

This is the accepted manuscript made available via CHORUS. The article has been published as:

Resummation of Goldstone boson contributions to the MSSM effective potential

Nilanjana Kumar and Stephen P. Martin

Phys. Rev. D **94**, 014013 — Published 12 July 2016

DOI: [10.1103/PhysRevD.94.014013](https://doi.org/10.1103/PhysRevD.94.014013)

Resummation of Goldstone boson contributions to the MSSM effective potential

Nilanjana Kumar and Stephen P. Martin

Department of Physics, Northern Illinois University, DeKalb IL 60115

We discuss the resummation of the Goldstone boson contributions to the effective potential of the Minimal Supersymmetric Standard Model (MSSM). This eliminates the formal problems of spurious imaginary parts and logarithmic singularities in the minimization conditions when the tree-level Goldstone boson squared masses are negative or approach zero. The numerical impact of the resummation is shown to be almost always very small. We also show how to write the two-loop minimization conditions so that Goldstone boson squared masses do not appear at all, and so that they can be solved without iteration.

Contents

I. Introduction	2
II. Effective potential of the MSSM	3
III. Expansion of the 2-loop MSSM effective potential for small G^0, G^\pm	6
IV. Resummation of leading Goldstone contributions in MSSM	12
V. Minimization conditions for the resummed MSSM effective potential	13
A. Minimization conditions with Goldstone boson resummation	13
B. Reexpansion to eliminate $m_{H_u}^2$ and $m_{H_d}^2$	16
C. Reexpansion to eliminate μ and b	18
VI. Singularities and spurious imaginary parts for small and negative h^0	20
VII. Numerical example	21
VIII. Outlook	24
References	25

I. INTRODUCTION

The relations between vacuum expectation values (VEVs) of Higgs fields and Lagrangian parameters can be obtained from the effective potential [1]–[5]. It is also a useful tool to understand vacuum stability [6]–[21]. The effective potential $V(\phi)$ is equal to the tree-level potential, plus the sum of one-particle-irreducible connected vacuum graphs, computed using field-dependent masses and couplings. In the Standard Model the full one and two loop contributions to the effective potentials have been computed in ref. [22], with the 3-loop leading contributions involving the strong and Yukawa couplings found in ref. [23], and the 4-loop part at leading order in QCD in ref. [24]. In supersymmetry, the 2-loop effective potential has been found for a general theory in ref. [25], and specialized to the case of the Minimal Supersymmetric Standard Model (MSSM) in ref. [26], with partial results previously given in refs. [27–33].

In ref. [23], it was noted that there are two related problems involving the mass square of the Goldstone boson (G) in Standard Model. One is when G is negative. Due to the appearance of logarithms of G , V_{eff} is complex. Thus it appears to suffer from an instability [4] although no physical instability is present. The second problem occurs as $G \rightarrow 0$, where the effective potential suffers from a logarithmic singularity at three loop order and power law singularity after that [23]. Even though the first problem can be avoided by dropping the imaginary term by hand and the second problem is not too severe for numerical analysis, a way to avoid them using resummation was given in [34, 35]; see also [36–38]. In practice these methods can be applied to any other model in which Goldstone radiative corrections lead to terms with IR problems in the effective potential.

In this paper, we analyze this problem for the 2-loop MSSM effective potential, which also suffers from the same problem when the neutral (G^0) and charged (G^\pm) Goldstone bosons are close to zero or negative at a particular value of renormalization scale Q . In the case of the MSSM, the neutral and charged Goldstone boson squared masses are distinct, and there are two minimization conditions, arising from the first derivatives of the effective potential (V_{eff}) with respect to the two real neutral Higgs degrees of freedom, denoted v_u and v_d in this paper. These minimization conditions both have singularities when G^0 and G^\pm tend to zero, and have imaginary parts when they are negative. In this paper we show how these problems of principle are avoided by the resummation procedure, so that working consistently at 2-loop order the Goldstone boson squared masses do not appear at all in the minimization conditions. In practice, the numerical effect of the resummation turns out to be very small for almost all choices of the renormalization scale. We illustrate this with a numerical example.

