scale masses, one complex combination disappears from low energy masses and mixings. One should add to this set two (real) VEV ratios (one from the two SM singlets of 45_H and one for the up-type and down-type Higgs doublet VEV ratio from the $\overline{126}_H$), and an overall scale for the right-handed neutrino masses. We thus see that the model has 14 real parameters and 7 phases to fit 18 observed values among quark masses, quark mixings and CP violation, charged fermion masses, neutrino mass-squared differences and mixing angles. Thus these models are rather constrained, yet we show that excellent fits are obtained. It may be noted that the minimal supersymmetric $SO(10)$ models with two symmetric Yukawa coupling matrices involving 10_H and $\overline{126}_H$ have 12 real parameters and 7 phases that enter into the flavor sector.

The basic structure of Eq. (2.1) can be realized in several other ways. We study all such $SO(10)$ models in this paper. The Higgs field 45_H in Eq. (2.1) may be replaced by a 210_H. In this case, since the 210_H contains three SM singlet fields, there are two ratios of VEVs from the 210_H, which would increase the number of parameters by one. These models may be realized with or without low energy supersymmetry. In the non-SUSY models, the VEVs of 45_H and 210_H are real, while in SUSY models they are in general complex (thus increasing the phase parameters to 8). In the SUSY models we find that although the 210_H has two associated VEV ratios, only one of the two is independent, due to symmetry breaking constraints arising from the superpotential. In SUSY versions, additional fields other than $\overline{126}_H$ and 210_H used in the Yukawa sector are often required, in order to avoid new chiral supermultiplets that remain light and spoil unification of gauge couplings. A summary of the models that fit into this classification and studied here is given below. All models contain a $\overline{126}_H$ (plus a 126_H in the case of SUSY), in addition to the Higgs fields shown below.

A. Non-SUSY Model with $45_H + 54_H$

B. Non-SUSY Model with $210_H + 54_H$ or $210_H + 16_H$

C. SUSY Model with $45_H + 54_H + 16_H + \overline{16}_H$

D. SUSY Model with $210_H + 54_H$

E. SUSY Model with $210_H + 16_H + \overline{16}_H$

F. SUSY Model with $210_H + 54_H + 16_H + \overline{16}_H$

The VEV of the SM singlet in $\overline{126}_H$ will be found *a posteriori* to be around $10^{13} - 10^{14}$ GeV in all models. This has an effect on the choice of Higgs fields, especially in the SUSY models: Very simple Higgs systems used for GUT symmetry breaking would lead to certain sub-multiplets having mass of order 10^{11} GeV, which would spoil perturbative unification of gauge couplings in SUSY $SO(10)$. The choice of "other Higgs fields" shown above are in part guided by this not happening. Furthermore, in some simplistic SUSY cases, the Higgs doublet mass matrix becomes proportional to other color sector mass matrix. Making a pair of Higgs doublets light would then lead to making a pair of colored states light as well, which affects perturbative unification. Such cases are avoided in the scenarios shown above. In each of the models listed above, seesaw mechanism may be realized via either type-I or type-II chain. Such sub-classes will be denoted by a label I or II when needed. Thus **AI** would refer to type-I seesaw in Model **A**, and likewise **AII** for type-II seesaw in the same model.

Models **A** and **B** are nonsupersymmetric, while models **C**–**F** are supersymmetric. For model **A**, in addition to 45_H, a 54_H is needed to break $SO(10)$ down to the SM without going through an intermediate $SU(5)$-symmetric limit. In Model **B** which uses a 210_H, an additional field, either a 54_H or a 16_H is needed for the following reason. As noted already, 126_H acquires a VEV of order $10^{13} - 10^{14}$ GeV, which can be ignored for the study of GUT symmetry breaking at around 10^{16} GeV. A single 210_H would break $SO(10)$ down to one of its maximal little groups, such as $SU(5) \times U(1)$, $SU(4)_C \times SU(2)_L \times SU(2)_R$ etc. The fermion mass matrix would then reflect this unbroken symmetry, which is not realistic in the light fermion spectrum. Addition of a 54_H (or a 16_H) with a GUT VEV would reduce the surviving symmetry and help with realistic fermion masses. For SUSY $SO(10)$, it is not a viable model if the symmetry is only broken by $45_H + 54_H$, since in this case, the Higgs doublet (1,2,1/2) and the Higgs octet (8,2,1/2) mass matrices become identical. So one cannot make the MSSM doublet fields light without also making the octet fields light. To break this degeneracy one needs to extend the Higgs sector. For this purpose in model **C**, we enlarge the Higgs sector by adding $16_H + \overline{16}_H$. SUSY $SO(10)$ model with $210_H + 126_H + \overline{126}_H$ is also not a consistent model, because with the requirement $v_R \sim 10^{13-14}$ GeV, the octet $(8, 3, 0)$ Higgs field becomes light with a mass of order $\sim 10^{10-11}$ GeV, so the theory does not remain perturbative up to the GUT scale. Thus, in order to avoid this, in model **D**, we include 54_H Higgs and in model **E**, we include a $16_H + \overline{16}_H$. It will be shown later in Sec. V that, in all these SUSY $SO(10)$ models with a 210_H, there is only one independent VEV ratio involving the 210_H field, owing to symmetry breaking constraints. Including more Higgs multiplets, one can break such relationships among VEVs which can lead to two independent VEV ratios for 210_H. We also consider this general case which is labeled as model **F**, where

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in addition to 210_H, one has both 54_H and $16_H + \overline{16}_H$ (or some unspecified) multiplets. It is to be mentioned that, we do not consider any model where both the 45_H and 210_H are present simultaneously, which would lead to more parameters and thus less predictions in the fermion sector. Details of the symmetry breaking schemes will be explained further in Sec. V.

3 The set-up and formalism

All models we study have one vector like $16 + \overline{16}$ pair plus 3 generations of chiral 16's. Their mass terms and couplings to a 45_H given in Eq. (2.1) can be expanded to yield

$$
\begin{aligned}
\overline{16}\,(m_a + \eta_a 45_H)\,16_a \;=\;& \bar{L}\,(m_a + \eta_a(3v_1))\,L_a + \bar{Q}\,(m_a + \eta_a(-v_1))\,Q_a \\
& + e_a^c\,(m_a + \eta_a(-3v_1 - v_2))\,\bar{e}^c + \nu_a^c\,(m_a + \eta_a(-3v_1 + v_2))\,\bar{\nu}^c \\
& + d_a^c\,(m_a + \eta_a(v_1 - v_2))\,\bar{d}^c + u_a^c\,(m_a + \eta_a(v_1 + v_2))\,\bar{u}^c,
\end{aligned} \tag{3.2}
$$

where $a = 1, \ldots, 4$ and

$$
v_1 = \langle 45_H \rangle_{(1,1,15)} \quad , \quad v_2 = \langle 45_H \rangle_{(1,3,1)}. \tag{3.3}
$$

These are the SM singlet components of 45_H which acquire GUT scale VEVs denoted here as $v_{1,2}$.

The mass terms are of the general form

$$
\bar{\psi} M_a \psi_a. \tag{3.4}
$$

Although by redefining the phases of ψ_a we can make all these M_a real, we will keep them complex in general. Then we project to the heavy states as usual by

$$
\psi_a \to U_{ab} \psi_b, \tag{3.5}
$$

with

$$
U \;=\; \begin{pmatrix} \Lambda & \Lambda x^* \\ -x^T \Lambda & \bar{\Lambda} \end{pmatrix} \qquad (U^\dagger = U^{-1}) \tag{3.6}
$$

$$
\Lambda \;=\; 1 - \frac{x^* x^T}{\sqrt{1 + |x|^2}(\sqrt{1 + |x|^2} + 1)} \qquad (\Lambda^\dagger = \Lambda) \tag{3.7}
$$

$$
x^T \;=\; \frac{1}{M_4}(M_1, M_2, M_3) \quad , \quad \bar{\Lambda} = \frac{1}{\sqrt{1 + |x|^2}}. \tag{3.8}
$$

To this we add the Yukawa couplings to $\overline{126}_H$. Although we are free to choose this 4×4 Yukawa matrix to be diagonal and real (in the original basis, i.e. before (3.5)), we will keep it to be complex symmetric and choose a convenient basis later on. The $\overline{16}$ has coupling to the 126_H, but this will turn out to not affect light fermion masses. The relevant Yukawa couplings are (see Eq. (2.1))

$$
16_a\,\mathcal{Y}_{ab}\,\overline{126}_H\,16_b + \overline{16}\,\bar{y}\,126_H\,\overline{16}. \tag{3.9}
$$

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In this original basis we put all together:

$$
\begin{pmatrix} d_a^c & \bar{d} \end{pmatrix}
\begin{pmatrix}
\mathcal{Y}_{ab} v_d & m_a + \eta_a(v_1 - v_2) \\
m_b + \eta_b(-v_1) & \bar{y} v_d
\end{pmatrix}
\begin{pmatrix} d_b \\ \bar{d}^c \end{pmatrix}
$$

$$
+ \begin{pmatrix} u_a^c & \bar{u} \end{pmatrix}
\begin{pmatrix}
\mathcal{Y}_{ab} v_u & m_a + \eta_a(v_1 + v_2) \\
m_b + \eta_b(-v_1) & \bar{y} v_u
\end{pmatrix}
\begin{pmatrix} u_b \\ \bar{u}^c \end{pmatrix}
$$

$$
+ \begin{pmatrix} e_a^c & \bar{e} \end{pmatrix}
\begin{pmatrix}
-3\mathcal{Y}_{ab} v_d & m_a + \eta_a(-3v_1 - v_2) \\
m_b + \eta_b(3v_1) & -3\bar{y} v_d
\end{pmatrix}
\begin{pmatrix} e_b \\ \bar{e}^c \end{pmatrix} \tag{3.10}
$$

$$
+ \begin{pmatrix} \nu_a^c & \bar{\nu} \end{pmatrix}
\begin{pmatrix}
-3\mathcal{Y}_{ab} v_u & m_a + \eta_a(-3v_1 + v_2) \\
m_b + \eta_b(3v_1) & -3\bar{y} v_u
\end{pmatrix}
\begin{pmatrix} \nu_b \\ \bar{\nu}^c \end{pmatrix}
$$

$$
+ \frac{1}{2} \begin{pmatrix} \nu_a^c & \bar{\nu} \end{pmatrix}
\begin{pmatrix}
\mathcal{Y}_{ab} v_R & 0 \\
0 & \bar{y} \bar{v}_L
\end{pmatrix}
\begin{pmatrix} \nu_b^c \\ \bar{\nu} \end{pmatrix}
+ \frac{1}{2} \begin{pmatrix} \nu_a & \bar{\nu}^c \end{pmatrix}
\begin{pmatrix}
\mathcal{Y}_{ab} v_L & 0 \\
0 & \bar{y} \bar{v}_R
\end{pmatrix}
\begin{pmatrix} \nu_b \\ \bar{\nu}^c \end{pmatrix},
$$

where

$$
\begin{aligned}
v_R &= \langle \overline{126}_H \rangle_{(1,3,10)} & , & \quad v_L = \langle \overline{126}_H \rangle_{(3,1,\overline{10})} \\
\bar{v}_R &= \langle 126_H \rangle_{(1,3,\overline{10})} & , & \quad \bar{v}_L = \langle 126_H \rangle_{(3,1,10)} \\
v_u &= \langle \overline{126}_H \rangle_{(2,2,15)_u} & , & \quad v_d = \langle \overline{126}_H \rangle_{(2,2,15)_d}.
\end{aligned} \tag{3.11}
$$

Here v_R and $\bar{v}_R$ are close to, but somewhat below the GUT scale, while $v_{u,d}$ are the VEVs of the electroweak Higgs doublets arising from the $\overline{126}_H$. v_L and $\bar{v}_L$ denote the induced VEVs of the $SU(2)_L$ triplets from 126_H and $\overline{126}_H$. In non-supersymmetric models, we have $\bar{v}_R = v_R^*$, $v_d = v_u^*$ and $\bar{v}_L = v_L^*$.

After the transformation given in Eq. (3.5) the matrices Eq. (3.10) become

$$
\rightarrow \begin{pmatrix} d_a^c & \bar{d} \end{pmatrix}
\begin{pmatrix}
(U_{d^c}^T)_{ae} \mathcal{Y}_{ef} v_d (U_Q)_{fb} & M_{d^c} \delta_{a4} \\
M_Q \delta_{b4} & \bar{y} v_d
\end{pmatrix}
\begin{pmatrix} d_b \\ \bar{d}^c \end{pmatrix}
$$

$$
+ \begin{pmatrix} u_a^c & \bar{u} \end{pmatrix}
\begin{pmatrix}
(U_{u^c}^T)_{ae} \mathcal{Y}_{ef} v_u (U_Q)_{fb} & M_{u^c} \delta_{a4} \\
M_Q \delta_{b4} & \bar{y} v_u
\end{pmatrix}
\begin{pmatrix} u_b \\ \bar{u}^c \end{pmatrix}
$$

$$
+ \begin{pmatrix} e_a^c & \bar{e} \end{pmatrix}
\begin{pmatrix}
(U_{e^c}^T)_{ae} (-3) \mathcal{Y}_{ef} v_d (U_L)_{fb} & M_{e^c} \delta_{a4} \\
M_L \delta_{b4} & -3\bar{y} v_d
\end{pmatrix}
\begin{pmatrix} e_b \\ \bar{e}^c \end{pmatrix} \tag{3.12}
$$

$$
+ \begin{pmatrix} \nu_a^c & \bar{\nu} \end{pmatrix}
\begin{pmatrix}
(U_{\nu^c}^T)_{ae} (-3) \mathcal{Y}_{ef} v_u (U_L)_{fb} & M_{\nu^c} \delta_{a4} \\
M_L \delta_{b4} & -3\bar{y} v_u
\end{pmatrix}
\begin{pmatrix} \nu_b \\ \bar{\nu}^c \end{pmatrix}
$$

$$
+ \frac{1}{2} \begin{pmatrix} \nu_a^c & \bar{\nu} \end{pmatrix}
\begin{pmatrix}
(U_{\nu^c}^T)_{ae} \mathcal{Y}_{ef} v_R (U_{\nu^c})_{fb} & 0 \\
0 & \bar{y} \bar{v}_L
\end{pmatrix}
\begin{pmatrix} \nu_b^c \\ \bar{\nu} \end{pmatrix}
$$

$$
+ \frac{1}{2} \begin{pmatrix} \nu_a & \bar{\nu}^c \end{pmatrix}
\begin{pmatrix}
(U_L^T)_{ae} \mathcal{Y}_{ef} v_L (U_L)_{fb} & 0 \\
0 & \bar{y} \bar{v}_R
\end{pmatrix}
\begin{pmatrix} \nu_b \\ \bar{\nu}^c \end{pmatrix}.
$$

To get the light fermion mass matrices defined as

$$
\mathcal{L} = d^{cT} M_D d + u^{cT} M_U u + e^{cT} M_E e + \frac{1}{2} \nu^T M_N \nu + h.c. \tag{3.13}
$$

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we have to project to the light generations. In doing so we need to evaluate ($\mathcal{Y}$ is a 4×4 matrix, while Y is its 3×3 submatrix)

$$
\begin{aligned}
U_1^T \mathcal{Y} U_2 &= \begin{pmatrix} \Lambda_1^T & -\Lambda_1^T x_1 \\ x_1^\dagger \Lambda_1^T & \bar{\Lambda}_1 \end{pmatrix} \begin{pmatrix} Y & y \\ y^T & y_4 \end{pmatrix} \begin{pmatrix} \Lambda_2 & \Lambda_2 x_2^* \\ -x_2^T \Lambda_2 & \bar{\Lambda}_2 \end{pmatrix} \\
&= \begin{pmatrix} \Lambda_1^T(Y - y x_2^T - x_1 y^T + y_4 x_1 x_2^T)\Lambda_2 & \Lambda_1^T(Y x_2^* + y - x_1 y^T x_2^* - y_4 x_1)\bar{\Lambda}_2 \\ \bar{\Lambda}_1(x_1^\dagger Y + y^T - x_1^\dagger y x_2^T - y_4 x_2^T)\Lambda_2 & \bar{\Lambda}_1(x_1^\dagger Y x_2^* + y^T x_2^* + x_1^\dagger Y + y_4)\bar{\Lambda}_2 \end{pmatrix},
\end{aligned}
\tag{3.14}
$$

where we used $\Lambda x^* = \bar{\Lambda} x^*$.

