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$\cos(4\phi)$ azimuthal anisotropy in small- x DIS dijet production beyond the leading power TMD limit

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We determine the first correction to the quadrupole operator in high-energy QCD beyond the TMD limit of Weizsäcker-Williams and linearly polarized gluon distributions. These functions give rise to isotropic resp. $\sim \cos 2\phi$ angular distributions in DIS dijet production. On the other hand, the correction produces a $\sim \cos 4\phi$ angular dependence which is suppressed by one additional power of the dijet transverse momentum scale (squared) P^2 .

I. INTRODUCTION

We consider (inclusive) production of a $q\bar{q}$ dijet at leading order in high-energy (small- x) Deep Inelastic Scattering (DIS) of an electron off a proton or nucleus. The average transverse momentum of the jets is denoted as P and the transverse momentum imbalance is q . In the “correlation limit” of roughly back to back jets [1] one has $P^2 \gg q^2$. In this limit the leading contribution (in terms of powers of $1/P^2$) to the cross section can be obtained from Transverse Momentum Dependent (TMD) factorization. For a recent review of TMD factorization see Ref. [2]. It predicts a distribution for linearly polarized gluons in an unpolarized target [3, 4] which gives rise to $\sim \cos 2\phi$ asymmetries in dijet production [5–7] and in other processes [8, 9]. The azimuthal angle ϕ is the angle between the transverse momentum vectors P and q^\perp . At small x the distribution of linearly polarized gluons $xh^{(1)}(x, q^2)$ is expected to be comparable in magnitude to the conventional Weizsäcker-Williams gluon distribution $xG^{(1)}(x, q^2)$ for q^2 of order the saturation momentum scale Q_s^2 of the target [10]. These could be measured at a future electron-ion collider (EIC) [11]. However, the experimental collaborations have requested an estimate [12] of $\langle \cos 4\phi \rangle$ since this may constitute a background for $\langle \cos 2\phi \rangle$ which is generated by $xh^{(1)}(x, q^2)$.

Corrections to TMD factorization appear beyond the leading order in $1/P^2$. In what follows we derive the operator form of the correction in Eqs. (9,10,11), and explicit expressions for the expectation values in a large- N_c Gaussian theory in Eq. (35), and we show that it leads to a new $\sim \cos 4\phi$ azimuthal harmonic.

The remainder of the paper is organized as follows. In section II we derive the operator corresponding to the correction to the (leading power) TMD approximation of the quadrupole. In sec. III we use a Gaussian (and large- N_c) approximation to obtain explicit expressions of this correction in terms of the two-point function of the Gaussian theory. The correction to the dijet cross section is worked out in sec. IV where we also provide a first qualitative estimate of the magnitude of $\langle \cos 4\phi \rangle$ relative to $\langle \cos 2\phi \rangle$. We close with a brief summary in sec. V.

II. EXTRACTING THE AZIMUTHAL ANGULAR COMPONENTS OF THE QUADRUPOLE OPERATOR

The production of a quark anti-quark dijet at small x in DIS involves the following expectation value of Wilson lines [1]:

$$\mathcal{Q}_x(x_1, x_2; x'_2, x'_1) = 1 + S_x^{(4)}(x_1, x_2; x'_2, x'_1) - S_x^{(2)}(x_1, x_2) - S_x^{(2)}(x'_2, x'_1), \quad (1)$$

where

$$S_x^{(2)}(x_1, x_2) = S_x^{(2)}((x_1 - x_2)^2) \equiv \frac{1}{N_c} \langle \text{Tr} V^\dagger(x_2) V(x_1) \rangle_x \quad (2)$$

¹ To avoid cluttering of notation we do not write vector arrows on 2d vectors. In this paper essentially all transverse coordinates and momenta are 2d vectors and their magnitudes are written as $|P|$ or $\sqrt{P^2}$ etc.

is the dipole S-matrix evolved to light-cone momentum fraction x ; we omit this subscript on the $\langle \cdot \rangle$ field configuration averages from now on. The field of the target is taken in covariant gauge. Also, x_1 and x_2 denote the 2d transverse coordinates of the fundamental Wilson lines corresponding to the quark and the anti-quark, respectively. The saturation momentum scale where the fields of the target become non-linear is conventionally defined implicitly through $S^{(2)}(2/Q_s^2) = 1/\sqrt{e}$.

The quadrupole operator is given by a single trace over four Wilson lines,

$$S^{(4)}(x_1, x_2; x'_2, x'_1) \equiv \frac{1}{N_c} \langle \text{Tr} V^\dagger(x_2) V(x_1) V^\dagger(x'_1) V(x'_2) \rangle . \quad (3)$$

$\mathcal{Q}_x(x_1, x_2; x'_2, x'_1)$ vanishes in the coincidence limits $x_1 \rightarrow x_2$ or $x'_1 \rightarrow x'_2$.