II. EFFECTIVE POTENTIAL OF THE MSSM

The scalar potential of the Minimal Supersymmetric Standard Model are very much sensitive to higher order corrections, so the minimization conditions for the scalar potential also depend very significantly on radiative corrections. The complete 2-loop effective potential of the MSSM has been given in [25, 26]. We follow those works for conventions and notations, in particular for the Lagrangian parameters (also as specified in [39]) and mixing parameters, and for 1-loop and 2-loop integral functions. Also, we follow the notation of using the name of a particle to represent its squared mass in formulas, for example

$$Z = \frac{1}{2}(g^2 + g'^2)(v_u^2 + v_d^2), \quad W = \frac{1}{2}g^2(v_u^2 + v_d^2), \quad (2.1)$$

$$t = y_t^2 v_u^2, \quad b = y_b^2 v_d^2, \quad \tau = y_\tau^2 v_d^2. \quad (2.2)$$

The MSSM effective potential can be written as

$$V_{\text{eff}} = V^{(0)} + \Delta V, \quad (2.3)$$

$$\Delta V = \frac{1}{16\pi^2}V^{(1)} + \frac{1}{(16\pi^2)^2}V^{(2)} + \frac{1}{(16\pi^2)^3}V^{(3)} + \dots, \quad (2.4)$$

where $V^{(0)}$ is the tree-level MSSM effective potential, expressed as

$$V^{(0)} = (|\mu|^2 + m_{H_u}^2)v_u^2 + (|\mu|^2 + m_{H_d}^2)v_d^2 - 2bv_uv_d + \frac{1}{8}(g^2 + g'^2)(v_u^2 - v_d^2)^2. \quad (2.5)$$

Here μ , the Higgs supersymmetric mass parameter, can have an arbitrary phase. The Higgs fields also have soft supersymmetry-breaking squared-mass running parameters $m_{H_u}^2$, $m_{H_d}^2$, and b . The first two of these are definitely real, and by convention b is taken to be real at the renormalization scale Q at which the effective potential is to be minimized. There are two gauge-eigenstate complex scalar doublet Higgs fields $H_u = (H_u^+, H_u^0)$ and $H_d = (H_d^0, H_d^-)$. The electrically neutral components have VEVs v_u and v_d , which are taken to be real and positive by convention. In general, $V^{(0)}$ also contains a constant vacuum energy term, necessary for renormalization group invariance [40–42], but we do not include it here because it plays no direct role in the following.

The gauge-eigenstate fields can be expressed in terms of the tree-level squared-mass eigenstate fields as

$$\begin{pmatrix} H_u^0 \\ H_d^0 \end{pmatrix} = \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}}R_\alpha \begin{pmatrix} h^0 \\ H^0 \end{pmatrix} + \frac{i}{\sqrt{2}}R_{\beta_0} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} \quad (2.6)$$

and

$$\begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix} = R_{\beta_{\pm}} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix}, \quad (2.7)$$

G^0 and G^{\pm} are Nambu-Goldstone fields, and h^0 , H^0 , A^0 , and H^{\pm} are the Higgs tree-level mass eigenstate fields, and v_u and v_d are the classical fields on which the masses and couplings entering the effective potential depends. The orthogonal matrices that accomplish the squared-mass diagonalizations are written

$$R_{\beta_0} = \begin{pmatrix} s_{\beta_0} & c_{\beta_0} \\ -c_{\beta_0} & s_{\beta_0} \end{pmatrix}, \quad R_{\beta_{\pm}} = \begin{pmatrix} s_{\beta_{\pm}} & c_{\beta_{\pm}} \\ -c_{\beta_{\pm}} & s_{\beta_{\pm}} \end{pmatrix}, \quad (2.8)$$

$$R_{\alpha} = \begin{pmatrix} c_{\alpha} & s_{\alpha} \\ -s_{\alpha} & c_{\alpha} \end{pmatrix}, \quad (2.9)$$

where we use the abbreviations $c_{\beta_0} = \cos(\beta_0)$ and $s_{\beta_0} = \sin(\beta_0)$, etc. In the following, we also write, for example, $s_{2\alpha}$ and $c_{2\alpha}$ for $\sin(2\alpha)$ and $\cos(2\alpha)$, respectively. Unlike the case in the ordinary Standard Model, the squared masses of the charged and neutral Goldstone bosons in the MSSM are not equal at tree level. They are given by

$$G^0 = |\mu|^2 + \frac{1}{2}(m_{H_u}^2 + m_{H_d}^2) - \frac{1}{2} \left\{ [m_{H_u}^2 - m_{H_d}^2 + \frac{(g^2 + g'^2)}{2}(v_u^2 - v_d^2)]^2 + 4b^2 \right\}^{1/2}, \quad (2.10)$$

$$\begin{aligned} G^{\pm} &= |\mu|^2 + \frac{1}{2}(m_{H_u}^2 + m_{H_d}^2) + \frac{g^2}{4}(v_u^2 + v_d^2) \\ &\quad - \frac{1}{2} \left\{ [m_{H_u}^2 - m_{H_d}^2 + \frac{g'^2}{2}(v_u^2 - v_d^2)]^2 + (2b + g^2 v_u v_d)^2 \right\}^{1/2}. \end{aligned} \quad (2.11)$$