For charged fermions this is enough, and we get (mass matrices are defined as $\psi^c M_\Psi \psi$)

$$
\begin{aligned}
M_D &= v_d \Lambda_{d^c}^T \left(Y - y x_Q^T - x_{d^c} y^T + y_4 x_{d^c} x_Q^T\right) \Lambda_Q & (3.15) \\
M_U &= v_u \Lambda_{u^c}^T \left(Y - y x_Q^T - x_{u^c} y^T + y_4 x_{u^c} x_Q^T\right) \Lambda_Q & (3.16) \\
M_E &= -3 v_d \Lambda_{e^c}^T \left(Y - y x_L^T - x_{e^c} y^T + y_4 x_{e^c} x_L^T\right) \Lambda_L. & (3.17)
\end{aligned}
$$

For neutrinos things are slightly more involved, since there are two kinds of heavy neutrinos, the usual right-handed ones, plus the new vector-like ones. The full symmetric Majorana mass matrix is 10×10. However, in the leading order in $y v_R / M_{L,\nu^c}$ (M_{L,ν^c} denote the masses of vector-like leptons), the situation returns to ordinary with

$$
\begin{aligned}
M_{\nu_D} &= -3 v_u \Lambda_{\nu^c}^T (Y - y x_L^T - x_{\nu^c} y^T + y_4 x_{\nu^c} x_L^T)\Lambda_L & (3.18) \\
M_{\nu_R} &= v_R \Lambda_{\nu^c}^T (Y - y x_{\nu^c}^T - x_{\nu^c} y^T + y_4 x_{\nu^c} x_{\nu^c}^T)\Lambda_{\nu^c} & (3.19) \\
M_{\nu_L} &= v_L \Lambda_L^T (Y - y x_L^T - x_L y^T + y_4 x_L x_L^T)\Lambda_L, & (3.20)
\end{aligned}
$$

so that as usual by using the seesaw [17] formula we arrive at the 3×3 light neutrino mass matrix as

$$
M_N = M_{\nu_L} - M_{\nu_D}^T M_{\nu_R}^{-1} M_{\nu_D}.
\tag{3.21}
$$

If the approximation $y v_R / M_{L,\nu^c} \ll 1$ is not good, we write the full symmetric matrix for $(\nu_i, \nu_i^c, \nu_4, \bar{\nu}^c, \nu_4^c, \bar{\nu})$:

$$
\begin{pmatrix}
(U_L^T(v_L \mathcal{Y})U_L)_{ij} & (U_L^T(-3v_u \mathcal{Y})U_{\nu^c})_{ij} & (U_L^T(v_L \mathcal{Y})U_L)_{i4} & 0 & (U_L^T(-3v_u \mathcal{Y})U_{\nu^c})_{i4} & 0 \\
(U_{\nu^c}^T(-3v_u \mathcal{Y})U_L)_{ij} & (U_{\nu^c}^T(v_R \mathcal{Y})U_{\nu^c})_{ij} & (U_{\nu^c}^T(-3v_u \mathcal{Y})U_L)_{i4} & 0 & (U_{\nu^c}^T(v_R \mathcal{Y})U_{\nu^c})_{i4} & 0 \\
(U_L^T(v_L \mathcal{Y})U_L)_{4j} & (U_L^T(-3v_u \mathcal{Y})U_{\nu^c})_{4j} & (U_L^T(v_L \mathcal{Y})U_L)_{44} & 0 & (U_L^T(-3v_u \mathcal{Y})U_{\nu^c})_{44} & M_L \\
0 & 0 & 0 & \bar{v}_R y & M_{\nu^c} & -3v_u y \\
(U_{\nu^c}^T(-3v_u \mathcal{Y})U_L)_{4j} & (U_{\nu^c}^T(v_R \mathcal{Y})U_{\nu^c})_{4j} & (U_{\nu^c}^T(-3v_u \mathcal{Y})U_L)_{44} & M_{\nu^c} & (U_{\nu^c}^T(v_R \mathcal{Y})U_{\nu^c})_{44} & 0 \\
0 & 0 & M_L & -3v_u y & 0 & \bar{v}_L y
\end{pmatrix}.
\tag{3.22}
$$

One can integrate out ν_4 and $\bar{\nu}$ without any trace, since they mix through a large M_L, but otherwise feel just the small VEVs. What remains is for $(\nu_i, \nu_i^c, \bar{\nu}^c, \nu_4^c)$:

$$
\begin{pmatrix}
(U_L^T(v_L \mathcal{Y})U_L)_{ij} & (U_L^T(-3v_u \mathcal{Y})U_{\nu^c})_{ij} & 0 & (U_L^T(-3v_u \mathcal{Y})U_{\nu^c})_{i4} \\
(U_{\nu^c}^T(-3v_u \mathcal{Y})U_L)_{ij} & (U_{\nu^c}^T(v_R \mathcal{Y})U_{\nu^c})_{ij} & 0 & (U_{\nu^c}^T(v_R \mathcal{Y})U_{\nu^c})_{i4} \\
0 & 0 & \bar{v}_R y & M_{\nu^c} \\
(U_{\nu^c}^T(-3v_u \mathcal{Y})U_L)_{4j} & (U_{\nu^c}^T(v_R \mathcal{Y})U_{\nu^c})_{4j} & M_{\nu^c} & (U_{\nu^c}^T(v_R \mathcal{Y})U_{\nu^c})_{44}
\end{pmatrix}.
\tag{3.23}
$$

This has again the form

$$\begin{pmatrix} M_{\nu_L} & M_{\nu_D}^T \\ M_{\nu_D} & M_{\nu_R} \end{pmatrix}. \tag{3.24}$$

and thus Eq. (3.21) applies with M_{ν_L} given by Eq. (3.20), but now for 5 right-handed neutrinos with a 5×3 matrix M_{ν_D} and a 5×5 symmetric matrix M_{ν_R}:

$$M_{\nu_D} = (-3v_u) \begin{pmatrix} \Lambda_{\nu^c}(Y - yx_L^T - x_{\nu^c}y^T + y_4 x_{\nu^c}x_L^T)\Lambda_L \\ 0 \\ \bar{\Lambda}_{\nu^c}(x_{\nu^c}^T Y + y^T - x_{\nu^c}^T yx_L^T - x_L^T y_4)\Lambda_L \end{pmatrix} \tag{3.25}$$

$$M_{\nu_R} = \begin{pmatrix} v_R \Lambda_{\nu^c}(Y - yx_{\nu^c}^T - x_{\nu^c}y^T + y_4 x_{\nu^c}x_{\nu^c}^T)\Lambda_{\nu^c} & 0 & v_R \Lambda_{\nu^c}(Yx_{\nu^c} + y - x_{\nu^c}y^T x_{\nu^c} - y_4 x_{\nu^c})\bar{\Lambda}_{\nu^c} \\ 0 & \bar{v}_R y & M_{\nu^c} \\ v_R \bar{\Lambda}_{\nu^c}(x_{\nu^c}^T Y + y^T - x_{\nu^c}^T yx_{\nu^c}^T - y_4 x_{\nu^c}^T)\Lambda_{\nu^c} & M_{\nu^c} & v_R \bar{\Lambda}_{\nu^c}(x_{\nu^c}^T Yx_{\nu^c} + y^T x_{\nu^c} + x_{\nu^c}^T y + y_4)\bar{\Lambda}_{\nu^c} \end{pmatrix}. \tag{3.26}$$

To conclude, let's write down explicitly the various $\vec{x}$'s:

$$\vec{x}_L = \frac{\vec{m} + \vec{\eta}(3v_1)}{m_4 + \eta_4(3v_1)} \quad , \quad \vec{x}_Q = \frac{\vec{m} + \vec{\eta}(-v_1)}{m_4 + \eta_4(-v_1)}, \tag{3.27}$$

$$\vec{x}_{e^c} = \frac{\vec{m} + \vec{\eta}(-3v_1 - v_2)}{m_4 + \eta_4(-3v_1 - v_2)} \quad , \quad \vec{x}_{\nu^c} = \frac{\vec{m} + \vec{\eta}(-3v_1 + v_2)}{m_4 + \eta_4(-3v_1 + v_2)}, \tag{3.28}$$

$$\vec{x}_{d^c} = \frac{\vec{m} + \vec{\eta}(v_1 - v_2)}{m_4 + \eta_4(v_1 - v_2)} \quad , \quad \vec{x}_{u^c} = \frac{\vec{m} + \vec{\eta}(v_1 + v_2)}{m_4 + \eta_4(v_1 + v_2)}. \tag{3.29}$$

Defining

$$\vec{x} = \frac{\vec{m}}{m_4} \quad , \quad u_{1,2} = \eta_4 \frac{v_{1,2}}{m_4} \quad , \quad \vec{z} = \frac{\vec{\eta}}{\eta_4}, \tag{3.30}$$

we can rewrite the above as

$$\vec{x}_L = \frac{\vec{x} + \vec{z}(3u_1)}{1 + (3u_1)} \quad , \quad \vec{x}_Q = \frac{\vec{x} + \vec{z}(-u_1)}{1 + (-u_1)},$$

$$\vec{x}_{e^c} = \frac{\vec{x} + \vec{z}(-3u_1 - u_2)}{1 + (-3u_1 - u_2)} \quad , \quad \vec{x}_{\nu^c} = \frac{\vec{x} + \vec{z}(-3u_1 + u_2)}{1 + (-3u_1 + u_2)}, \tag{3.31}$$

$$\vec{x}_{d^c} = \frac{\vec{x} + \vec{z}(u_1 - u_2)}{1 + (u_1 - u_2)} \quad , \quad \vec{x}_{u^c} = \frac{\vec{x} + \vec{z}(u_1 + u_2)}{1 + (u_1 + u_2)}.$$

To get the masses and mixings we change the basis

$$x \to X_L x \quad , \quad x^c \to X_R^* x^c \tag{3.32}$$

for $x = d, u, e, \nu$ and $X = D, U, E, N$. This means that (for $X = N$, $X_R = X_L^*$)

$$M_X = X_R M_X^d X_L^\dagger \tag{3.33}$$

so that the CKM and PMNS matrices are defined as

$$V_{CKM} = U_L^\dagger D_L \tag{3.34}$$

$$V_{PMNS} = E_L^\dagger N_L. \tag{3.35}$$

So far we have been very general. However, there are redundancies that are present, which should be removed for an efficient numerical fitting algorithm. In the next section we shall choose a specific basis, which may appear at first to be less intuitive but which is well-suited for our numerical minimization. There are two obvious basis choices, one where $\mathcal{Y}_{ab}$ is diagonal, and a second one where the vectors m_a and η_a have simple forms. It is the second one that is used in the next section. For further use we give here the relations between the two sets of parameters.

$$\vec{x} \;=\; (0, 0, \tan\theta e^{-i\phi}) \tag{3.36}$$

$$\vec{z} \;=\; (0, 0, 0) \tag{3.37}$$

$$Y_{ij} \;=\; a_{ij} \tag{3.38}$$

$$y \;=\; (a_{41}, a_{42}, a_{43}) \tag{3.39}$$

$$y_4 \;=\; a_{44} \tag{3.40}$$

and

$$u_1 \;=\; -\frac{T e^{-i\phi}}{\cos\theta} \frac{\epsilon}{5} \tag{3.41}$$

$$u_2 \;=\; -\frac{T e^{-i\phi}}{\cos\theta} \left(1 + \frac{3\epsilon}{5}\right). \tag{3.42}$$

3.1 210_H instead of 45_H

If the 45_H is replaced by a 210_H, we simply change Eq. (3.2) into:

$$
\begin{aligned}
\overline{16}\,(m_a + \eta_a 210)\,16_a \;=\;\; & \bar{L}\,(m_a + \eta_a(\phi_1 - 3\phi_2))\,L_a \\
+\;\; & \bar{Q}\,(m_a + \eta_a(\phi_1 + \phi_2))\,Q_a \\
+\;\; & e_a^c\,(m_a + \eta_a(-\phi_1 - 3\phi_2 + 3\phi_3))\,\bar{e}^c \\
+\;\; & \nu_a^c\,(m_a + \eta_a(-\phi_1 - 3\phi_2 - 3\phi_3))\,\bar{\nu}^c \\
+\;\; & d_a^c\,(m_a + \eta_a(-\phi_1 + \phi_2 - \phi_3))\,\bar{d}^c \\
+\;\; & u_a^c\,(m_a + \eta_a(-\phi_1 + \phi_2 + \phi_3))\,\bar{u}^c
\end{aligned}
\tag{3.43}
$$

where

$$\phi_1 = \langle 210_H \rangle_{(1,1,1)} \quad,\quad \phi_2 = \langle 210_H \rangle_{(1,1,15)} \quad,\quad \phi_3 = \langle 210_H \rangle_{(1,3,15)} \tag{3.44}$$

are the VEVs of the three SM singlets of 210_H.

This then changes Eq. (3.31) into

$$
\begin{aligned}
\vec{x}_L &= \frac{\vec{x} + \vec{z}(u_1 - 3u_2)}{1 + (u_1 - 3u_2)} \quad,\quad \vec{x}_Q = \frac{\vec{x} + \vec{z}(u_1 + u_2)}{1 + (u_1 + u_2)}, \\[4pt]
\vec{x}_{e^c} &= \frac{\vec{x} + \vec{z}(-u_1 - 3u_2 + 3u_3)}{1 + (-u_1 - 3u_2 + 3u_3)} \quad,\quad \vec{x}_{\nu^c} = \frac{\vec{x} + \vec{z}(-u_1 - 3u_2 - 3u_3)}{1 + (-u_1 - 3u_2 - 3u_3)}, \\[4pt]
\vec{x}_{d^c} &= \frac{\vec{x} + \vec{z}(-u_1 + u_2 - u_3)}{1 + (-u_1 + u_2 - u_3)} \quad,\quad \vec{x}_{u^c} = \frac{\vec{x} + \vec{z}(-u_1 + u_2 + u_3)}{1 + (-u_1 + u_2 + u_3)},
\end{aligned}
\tag{3.45}
$$

where now

$$u_{1,2,3} = \eta_4 \frac{\phi_{1,2,3}}{m_4}. \tag{3.46}$$

For correspondence with the specific basis chosen in the next section, we still have Eqs. (3.36)-(3.40), but Eqs. (3.41)-(3.42) are replaced by

$$u_1 = \frac{Te^{-i\phi}}{\cos\theta} \tag{3.47}$$

$$u_2 = \frac{Te^{-i\phi}}{\cos\theta}\frac{\epsilon_1}{\sqrt{3}} \tag{3.48}$$

$$u_3 = \frac{Te^{-i\phi}}{\cos\theta}\epsilon_2\sqrt{\frac{2}{3}}. \tag{3.49}$$

4 Analysis in a specific basis

The general formulas given in the previous section for the light fermion mass matrices have built-in redundancies. Here we choose a specific basis where these redundancies are removed. We choose a basis where the four-vectors in Eq. (2.1) have simple forms:

$$\eta_a = (0,0,0,1)\,b, \quad m_a = (0,0,\sin\theta, e^{i\phi}\cos\theta)\,M\;. \tag{4.50}$$

These simple forms are achieved by 4×4 family rotation, which makes the vector $\vec{\eta}$ to have the form shown, and a subsequent 3×3 family rotation that brings the vector $\vec{m}$ to this form. A further rotation in the first two family space can be made, we choose this rotation to make the 4×4 Yukawa matrix, denoted as a_{ij} in this specific basis, to be diagonal in the 1-2 subspace, i.e., $a_{12} = a_{21} = 0$. The correspondence given in Eqs. (3.36)-(3.40) as well as Eqs. (3.41)-(3.42) for the case of 45_H arise from this choice of basis. (The parameters T and ϵ will be defined shortly.) Let us denote $\Phi = 45_H$ or 210_H and the VEV of Φ to be $\langle\Phi\rangle = \Omega$ which has two components (for $\Phi = 45_H$) or three components (for $\Phi = 210_H$). The Yukawa Lagrangian in this specific basis takes the form:

$$\mathcal{L}_{\text{Yuk}} = \sum_{i,j=1}^{4} a_{ij}\,16_i\,16_j\,\overline{126}_H + \bar{y}\,\overline{16}\,\overline{16}\,126_H + b\,\overline{16}\,16_4\,\Phi + M\,\overline{16}\,(\sin\theta\,16_3 + e^{i\phi}\cos\theta\,16_4). \tag{4.51}$$

The effective mass terms that arise after the VEV of Φ is inserted would depend on the VEV ratio of the two SM singlets in 45_H and on two VEV ratios of the three SM singlets in the case of 210_H. For the former, we can define an unbroken charge Q, which is not the electric charge, but a linear combination of hypercharge Y and the $U(1)_X$ charge contained in $SO(10) \to SU(5) \times U(1)_X$ – the 45_H leaves this charge Q unbroken. A parameter ϵ can be introduced in terms of which the unbroken charge Q can be defined for each of the SM fermions [37]:

$$Q = -\frac{1}{5}X + \frac{6(\epsilon+1)}{5}\frac{Y}{2} = 2I_{3R} + \frac{6\epsilon}{5}\frac{Y}{2}, \tag{4.52}$$

where X is normalized so that $X_{10\in16} = 1$, $X_{\bar{5}\in16} = -3$ and $X_{1\in16} = 5$. Thus the charges of fermions $\in 16$ of $SO(10)$ for the case of 45_H are:

$$Q_{u,d} = \frac{1}{5}\epsilon; \ Q_{u^c} = -1 - \frac{4}{5}\epsilon; \ Q_{d^c} = 1 + \frac{2}{5}\epsilon;$$
$$Q_{e,\nu} = -\frac{3}{5}\epsilon; \ Q_{e^c} = 1 + \frac{6}{5}\epsilon; \ Q_{\nu^c} = -1. \tag{4.53}$$

For 210_H case the fermion charges are given in terms of two parameters $\epsilon_{1,2}$:

$$Q_{u,d} = 1 + \frac{\epsilon_1}{\sqrt{3}}; \ Q_{u^c} = -1 + \frac{\epsilon_1}{\sqrt{3}} + \sqrt{\frac{2}{3}}\epsilon_2; \ Q_{d^c} = -1 + \frac{\epsilon_1}{\sqrt{3}} - \sqrt{\frac{2}{3}}\epsilon_2;$$
$$Q_{e,\nu} = 1 - \sqrt{3}\epsilon_1; \ Q_{e^c} = -1 - \sqrt{3}\epsilon_1 + \sqrt{6}\epsilon_2; \ Q_{\nu^c} = -1 - \sqrt{3}\epsilon_1 - \sqrt{6}\epsilon_2. \tag{4.54}$$

These charges are obtained from Eq. (3.43) by setting $\phi_1 = 1$, $\phi_2 = \epsilon_1/\sqrt{3}$ and $\phi_3 = \sqrt{2/3}\,\epsilon_2$.