In the so-called ‘‘correlation limit’’ [1, 7] of roughly back to back jets it is useful to introduce

$$u = x_1 - x_2; \quad v = \frac{1}{2}(x_1 + x_2) , \quad (4)$$

and similar for the primed coordinates. In this limit one expands \mathcal{Q} in powers of u and u' ,

$$\mathcal{Q} = u_i u'_j \mathcal{G}^{i,j}(v, v') + u_i u'_j u'_k u'_l \mathcal{G}^{i,jkl}(v, v') + u_i u_j u_k u'_l \mathcal{G}^{ijkl}(v, v') + u_i u_j u'_k u'_l \mathcal{G}^{ij,kl}(v, v') + \dots . \quad (5)$$

Ref. [7] performed the expansion to order $\mathcal{O}(uu')$ from where one obtains the Weizsäcker-Williams (WW) gluon distribution. It is proportional to

$$\mathcal{G}^{i,j}(v, v') = -\frac{1}{N_c} \langle \text{Tr} V^\dagger(v) \partial_i V(v) V^\dagger(v') \partial_j V(v') \rangle . \quad (6)$$

This is a two-point correlator of the target field transformed to light-cone gauge and so defines a gluon distribution. Its Fourier transform,

$$xG_{\text{WW}}^{ij}(x, q) \equiv \frac{2N_c}{\alpha_s} \mathcal{G}^{i,j}(q) = -\frac{2}{\alpha_s} \int \frac{d^2v}{(2\pi)^2} \frac{d^2v'}{(2\pi)^2} e^{-iq \cdot (v-v')} \langle \text{Tr} V^\dagger(v) \partial_i V(v) V^\dagger(v') \partial_j V(v') \rangle \quad (7)$$

can be projected onto its diagonal and traceless parts

$$xG_{\text{WW}}^{ij}(x, q) = \frac{1}{2} \delta_{ij} xG^{(1)}(x, q^2) - \frac{1}{2} \left(\delta_{ij} - 2 \frac{q_i q_j}{q^2} \right) xh^{(1)}(x, q^2) . \quad (8)$$

The conventional WW gluon distribution $xG^{(1)}(x, q^2)$ leads to a dijet cross section which is isotropic in ϕ , i.e. in the angle between the dijet transverse momentum imbalance q and the average transverse momentum P .

In Eq. (8) the distribution of linearly polarized gluons is denoted as $xh^{(1)}(x, q^2)$. This function has been computed within the McLerran-Venugopalan (MV) model of semi-classical gluon fields [13] in Refs. [6, 7], and its QCD quantum evolution to small- x has been determined in Ref. [10]. A non-vanishing $xh^{(1)}(x, q^2)$ gives rise to a $\sim \cos(2\phi)$ azimuthal anisotropy of the dijet cross section which is long range in the rapidity asymmetry of the dijet [10].

In this paper we extend the expansion to fourth order in u and/or u' as indicated in Eq. (5). At quartic order,

$$\mathcal{G}^{ij,mn}(v, v') = \frac{1}{16N_c} \langle \text{Tr} [V^\dagger(v) \partial_i \partial_j V(v) + (\partial_i \partial_j V^\dagger(v)) V(v)] [(\partial_m \partial_n V^\dagger(v')) V(v') + V^\dagger(v') \partial_m \partial_n V(v')] \rangle , \quad (9)$$

$$\mathcal{G}^{ijm,n}(v, v') = -\frac{1}{24N_c} \langle \text{Tr} [V^\dagger(v) \partial_i \partial_j \partial_m V(v) + 3(\partial_i \partial_j V^\dagger(v)) \partial_m V(v)] V^\dagger(v') \partial_n V(v') \rangle , \quad (10)$$

$$\mathcal{G}^{n,ijm}(v, v') = -\frac{1}{24N_c} \langle \text{Tr} [V^\dagger(v) \partial_n V(v)] [V^\dagger(v') \partial_i \partial_j \partial_m V(v') + 3(\partial_i \partial_j V^\dagger(v')) \partial_m V(v')] \rangle . \quad (11)$$

These expressions have been simplified by taking advantage of the symmetries in Eq. (5). Their Fourier transforms are performed as for the WW distribution in Eq. (7) above and the resulting tensors can be decomposed as follows:

$$\frac{2N_c}{\alpha_s} \mathcal{G}^{ijmn}(x, q^2) = \mathfrak{P}_1^{ijkm} \Phi_0(x, q^2) + \mathfrak{P}_2^{ijkm} \Phi_1(x, q^2) - \mathfrak{P}_3^{ijkm} \Phi_2(x, q^2) , \quad (12)$$

where

$$\mathcal{G}^{ijmn}(x, q^2) = \mathcal{G}^{i,jmn}(x, q^2) + \mathcal{G}^{ijm,n}(x, q^2) - \frac{2}{3}\mathcal{G}^{ij,mn}(x, q^2) , \quad (13)$$

$$\mathfrak{P}_1^{ijmn} = \frac{1}{2\sqrt{6}} (\delta_{ij}\delta_{mn} + \delta_{im}\delta_{jn} + \delta_{jm}\delta_{in}) , \quad (14)$$

$$\mathfrak{P}_2^{ijmn} = -\frac{1}{6\sqrt{2}} (\delta_{ij}\Pi_{mn} + \delta_{mn}\Pi_{ij} + \delta_{im}\Pi_{jn} + \delta_{jn}\Pi_{im} + \delta_{jm}\Pi_{in} + \delta_{in}\Pi_{jm}) , \quad (15)$$

$$\mathfrak{P}_3^{ijmn} = -\frac{1}{6\sqrt{2}} (\delta_{ij}\delta_{mn} + \delta_{im}\delta_{jn} + \delta_{jm}\delta_{in} - 2(\Pi_{ij}\Pi_{mn} + \Pi_{im}\Pi_{jn} + \Pi_{jm}\Pi_{in})) , \quad (16)$$

$$\Pi_{ij} = \delta_{ij} - \frac{2q_i q_j}{q^2} . \quad (17)$$

The function $\mathcal{G}^{ijmn}(x, q^2)$ as introduced in Eq. (13) appears in the dijet cross section, see section IV below.

The projectors are normalized so that $\mathfrak{P}_i^2 = 1$ for $i = 1, 2, 3$ and they satisfy

$$P_i P_j P_m P_n \mathfrak{P}_1^{ijmn} = \frac{1}{2} \sqrt{\frac{3}{2}} P^4 , \quad (18)$$

$$P_i P_j P_m P_n \mathfrak{P}_2^{ijmn} = \frac{1}{\sqrt{2}} P^4 \cos 2\phi , \quad (19)$$

$$P_i P_j P_m P_n \mathfrak{P}_3^{ijmn} = \frac{1}{2\sqrt{2}} P^4 \cos 4\phi . \quad (20)$$

Hence, the parity of \mathfrak{P}_i under $\phi \rightarrow \phi + \pi/2$ is $(-)^{i-1}$.

In what follows we shall focus on $\Phi_2(x, q^2)$ which determines the amplitude of the $\sim \cos 4\phi$ contribution to dijet production,

$$\Phi_2(x, q^2) = -\frac{2N_c}{\alpha_s} \mathfrak{P}_3^{ijmn} \mathcal{G}^{ijmn}(x, q^2) . \quad (21)$$

The first two terms from Eq. (12) only contribute corrections (suppressed by $\sim 1/P^2$) to the isotropic and ‘‘elliptic’’ ($\sim \cos 2\phi$) contributions.

Equation (21) is the final result of this section. It expresses the correlation function $\Phi_2(x, q^2)$ which determines the $\sim \cos 4\phi$ asymmetry in terms of a combination of correlation functions of Wilson lines written in Eqs. (9,10,11).

III. GAUSSIAN APPROXIMATION

In this section we compute the correlator $\Phi_2(x, q^2)$ analytically in the Gaussian and large- N_c approximations. The Gaussian theory is believed to be a good approximation at small x [14] unless, perhaps, the contribution from so-called ‘‘pomeron loops’’ is large [15]. This has been confirmed explicitly by a numerical analysis [16]. Note, however, that Ref. [16] did not test configurations corresponding to large $v - v'$ and small u, u' as required for the present analysis.

At a Gaussian fixed point the theory is defined in terms of the two-point function

$$g^2 \langle A^{-a}(z_1^+, z_1) A^{-b}(z_2^+, z_2) \rangle = \delta^{ab} \delta(z_1^+ - z_2^+) \mu^2(z^+) L_{z_1 z_2} , \quad (22)$$

$$L_{z_1 z_2} = g^4 \int d^2 z G_0(z_1 - z) G_0(z_2 - z) , \quad (23)$$

$$G_0(z) = \int \frac{d^2 k}{(2\pi)^2} \frac{1}{k^2 + \Lambda_{\text{IR}}^2} e^{ik \cdot z} = \frac{1}{4\pi} \ln \frac{1}{z^2 \Lambda_{\text{IR}}^2} . \quad (24)$$

Λ_{IR} regularizes the long-distance 2d Coulomb singularity and we restrict to $z^2 \Lambda_{\text{IR}}^2 \ll 1$. This leads to the dipole S-matrix

$$S^{(2)}(x_1, x_2) = \exp \left(-\frac{1}{2} C_F \Gamma((x_1 - x_2)^2) \right) , \quad (25)$$

where

$$\Gamma(r^2) = 2 (L(0) - L(r^2)) . \quad (26)$$

In the large- N_c limit \mathcal{Q} as defined in (5) can be written in the Gaussian theory as [1, 17]

$$\mathcal{Q}^G = 1 + e^{-\frac{C_F}{2}[\Gamma(x_1-x_2)+\Gamma(x'_2-x'_1)]} - e^{-\frac{C_F}{2}[\Gamma(x_1-x_2)]} - e^{-\frac{C_F}{2}[\Gamma(x'_2-x'_1)]} \quad (27)$$

$$- \frac{\Gamma(x_1-x'_1) - \Gamma(x_1-x'_2) + \Gamma(x_2-x'_2) - \Gamma(x_2-x'_1)}{\Gamma(x_1-x'_1) - \Gamma(x_1-x_2) + \Gamma(x_2-x'_2) - \Gamma(x'_2-x'_1)} \left(e^{-\frac{C_F}{2}[\Gamma(x_1-x_2)+\Gamma(x'_2-x'_1)]} - e^{-\frac{C_F}{2}[\Gamma(x_1-x'_1)+\Gamma(x'_2-x_2)]} \right).$$