For non-SUSY case, $Q_f = Q_f^*$ as Φ is a real field in this case, while Q_f is complex in the case of SUSY. Now, writing $b\,\overline{16}\,16_4\,\langle\Phi\rangle = b\Omega(\overline{f}Q_f f_4 + \overline{f}^c Q_{f^c} f_4^c)$, the last two terms of the Yukawa Lagrangian in Eq. (4.51) can be written as

$$\mathcal{L}_{\text{Yuk}} \supseteq \overline{f}[(M\sin\theta)f_3 + (Me^{i\phi}\cos\theta + b\Omega Q_f)f_4] + \overline{f}^c[(M\sin\theta)f_3^c + (Me^{i\phi}\cos\theta + b\Omega Q_{f^c})f_4^c]. \tag{4.55}$$

Then defining

$$T \equiv b\Omega/M; \ N_{f,f^c} \equiv \sqrt{1 + T^2|Q_{f,f^c}|^2 + T\cos\theta(e^{-i\phi}Q_{f,f^c} + e^{i\phi}Q_{f,f^c}^*)}, \tag{4.56}$$

the heavy (GUT scale) fields $(\hat{f}_4, \hat{f}_4^c)$ and the light SM fields $(\hat{f}_3, \hat{f}_3^c)$ can be identified as

$$\frac{(\sin\theta)f_3 + (e^{i\phi}\cos\theta + TQ_f)f_4}{N_f} \equiv \hat{f}_4 \ ; \ \frac{(e^{-i\phi}\cos\theta + TQ_f^*)f_3 - (\sin\theta)f_4}{N_f} \equiv \hat{f}_3; \tag{4.57}$$

$$\frac{(\sin\theta)f_3^c + (e^{i\phi}\cos\theta + TQ_{f^c})f_4^c}{N_{f^c}} \equiv \hat{f}_4^c \ ; \ \frac{(e^{-i\phi}\cos\theta + TQ_{f^c}^*)f_3^c - (\sin\theta)f_4^c}{N_{f^c}} \equiv \hat{f}_3^c. \tag{4.58}$$

These expressions are valid for $f = u, d, e, \nu$ and $f^c = u^c, d^c, e^c, \nu^c$. Then from the full Yukawa Lagrangian one can compute the charged fermion and Dirac neutrino mass matrices for the light fermions written as $f^c M_f f$ as:

$$M_f^T = v_f k_f \begin{bmatrix} a_{11}^f & 0 & a_{13}^f \\ 0 & a_{22}^f & a_{23}^f \\ a_{31}^f & a_{32}^f & a_{33}^f \end{bmatrix}, \tag{4.59}$$

where $f = U, D, E, \nu_D$, $v_e = v_d$, $v_\nu = v_u$, $k_{u,d} = 1$ and $k_{e,\nu} = -3$. We define the ratio $v_u/v_d \equiv r$. Note that this ratio is not exactly equal to $\tan\beta$ of MSSM, but is closely related to it. If we ignore the mixing of the up and down-type Higgs doublets from $\overline{126}_H$ with other doublets present in the theory, r would be equal to $\tan\beta$ in MSSM. The following relations are then readily obtained:

$$a_{11}^f = a_{11} \, , \tag{4.60}$$

$$a_{13}^f = \frac{a_{13}(e^{i\phi}\cos\theta + TQ_{f^c}) - a_{14}\sin\theta}{N_{f^c}} \, , \tag{4.61}$$

$$a_{22}^f = a_{22} \, , \tag{4.62}$$

$$a_{23}^f = \frac{a_{23}(e^{i\phi}\cos\theta + TQ_{f^c}) - a_{24}\sin\theta}{N_{f^c}} \, , \tag{4.63}$$

$$a_{31}^f = \frac{a_{13}(e^{i\phi}\cos\theta + TQ_f) - a_{14}\sin\theta}{N_f} \, , \tag{4.64}$$

$$a_{32}^f = \frac{a_{23}(e^{i\phi}\cos\theta + TQ_f) - a_{24}\sin\theta}{N_f} \, , \tag{4.65}$$

$$a_{33}^f = \frac{a_{33}(e^{i\phi}\cos\theta + TQ_f)(e^{i\phi}\cos\theta + TQ_{f^c}) + a_{44}\sin^2\theta - a_{34}\sin\theta[2e^{i\phi}\cos\theta + T(Q_f + Q_{f^c})]}{N_f N_{f^c}} \, . \tag{4.66}$$

Note that a rotation in the 1-2 sector has been made which makes $a_{12}^f = a_{21}^f = a_{12} = 0$. These mass matrices are not symmetric, since $a_{ij}^f \neq a_{ji}^f$, although the original matrix obeyes $a_{ij} = a_{ji}$. These four mass matrices for $f = U, D, D, \nu_D$ are given in terms of the parameters $\epsilon, T, \theta, \phi$ and a_{ij} (with $i, j = 1 - 4$ and $a_{12} = a_{21} = 0$). We choose to take elements of M_E to be independent. One can then solve for a_{13} and a_{14} in terms of a_{13}^e and a_{31}^e; similarly a_{23} and a_{24} in terms of a_{23}^e and a_{32}^e. From Eqs.(4.61), (4.63), (4.64), (4.65) and (4.53) one sees that this is a valid choice provided that $\epsilon \neq -5/9$ for $\Phi = 45_H$. (If $\epsilon = -5/9$, $a_{13}^e = a_{31}^e$ and $a_{23}^e = a_{32}^e$, which does not lead to realistic fermion masses.) Similarly for the case of $\Phi = 210_H$, the restriction is $\epsilon_2 \neq 0$ is required as can be seen from Eq. (4.54). All these mass matrices have the same 1-2 sector and one can choose $a_{11} = a_{11}^e$ and $a_{22} = a_{22}^e$. In addition, $a_{33}^e, a_{33}^u, a_{33}^d$ depend on 3 independent parameters a_{33}, a_{34}, a_{44} that appear only in the (3,3) sector of the light mass matrices. Since this linear system is invertible, one can treat $a_{33}^e, a_{33}^u, a_{33}^d$ as independent parameters. The (3,3) element of the right-handed neutrino Majorana matrix is then not free, and is determined in terms of $a_{33}^e, a_{33}^u, a_{33}^d$. Expressions for a_{ij} in terms of the independent parameters chosen are given in Appendix A .

The elements of M_E are independent parameters. We can express M_U and M_D in terms of $T, \theta, \phi, a_{33}^u, a_{33}^d, a_{ij}^e$ and ϵ (or $\epsilon_{1,2}$) for the case of 45_H (or 210_H), so in this basis the charged fermion mass matrices are:

$$M_E^T = -3v_d \begin{bmatrix} a_{11}^e & 0 & a_{13}^e \\ 0 & a_{22}^e & a_{23}^e \\ a_{31}^e & a_{32}^e & a_{33}^e \end{bmatrix} \; ; \tag{4.67}$$

$$M_U^T = v_u \begin{bmatrix} a_{11}^e & 0 & \frac{a_{13}^e N_{e^c}(Q_e - Q_{u^c}) + a_{31}^e N_e(-Q_{e^c} + Q_{u^c})}{N_{u^c}(Q_e - Q_{e^c})} \\ 0 & a_{22}^e & \frac{a_{23}^e N_{e^c}(Q_e - Q_{u^c}) + a_{32}^e N_e(-Q_{e^c} + Q_{u^c})}{N_{u^c}(Q_e - Q_{e^c})} \\ \frac{a_{13}^e N_{e^c}(Q_e - Q_u) + a_{31}^e N_e(-Q_{e^c} + Q_u)}{N_u(Q_e - Q_{e^c})} & \frac{a_{23}^e N_{e^c}(Q_e - Q_u) + a_{32}^e N_e(-Q_{e^c} + Q_u)}{N_u(Q_e - Q_{e^c})} & a_{33}^u \end{bmatrix} \; ; \tag{4.68}$$

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$$M_D^T = v_d \begin{bmatrix} a_{11}^e & 0 & \frac{a_{13}^e N_{e^c}(Q_e - Q_{d^c}) + a_{31}^e N_e(-Q_{e^c} + Q_{d^c})}{N_{d^c}(Q_e - Q_{e^c})} \\ 0 & a_{22}^e & \frac{a_{23}^e N_{e^c}(Q_e - Q_{d^c}) + a_{32}^e N_e(-Q_{e^c} + Q_{d^c})}{N_{d^c}(Q_e - Q_{e^c})} \\ \frac{a_{13}^e N_{e^c}(Q_e - Q_u) + a_{31}^e N_e(-Q_{e^c} + Q_u)}{N_u(Q_e - Q_{e^c})} & \frac{a_{23}^e N_{e^c}(Q_e - Q_u) + a_{32}^e N_e(-Q_{e^c} + Q_u)}{N_u(Q_e - Q_{e^c})} & a_{33}^d \end{bmatrix} .$$

$$(4.69)$$

Since $Q_u = Q_d$, we have $a_{31}^d = a_{31}^u$ and $a_{32}^d = a_{32}^u$, see Eqs. (4.64) and (4.65).

Now, the rotation that was made in the 1-2 sector to set $a_{12} = 0$ simultaneously can make a_{11}^e and a_{22}^e real. This rotation will alter the column $\{(M_E)_{13}, (M_E)_{23}\}^T$ and the row $\{(M_E)_{31}, (M_E)_{32}\}$ in such a way that the forms of M_U Eq. (4.68) and M_D Eq. (4.69) are preserved. All parameters are complex, except that one among $a_{33}^{u,d,e}$ can be made real (we choose a_{33}^d to be real), and that T can be chosen real. So the parameter set is

$$\{\epsilon, r, T, \theta, \phi, a_{11}^e, a_{22}^e, a_{13}^e, a_{31}^e, a_{23}^e, a_{32}^e, a_{33}^e, a_{33}^u, a_{33}^d\} \text{ for } 45_H \text{ or}$$

$$\{\epsilon_1, \epsilon_2, r, T, \theta, \phi, a_{11}^e, a_{22}^e, a_{13}^e, a_{31}^e, a_{23}^e, a_{32}^e, a_{33}^e, a_{33}^u, a_{33}^d\} \text{ for } 210_H.$$

Of these sets, $\{a_{13}^e, a_{31}^e, a_{23}^e, a_{32}^e, a_{33}^e, a_{33}^u\}$ are complex (with a_{33}^d chosen to be real). For $\Phi = 45_H$, there are 13 magnitudes and 7 phases (in total 20 parameters) for non-SUSY case. In the case of SUSY, ϵ is complex, so one additional phase enters (for a total 21 parameters). For $\Phi = 210_H$ in the SUSY context with minimal Higgs content, ϵ_1 and ϵ_2 are not independent of each other (see later), so there are again 13 magnitudes and 8 phases (in total 21 parameters). Later we will also consider a case with non-minimal Higgs sector where both these VEV ratios $\epsilon_{1,2}$ can be in general independent of each other. In the neutrino sector (discussed in the next subsection) the mass matrix is given by these same parameters except for an overall scale ($v_{R,L}$ for type-I and type-II seesaw scenarios respectively) that adds one new parameter.

4.1 The neutrino sector

4.1.1 Type-I seesaw

To write down the mass matrix in the neutrino sector, we make the assumption that $M, b\Omega \gg v_R$, which is a valid approximation provided that $M, b\Omega \sim M_{GUT} \sim 10^{16}$ GeV. Note that in order to generate light neutrino masses by using the seesaw mechanism, one roughly needs $v_R \sim 10^{12-14}$ GeV. In this approximation, no new parameter comes into play in the neutrino mass matrix except the scale v_R. For type-I seesaw mechanism the Dirac neutrino mass matrix can be read off from Eq. (4.59):

$$M_{\nu_D}^T = -3 v_u \begin{bmatrix} a_{11}^e & 0 & \frac{a_{13}^e N_{e^c}(Q_e - Q_{\nu^c}) + a_{31}^e N_e(-Q_{e^c} + Q_{\nu^c})}{N_{\nu^c}(Q_e - Q_{e^c})} \\ 0 & a_{22}^e & \frac{a_{23}^e N_{e^c}(Q_e - Q_{\nu^c}) + a_{32}^e N_e(-Q_{e^c} + Q_{\nu^c})}{N_{\nu^c}(Q_e - Q_{e^c})} \\ a_{31}^e & a_{32}^e & a_{33}^\nu \end{bmatrix} .$$

$$(4.70)$$

Since $Q_e = Q_\nu$, $a_{31}^\nu = a_{31}^e$ and $a_{32}^\nu = a_{32}^e$. The expressions for a_{33}^ν are given in Eqs. (B.131) and (B.132) for $\Phi = 45_H$ and 210_H respectively in Appendix B . One can derive the right-handed neutrino Majorana mass matrix to be

$$\frac{M_{\nu_R}}{v_R} = \begin{bmatrix} a_{11}^e & 0 & \frac{a_{13}^e N_{e^c}(Q_e - Q_{\nu^c}) + a_{31}^e N_e(-Q_{e^c}+Q_{\nu^c})}{N_{\nu^c}(Q_e - Q_{e^c})} \\ 0 & a_{22}^e & \frac{a_{23}^e N_{e^c}(Q_e - Q_{\nu^c}) + a_{32}^e N_e(-Q_{e^c}+Q_{\nu^c})}{N_{\nu^c}(Q_e - Q_{e^c})} \\ \frac{a_{13}^e N_{e^c}(Q_e - Q_{\nu^c}) + a_{31}^e N_e(-Q_{e^c}+Q_{\nu^c})}{N_{\nu^c}(Q_e - Q_{e^c})} & \frac{a_{23}^e N_{e^c}(Q_e - Q_{\nu^c}) + a_{32}^e N_e(-Q_{e^c}+Q_{\nu^c})}{N_{\nu^c}(Q_e - Q_{e^c})} & a_{33}^R \end{bmatrix},$$

$$(4.71)$$

The expressions for a_{33}^R are given in Eqs. (B.137) and (B.138) for $\Phi = 45_H$ and 210_H respectively in Appendix B. Then, the light neutrino mass matrix in the type-I seesaw scenario is given by

$$M_N = -M_{\nu_D}^T M_{\nu_R}^{-1} M_{\nu_D}. \tag{4.72}$$

4.1.2 Type-II seesaw

In analogy to the the analysis done in Sec. 4.1.1 one can derive the type-II seesaw contributions to the the neutrino mass matrix by replacing $v_R \to v_L$ and $\nu^c \to \nu$. In this type-II seesaw scenario the neutrino mass matrix is then given by

$$M_{\nu_L} = v_L \begin{bmatrix} a_{11}^e & 0 & a_{31}^e \\ 0 & a_{22}^e & a_{32}^e \\ a_{31}^e & a_{32}^e & a_{33}^L \end{bmatrix}. \tag{4.73}$$

The expressions for a_{33}^L are given in Eqs. (B.143) and (B.144) for $\Phi = 45_H$ and 210_H respectively in Appendix B.

5 Symmetry breaking constraints

In all models studied here, there is no 10_H Higgs and matter fields couple to $126_H + \overline{126}_H$ and 45_H or 210_H scalars. There are considerations as outlined in Sec. II that would require additional Higgs fields to be present for consistent symmetry breaking. While there are no constraints on the VEV ratios when a 210_H is employed in the non-SUSY framework, these ratios are determined in the case of SUSY. We consider the various constraints on the symmetry breaking sector in this section.

5.1 Non-SUSY $SO(10)$ models A and B

Model **A** employs 126_H, 45_H and a 54_H. Breaking of $SO(10)$ down to SM via $SU(5)$ channel is not viable due to gauge coupling unification and proton decay limits. If only 45_H and 126_H (or 16_H) Higgs multiplets are used to break $SO(10)$, breaking takes place through the $SU(5)$-symmetric channel [38–40]. The other two breaking channels $SO(10) \to SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \to SM$ and $SO(10) \to SU(4)_c \times SU(2)_L \times U(1)_R \to SM$ do not have stable vacuum at the tree-level. Recently quantum corrections to the tree-level potential have been taken into account [41, 42] and the validity of such breaking channels has been shown. However, we do not rely on quantum corrections in this paper. This is why

the Higgs sector needs to be extended with a 54_H for consistent $SO(10)$ breaking down to SM [43,44]. Note that a Higgs system consisting of 126_H and 54_H is sufficient for symmetry breaking purposes if also a 10_H is used [45], but without the 10_H as in our case, a 45_H is necessary.

Since the SM Higgs doublet is part of the 126_H in this model, a question arises as to the negativity of its squared mass. Consistency of the GUT scale symmetry breaking would require all physical scalar squared masses to be positive, which includes the SM Higgs doublet. There must then be a source that turns this positive mass to negative value. It has been shown in Ref. [46] that indeed such a turn-around is possible, provided that some scalar from any GUT multiplet remains light and has non-negligible couplings to the SM Higgs doublet. The context in Ref. [46] is similar to our present case, where a 144_H of $SO(10)$ is used to break the GUT symmetry as well as the electroweak symmetry. Since our present non-SUSY model has an intermediate scale, we expect some of the scalars to survive down to the intermediate scale, which would enable turning the Higgs mass-squared to negative value so as to trigger electroweak symmetry breaking.

In Model **B** we employ a 210_H in addition to the 126_H. This is not however sufficient for our purpose. Since the VEV of 126_H is much smaller than the GUT scale, a single 210_H would break the GUT symmetry to one of its maximal little groups, such as $SU(5) \times U(1)$ or $SU(4)_c \times SU(2)_L \times SU(2)_R$ [47]. The fermion mass matrices will then carry traces of this unbroken symmetry, which would lead to unwanted mass relations. This is why we extend the scalar sector by adding a 54_H or 16_H. For non-SUSY $SO(10)$ model with Higgs multiplets $210_H + 54_H$, since $54^2 \ni 1_s + 54_s + 770_s$ and $210^2 \ni 1_s + 54_s + 770_s$, the scalar potential contains 2 non-trivial quartic couplings between $210_H - 54_H$. In addition, 210_H has 3 non-trivial quartic couplings and 54_H has one cubic and one non-trivial quartic couplings. This counting of non-trivial couplings dictates that in general the two VEV ratios $\epsilon_{1,2}$ from the 210_H are free parameters. Similar argument can be provided if 54_H is replaced by 16_H Higgs.

5.2 SUSY $SO(10)$ Models C–F

The Higgs sector of Model **D** consists of $210_H + 54_H + 126_H + \overline{126}_H$. This system is a subset of the SUSY $SO(10)$ models studied in Ref. [48]. The relevant part of the superpotential with only 210_H, 54_H and $126_H + \overline{126}_H$ is:

$$
\begin{aligned}
W = {} & \frac{1}{2} m_1 \Phi^2 + m_2 \Delta \overline{\Delta} + \frac{1}{2} m_5 E^2 + \lambda_1 \Phi^3 + \lambda_8 E^3 \\
& + \lambda_2 \Phi \Delta \overline{\Delta} + \lambda_{10} \Phi^2 E + \lambda_{11} \Delta^2 E + \lambda_{12} \overline{\Delta}^2 E.
\end{aligned}
\tag{5.74}
$$

Since the VEV of 126_H is required to be in the intermediate scale $\sim 10^{13-14}$ GeV range from a fit to light neutrino masses arising via the seesaw mechanism, in this analysis of the superpotential one can neglect the contribution coming from this field as the other scalars

$210_H + 54_H$ will get much larger VEVs of order the GUT scale $\sim 10^{16}$ GeV. Then the relevant stationary equations are

$$0 = m_1 V_1 + \frac{\lambda_1}{2\sqrt{6}} V_3^2 + \sqrt{\frac{3}{5}} V_1 V_E,$$

$$0 = m_1 V_2 + \frac{\lambda_1}{3\sqrt{2}} (V_2^2 + V_3^2) - \frac{2\lambda_{10}}{\sqrt{15}} V_2 V_E,$$

$$0 = m_1 V_3 + \frac{\lambda_1}{\sqrt{6}} V_1 V_3 + \frac{\sqrt{2}\lambda_1}{3} V_2 V_3 + \frac{\lambda_{10}}{2\sqrt{15}} V_3 V_E, \tag{5.75}$$

$$0 = m_5 V_E + \frac{\sqrt{3}\lambda_8}{2\sqrt{5}} V_E^2 + \frac{\sqrt{3}\lambda_{10}}{2\sqrt{5}} V_1^2 - \frac{\lambda_{10}}{\sqrt{15}} V_2^2 + \frac{\lambda_{10}}{4\sqrt{15}} V_3^2.$$

Here the $V_1 = \langle(1,1,1)\rangle$, $V_2 = \langle(1,1,15)\rangle$ and $V_3 = \langle(1,3,15)\rangle$ are the VEVs of $\Phi(210_H)$ and the 54_H VEV is $V_E = \langle(1,1,1)\rangle$ under the Pati-Salam group $SU(2)_L \times SU(2)_R \times SU(4)_C$ decomposition. Compared to Eqs. (3.44), here a different normalization is used and one can make the identifications $V_1 = \phi_1, V_2 = \sqrt{3}\phi_2, V_3 = \sqrt{3/2}\phi_3$.

The last relation in Eq. (5.75) can be solved for the free mass parameter m_5. Taking differences of the other three twice, we obtain two independent solutions,

$$V_1 = -\frac{\sqrt{3}V_2}{2} \quad \text{or,} \quad V_1 = \frac{V_3^2}{2\sqrt{3}V_2}. \tag{5.76}$$

These correspond to the VEV ratios ($\epsilon_1 = V_2/V_1, \epsilon_2 = V_3/V_1$) given as

$$\epsilon_1 = -\frac{2}{\sqrt{3}} \quad \text{or,} \quad \epsilon_1 = \frac{\epsilon_2^2}{2\sqrt{3}}. \tag{5.77}$$

While studing the fermions masses and mixing numerically, we will consider both these cases. These models are labelled as $\mathbf{D^a}$ for the solution $\epsilon_1 = -\frac{2}{\sqrt{3}}$ and $\mathbf{D^b}$ for solution $\epsilon_1 = \frac{\epsilon_2^2}{2\sqrt{3}}$.

In Model $\mathbf{E}$, we use a 210_H along with a $16_H + \overline{16}_H$ for symmetry breaking purpose. These fields are in addition to the $126_H + \overline{126}_H$ fields present. Just like the previous case, since the $SO(10)$ breaking VEV of the Higgs scalars 210_H and $16_H + \overline{16}_H$ are $\sim M_{GUT}$, one can neglect the terms involving the scalar 126_H which has a much lower VEV. The form of the superpotantial is identical to Eq. (5.74) with the $126_H(\overline{126}_H)$ replaced by $16_H(\overline{16}_H)$. Denoting the $16_H(\overline{16}_H)$ VEV as $V_\psi(\overline{V}_\psi)$ and its mass by m_ψ, the relevant stationary equations in this case are

$$0 = m_1 V_1 + \frac{\lambda_1}{2\sqrt{6}} V_3^2 + \frac{\lambda_2}{10\sqrt{6}} V_\psi \overline{V}_\psi,$$

$$0 = m_1 V_2 + \frac{\lambda_1}{3\sqrt{2}} (V_2^2 + V_3^2) + \frac{\lambda_2}{10\sqrt{2}} V_\psi \overline{V}_\psi,$$

$$0 = m_1 V_3 + \frac{\lambda_1}{\sqrt{6}} V_1 V_3 + \frac{\sqrt{2}\lambda_1}{3} V_2 V_3 + \frac{\lambda_2}{10} V_\psi \overline{V}_\psi, \tag{5.78}$$

$$0 = V_\psi \overline{V}_\psi [m_\psi + \frac{\lambda_2}{10\sqrt{6}} V_1 + \frac{\lambda_2}{10\sqrt{2}} V_2 + \frac{\lambda_2}{10} V_3].$$

There are two different solutions of this system of stationary equations

$$V_1 = \frac{V_3}{\sqrt{6}}, \quad V_2 = \frac{V_3}{\sqrt{2}};$$

$$\text{or,} \quad V_1 = \frac{-36m_1^2 V_3 + 5V_3^3\lambda_1^2}{\sqrt{6}(-6m_1 + V_3\lambda_1)^2}, \quad V_2 = -\frac{-36m_1^2 + 12m_1 V_3\lambda_1 + V_3^2\lambda_1^2}{\sqrt{2}\lambda_1(-6m_1 + V_3\lambda_1)}. \tag{5.79}$$

So the VEV ratios are given by

$$\epsilon_1 = \sqrt{3}, \quad \epsilon_2 = \sqrt{6};$$

$$\text{or,} \quad \epsilon_1 = \frac{\sqrt{3}(-6+\epsilon)(-36+12\epsilon+\epsilon^2)}{\epsilon(36-5\epsilon^2)}, \quad \epsilon_2 = \frac{\sqrt{6}(-6+\epsilon)^2}{-36+5\epsilon^2} \;; \quad \text{with } \epsilon \equiv V_3\frac{\lambda_1}{m_1}, \tag{5.80}$$

where ϵ is a free parameter. We discard the first solution since this corresponds to $SU(5)$-symmetric case. The surviving model will be labeled **E**.

By adding more Higgs multiplets in either of the models **D** or **E**, as for example $16_H + \overline{16}_H$ or adding another 54_H to model **D**, these relations for VEV ratios can be made invalid and $\epsilon_{1,2}$ can be made independent parameters. We will also study this general case. We choose to add $16_H + \overline{16}_H$ in model **D** and 54_H in model **E** and label these classes of model as **F**. Finally, for SUSY model **C**, consisting of $126_H + \overline{126}_H + 45_H + 54_H + 16_H + \overline{16}_H$, we stress that the $16_H + \overline{16}_H$ are needed for successfully tuning the MSSM doublets light without making simultaneously any other submultiplet light. The parameter ϵ is arbitrary in this case.

6 Numerical analysis of fermion masses and mixings

In this section we show our fit results of fermion masses and mixings for different $SO(10)$ models described in Sections II and V. We do the fitting for both non-SUSY and SUSY cases, each with type-I and type-II seesaw scenarios. For optimization purpose we do a χ^2-analysis. The pull and χ^2-function are defined as:

$$P_i = \frac{O_{i\,\text{th}} - E_{i\,\text{exp}}}{\sigma_i}, \tag{6.81}$$

$$\chi^2 = \sum_i P_i^2, \tag{6.82}$$

where σ_i represent experimental 1σ uncertainty and $O_{i\,\text{th}}$, $E_{i\,\text{exp}}$ and P_i represent the theoretical prediction, experimental central value and pull of observable i. We fit the values of the observables at the GUT scale, $M_{GUT} = 2 \times 10^{16}$ GeV. To get the GUT scale values of the observables, for non-SUSY case, we take the central values at the M_Z scale from Table-1 of Ref. [49] and use the renormalization group equation (RGE) running factors given in Ref. [50] to get the GUT scale inputs. For the associated one sigma uncertainties of the observables at the GUT scale, we keep the same percentage uncertainty with respect to the central value of each quantity as that at the M_Z scale. For SUSY case, the low scale values of the observables are taken from Table-2 of [49] at $\mu = 1$ TeV where the values are

converted to the $\overline{\text{DR}}$ scheme and then using the renormalization group equation running for MSSM [51,52] we get the GUT scale inputs. For all different SUSY $SO(10)$ models, we do the fitting for $\tan\beta = 10$. For the charged lepton masses, a relative uncertainty of 0.1% is assumed in order to take into account the theoretical uncertainties arising for example from threshold effects. The inputs in the neutrino sector are taken from Ref. [53]. For neutrino observables, we do not run the RGE from low scale to the GUT scale, which is a relatively small effect, except for an overall rescaling on the neutrino masses that can be absorbed in the corresponding scale v_R or v_L. In the case of inverted hierarchical neutrino mass spectrum, RGE effects can be important, whereas for all our cases the spectrum turns out to be normal hierarchical. Since the right-handed neutrino masses are extremely heavy, threshold corrections might also have effects on the neutrino observables if the Dirac neutrino matrix elements are of order one, but in our case the elements are much smaller than one. All these inputs are shown in the tables where the fit results are presented. Below we present our best fit results and the corresponding parameters for different $SO(10)$ GUT models as discussed above.

Model A: Non-SUSY $SO(10)$: $45_H + 54_H + \overline{126}_H$

Masses (in GeV) and Mixing parameters	Inputs (at $\mu = M_{GUT}$)	Fitted values (**AI**) (at $\mu = M_{GUT}$)	pulls (**AI**)	Fitted values (**AII**) (at $\mu = M_{GUT}$)	pulls (**AII**)
$m_u/10^{-3}$	0.437±0.147	0.441	0.03	0.469	0.21
m_c	0.236±0.007	0.236	0.003	0.236	0.02
m_t	73.82±0.64	73.82	0.01	73.81	-0.01
$m_d/10^{-3}$	1.12±0.11	1.14	0.16	1.12	-0.01
$m_s/10^{-3}$	21.93±1.07	21.82	-0.10	21.98	0.04
m_b	0.987±0.008	0.987	-0.003	0.987	-0.003
$m_e/10^{-3}$	0.469658±0.000469	0.469649	-0.01	0.469757	0.21
$m_\mu/10^{-3}$	99.1474±0.0991	99.1555	0.08	99.0913	-0.56
m_τ	1.68551±0.00168	1.68542	-0.05	1.68602	0.29
$|V_{us}|/10^{-2}$	22.54±0.06	22.53	-0.01	22.54	0.005
$|V_{cb}|/10^{-2}$	4.856±0.06	4.856	0.001	4.853	-0.03
$|V_{ub}|/10^{-2}$	0.420±0.013	0.420	0.07	0.420	0.02
δ_{CKM}	1.207±0.054	1.205	-0.03	1.205	-0.03
$\Delta m_{sol}^2/10^{-5}(\text{eV}^2)$	7.56±0.24	7.56	0.01	7.54	-0.06
$\Delta m_{atm}^2/10^{-3}(\text{eV}^2)$	2.41±0.08	2.40	-0.004	2.41	0.05
$\sin^2\theta_{12}^{PMNS}$	0.308±0.017	0.308	0.01	0.302	-0.29
$\sin^2\theta_{23}^{PMNS}$	0.387±0.0225	0.388	0.03	0.396	0.42
$\sin^2\theta_{13}^{PMNS}$	0.0241±0.0025	0.0238	-0.11	0.0239	-0.04

Table 1: Fitted values of the observables correspond to $\chi^2 = 7 \cdot 10^{-2}$ and 0.78 for models **AI** and **AII** respectively. These fittings correspond to $|a_{ij}|_{max} = |a_{44}| = 1.9$ and 3.3 for the type-I and type-II cases respectively (see text for details). For the charged lepton masses, a relative uncertainty of 0.1% is assumed in order to take into account the theoretical uncertainties arising for example from threshold effects.

Quantity	Predicted Value (**AI**)	Predicted Value (**AII**)
$\{m_1, m_2, m_3\}$ (in eV)	$\{3.72 \cdot 10^{-3}, 9.45 \cdot 10^{-3}, 4.99 \cdot 10^{-2}\}$	$\{4.38 \cdot 10^{-3}, 9.72 \cdot 10^{-3}, 5.00 \cdot 10^{-2}\}$
$\{\delta^{PMNS}, \alpha_{21}^{PMNS}, \alpha_{31}^{PMNS}\}$	$\{120.03°, 144.92°, -168.49°\}$	$\{104.80°, 159.32°, 95.05°\}$
$\{m_{cos}, m_\beta, m_{\beta\beta}\}$ (in eV)	$\{6.31 \cdot 10^{-2}, 6.55 \cdot 10^{-3}, 1.22 \cdot 10^{-3}\}$	$\{6.42 \cdot 10^{-2}, 7.05 \cdot 10^{-3}, 2.24 \cdot 10^{-4}\}$
$\{M_1, M_2, M_3\}$ (in GeV)	$\{8.65 \cdot 10^7, 2.66 \cdot 10^{10}, 6.99 \cdot 10^{11}\}$	-

Table 2: Predictions of the models **A**. m_i are the light neutrino masses, M_i are the right-handed neutrino masses, $\alpha_{21,31}$ are the Majorana phases following the PDG parametrization, $m_{cos} = \sum_i m_i$, $m_\beta = \sum_i |U_{ei}|^2 m_i$ is the effective mass parameter for beta-decay and $m_{\beta\beta} = |\sum_i U_{ei}^2 m_i|$ is the effective mass parameter for neutrinoless double beta decay.

The fit results and the predictions for model **A** are shown in Table 1 and 2 respectively. For model **AI** (Model **A** with type-I seesaw) the parameter set is:

$$\{a_{33}^u, a_{33}^d, \epsilon, T, \theta, \phi, v_R, r\} = \{0.415986 + 0.0944114i, 0.0246549, -1.24753, 8.68487,$$
$$0.560999, -0.0127783, 1.58339 \cdot 10^{13} \text{GeV}, 6.76689\}$$

and

$$a_{ij}^e = 10^{-2} \begin{pmatrix} 0.0959072 & 0 & -1.47328 - 0.508307i \\ 0 & -0.00693205 & -0.302045 - 0.119282i \\ 0.149467 + 0.0128315i & 0.0534903 - 0.0345252i & 0.461306 - 1.4512i \end{pmatrix}.$$
$$(6.83)$$

For model **AII** (Model **A** with type-II seesaw) the parameter set is:

$$\{a_{33}^u, a_{33}^d, \epsilon, T, \theta, \phi, v_L, r\} = \{0.152744 + 0.399269i, -0.0244755, -0.393925, 11.4001, 0.560999,$$
$$0.105066, 1.69937 \cdot 10^{-8} \text{GeV}, 6.75824\}$$

and

$$a_{ij}^e = 10^{-2} \begin{pmatrix} 0.127684 & 0 & -0.0742479 + 0.0532305i \\ 0 & -0.00055042 & 0.0264824 + 0.0152045i \\ 0.136072 + 0.0070994i & 0.0582979 + 0.00164043i & -0.398502 - 2.1619i \end{pmatrix}.$$
$$(6.84)$$

In performing such optimization, solutions with lower values of χ^2 exist but we are only interested in the solutions for which the original couplings a_{ij} are also in the perturbative range. In the optimization process we restrict ourselves to the case of $(a_{ij})_{max} \lesssim 2$. For all the solutions that are presented, we did find good fits with this cut-off except for model **AII** where $|a_{44}| = 3.3$ as can be seen from Eq. C.150 . The original coupling matrices a_{ij} can be computed with the parameter sets that result due to the minimization process. For all the fits to the different models presented in this work, these matrices are shown in Appendix C.