We now express x_1, x_2, x'_1, x'_2 in terms of u, u', v, v' and expand in powers of u and u' . The leading contribution at quadratic order is

$$\mathcal{G}^{i,j}(r^2) = \left(1 - [S^{(2)}(r^2)]^2\right) \left(\delta_{ij} \frac{\Gamma^{(1)}(r^2)}{\Gamma(r^2)} + 2r_i r_j \frac{\Gamma^{(2)}(r^2)}{\Gamma(r^2)} \right), \quad (28)$$

where

$$\Gamma^{(n)}(r^2) = \frac{d^n \Gamma(r^2)}{d(r^2)^n}, \quad r \equiv v - v'. \quad (29)$$

From this one obtains the gluon distributions

$$xh^{(1)}(x, q^2) = \frac{2N_c}{\alpha_s} \Pi_{ij} \mathcal{G}^{i,j}(x, q^2) = \frac{4N_c}{\alpha_s} \frac{S_\perp}{(2\pi)^3} \int d|r| |r|^3 J_2(|q||r|) \left(1 - [S^{(2)}(r^2)]^2\right) \frac{\Gamma^{(2)}(r^2)}{\Gamma(r^2)} \quad (30)$$

and

$$xG^{(1)}(x, q^2) = \frac{2N_c}{\alpha_s} \delta_{ij} \mathcal{G}^{i,j}(x, q^2) = \frac{4N_c}{\alpha_s} \frac{S_\perp}{(2\pi)^3} \int d|r| |r| J_0(|q||r|) \left(1 - [S^{(2)}(r^2)]^2\right) \left(\frac{\Gamma^{(1)}(r^2)}{\Gamma(r^2)} + r^2 \frac{\Gamma^{(2)}(r^2)}{\Gamma(r^2)} \right). \quad (31)$$

S_\perp denotes a transverse area.

In the MV model, in leading $\log 1/r^2 \Lambda_{\text{IR}}^2 \gg 1$ approximation,

$$\Gamma(r^2) = \frac{Q_s^2}{4C_F} r^2 \log \frac{1}{r^2 \Lambda_{\text{IR}}^2}, \quad (32)$$

where Q_s denotes the saturation momentum. Note that the logarithmic factor in $\Gamma(r^2)$ ensures that the Fourier transform of the dipole S-matrix is a power-law at high momentum, rather than a Gaussian; it also leads to a non-vanishing second derivative of $\Gamma(r^2)$ w.r.t. r^2 to generate the distribution of linearly polarized gluons, $xh^{(1)}(x, q^2)$:

$$xh^{(1)}(x, q^2) = \frac{N_c S_\perp}{2\pi^3 \alpha_s} \int d|r| |r| J_2(|r||q|) \left[1 - \exp\left(-\frac{Q_s^2 r^2}{4} \log \frac{1}{r^2 \Lambda_{\text{IR}}^2}\right) \right] \frac{1}{r^2 \log \frac{1}{r^2 \Lambda_{\text{IR}}^2}}, \quad (33)$$

$$xG^{(1)}(x, q^2) = \frac{N_c S_\perp}{2\pi^3 \alpha_s} \int d|r| |r| J_0(|r||q|) \left[1 - \exp\left(-\frac{Q_s^2 r^2}{4} \log \frac{1}{r^2 \Lambda_{\text{IR}}^2}\right) \right] \frac{1}{r^2} \quad (34)$$

which has been obtained previously in Refs. [6, 7].

At fourth order in u and/or u' we find the following additional contribution to \mathcal{Q}^G :

$$\begin{aligned} & \frac{1}{4} C_F^2 \Gamma(u^2) \Gamma(u'^2) + \left[1 - [S^{(2)}(r^2)]^2 \right] \left[u \cdot u' \left[\frac{1}{4} (u^2 + u'^2) \frac{\Gamma^{(2)}(r^2)}{\Gamma(r^2)} + \frac{1}{2} ((u \cdot r)^2 + (u' \cdot r)^2) \frac{\Gamma^{(3)}(r^2)}{\Gamma(r^2)} \right] \right. \\ & \quad \left. + u \cdot r u' \cdot r \left[\frac{1}{2} (u^2 + u'^2) \frac{\Gamma^{(3)}(r^2)}{\Gamma(r^2)} + \frac{1}{3} ((u \cdot r)^2 + (u' \cdot r)^2) \frac{\Gamma^{(4)}(r^2)}{\Gamma(r^2)} \right] \right] \\ & + \frac{u \cdot u' \Gamma^{(1)}(r^2) + 2u \cdot r u' \cdot r \Gamma^{(2)}(r^2)}{2\Gamma^2(r^2)} \left[-(\Gamma(u^2) + \Gamma(u'^2)) \left([S^{(2)}(r^2)]^2 - 1 + C_F \Gamma(r^2) \right) \right. \\ & \quad \left. + \left([S^{(2)}(r^2)]^2 (1 + C_F \Gamma(r^2)) - 1 \right) \left(\frac{1}{2} (u - u')^2 \Gamma^{(1)}(r^2) + (u \cdot r - u' \cdot r)^2 \Gamma^{(2)}(r^2) \right) \right]. \quad (35) \end{aligned}$$