In Table 2, the predicted quantities correspond to the best fit values. For example, for model **AI**, the predicted value of the Dirac type CP violating phase in the neutrino sector is $\delta^{PMNS} = 2\pi/3$. The fit result presented in this case is very good since $\chi^2 = 7 \cdot 10^{-2}$. We have investigated the robustness of the predicted value of δ_{PMNS} and found it to be not

very robust. Since the χ^2 for the best fit is extremely small, it is quite fine to deviate from the minimum χ^2 are still find acceptable fits. We find that the variation of δ_{PMNS} from the predicted value can be quite large. In Fig. 1, we show the variation of δ^{PMNS} with χ^2/n_{obs}. Most of the fit results presented in this work have small total χ^2, so this conclusion on the robustness of δ^{PMNS} prediction is valid for the other models as well. We present the variation plot only for model **AI**.
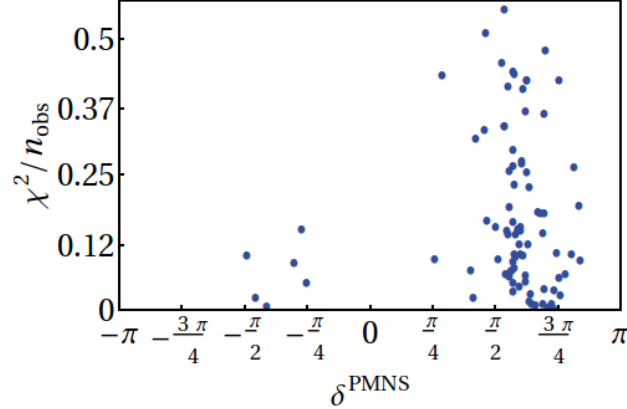

Figure 1: Variation of δ^{PMNS} with χ^2/n_{obs} for the model **AI**. In plotting this, we restrict to the regime for which $\chi^2 \leq 10$.

Model B: Non-SUSY $SO(10)$: $210_H + 54_H + \overline{126}_H$ (or $210_H + 16_H + \overline{126}_H$)

The fit results and the predictions for models **B** are shown in Table 3 and 4 respectively. The parameter set for model **BI** is:

$$\{a^u_{33}, a^d_{33}, \epsilon_1, \epsilon_2, T, \theta, \phi, v_R, r\} = \{-0.0380751 - 0.424441i, 0.0244949, 1.61753, 1.67225, 0.764487,$$
$$0.541654, -2.91319, 2.23915 \cdot 10^{13} \text{GeV}, 6.74578\}$$

and

$$a^e_{ij} = 10^{-2} \begin{pmatrix} -0.122115 & 0 & 0.899426 + 1.16951i \\ 0 & 0.00569753 & -0.104101 - 0.15069i \\ -0.175821 - 0.103765i & 0.0325028 + 0.0638096i & 1.46544 + 0.663581i \end{pmatrix}.$$
$$(6.85)$$

The parameter set for model **BII** is:

$$\{a^u_{33}, a^d_{33}, \epsilon_1, \epsilon_2, T, \theta, \phi, v_R, r\} = \{0.174446 + 0.389832i, 0.0244585, 1.07061, 0.666248, 0.526787,$$
$$0.713998, 0.295856, 1.32386 \cdot 10^{-8} \text{GeV}, 6.74635\}$$

and

$$a^e_{ij} = 10^{-2} \begin{pmatrix} 0.00424453 & 0 & 0.130198 - 0.0532261i \\ 0 & 0.0963929 & -0.386912 - 0.75915i \\ -0.0711623 - 0.0235054i & 0.0531238 + 0.181145i & -1.64856 - 1.16034i \end{pmatrix}.$$
$$(6.86)$$

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Masses (in GeV) and Mixing parameters	Inputs (at $\mu = M_{GUT}$)	Fitted values (**B I**) (at $\mu = M_{GUT}$)	pulls (**B I**)	Fitted values (**B II**) (at $\mu = M_{GUT}$)	pulls (**B II**)
$m_u/10^{-3}$	0.437±0.147	0.436	-0.0007	0.437	0.0002
m_c	0.236±0.007	0.236	0.006	0.236	-0.00009
m_t	73.82±0.64	73.82	0.003	73.82	-0.00005
$m_d/10^{-3}$	1.12±0.11	1.12	0.0	1.12	-0.0005
$m_s/10^{-3}$	21.93±1.07	21.95	0.01	21.93	-0.0003
m_b	0.987±0.008	0.987	0.005	0.987	0.0003
$m_e/10^{-3}$	0.469658±0.000469	0.469654	-0.008	0.469658	-0.0004
$m_\mu/10^{-3}$	99.1474±0.0991	99.1412	-0.06	99.1476	0.002
m_τ	1.68551±0.00168	1.68555	0.02	1.68551	-0.002
$|V_{us}|/10^{-2}$	22.54±0.06	22.54	0.0009	22.54	-0.00004
$|V_{cb}|/10^{-2}$	4.856±0.06	4.856	0.0001	4.856	0.0002
$|V_{ub}|/10^{-2}$	0.420±0.013	0.419	-0.001	0.419	-0.0001
δ_{CKM}	1.207±0.054	1.207	0.003	1.207	0.0005
$\Delta m^2_{sol}/10^{-5}(\text{eV}^2)$	7.56±0.24	7.55	-0.001	7.56	0.00005
$\Delta m^2_{atm}/10^{-3}(\text{eV}^2)$	2.41±0.08	2.40	0.004	2.41	0.0001
$\sin^2\theta_{12}^{PMNS}$	0.308±0.017	0.307	-0.004	0.307	-0.0003
$\sin^2\theta_{23}^{PMNS}$	0.387±0.0225	0.387	-0.002	0.387	0.00004
$\sin^2\theta_{13}^{PMNS}$	0.0241±0.0025	0.0241	0.01	0.0241	0.00009

Table 3: Best fit values of the observables correspond to $\chi^2 = 5 \cdot 10^{-3}$ and $1 \cdot 10^{-5}$ for models **BI** and **BII** respectively. These fittings correspond to $|a_{ij}|_{max} = |a_{44}| = 0.56$ and 0.26 for the type-I and type-II cases respectively. For the charged lepton masses, a relative uncertainty of 0.1% is assumed in order to take into account the theoretical uncertainties arising for example from threshold effects.

Quantity	Predicted Value (**BI**)	Predicted Value (**BII**)
$\{m_1, m_2, m_3\}$ (in eV)	$\{2.58 \cdot 10^{-3}, 9.07 \cdot 10^{-3}, 4.99 \cdot 10^{-2}\}$	$\{2.61 \cdot 10^{-3}, 9.07 \cdot 10^{-3}, 4.99 \cdot 10^{-2}\}$
$\{\delta^{PMNS}, \alpha_{21}^{PMNS}, \alpha_{31}^{PMNS}\}$	$\{-38.38°, 175.84°, -131.48°\}$	$\{-65.38°, -158.28°, -96.19°\}$
$\{m_{cos}, m_\beta, m_{\beta\beta}\}$ (in eV)	$\{6.15 \cdot 10^{-2}, 5.67 \cdot 10^{-3}, 8.33 \cdot 10^{-4}\}$	$\{6.16 \cdot 10^{-2}, 5.69 \cdot 10^{-3}, 3.93 \cdot 10^{-4}\}$
$\{M_1, M_2, M_3\}$ (in GeV)	$\{5.47 \cdot 10^8, 3.48 \cdot 10^{10}, 5.73 \cdot 10^{11}\}$	-

Table 4: Predictions of models **B**. m_i are the light neutrino masses, M_i are the right handed neutrino masses, $\alpha_{21,31}$ are the Majorana phases following the PDG parametrization, $m_{cos} = \sum_i m_i$, $m_\beta = \sum_i |U_{ei}|^2 m_i$ is the effective mass parameter for beta-decay and $m_{\beta\beta} = |\sum_i U_{ei}^2 m_i|$ is the effective mass parameter for neutrinoless double beta decay.

Model C: SUSY $SO(10)$: $45_H + 54_H + 16_H + \overline{16}_H + 126_H + \overline{126}_H$

The fit results and the predictions for models **C** are shown in Table 5 and 6 respectively. The parameter set for model **CI** is:

$$\{a_{33}^u, a_{33}^d, \epsilon, T, \theta, \phi, v_R, r\} = \{-0.22531 + 0.467722i, 0.0317632, -1.10245 + 0.269791i,$$
$$5.51352, 0.560999, 0.448339, 1.39544 \cdot 10^{13}\text{GeV}, 9.14437\}$$

and

$$a_{ij}^e = 10^{-2} \begin{pmatrix} 0.113952 & 0 & -1.52545 - 0.861024i \\ 0 & 0.0066433 & 0.103718 - 0.25156i \\ 0.103242 - 0.07705i & 0.0528824 + 0.0275714i & -0.300714 - 1.23388i \end{pmatrix}. \tag{6.87}$$

The parameter set for model **CII** is:

$$\{a_{33}^u, a_{33}^d, \epsilon, T, \theta, \phi, v_L, r\} = \{0.0307775 + 0.518792i, -0.0317378, -0.75526 - 0.237386i,$$
$$4.66699, 0.713998, -0.0578946, 1.55237 \cdot 10^{-8} \text{GeV}, 9.1424\}$$

and

$$a_{ij}^e = 10^{-2} \begin{pmatrix} -0.104302 & 0 & 0.88437 - 0.647303i \\ 0 & 0.00422339 & 0.0628707 - 0.166004i \\ -0.0805026 - 0.140317i & -0.0449238 - 0.0524454i & -0.793384 + 1.69782i \end{pmatrix}. \tag{6.88}$$

Masses (in GeV) and Mixing parameters	Inputs (at $\mu = M_{GUT}$)	Fitted values (**CI**) (at $\mu = M_{GUT}$)	pulls (**CI**)	Fitted values (**CII**) (at $\mu = M_{GUT}$)	pulls (**CII**)
$m_u/10^{-3}$	0.502±0.155	0.502	0.001	0.501	-0.0005
m_c	0.245±0.007	0.245	0.002	0.245	0.001
m_t	90.28±0.90	90.28	-0.0005	90.28	-0.002
$m_d/10^{-3}$	0.839±0.084	0.838	-0.006	0.839	-0.001
$m_s/10^{-3}$	16.62±0.90	16.62	-0.00005	16.62	0.002
m_b	0.938±0.009	0.938	-0.001	0.938	-0.001
$m_e/10^{-3}$	0.344021±0.000344	0.344021	0.0001	0.344018	-0.008
$m_\mu/10^{-3}$	72.6256±0.0726	72.6273	0.02	72.6240	-0.02
m_τ	1.24038±0.00124	1.24036	-0.01	1.24038	-0.001
$|V_{us}|/10^{-2}$	22.53±0.07	22.53	0.002	22.53	0.001
$|V_{cb}|/10^{-2}$	3.934±0.06	3.933	-0.001	3.934	0.001
$|V_{ub}|/10^{-2}$	0.340±0.011	0.340	-0.001	0.340	-0.004
δ_{CKM}	1.208±0.054	1.208	0.004	1.208	0.001
$\Delta m_{sol}^2/10^{-5}(\text{eV}^2)$	7.56±0.24	7.56	0.001	7.55	-0.001
$\Delta m_{atm}^2/10^{-3}(\text{eV}^2)$	2.41±0.08	2.41	0.001	2.40	-0.0006
$\sin^2\theta_{12}^{PMNS}$	0.308±0.017	0.308	0.005	0.308	0.003
$\sin^2\theta_{23}^{PMNS}$	0.387±0.0225	0.387	-0.0001	0.387	-0.002
$\sin^2\theta_{13}^{PMNS}$	0.0241±0.0025	0.0240	-0.002	0.0240	-0.003

Table 5: Best fit result for models **C** with inputs correspond to $\tan\beta = 10$. The fitted values correspond to $\chi^2 = 7 \cdot 10^{-4}$ for model **CI** and $6 \cdot 10^{-4}$ for model **CII**. These fittings correspond to $|a_{ij}|_{max} = |a_{44}| = 1.5$ and 1.03 for the type-I and type-II cases respectively. For the charged lepton masses, a relative uncertainty of 0.1% is assumed in order to take into account the theoretical uncertainties arising for example from threshold effects.

Quantity	Predicted Value (**CI**)	Predicted Value (**CII**)
$\{m_1, m_2, m_3\}$ (in eV)	$\{5.36 \cdot 10^{-3}, 1.02 \cdot 10^{-2}, 5.01 \cdot 10^{-2}\}$	$\{3.68 \cdot 10^{-3}, 9.44 \cdot 10^{-3}, 4.99 \cdot 10^{-2}\}$
$\{\delta^{PMNS}, \alpha_{21}^{PMNS}, \alpha_{31}^{PMNS}\}$	$\{157.92°, 158.41°, -104.87°\}$	$\{85.15°, 165.93°, 138.22°\}$
$\{m_{cos}, m_\beta, m_{\beta\beta}\}$ (in eV)	$\{6.57 \cdot 10^{-2}, 7.90 \cdot 10^{-3}, 1.35 \cdot 10^{-3}\}$	$\{6.31 \cdot 10^{-2}, 6.53 \cdot 10^{-3}, 7.55 \cdot 10^{-4}\}$
$\{M_1, M_2, M_3\}$ (in GeV)	$\{1.91 \cdot 10^8, 1.63 \cdot 10^{10}, 1.33 \cdot 10^{12}\}$	-

Table 6: Predictions of the models **C**. m_i are the light neutrino masses, M_i are the right handed neutrino masses, $\alpha_{21,31}$ are the Majorana phases following the PDG parametrization, $m_{cos} = \sum_i m_i$, $m_\beta = \sum_i |U_{ei}|^2 m_i$ is the effective mass parameter for beta-decay and $m_{\beta\beta} = |\sum_i U_{ei}^2 m_i|$ is the effective mass parameter for neutrinoless double beta decay.

Model D: SUSY $SO(10)$: $210_H + 54_H + 126_H + \overline{126}_H$

The fit results and the predictions for model $\mathbf{D^a I}$ are shown in Table 7 and 8 respectively. The parameter set for this fit of model $\mathbf{D^a I}$ is:

$$\{a_{33}^u, a_{33}^d, \epsilon_2, T, \theta, \phi, v_R, r\} = \{-0.343904 + 0.38917i, 0.0318629, -5.89976 - 0.158839i, 0.77736,$$
$$0.532473, 2.76646, 1.95768 \cdot 10^{13}\text{GeV}, 9.15103\}$$

and

$$a_{ij}^e = 10^{-10} \begin{pmatrix} -0.00696426 & 0 & 0.289526 - 0.387539i \\ 0 & -0.0703536 & 1.4192 + 0.447705i \\ 0.00893467 - 0.0548221i & 0.11474 + 0.140445i & 1.06975 - 1.07627i \end{pmatrix}.$$
$$(6.89)$$

The fit results and the predictions for models $\mathbf{D^b}$ are shown in Table 9 and 10 respectively. The parameter set for $\mathbf{D^b I}$ is:

$$\{a_{33}^u, a_{33}^d, \epsilon_2, T, \theta, \phi, v_R, r\} = \{-0.416619 - 0.310425i, -0.0317247, 3.76592 + 0.0145385i, 0.310345,$$
$$2.84818, 0.132797, 2.21257 \cdot 10^{13}\text{GeV}, 9.14124\}$$

and

$$a_{ij}^e = 10^{-2} \begin{pmatrix} 0.0964978 & 0 & -0.0230964 + 1.18352i \\ 0 & -0.00493562 & 0.00684639 - 0.202567i \\ -0.0394903 - 0.200904i & 0.055507 + 0.0481135i & 0.753644 + 1.64867i \end{pmatrix}.$$
$$(6.90)$$

And the parameter set for model $\mathbf{D^b II}$ is:

$$\{a_{33}^u, a_{33}^d, \epsilon_2, T, \theta, \phi, v_L, r\} = \{-0.365477 - 0.36971i, 0.0316996, 3.53671 - 0.311658i, 0.343597,$$
$$-2.95969, -0.131357, 1.58947 \cdot 10^{-8}\text{GeV}, 9.14446\}$$

and

$$a_{ij}^e = 10^{-2} \begin{pmatrix} 0.00324628 & 0 & -0.026757 + 0.083972i \\ 0 & -0.148375 & 0.450875 + 0.843973i \\ -0.0190056 - 0.0577497i & -0.129264 - 0.0587799i & 1.86523 + 0.58344i \end{pmatrix}.$$
$$(6.91)$$

Masses (in GeV) and Mixing parameters	Inputs (at $\mu = M_{GUT}$)	Fitted values ($\mathbf{D^a I}$) (at $\mu = M_{GUT}$)	pulls ($\mathbf{D^a I}$)
$m_u/10^{-3}$	0.502±0.155	0.520	0.12
m_c	0.245±0.007	0.243	-0.20
m_t	90.28±0.90	90.17	-0.11
$m_d/10^{-3}$	0.839±0.084	0.967	1.51
$m_s/10^{-3}$	16.62±0.90	16.49	-0.14
m_b	0.938±0.009	0.939	0.14
$m_e/10^{-3}$	0.344021±0.000344	0.343834	-0.54
$m_\mu/10^{-3}$	72.6256±0.0726	72.4978	-1.75
m_τ	1.24038±0.00124	1.23997	-0.32
$|V_{us}|/10^{-2}$	22.53±0.07	22.53	-0.09
$|V_{cb}|/10^{-2}$	3.934±0.06	3.920	-0.22
$|V_{ub}|/10^{-2}$	0.340±0.011	0.341	0.10
δ_{CKM}	1.208±0.054	1.192	-0.28
$\Delta m^2_{sol}/10^{-5}(\text{eV}^2)$	7.56±0.24	7.52	-0.15
$\Delta m^2_{atm}/10^{-3}(\text{eV}^2)$	2.41±0.08	2.42	0.13
$\sin^2\theta_{12}^{PMNS}$	0.308±0.017	0.290	-1.00
$\sin^2\theta_{23}^{PMNS}$	0.387±0.0225	0.399	0.55
$\sin^2\theta_{13}^{PMNS}$	0.0241±0.0025	0.0235	-0.20

Table 7: Fitting result for model $\mathbf{D^a I}$ with inputs correspond to $\tan\beta = 10$. The fitted values correspond to $\chi^2 = 7.4$ for type-I. It should be mentioned that, among all the fit results presented in this work, this specific fit has the largest value of χ^2 which is 7.4 for 18 observables. This fit correspond to $|a_{ij}|_{max} = |a_{44}| = 1.55$. For the charged lepton masses, a relative uncertainty of 0.1% is assumed in order to take into account theoretical uncertainties arising for example from threshold effects. We did not find any acceptable fit within the perturbative range for model $\mathbf{D^a II}$.

Quantity	Predicted Value ($\mathbf{D^a I}$)
$\{m_1, m_2, m_3\}$ (in eV)	$\{1.58 \cdot 10^{-3}, 8.81 \cdot 10^{-3}, 4.99 \cdot 10^{-2}\}$
$\{\delta^{PMNS}, \alpha_{21}^{PMNS}, \alpha_{31}^{PMNS}\}$	$\{85.64°, 139.76°, 149.60°\}$
$\{m_{cos}, m_\beta, m_{\beta\beta}\}$ (in eV)	$\{6.03 \cdot 10^{-2}, 4.78 \cdot 10^{-3}, 1.21 \cdot 10^{-3}\}$
$\{M_1, M_2, M_3\}$ (in GeV)	$\{8.89 \cdot 10^7, 2.14 \cdot 10^{10}, 2.63 \cdot 10^{12}\}$

Table 8: Predictions of the model $\mathbf{D^a I}$. m_i are the light neutrino masses, M_i are the right handed neutrino masses, $\alpha_{21,31}$ are the Majorana phases following the PDG parametrization, $m_{cos} = \sum_i m_i$, $m_\beta = \sum_i |U_{ei}|^2 m_i$ is the effective mass parameter for beta-decay and $m_{\beta\beta} = |\sum_i U_{ei}^2 m_i|$ is the effective mass parameter for neutrinoless double beta decay.

Model E: SUSY $SO(10)$: $210_H + 16_H + \overline{16}_H + 126_H + \overline{126}_H$

The fit results and the predictions for models $\mathbf{E}$ are shown in Table 11 and 12 respectively. For model $\mathbf{EI}$, the parameter set is:

Masses (in GeV) and Mixing parameters	Inputs (at $\mu = M_{GUT}$)	Fitted values ($\mathbf{D^b I}$) (at $\mu = M_{GUT}$)	pulls ($\mathbf{D^b I}$)	Fitted values ($\mathbf{D^b II}$) (at $\mu = M_{GUT}$)	pulls ($\mathbf{D^b II}$)
$m_u/10^{-3}$	0.502±0.155	0.501	-0.0006	0.502	0.001
m_c	0.245±0.007	0.245	-0.004	0.245	0.003
m_t	90.28±0.90	90.28	0.002	90.28	-0.00009
$m_d/10^{-3}$	0.839±0.084	0.839	0.001	0.838	-0.004
$m_s/10^{-3}$	16.62±0.90	16.62	-0.001	16.62	-0.0001
m_b	0.938±0.009	0.938	-0.001	0.938	0.001
$m_e/10^{-3}$	0.344021±0.000344	0.344016	-0.01	0.344019	-0.007
$m_\mu/10^{-3}$	72.6256±0.0726	72.6279	0.03	72.62249	-0.01
m_τ	1.24038±0.00124	1.24035	-0.02	1.24039	0.004
$|V_{us}|/10^{-2}$	22.53±0.07	22.53	0.0004	22.53	-0.0003
$|V_{cb}|/10^{-2}$	3.934±0.06	3.934	0.002	3.933	-0.0005
$|V_{ub}|/10^{-2}$	0.340±0.011	0.340	-0.001	0.340	-0.0005
δ_{CKM}	1.208±0.054	1.208	0.002	1.208	-0.001
$\Delta m^2_{sol}/10^{-5}(\text{eV}^2)$	7.55±0.24	7.56	-0.0004	7.55	-0.0003
$\Delta m^2_{atm}/10^{-3}(\text{eV}^2)$	2.41±0.08	2.41	0.0008	2.40	-0.0003
$\sin^2\theta^{PMNS}_{12}$	0.308±0.017	0.308	-0.001	0.308	-0.0003
$\sin^2\theta^{PMNS}_{23}$	0.387±0.0225	0.387	0.0007	0.387	0.001
$\sin^2\theta^{PMNS}_{13}$	0.0241±0.0025	0.0241	0.001	0.02409	-0.001

Table 9: Fitting result for model $\mathbf{D^b}$ with inputs correspond to $\tan\beta = 10$. The fitted values correspond to $\chi^2 = 1.9 \cdot 10^{-3}$ and $2 \cdot 10^{-4}$ for models $\mathbf{D^b I}$ and $\mathbf{D^b II}$ respectively. These fits correspond to $|a_{ij}|_{max} = |a_{44}| = 0.81$ and 0.99 for the two cases respectively. For the charged lepton masses, a relative uncertainty of 0.1% is assumed in order to take into account theoretical uncertainties arising for example from threshold effects.

Quantity	Predicted Value ($\mathbf{D^b I}$)	Predicted Value ($\mathbf{D^b II}$)
$\{m_1, m_2, m_3\}$ (in eV)	$\{2.20 \cdot 10^{-3}, 8.96 \cdot 10^{-3}, 4.99 \cdot 10^{-2}\}$	$\{4.72 \cdot 10^{-3}, 9.89 \cdot 10^{-3}, 5.00 \cdot 10^{-2}\}$
$\{\delta^{PMNS}, \alpha^{PMNS}_{21}, \alpha^{PMNS}_{31}\}$	$\{50.24°, 169.13°, 111.61°\}$	$\{66.63°, 161.63°, 0.41°\}$
$\{m_{cos}, m_\beta, m_{\beta\beta}\}$ (in eV)	$\{6.10 \cdot 10^{-2}, 5.38 \cdot 10^{-3}, 7.40 \cdot 10^{-4}\}$	$\{6.47 \cdot 10^{-2}, 7.37 \cdot 10^{-3}, 4.54 \cdot 10^{-4}\}$
$\{M_1, M_2, M_3\}$ (in GeV)	$\{9.40 \cdot 10^8, 3.13 \cdot 10^{10}, 2.44 \cdot 10^{11}\}$	-

Table 10: Predictions of models $\mathbf{D^b}$. m_i are the light neutrino masses, M_i are the right handed neutrino masses, $\alpha_{21,31}$ are the Majorana phases following the PDG parametrization, $m_{cos} = \sum_i m_i$, $m_\beta = \sum_i |U_{ei}|^2 m_i$ is the effective mass parameter for beta-decay and $m_{\beta\beta} = |\sum_i U^2_{ei} m_i|$ is the effective mass parameter for neutrinoless double beta decay.

$$\{a^u_{33}, a^d_{33}, \epsilon, T, \theta, \phi, v_R, r\} = \{0.0873809 + 0.511807i, 0.0316596, 3.21783 + 0.31637i, 0.762371,$$
$$0.747998, 2.38528, 2.26917 \cdot 10^{13}\text{GeV}, 9.13917\}$$

and

$$a^e_{ij} = 10^{-2} \begin{pmatrix} 0.00565532 & 0 & 0.0242668 + 0.230491i \\ 0 & 0.10865 & 1.42287 - 0.445238i \\ -0.0636824 - 0.00136495i & -0.154896 + 0.137372i & -1.56252 + 0.079592i \end{pmatrix}.$$

(6.92)

For model **EII**, the parameter set is:

$$\{a^u_{33}, a^d_{33}, \epsilon, T, \theta, \phi, v_L, r\} = \{-0.43609 + 0.282193i, -0.0316974, 3.21172 + 0.154721i, 0.54795,$$
$$0.682955, 2.41863, 1.26299 \cdot 10^{-8} \text{GeV}, 9.1442\}$$

and

$$a^e_{ij} = 10^{-2} \begin{pmatrix} -0.00491523 & 0 & 0.0991935 + 0.158945i \\ 0 & -0.0989956 & 1.00507 - 0.775818i \\ -0.0677936 - 0.0254017i & 0.0688011 + 0.186132i & -1.33307 + 1.14438i \end{pmatrix}.$$

$$(6.93)$$

Masses (in GeV) and Mixing parameters	Inputs (at $\mu = M_{GUT}$)	Fitted values (**EI**) (at $\mu = M_{GUT}$)	pulls (**EI**)	Fitted values (**EII**) (at $\mu = M_{GUT}$)	pulls (**EII**)
$m_u/10^{-3}$	0.502±0.155	0.501	-0.001	0.502	0.0005
m_c	0.245±0.007	0.245	-0.007	0.245	0.001
m_t	90.28±0.90	90.28	0.001	90.28	-0.002
$m_d/10^{-3}$	0.839±0.084	0.839	0.001	0.839	-0.0005
$m_s/10^{-3}$	16.62±0.90	16.62	0.00009	16.62	-0.0001
m_b	0.938±0.009	0.938	-0.0002	0.938	-0.001
$m_e/10^{-3}$	0.344021±0.000344	0.344022	0.004	0.344023	0.005
$m_\mu/10^{-3}$	72.6256±0.0726	72.6250	-0.007	72.62641	0.01
m_τ	1.24038±0.00124	1.24036	-0.01	1.24037	-0.009
$\|V_{us}\|/10^{-2}$	22.53±0.07	22.53	0.001	22.53	-0.0001
$\|V_{cb}\|/10^{-2}$	3.934±0.06	3.934	0.005	3.933	-0.0003
$\|V_{ub}\|/10^{-2}$	0.340±0.011	0.340	-0.007	0.340	0.0006
δ_{CKM}	1.208±0.054	1.208	0.007	1.208	0.004
$\Delta m^2_{sol}/10^{-5}(\text{eV}^2)$	7.56±0.24	7.56	0.001	7.55	-0.0002
$\Delta m^2_{atm}/10^{-3}(\text{eV}^2)$	2.41±0.08	2.409	-0.0007	2.41	0.0003
$\sin^2\theta^{PMNS}_{12}$	0.308±0.017	0.308	0.006	0.307	-0.002
$\sin^2\theta^{PMNS}_{23}$	0.387±0.0225	0.387	0.002	0.387	0.0008
$\sin^2\theta^{PMNS}_{13}$	0.0241±0.0025	0.0240	0.0001	0.0241	0.001

Table 11: Fitting result for models **E** with inputs correspond to $\tan\beta = 10$. The fitted values correspond to $\chi^2 = 4 \cdot 10^{-4}$ for model **EI** and $2 \cdot 10^{-4}$ for model **EII** respectively. These fittings correspond to $|a_{ij}|_{max} = |a_{44}| = 0.76$ and 0.89 for the type-I and type-II cases respectively. For the charged lepton masses, a relative uncertainty of 0.1% is assumed in order to take into account theoretical uncertainties arising for example from threshold effects.

Model F: SUSY $SO(10)$: $210_H + 54_H + 16_H + \overline{16}_H + 126_H + \overline{126}_H$

The fit results and the predictions for models **F** are shown in Table 14 and 13 respectively. The parameter set for model **FI** is:

$$\{a^u_{33}, a^d_{33}, \epsilon_1, \epsilon_2, T, \theta, \phi, v_R, r\} = \{-0.508413 + 0.106596i, 0.0317542, 1.21369 + 0.393457i, 1.11752$$
$$+ 1.12726i, 0.652924, 0.682955, -2.69221, 2.17249 \cdot 10^{13} \text{GeV}, 9.14433\}$$

and

Quantity	Predicted Value (**EI**)	Predicted Value (**EII**)
$\{m_1, m_2, m_3\}$ (in eV)	$\{2.06 \cdot 10^{-3}, 8.93 \cdot 10^{-3}, 4.98 \cdot 10^{-2}\}$	$\{2.46 \cdot 10^{-3}, 9.03 \cdot 10^{-3}, 4.99 \cdot 10^{-2}\}$
$\{\delta^{PMNS}, \alpha_{21}^{PMNS}, \alpha_{31}^{PMNS}\}$	$\{46.84°, -178.55°, 141.46°\}$	$\{-53.69°, -172.46°, -123.70°\}$
$\{m_{cos}, m_\beta, m_{\beta\beta}\}$ (in eV)	$\{6.08 \cdot 10^{-2}, 5.28 \cdot 10^{-3}, 9.54 \cdot 10^{-4}\}$	$\{6.14 \cdot 10^{-2}, 5.58 \cdot 10^{-3}, 7.05 \cdot 10^{-4}\}$
$\{M_1, M_2, M_3\}$ (in GeV)	$\{2.79 \cdot 10^8, 2.15 \cdot 10^{10}, 1.82 \cdot 10^{12}\}$	-

Table 12: Predictions of models **E**. m_i are the light neutrino masses, M_i are the right handed neutrino masses, $\alpha_{21,31}$ are the Majorana phases following the PDG parametrization, $m_{cos} = \sum_i m_i$, $m_\beta = \sum_i |U_{ei}|^2 m_i$ is the effective mass parameter for beta-decay and $m_{\beta\beta} = |\sum_i U_{ei}^2 m_i|$ is the effective mass parameter for neutrinoless double beta decay.

$$a_{ij}^e = 10^{-2} \begin{pmatrix} -0.101322 & 0 & 1.50945 + 0.641937i \\ 0 & 0.00628798 & -0.311418 - 0.017807i \\ -0.0659206 - 0.186996i & -0.0254875 + 0.0490826i & 0.979206 + 0.994616i \end{pmatrix}.$$

$$(6.94)$$

And the parameter set for model **FII** is:

$$\{a_{33}^u, a_{33}^d, \epsilon_1, \epsilon_2, T, \theta, \phi, v_L, r\} = \{-0.0175831 - 0.518919i, -0.0317748, 1.13488 - 0.537296i,$$
$$0.934779 - 0.810325i, 0.577852, 0.541654, 2.37836, 1.17202 \cdot 10^{-8}\text{GeV}, 9.14329\}$$

and

$$a_{ij}^e = 10^{-2} \begin{pmatrix} 0.00631171 & 0 & -0.244096 - 0.0355119i \\ 0 & -0.106855 & 1.42499 - 0.0405503i \\ -0.00203514 - 0.0627216i & -0.159398 + 0.138939i & 1.56233 + 0.439752i \end{pmatrix}.$$

$$(6.95)$$

Quantity	Predicted Value (**FI**)	Predicted Value (**FII**)
$\{m_1, m_2, m_3\}$ (in eV)	$\{1.84 \cdot 10^{-3}, 8.88 \cdot 10^{-3}, 4.98 \cdot 10^{-2}\}$	$\{2.00 \cdot 10^{-3}, 8.92 \cdot 10^{-3}, 4.98 \cdot 10^{-2}\}$
$\{\delta^{PMNS}, \alpha_{21}^{PMNS}, \alpha_{31}^{PMNS}\}$	$\{-60.72°, -175.43°, -164.89°\}$	$\{44.97°, 179.45°, 133.12°\}$
$\{m_{cos}, m_\beta, m_{\beta\beta}\}$ (in eV)	$\{6.06 \cdot 10^{-2}, 5.11 \cdot 10^{-3}, 1.17 \cdot 10^{-3}\}$	$\{6.08 \cdot 10^{-2}, 5.23 \cdot 10^{-3}, 9.61 \cdot 10^{-4}\}$
$\{M_1, M_2, M_3\}$ (in GeV)	$\{3.92 \cdot 10^8, 1.97 \cdot 10^{10}, 1.27 \cdot 10^{12}\}$	-

Table 13: Predictions of models **F**. m_i are the light neutrino masses, M_i are the right handed neutrino masses, $\alpha_{21,31}$ are the Majorana phases following the PDG parametrization, $m_{cos} = \sum_i m_i$, $m_\beta = \sum_i |U_{ei}|^2 m_i$ is the effective mass parameter for beta-decay and $m_{\beta\beta} = |\sum_i U_{ei}^2 m_i|$ is the effective mass parameter for neutrinoless double beta decay.