This is the complete power-suppressed correction to \mathcal{Q}^G . The terms proportional to $r_i r_j r_m r_n$ which project onto $\sim \cos 4\phi$ are

$$\mathcal{G}^{ij,mn} = r_i r_j r_m r_n 2 \left(\frac{\Gamma^{(2)}(r^2)}{\Gamma(r^2)} \right)^2 \left[1 - [S^{(2)}(r^2)]^2 (1 + C_F \Gamma(r^2)) \right], \quad (36)$$

$$\mathcal{G}^{ijm,n} = \mathcal{G}^{i,jmn} = r_i r_j r_m r_n \frac{1}{3} \frac{\Gamma^{(4)}(r^2)}{\Gamma(r^2)} \left(1 - [S^{(2)}(r^2)]^2 \right) - \frac{1}{2} \mathcal{G}^{ij,mn} \quad (37)$$

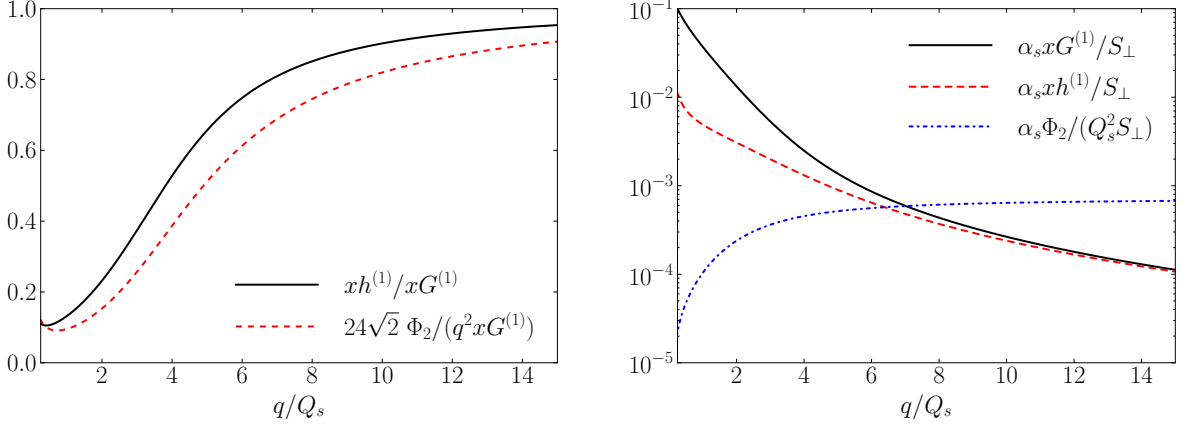


FIG. 1: The functions $xG^{(1)}(q^2)$, $xh^{(1)}(q^2)$ and $\Phi_2(q^2)$ in the MV model. These functions determine the amplitudes of the $\cos 2n\phi$ contributions to the dijet angular distributions for $n = 0, 1, 2$, respectively. See text for details.

and so

$$\mathcal{G}^{ijmn}(r) = \mathcal{G}^{i,jmn} + \mathcal{G}^{ijm,n} - \frac{2}{3}\mathcal{G}^{ij,mn} \quad (38)$$

$$= r_i r_j r_m r_n \frac{2}{3} \left[\frac{\Gamma^{(4)}(r^2)}{\Gamma(r^2)} \left(1 - [S^{(2)}(r^2)]^2 \right) - 5 \left(\frac{\Gamma^{(2)}(r^2)}{\Gamma(r^2)} \right)^2 \left[1 - [S^{(2)}(r^2)]^2 (1 + C_F \Gamma(r^2)) \right] \right]. \quad (39)$$

Performing a Fourier transform like in Eq. (7) and projecting with \mathfrak{P}_3 we extract

$$\begin{aligned} \Phi_2(x, q^2) &= -\frac{2N_c}{\alpha_s} \mathfrak{P}_3^{ijmn} \mathcal{G}^{ijmn}(x, q^2) \\ &= -\frac{N_c}{\sqrt{2} 3\pi\alpha_s} \frac{S_\perp}{(2\pi)^2} \int d|r| J_4(|r||q|) |r|^5 \\ &\quad \times \left[\frac{\Gamma^{(4)}(r^2)}{\Gamma(r^2)} \left(1 - [S^{(2)}(r^2)]^2 \right) - 5 \left(\frac{\Gamma^{(2)}(r^2)}{\Gamma(r^2)} \right)^2 \left[1 - [S^{(2)}(r^2)]^2 (1 + C_F \Gamma(r^2)) \right] \right]. \quad (40) \end{aligned}$$