7 $d = 5$ proton decay

Since the flavor dynamics occurs at the GUT scale in this class of models, the best hope for testing this idea is by studying proton decay, in particular, its branching ratios into different modes. While such an analysis can be done for both non-SUSY and SUSY models, here we confine our discussion to the more dominant $d = 5$ decay modes in SUSY mediated by the color-triplet Higgsinos.

Masses (in GeV) and Mixing parameters	Inputs (at $\mu = M_{GUT}$)	Fitted values (**FI**) (at $\mu = M_{GUT}$)	pulls (**FI**)	Fitted values (**FII**) (at $\mu = M_{GUT}$)	pulls (**FII**)
$m_u/10^{-3}$	0.502±0.155	0.501	-0.003	0.501	-0.0005
m_c	0.245±0.007	0.245	0.006	0.245	0.001
m_t	90.28±0.90	90.28	0.003	90.28	0.001
$m_d/10^{-3}$	0.839±0.084	0.839	0.004	0.839	0.001
$m_s/10^{-3}$	16.62±0.90	16.62	-0.001	16.62	0.001
m_b	0.938±0.009	0.938	0.0001	0.938	-0.0001
$m_e/10^{-3}$	0.344021±0.000344	0.344022	0.001	0.344022	0.002
$m_\mu/10^{-3}$	72.6256±0.0726	72.6237	-0.02	72.62539	-0.002
m_τ	1.24038±0.00124	1.24039	0.007	1.24038	0.0003
$|V_{us}|/10^{-2}$	22.53±0.07	22.53	0.0002	22.53	0.0001
$|V_{cb}|/10^{-2}$	3.934±0.06	3.933	-0.001	3.934	0.0001
$|V_{ub}|/10^{-2}$	0.340±0.011	0.340	-0.007	0.340	-0.001
δ_{CKM}	1.208±0.054	1.208	0.001	1.208	0.004
$\Delta m^2_{sol}/10^{-5}(\text{eV}^2)$	7.56±0.24	7.56	0.00003	7.55	-0.0002
$\Delta m^2_{atm}/10^{-3}(\text{eV}^2)$	2.41±0.08	2.41	0.0005	2.41	0.0001
$\sin^2\theta_{12}^{PMNS}$	0.308±0.017	0.308	0.0004	0.308	0.0004
$\sin^2\theta_{23}^{PMNS}$	0.387±0.0225	0.387	-0.001	0.387	0.001
$\sin^2\theta_{13}^{PMNS}$	0.0241±0.0025	0.0240	-0.0009	0.02409	-0.002

Table 14: Fitting result for models **F** with inputs correspond to $\tan\beta = 10$. The fitted values correspond to $\chi^2 = 9 \cdot 10^{-4}$ and $3 \cdot 10^{-5}$ for models **FI** and **FII** respectively. These fittings correspond to $|a_{ij}|_{max} = |a_{44}| = 0.67$ and 1.08 for the type-I and type-II cases respectively. For the charged lepton masses, a relative uncertainty of 0.1% is assumed in order to take into account theoretical uncertainties arising for example from threshold effects.

We will bound ourselves to the (presumably) dominant $d = 5$ (charged) wino mediated mode, so that only $SU(2)_L$ non-singlets will appear in the effective operators:

$$W \propto (Y_{QQ})_{ij} (Y_{QL})_{kl} (Q_i Q_j)(Q_k L_l) \tag{7.96}$$

with

$$Y_{QQ} = \Lambda_Q^T \left(Y - y x_Q^T - x_Q y^T + y_4 x_Q x_Q^T\right) \Lambda_Q \tag{7.97}$$

$$Y_{QL} = \Lambda_Q^T \left(Y - y x_L^T - x_Q y^T + y_4 x_Q x_L^T\right) \Lambda_L \tag{7.98}$$

We have to project them to the mass eigenstates defined by the unitary matrices $X = U, D, E, N$ which diagonalize the mass matrices as

$$M_X = X_R M_X^d X_L^\dagger \tag{7.99}$$

We will use the notation $(X = U, D)$

$$Y_{XZ} = X_L^T Y_{QQ} Z_L \quad (\text{for } Z = U, D) \tag{7.100}$$

$$= X_L^T Y_{QL} Z_L \quad (\text{for } Z = E, N) \tag{7.101}$$

After 1-loop $\tilde{w}^\pm$ dressing and assuming degeneracy and negligible left-right sfermion mixing the normalized amplitudes for different channels [54] are, in the mass eigenbasis,

$$A(K^+\bar{\nu}_l) = \langle K^+|\,(ud)_L\,s_L|p\rangle\,[(Y_{UD})_{11}\,(Y_{DN})_{2l} - (Y_{DD})_{21}\,(Y_{UN})_{1l}]$$

$$+ \langle K^+|\,(us)_L\,d_L|p\rangle\,[(Y_{UD})_{12}\,(Y_{DN})_{1l} - (Y_{DD})_{12}\,(Y_{UN})_{1l}] \qquad (7.102)$$

$$A(\pi^+\bar{\nu}_l) = \langle \pi^+|\,(ud)_L\,d_L|p\rangle\,[(Y_{UD})_{11}\,(Y_{DN})_{1l} - (Y_{DD})_{11}\,(Y_{UN})_{1l}] \qquad (7.103)$$

$$A(K^0 e_l^+) = \langle K^0|\,(us)_L\,u_L|p\rangle\,[(Y_{UU})_{11}\,(Y_{DE})_{2l} - (Y_{UD})_{12}\,(Y_{UE})_{1l}] \qquad (7.104)$$

$$A(\pi^0 e_l^+) = \langle \pi^0|\,(ud)_L\,u_L|p\rangle\,[(Y_{UU})_{11}\,(Y_{DE})_{1l} - (Y_{UD})_{11}\,(Y_{UE})_{1l}] \qquad (7.105)$$

$$A(\eta e_l^+) = \langle \eta|\,(ud)_L\,u_L|p\rangle\,[(Y_{UU})_{11}\,(Y_{DE})_{1l} - (Y_{UD})_{11}\,(Y_{UE})_{1l}] \qquad (7.106)$$

where the numerical values (with maximal error around 30%) of the hadron matrix elements can be found in [55].

The unitary matrices X and the Yukawa matrix elements $Y_{QQ,QL}$ are outputs of each successful fit done. As an example, for model $\mathbf{D^aI}$ we find

$$Y_{QQ} = \begin{pmatrix} -0.0000696426 & 0 & -0.0105713 + 0.00524935i \\ 0 & -0.000703536 & -0.0237115 - 0.0274144i \\ -0.0105713 + 0.00524935i & -0.0237115 - 0.0274144i & -1.05171 - 0.204611i \end{pmatrix}$$

$$(7.107)$$

$$Y_{QL} = \begin{pmatrix} -0.0000696426 & 0 & -0.0000232394 - 0.000554968i \\ 0 & -0.000703536 & 0.00140745 + 0.00114372i \\ -0.0105713 + 0.00524935i & -0.0237115 - 0.0274144i & 0.00550524 - 0.00420826i \end{pmatrix}$$

$$(7.108)$$

$$U_L = \begin{pmatrix} 0.947932 + 0.154511i & 0.0250533 - 0.277159i & -0.00483953 - 0.00916537i \\ 0.0236485 + 0.277423i & -0.948101 + 0.150221i & -0.0314764 + 0.00505612i \\ -0.00438488 - 0.00319527i & 0.0288651 + 0.0161593i & -0.895175 - 0.444452i \end{pmatrix}$$

$$(7.109)$$

$$D_L = \begin{pmatrix} 0.44376 + 0.785783i & -0.114306 - 0.415343i & -0.00683343 + 0.0000682437i \\ -0.135958 + 0.408742i & -0.484535 + 0.761308i & 0.00448262 - 0.00785931i \\ -0.00402935 - 0.00717747i & -0.00772054 - 0.00109752i & -0.895597 - 0.444722i \end{pmatrix}$$

$$(7.110)$$

$$E_L = \begin{pmatrix} -0.914868 + 0.192948i & -0.182083 - 0.209497i & -0.00368156 + 0.220751i \\ 0.16774 - 0.285354i & -0.612359 - 0.228437i & 0.639383 + 0.233362i \\ -0.0189384 - 0.125958i & -0.00319539 + 0.704116i & -0.0568573 + 0.696242i \end{pmatrix}$$

$$(7.111)$$

$$N_L = \begin{pmatrix} -0.502397 + 0.721475i & -0.139083 - 0.437348i & 0.047407 + 0.119192i \\ -0.43546 + 0.122682i & -0.243261 + 0.682259i & 0.24818 - 0.45725i \\ -0.0953668 + 0.115327i & 0.0333823 + 0.513435i & -0.0502508 + 0.842822i \end{pmatrix}$$

$$(7.112)$$

After squaring (7.102)-(7.106) and multiplying by the appropriate phase space factor (m_P, m_L, m_p are the pseudo-scalar, lepton and proton mass, respectively)

$$\left(1 - 2\left(\frac{m_P^2 + m_L^2}{m_p^2}\right) + \left(\frac{m_P^2 - m_L^2}{m_p^2}\right)^2\right) \tag{7.113}$$

one can calculate the branching fractions for different channels (for neutrino final states we sum over all 3 flavors), the results are given for the different models in table 15. While as expected, the $K^+\bar{\nu}$ mode dominates, other sub-leading modes, notably $p \to \pi^+\overline{\nu}$, can be used to test and distinguish various models.

	CI	**CII**	**D$^\text{a}$ I**	**D$^\text{b}$I**	**D$^\text{b}$II**	**EI**	**EII**	**FI**	**FII**
$K^+\bar{\nu}$	88.39	94.36	50.39	92.71	75.26	89.03	77.91	94.78	90.65
$\pi^+\bar{\nu}$	10.85	5.55	48.33	7.12	24.62	10.48	21.58	4.95	9.17
$K^0 e^+$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$K^0 \mu^+$	0.35	0.04	0.49	0.08	0.05	0.23	0.21	0.13	0.09
$\pi^0 e^+$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\pi^0 \mu^+$	0.34	0.04	0.66	0.08	0.06	0.21	0.25	0.12	0.08
ηe^+	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\eta \mu^+$	0.06	0.01	0.12	0.01	0.01	0.04	0.05	0.02	0.01

Table 15: Branching ratios for the main decay modes of the proton mediated by colored Higgsinos in SUSY $SO(10)$ models with successful fermion fits.

8 Conclusion

We have presented in this paper a new class of $SO(10)$ models that can successfully address the flavor puzzle. The key ingredient of our models is the absence of 10_H that is conventionally used in most $SO(10)$ models. Its absence is compensated by the introduction of a vector-like family in the $16 + \overline{16}$ representation. The Yukawa sector of these models has just a single 4×4 matrix, along with two four-vectors. As a consequence, there are only 14 flavor parameters and 7 phases to fit all fermion masses and mixings, including the neutrino sector.

While the Yukawa system is highly nonlinear, by numerical optimization we have found excellent fits to the fermion observables in a variety of models. A $\overline{126}_H$ is present in all models, to generate large right-handed neutrino Majorana masses as well as to provide the SM Higgs doublet. The vector-like fermions have couplings to either a 45_H or a 210_H that is used to complete the symmetry breaking. A total of six models, four supersymmetric and two non-supersymmetric, have been studied. In each case type-I or type-II seesaw mechanism was analyzed. In one case (Model **D**) with SUSY, minimization of the Higgs potential led to a two-fold solution set, with each providing an excellent fit to flavor observables.

While this class of high scale models cannot be easily tested at collider experiments, proton decay provides an avenue to probe such models. We have investigate the branching

ratios for proton decay in the SUSY models, with the results presented in Table 15. While it is an ambitious goal to test flavor models in proton decay discovery, even without such a discovery it is heartening to learn that a large class of models can shed light on the various puzzles of fermion masses observed in nature. In particular, starting from a highly symmetrical quark and lepton sector these models produce large neutrino mixing simultaneously with small quark mixing, a highly nontrivial achievement.

Acknowledgments

Appendices

A Expressions for a_{ij}

In this Appendix we give expressions for a_{ij} used in the numerical analysis.

$$a_{13} = \frac{N_{e^c} a_{13}^e - N_e a_{31}^e}{T(Q_{e^c} - Q_e)} \, , \tag{A.114}$$

$$a_{14} = \frac{N_{e^c} a_{13}^e (e^{i\phi} \cos\theta + T Q_e) - N_e a_{31}^e (e^{i\phi} \cos\theta + T Q_{e^c})}{\sin\theta T(Q_{e^c} - Q_e)} \, , \tag{A.115}$$

$$a_{23} = \frac{N_{e^c} a_{23}^e - N_e a_{32}^e}{T(Q_{e^c} - Q_e)} \, , \tag{A.116}$$

$$a_{24} = \frac{N_{e^c} a_{23}^e (e^{i\phi} \cos\theta + T Q_e) - N_e a_{32}^e (e^{i\phi} \cos\theta + T Q_{e^c})}{\sin\theta T(Q_{e^c} - Q_e)} \, , \tag{A.117}$$

$$a_{33} = (a_{33}^u N_u N_{u^c} C_1 + a_{33}^d N_d N_{d^c} C_2 + a_{33}^e N_e N_{e^c} C_3)/(T^2 D_1) \, , \tag{A.118}$$

$$a_{34} = (a_{33}^u N_u N_{u^c} C_4 + a_{33}^d N_d N_{d^c} C_5 - a_{33}^e N_e N_{e^c} C_6)/(T^2 D_1) \, , \tag{A.119}$$

$$a_{44} = (a_{33}^u N_u N_{u^c} C_7 + a_{33}^d N_d N_{d^c} C_8 - a_{33}^e N_e N_{e^c} C_9)/(T^2 D_1) \, , \tag{A.120}$$

with

$$C_1 = Q_{d^c} - Q_{e^c} + Q_d - Q_e; \tag{A.121}$$

$$C_2 = Q_{e^c} - Q_{u^c} + Q_e - Q_u; \tag{A.122}$$

$$C_3 = -Q_{d^c} + Q_{u^c} - Q_d + Q_u; \tag{A.123}$$

$$C_4 = \csc\theta \left(e^{i\phi}\cos\theta \left(Q_{d^c} - Q_{e^c} \right) + Q_d \left(TQ_{d^c} + e^{i\phi}\cos\theta \right) - Q_e \left(TQ_{e^c} + e^{i\phi}\cos\theta \right) \right); \tag{A.124}$$

$$C_5 = \csc\theta \left(Q_e \left(TQ_{e^c} + e^{i\phi}\cos\theta \right) + e^{i\phi}\cos\theta \left(Q_{e^c} - Q_{u^c} \right) - Q_u \left(TQ_{u^c} + e^{i\phi}\cos\theta \right) \right); \tag{A.125}$$

$$C_6 = \csc\theta \left(Q_d \left(TQ_{d^c} + e^{i\phi}\cos\theta \right) + e^{i\phi}\cos\theta \left(Q_{d^c} - Q_{u^c} \right) - Q_u \left(TQ_{u^c} + e^{i\phi}\cos\theta \right) \right); \tag{A.126}$$

$$\begin{aligned}
C_7 = {}& T^2 Q_d Q_e \csc^2\theta\, Q_{d^c} - T^2 Q_d Q_e \csc^2\theta\, Q_{e^c} + T^2 Q_d \csc^2\theta\, Q_{d^c} Q_{e^c} - T^2 Q_e \csc^2\theta\, Q_{d^c} Q_{e^c} \\
&+ e^{2i\phi}\cot^2\theta\, Q_{d^c} + 2T e^{i\phi} Q_d \cot\theta \csc\theta\, Q_{d^c} - e^{2i\phi}\cot^2\theta\, Q_{e^c} \\
&- 2T e^{i\phi} Q_e \cot\theta \csc\theta\, Q_{e^c} + e^{2i\phi} Q_d \cot^2\theta - e^{2i\phi} Q_e \cot^2\theta;
\end{aligned} \tag{A.127}$$

$$\begin{aligned}
C_8 = {}& e^{2i\phi}\cot^2\theta\, Q_{e^c} + T^2 Q_e \csc^2\theta\, Q_u Q_{e^c} - T^2 Q_e \csc^2\theta\, Q_u Q_{u^c} + T^2 Q_e \csc^2\theta\, Q_{e^c} Q_{u^c} \\
&- T^2 \csc^2\theta\, Q_u Q_{e^c} Q_{u^c} + 2T e^{i\phi} Q_e \cot\theta \csc\theta\, Q_{e^c} - 2T e^{i\phi}\cot\theta \csc\theta\, Q_u Q_{u^c} \\
&- e^{2i\phi}\cot^2\theta\, Q_{u^c} + e^{2i\phi} Q_e \cot^2\theta - e^{2i\phi}\cot^2\theta\, Q_u;
\end{aligned} \tag{A.128}$$

$$\begin{aligned}
C_9 = {}& e^{2i\phi}\cot^2\theta\, Q_{d^c} + T^2 Q_d \csc^2\theta\, Q_u Q_{d^c} - T^2 Q_d \csc^2\theta\, Q_u Q_{u^c} + T^2 Q_d \csc^2\theta\, Q_{d^c} Q_{u^c} \\
&- T^2 \csc^2\theta\, Q_u Q_{d^c} Q_{u^c} + 2T e^{i\phi} Q_d \cot\theta \csc\theta\, Q_{d^c} - 2T e^{i\phi}\cot\theta \csc\theta\, Q_u Q_{u^c} \\
&- e^{2i\phi}\cot^2\theta\, Q_{u^c} + e^{2i\phi} Q_d \cot^2\theta - e^{2i\phi}\cot^2\theta\, Q_u;
\end{aligned} \tag{A.129}$$

$$\begin{aligned}
D_1 = {}& Q_d Q_e Q_{d^c} + Q_d Q_{d^c} Q_{e^c} - Q_e Q_{d^c} Q_{e^c} - Q_d Q_e Q_{e^c} - Q_d Q_u Q_{d^c} - Q_d Q_{d^c} Q_{u^c} \\
&+ Q_u Q_{d^c} Q_{u^c} + Q_d Q_u Q_{u^c} + Q_e Q_u Q_{e^c} - Q_e Q_u Q_{u^c} + Q_e Q_{e^c} Q_{u^c} - Q_u Q_{e^c} Q_{u^c}. \tag{A.130}
\end{aligned}$$

B Expressions for a_{33}^ν, a_{33}^R and a_{33}^L

In this appendix, we give the expressions for $a_{33}^\nu, a_{33}^R, a_{33}^L$ for both the $\Phi = 45_H$ and 210_H cases. Using Eqs.(4.66), (A.118), (A.119) and (A.120) it is straightforward to find for the 45_H-case:

$$a_{33}^\nu = a_{33}^u \frac{N_u N_{u^c}}{N_\nu N_{\nu^c}} + a_{33}^d \frac{N_d N_{d^c}}{N_\nu N_{\nu^c}} \frac{1 + \epsilon/5}{1 + \epsilon} - a_{33}^e \frac{N_e N_{e^c}}{N_\nu N_{\nu^c}} \frac{1 + \epsilon/5}{1 + \epsilon}. \tag{B.131}$$

And for the case of 210_H we find:

$$a_{33}^\nu = a_{33}^u \frac{N_u N_{u^c}}{N_\nu N_{\nu^c}} \frac{C_{10}}{D_2} + a_{33}^d \frac{N_d N_{d^c}}{N_\nu N_{\nu^c}} \frac{C_{11}}{D_2} + a_{33}^e \frac{N_e N_{e^c}}{N_\nu N_{\nu^c}} \frac{C_{12}}{D_2}, \tag{B.132}$$

with

$$C_{10} = 3\left(8\sqrt{6}\epsilon_1^2 - 4\left(2\sqrt{3}\epsilon_2 + 3\sqrt{2}\right)\epsilon_1 + \epsilon_2\left(\sqrt{6}\epsilon_2 + 6\right)\right), \tag{B.133}$$

$$C_{11} = 3\left(-8\sqrt{6}\epsilon_1^2 + 12\sqrt{2}\epsilon_1 + \epsilon_2\left(\sqrt{6}\epsilon_2 - 6\right)\right), \tag{B.134}$$

$$C_{12} = 8\sqrt{6}\epsilon_1^2 + 4\left(2\sqrt{3}\epsilon_2 + 3\sqrt{2}\right)\epsilon_1 - 3\epsilon_2\left(\sqrt{6}\epsilon_2 + 2\right), \tag{B.135}$$

$$D_2 = \left(4\epsilon_1 - \sqrt{2}\epsilon_2\right)\left(2\sqrt{6}\epsilon_1 - 3\sqrt{3}\epsilon_2 + 3\sqrt{2}\right). \tag{B.136}$$

Using Eqs. (A.118), (A.119) and (A.120) for the 45_H-case we have:

$$a_{33}^R = \frac{3}{2}a_{33}^u \frac{N_u N_{u^c}}{N_{\nu^c}^2}\frac{1+\epsilon/5}{1+3\epsilon/5} - \frac{5}{4}a_{33}^e \frac{N_e N_{e^c}}{N_{\nu^c}^2}\frac{(1+\epsilon/5)^2}{\epsilon(1+\epsilon)} + \frac{5}{4}a_{33}^d \frac{N_d N_{d^c}}{N_{\nu^c}^2}\frac{1+\frac{3}{5}\epsilon+\frac{3}{25}\epsilon^2+\frac{33}{125}\epsilon^3}{\epsilon(1+\epsilon)(1+3\epsilon/5)}. \tag{B.137}$$

And for 210_H-case we have:

$$a_{33}^R = a_{33}^u \frac{N_u N_{u^c}}{N_{\nu^c}^2}\frac{C_{13}}{D_3} - a_{33}^d \frac{N_d N_{d^c}}{N_{\nu^c}^2}\frac{C_{14}}{D_3} + a_{33}^e \frac{N_e N_{e^c}}{N_{\nu^c}^2}\frac{C_{15}}{D_3}, \tag{B.138}$$

with
$$C_{13} = 18\sqrt{3}\epsilon_2^4 + 36\sqrt{2}\epsilon_2^3 - 45\sqrt{6}\epsilon_1\epsilon_2^3 + 12\sqrt{3}\epsilon_2^2 - 156\epsilon_1\epsilon_2^2 + 72\sqrt{6}\epsilon_1^3\epsilon_2 + 12\sqrt{2}\epsilon_1^2\epsilon_2 - 30\sqrt{6}\epsilon_1\epsilon_2$$
$$+ 48\epsilon_1^3 + 24\sqrt{3}\epsilon_1^2, \tag{B.139}$$

$$C_{14} = 9\sqrt{3}\epsilon_2^4 + 18\sqrt{2}\epsilon_2^3 - 45\sqrt{6}\epsilon_1\epsilon_2^3 + 72\sqrt{3}\epsilon_1^2\epsilon_2^2 + 6\sqrt{3}\epsilon_2^2 - 96\epsilon_1\epsilon_2^2 + 72\sqrt{6}\epsilon_1^3\epsilon_2 + 36\sqrt{2}\epsilon_1^2\epsilon_2$$
$$- 18\sqrt{6}\epsilon_1\epsilon_2 + 48\epsilon_1^3 + 24\sqrt{3}\epsilon_1^2, \tag{B.140}$$

$$C_{15} = 16\sqrt{6}\epsilon_1^3 + 8\left(5\sqrt{3}\epsilon_2 + 6\sqrt{2}\right)\epsilon_1^2 + 6\left(\sqrt{6}\epsilon_2^2 + 8\epsilon_2 + 2\sqrt{6}\right)\epsilon_1 - 3\epsilon_2\left(3\sqrt{3}\epsilon_2^2 + 6\sqrt{2}\epsilon_2 + 2\sqrt{3}\right), \tag{B.141}$$

$$D_3 = 2\epsilon_1\left(4\epsilon_1 - \sqrt{2}\epsilon_2\right)\left(2\sqrt{6}\epsilon_1 - 3\sqrt{3}\epsilon_2 + 3\sqrt{2}\right). \tag{B.142}$$

For the case of 45_H we have:

$$a_{33}^L = \frac{-4}{5}a_{33}^e \frac{N_{e^c}}{N_\nu}\frac{\epsilon}{1+\epsilon} + \frac{1}{2}a_{33}^u \frac{N_u N_{u^c}}{N_\nu^2}\frac{5+9\epsilon}{5+3\epsilon} + \frac{1}{10}a_{33}^d \frac{N_d N_{d^c}}{N_\nu^2}\frac{25+50\epsilon+9\epsilon^2}{(1+\epsilon)(5+3\epsilon)}. \tag{B.143}$$

And for the case of 210_H we have:

$$a_{33}^L = a_{33}^u \frac{N_u N_{u^c}}{N_\nu^2}\frac{C_{16}}{D_4} + a_{33}^d \frac{N_d N_{d^c}}{N_\nu^2}\frac{C_{17}}{D_4} + a_{33}^e \frac{N_e N_{e^c}}{N_\nu^2}\frac{C_{18}}{D_4}, \tag{B.144}$$

with
$$C_{16} = 3\left(8\left(\sqrt{6}\epsilon_2 - 2\right)\epsilon_1^2 - 4\left(2\sqrt{3}\epsilon_2^2 + \sqrt{2}\epsilon_2 - 2\sqrt{3}\right)\epsilon_1 + \epsilon_2\left(\sqrt{6}\epsilon_2^2 + 4\epsilon_2 - 2\sqrt{6}\right)\right), \tag{B.145}$$

$$C_{17} = 3\left(-8\left(\sqrt{6}\epsilon_2 - 2\right)\epsilon_1^2 + \left(12\sqrt{2}\epsilon_2 - 8\sqrt{3}\right)\epsilon_1 + \epsilon_2\left(\sqrt{6}\epsilon_2^2 - 8\epsilon_2 + 2\sqrt{6}\right)\right), \tag{B.146}$$

$$C_{18} = 8\sqrt{3}\epsilon_1\epsilon_2\left(2\sqrt{2}\epsilon_1 - \epsilon_2\right), \tag{B.147}$$

$$D_4 = 2\epsilon_2\left(\sqrt{2}\epsilon_2 - 4\epsilon_1\right)\left(-2\sqrt{6}\epsilon_1 + 3\sqrt{3}\epsilon_2 - 3\sqrt{2}\right). \tag{B.148}$$

C Numerical values of the original Yukawa couplings a_{ij}

In this appendix we present the original coupling matrices a_{ij} for all the different models. a_{ij} matrices are calculated from the fitted parameter sets.

Model **AI**:

$$a_{ij} = 10^{-2} \begin{pmatrix} 0.0959072 & 0 & 0.579907 + 0.173698i & 5.94255 + 2.20933i \\ 0 & -0.00693205 & 0.134449 + 0.0151987i & 1.11642 + 0.685207i \\ 0.579907 + 0.173698i & 0.134449 + 0.0151987i & 0.343854 + 2.01413i & 16.4068 + 0.693589i \\ 5.94255 + 2.20933i & 1.11642 + 0.685207i & 16.4068 + 0.693589i & 192.42 + 53.2691i \end{pmatrix}.$$

(C.149)

Model **AII**:

$$a_{ij} = 10^{-2} \begin{pmatrix} 0.127684 & 0 & -0.300661 + 0.102672i & -2.93065 + 0.584561i \\ 0 & -0.00055042 & -0.00799172 + 0.0297446i & -0.450085 + 0.185354i \\ -0.300661 + 0.102672i & -0.00799172 + 0.0297446i & 0.164493 - 2.14188i & 8.08605 + 16.5832i \\ -2.93065 + 0.584561i & -0.450085 + 0.185354i & 8.08605 + 16.5832i & 96.688 + 321.511i \end{pmatrix}.$$

(C.150)

Model **BI**:

$$a_{ij} = 10^{-2} \begin{pmatrix} -0.122115 & 0 & 0.711183 + 0.747142i & -1.99272 - 3.01439i \\ 0 & 0.00569753 & -0.0996008 - 0.168017i & 0.22039 + 0.476214i \\ 0.711183 + 0.747142i & -0.0996008 - 0.168017i & -1.6329 - 3.89159i & 2.08313 + 14.9094i \\ -1.99272 - 3.01439i & 0.22039 + 0.476214i & 2.08313 + 14.9094i & 10.1819 - 55.9358i \end{pmatrix}.$$

(C.151)

Model **BII**:

$$a_{ij} = 10^{-2} \begin{pmatrix} 0.00424453 & 0 & -0.739696 + 0.100859i & -0.261416 - 0.180202i \\ 0 & 0.0963929 & 1.59142 + 3.41719i & -0.546982 + 1.75381i \\ -0.739696 + 0.100859i & 1.59142 + 3.41719i & 35.6473 + 92.8117i & -22.3139 + 45.2696i \\ -0.261416 - 0.180202i & -0.546982 + 1.75381i & -22.3139 + 45.2696i & -25.416 + 7.30749i \end{pmatrix}.$$

(C.152)

Model **CI**:

$$a_{ij} = 10^{-2} \begin{pmatrix} 0.113952 & 0 & 0.491463 + 0.565896i & 3.76471 + 4.85316i \\ 0 & 0.0066433 & -0.0563066 + 0.107998i & -0.804669 + 0.719013i \\ 0.491463 + 0.565896i & -0.0563066 + 0.107998i & -1.4383 + 2.14192i & -6.53535 + 21.208i \\ 3.76471 + 4.85316i & -0.804669 + 0.719013i & -6.53535 + 21.208i & -59.2957 + 144.035i \end{pmatrix}.$$

(C.153)

Model **CII**:

$$a_{ij} = 10^{-2} \begin{pmatrix} -0.104302 & 0 & -0.23862 + 0.777714i & -1.41328 + 3.82589i \\ 0 & 0.00422339 & -0.0155148 + 0.115636i & 0.0287095 + 0.732858i \\ -0.23862 + 0.777714i & -0.0155148 + 0.115636i & -0.0699652 + 2.31316i & 0.347507 + 16.8414i \\ -1.41328 + 3.82589i & 0.0287095 + 0.732858i & 0.347507 + 16.8414i & -5.81915 + 103.489i \end{pmatrix}.$$

(C.154)

Model **D^{a}I**:

$$a_{ij} = 10^{-10} \begin{pmatrix} -0.00696426 & 0 & -0.245942 + 0.340282i & -0.981934 + 1.05034i \\ 0 & -0.0703536 & -1.2441 - 0.347061i & -3.9061 - 2.2744i \\ -0.245942 + 0.340282i & -1.2441 - 0.347061i & -11.3476 + 13.1125i & -38.737 + 28.5679i \\ -0.981934 + 1.05034i & -3.9061 - 2.2744i & -38.737 + 28.5679i & -137.986 + 71.4185i \end{pmatrix}.$$

(C.155)

Model $\mathbf{D}^{\mathbf{b}}\mathbf{I}$:

$$a_{ij} = 10^{-2} \begin{pmatrix} 0.0964978 & 0 & 0.0463804 + 0.612925i & 0.23948 - 4.0549i \\ 0 & -0.00493562 & -0.0692766 - 0.122098i & 0.0709726 + 0.757652i \\ 0.0463804 + 0.612925i & -0.0692766 - 0.122098i & -1.50677 - 1.28146i & 9.26908 + 9.93996i \\ 0.23948 - 4.0549i & 0.0709726 + 0.757652i & 9.26908 + 9.93996i & -60.1029 - 54.8455i \end{pmatrix}.$$

(C.156)

Model $\mathbf{D}^{\mathbf{b}}\mathbf{II}$:

$$a_{ij} = 10^{-2} \begin{pmatrix} 0.00324628 & 0 & 0.00619872 + 0.0924098i & 0.0581533 + 0.495523i \\ 0 & -0.148375 & 0.234817 + 0.306995i & 2.43678 + 3.11641i \\ 0.00619872 + 0.0924098i & 0.234817 + 0.306995i & -0.940024 - 1.19953i & -10.2704 - 9.213i \\ 0.0581533 + 0.495523i & 2.43678 + 3.11641i & -10.2704 - 9.213i & -63.9596 - 76.7709i \end{pmatrix}.$$

(C.157)

Model $\mathbf{EI}$:

$$a_{ij} = 10^{-2} \begin{pmatrix} 0.00565532 & 0 & -0.266188 + 0.0701303i & 0.518805 - 0.533918i \\ 0 & 0.10865 & 0.425618 + 1.82085i & -3.18677 - 2.87341i \\ -0.266188 + 0.0701303i & 0.425618 + 1.82085i & -14.7905 + 7.46579i & 14.094 - 32.9412i \\ 0.518805 - 0.533918i & -3.18677 - 2.87341i & 14.094 - 32.9412i & 19.4372 + 73.7643i \end{pmatrix}.$$

(C.158)

Model $\mathbf{EII}$:

$$a_{ij} = 10^{-2} \begin{pmatrix} -0.00491523 & 0 & -0.113267 + 0.379235i & -0.0710291 - 0.772947i \\ 0 & -0.0989956 & 1.91708 + 0.433922i & -4.21969 + 0.992353i \\ -0.113267 + 0.379235i & 1.91708 + 0.433922i & -14.9942 + 2.32607i & 31.5765 - 18.8751i \\ -0.0710291 - 0.772947i & -4.21969 + 0.992353i & 31.5765 - 18.8751i & -50.6035 + 73.4838i \end{pmatrix}.$$

(C.159)

Model $\mathbf{FI}$:

$$a_{ij} = 10^{-2} \begin{pmatrix} -0.101322 & 0 & 1.00796 - 1.0692i & -3.40853 + 1.66912i \\ 0 & 0.00628798 & -0.117644 + 0.20702i & 0.5911 - 0.454689i \\ 1.00796 - 1.0692i & -0.117644 + 0.20702i & -3.59013 + 11.2045i & 20.5757 - 20.2831i \\ -3.40853 + 1.66912i & 0.5911 - 0.454689i & 20.5757 - 20.2831i & -63.553 + 23.8071i \end{pmatrix}.$$

(C.160)

Model $\mathbf{FII}$:

$$a_{ij} = 10^{-2} \begin{pmatrix} 0.00631171 & 0 & -0.0913116 - 0.224948i & 0.708328 + 0.52168i \\ 0 & -0.106855 & 0.477564 + 1.57686i & -4.01776 - 3.01492i \\ -0.0913116 - 0.224948i & 0.477564 + 1.57686i & 12.5721 - 5.02818i & -18.1583 + 34.5582i \\ 0.708328 + 0.52168i & -4.01776 - 3.01492i & -18.1583 + 34.5582i & -22.102 - 106.709i \end{pmatrix}.$$

(C.161)

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