For the MV model, specifically,

$$\begin{aligned} \Phi_2(q^2) &= \frac{N_c}{\sqrt{2} 3\pi\alpha_s} \frac{S_\perp}{(2\pi)^2} \int \frac{d|r|}{|r|^3} J_4(|r||q|) \left[\frac{2}{\ln \frac{1}{r^2 \Lambda_{\text{IR}}^2}} \left\{ 1 - \exp \left(-\frac{Q_s^2 r^2}{4} \log \frac{1}{r^2 \Lambda_{\text{IR}}^2} \right) \right\} \right. \\ &\quad \left. + \frac{5}{\ln^2 \frac{1}{r^2 \Lambda_{\text{IR}}^2}} \left\{ 1 - \exp \left(-\frac{Q_s^2 r^2}{4} \log \frac{1}{r^2 \Lambda_{\text{IR}}^2} \right) \left[1 + \frac{Q_s^2 r^2}{4} \log \frac{1}{r^2 \Lambda_{\text{IR}}^2} \right] \right\} \right] \quad (41) \end{aligned}$$

For large $q \gg Q_s$ we have $\Phi_2(q^2) \rightarrow (N_c/\sqrt{2} 24\pi\alpha_s) (S_\perp/4\pi^2) Q_s^2$. For small $\Lambda_{\text{IR}} \ll q \ll Q_s$ we have $\Phi_2(q^2) \sim (N_c/\alpha_s \log Q_s^2/\Lambda_{\text{IR}}^2) S_\perp q^2$ with a coefficient that can be determined numerically.

Figure 1 shows the functions $xG^{(1)}(q^2)$, $xh^{(1)}(q^2)$ and $\Phi_2(q^2)$ in the MV model as written in Eqs. (33,34,41). In the numerical computations we have replaced $1/r^2 \Lambda_{\text{IR}}^2 \rightarrow e + 1/r^2 \Lambda_{\text{IR}}^2$ in the arguments of the logarithms to ensure that they are ≥ 1 for all r^2 . Also, we have used $Q_s/\Lambda_{\text{IR}} = 20$.

IV. DIJET CROSS SECTION IN DIS

At leading order the cross section for production of a $q\bar{q}$ dijet in DIS is given by [1]

$$\frac{d\sigma^{\gamma^*,L A \rightarrow q\bar{q}X}}{d^2k_1 dz_1 d^2k_2 dz_2} = N_c \alpha_{em} e_q^2 \delta(p^+ - k_1^+ - k_2^+) \int \frac{d^2u}{(2\pi)^2} \frac{d^2u'}{(2\pi)^2} \frac{d^2v}{(2\pi)^2} \frac{d^2v'}{(2\pi)^2} \times e^{-iP \cdot (u-u') - iq \cdot (v-v')} \mathcal{Q}(u, u', v, v') \sum_{\lambda\alpha\beta} \psi_{\alpha\beta}^{T,L\lambda}(u) \psi_{\alpha\beta}^{T,L\lambda*}(u'). \quad (42)$$

k_1 and k_2 denote the 2d transverse momenta of the quark and anti-quark, respectively, and $P = (k_1 - k_2)/2$, $q = k_1 + k_2$. We assume here that only the dijet is being detected while the azimuthal angle of the electron is integrated over. If the azimuthal angle of the electron can be measured then the dijet cross section could exhibit a more involved angular dependence [18].

In the ‘‘correlation limit’’ of roughly back to back jets $P^2 \gg q^2$. Using the $\gamma^* \rightarrow q\bar{q}$ splitting functions from the literature, e.g. Ref. [1], and expanding \mathcal{Q} to fourth order in u or u' we obtain

$$\frac{d\sigma^{\gamma^*,L A \rightarrow q\bar{q}X}}{d^2k_1 dz_1 d^2k_2 dz_2} = 2N_c \alpha_{em} e_q^2 (2\pi)^2 \delta(x_\gamma - z_1 - z_2) (z_2^2 + z_1^2) \int \frac{d^2u}{(2\pi)^2} \frac{d^2u'}{(2\pi)^2} e^{-iP \cdot (u-u')} \nabla K_0(\epsilon_f u) \cdot \nabla K_0(\epsilon_f u') \times [u_i u'_j \mathcal{G}^{i,j}(q) + u_i u'_j u'_k u'_l \mathcal{G}^{i,jkl}(q) + u_i u_j u_k u'_l \mathcal{G}^{ijk,l}(q) + u_i u_j u'_k u'_l \mathcal{G}^{ij,kl}(q)] , \quad (43)$$

$$\frac{d\sigma^{\gamma^*,L A \rightarrow q\bar{q}X}}{d^2k_1 dz_1 d^2k_2 dz_2} = 8N_c \alpha_{em} e_q^2 (2\pi)^2 \delta(x_\gamma - z_1 - z_2) z_1 z_2 \epsilon_f^2 \int \frac{d^2u}{(2\pi)^2} \frac{d^2u'}{(2\pi)^2} e^{-iP \cdot (u-u')} K_0(\epsilon_f u) K_0(\epsilon_f u') \times [u_i u'_j \mathcal{G}^{i,j}(q) + u_i u'_j u'_k u'_l \mathcal{G}^{i,jkl}(q) + u_i u_j u_k u'_l \mathcal{G}^{ijk,l}(q) + u_i u_j u'_k u'_l \mathcal{G}^{ij,kl}(q)] . \quad (44)$$

Here, $\epsilon_f^2 = z_1 z_2 Q^2$ with Q^2 the virtuality of the photon which is on the order of P^2 .

The integrals over u and u' can be performed using the formulas collected in the appendix. The leading (in powers of $1/P^2$) contributions proportional to $\cos 2n\phi$, for $n = 0, 1, 2$, can be summarized as

$$\frac{d\sigma^{\gamma^*,L A \rightarrow q\bar{q}X}}{d^2k_1 dz_1 d^2k_2 dz_2} = \alpha_s \alpha_{em} e_q^2 \delta(x_\gamma - z_1 - z_2) (z_1^2 + z_2^2) \left[\frac{P^4 + \epsilon_f^4}{(P^2 + \epsilon_f^2)^4} \left(xG^{(1)}(x, q^2) - \frac{2\epsilon_f^2 P^2}{P^4 + \epsilon_f^4} xh^{(1)}(x, q^2) \cos 2\phi + \mathcal{O}\left(\frac{1}{P^2}\right) \right) - \frac{48\epsilon_f^2 P^4}{\sqrt{2}(P^2 + \epsilon_f^2)^6} \Phi_2(x, q^2) \cos 4\phi \right] \quad (45)$$

$$\frac{d\sigma^{\gamma^*,L A \rightarrow q\bar{q}X}}{d^2k_1 dz_1 d^2k_2 dz_2} = 8\alpha_s \alpha_{em} e_q^2 \delta(x_\gamma - z_1 - z_2) z_1 z_2 \epsilon_f^2 \left[\frac{P^2}{(P^2 + \epsilon_f^2)^4} \left(xG^{(1)}(x, q^2) + xh^{(1)}(x, q^2) \cos 2\phi + \mathcal{O}\left(\frac{1}{P^2}\right) \right) + \frac{48P^4}{\sqrt{2}(P^2 + \epsilon_f^2)^6} \Phi_2(x, q^2) \cos 4\phi \right] . \quad (46)$$

Here, $\cos \phi = \hat{q} \cdot \hat{P}$. Note that the contribution $\sim \cos 4\phi$ is suppressed by $1/P^2$ relative to the isotropic and $\sim \cos 2\phi$ pieces which are due to the $xG^{(1)}(x, q^2)$ and $xh^{(1)}(x, q^2)$ TMDs.

Finally, we evaluate numerically the following angular averages for a longitudinally polarized photon:

$$\langle \cos 2\phi \rangle = \frac{1}{2} \frac{xh^{(1)}(q^2)}{xG^{(1)}(q^2)} , \quad \langle \cos 4\phi \rangle = \frac{24}{\sqrt{2} P^2} \left(\frac{P^2}{P^2 + \epsilon_f^2} \right)^2 \frac{\Phi_2(q^2)}{xG^{(1)}(q^2)} . \quad (47)$$

We employ the MV model expressions for $xG^{(1)}(q^2)$, $xh^{(1)}(q^2)$, and $\Phi_2(q^2)$ derived in the previous section. The results are shown in Fig. 2 assuming $\sqrt{P^2} = 4.5 Q_s$, $z = 0.5$ and $Q^2 = P^2$. They confirm that for $q^2 \ll P^2$ the

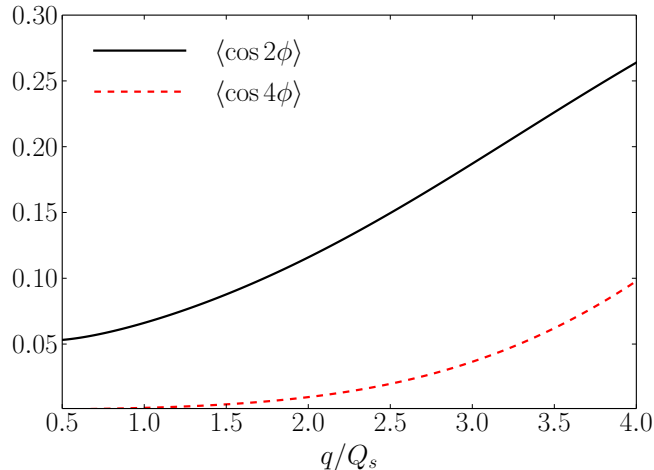


FIG. 2: $\langle \cos 2\phi \rangle$ and $\langle \cos 4\phi \rangle$ in $\gamma_L^* + A \rightarrow q + \bar{q}$ dijet production from the MV model. See text for details.

average $\cos 4\phi$ is substantially less than the average $\cos 2\phi$ although it may be measurable at a future high-energy electron-ion collider. For more quantitative estimates it is required, however, to account for small- x QCD evolution of these functions. This has been done in Ref. [10] for $xG^{(1)}(x, q^2)$ and $xh^{(1)}(x, q^2)$ and needs to be extended to $\Phi_2(x, q^2)$.

V. SUMMARY

In this paper we have considered the expansion of the quadrupole operator

$$S^{(4)}(x_1, x_2; x'_2, x'_1) \equiv \frac{1}{N_c} \langle \text{Tr} V^\dagger(x_2) V(x_1) V^\dagger(x'_1) V(x'_2) \rangle \quad (48)$$

about the coincidence limits $u \equiv x_1 - x_2 \rightarrow 0$, $u' \equiv x'_1 - x'_2 \rightarrow 0$. At quadratic order it becomes a two-point correlator of light-cone gauge fields [7],

$$u_i u'_j \left\langle \text{Tr} \left[V^\dagger(v) \frac{i}{g} \partial^i V(v) \right] \left[V^\dagger(v') \frac{i}{g} \partial^j V(v') \right] \right\rangle \quad (49)$$

which defines the Weizsäcker-Williams and linearly polarized gluon distributions. We have extended the expansion to fourth order in u/u' which leads to more involved correlators of Wilson lines and their derivatives, c.f. Eqs. (9,10,11). Furthermore, we have obtained explicit analytic expressions in a Gaussian, large N_c approximation for the specific correlation function denoted as $\Phi_2(x, q^2)$. This function gives rise to a $\sim \cos 4\phi$ azimuthal harmonic in dijet production. First qualitative estimates obtained within a specific Gaussian model (McLerran-Venugopalan model [13]) indicate that $\langle \cos 4\phi \rangle$ is much smaller than $\langle \cos 2\phi \rangle$ generated by the distribution of linearly polarized gluons $xh^{(1)}(x, q^2)$, at least in the nearly back to back “correlation limit” $P^2 \gg q^2$.

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Appendix: Useful Integrals

We start from the well known integral

$$\int \frac{d^2u}{(2\pi)^2} \exp(-iu \cdot P) K_0(\epsilon_f u) = \frac{1}{2\pi} \frac{1}{P^2 + \epsilon_f^2}. \quad (\text{A.1})$$

Taking a derivative with respect to P_i gives

$$\int \frac{d^2u}{(2\pi)^2} \exp(-iu \cdot P) u_i K_0(\epsilon_f u) = \frac{1}{2\pi i} \frac{2P_i}{(P^2 + \epsilon_f^2)^2}. \quad (\text{A.2})$$

Repeating this procedure one finds

$$\int \frac{d^2u}{(2\pi)^2} \exp(-iu \cdot P) u_i u_j K_0(\epsilon_f u) = \frac{1}{2\pi} \frac{2}{(P^2 + \epsilon_f^2)^2} \left[\delta_{ij} - \frac{4P_i P_j}{P^2 + \epsilon_f^2} \right] \quad (\text{A.3})$$

and

$$\int \frac{d^2u}{(2\pi)^2} \exp(-iu \cdot P) u_i u_j u_k K_0(\epsilon_f u) = \frac{1}{2\pi i} \frac{8}{(P^2 + \epsilon_f^2)^3} \left[P_i \delta_{jk} + P_j \delta_{ik} + P_k \delta_{ij} - \frac{6P_i P_j P_k}{P^2 + \epsilon_f^2} \right]. \quad (\text{A.4})$$

For transverse photon polarization we need

$$\int \frac{d^2u}{(2\pi)^2} \exp(-iu \cdot P) \frac{\partial}{\partial u_i} K_0(\epsilon_f u) = - \int \frac{d^2u}{(2\pi)^2} \left(\frac{\partial}{\partial u_i} \exp(-iu \cdot P) \right) K_0(\epsilon_f u) = - \frac{1}{2\pi i} \frac{P_l}{P^2 + \epsilon_f^2}, \quad (\text{A.5})$$

$$\int \frac{d^2u}{(2\pi)^2} \exp(-iu \cdot P) u_i \frac{\partial}{\partial u_l} K_0(\epsilon_f u) = - \frac{1}{2\pi} \frac{1}{P^2 + \epsilon_f^2} \left(\delta_{il} - \frac{2P_i P_l}{P^2 + \epsilon_f^2} \right), \quad (\text{A.6})$$

$$\int \frac{d^2u}{(2\pi)^2} \exp(-iu \cdot P) u_i u_j \frac{\partial}{\partial u_l} K_0(\epsilon_f u) = - \frac{1}{2\pi i} \frac{2}{(P^2 + \epsilon_f^2)^2} \left(\delta_{ij} P_l + \delta_{il} P_j + \delta_{jl} P_i - \frac{4P_i P_j P_l}{P^2 + \epsilon_f^2} \right), \quad (\text{A.7})$$

$$\int \frac{d^2u}{(2\pi)^2} \exp(-iu \cdot P) u_i u_j u_k \frac{\partial}{\partial u_l} K_0(\epsilon_f u) = - \frac{1}{2\pi} \frac{2}{(P^2 + \epsilon_f^2)^2} \times \left(\delta_{il} \delta_{jk} + \delta_{ij} \delta_{kl} + \delta_{ik} \delta_{jl} - 4 \frac{P_j P_k \delta_{il} + P_l P_k \delta_{ij} + P_i P_k \delta_{jl} + P_j P_l \delta_{ik} + P_i P_j \delta_{lk} + P_i P_l \delta_{jk}}{P^2 + \epsilon_f^2} + 24 \frac{P_i P_j P_l P_k}{(P^2 + \epsilon_f^2)^2} \right) \quad (\text{A.8})$$

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