The tree-level squared masses of the other Higgs fields are:

$$A^0 = |\mu|^2 + \frac{1}{2}(m_{H_u}^2 + m_{H_d}^2) + \frac{1}{2} \left\{ [m_{H_u}^2 - m_{H_d}^2 + \frac{(g^2 + g'^2)}{2}(v_u^2 - v_d^2)]^2 + 4b^2 \right\}^{1/2}, \quad (2.12)$$

$$\begin{aligned} H^{\pm} &= |\mu|^2 + \frac{1}{2}(m_{H_u}^2 + m_{H_d}^2) + \frac{g^2}{4}(v_u^2 + v_d^2) \\ &\quad + \frac{1}{2} \left\{ [m_{H_u}^2 - m_{H_d}^2 + \frac{g'^2}{2}(v_u^2 - v_d^2)]^2 + (2b + g^2 v_u v_d)^2 \right\}^{1/2}, \end{aligned} \quad (2.13)$$

$$\begin{aligned} h^0 &= |\mu|^2 + \frac{1}{2}(m_{H_u}^2 + m_{H_d}^2) + \frac{(g^2 + g'^2)}{4}(v_u^2 + v_d^2) \\ &\quad - \frac{1}{2} \left\{ [m_{H_u}^2 - m_{H_d}^2 + (g^2 + g'^2)(v_u^2 - v_d^2)]^2 + (2b + (g^2 + g'^2)v_u v_d)^2 \right\}^{1/2}, \end{aligned} \quad (2.14)$$

$$H^0 = |\mu|^2 + \frac{1}{2}(m_{H_u}^2 + m_{H_d}^2) + \frac{(g^2 + g'^2)}{4}(v_u^2 + v_d^2)$$

$$+\frac{1}{2}\left\{[m_{H_u}^2 - m_{H_d}^2 + (g^2 + g'^2)(v_u^2 - v_d^2)]^2 + (2b + (g^2 + g'^2)v_u v_d)^2\right\}^{1/2}. \quad (2.15)$$

The minimization conditions of the full effective potential can be written as

$$\frac{1}{2v_u} \frac{\partial V_{\text{eff}}}{\partial v_u} = \frac{1}{2v_d} \frac{\partial V_{\text{eff}}}{\partial v_d} = 0. \quad (2.16)$$

We define δ_u and δ_d by

$$\delta_u = \frac{1}{2v_u} \frac{\partial}{\partial v_u} \Delta V, \quad (2.17)$$

$$\delta_d = \frac{1}{2v_d} \frac{\partial}{\partial v_d} \Delta V, \quad (2.18)$$

so that at the minimum of the full effective potential

$$|\mu|^2 + m_{H_u}^2 - b \frac{v_d}{v_u} + \frac{(g^2 + g'^2)}{4} (v_u^2 - v_d^2) = -\delta_u, \quad (2.19)$$

$$|\mu|^2 + m_{H_d}^2 - b \frac{v_u}{v_d} + \frac{(g^2 + g'^2)}{4} (v_d^2 - v_u^2) = -\delta_d. \quad (2.20)$$

The minimum of the effective potential is not a minimum of the tree-level potential. For this reason, the angles β_0 and β_{\pm} for the rotations in the pseudo-scalar and charged Higgs sector are distinct from each other, and are also different from the angle β defined by

$$\tan \beta \equiv v_u/v_d. \quad (2.21)$$

Hence it is possible to write an exact relation between β_0 and β

$$\cot(2\beta_0) = \cot(2\beta) + \frac{\delta_d - \delta_u}{2b}. \quad (2.22)$$

An approximate relation can be obtained by expanding in terms of δ_u and δ_d :

$$\tan \beta_0 = \tan \beta + \frac{\delta_u - \delta_d}{b} s_{\beta}^2 + \frac{(\delta_u - \delta_d)^2}{8b^2} s_{2\beta}^3 + \dots \quad (2.23)$$

Similar relations between β_{\pm} and β can be achieved in a similar manner, and give the same result with the replacement of b by $b + g^2 v_u v_d/2$:

$$\cot(2\beta_{\pm}) = \cot(2\beta) + \frac{\delta_d - \delta_u}{2b + g^2 v_u v_d}, \quad (2.24)$$

$$\tan \beta_{\pm} = \tan \beta + \frac{\delta_u - \delta_d}{b + g^2 v_u v_d / 2} s_{\beta}^2 + \frac{(\delta_u - \delta_d)^2}{8(b + g^2 v_u v_d / 2)^2} s_{2\beta}^3 + \dots \quad (2.25)$$

Substituting eqs. (2.19) and (2.20) in eqs. (2.10) and (2.11) and expanding in δ_u and δ_d ,

$$G^0 = -\delta_u s_{\beta}^2 - \delta_d c_{\beta}^2 - \frac{(\delta_u - \delta_d)^2}{8b} s_{2\beta}^3 + \dots, \quad (2.26)$$

$$G^{\pm} = -\delta_u s_{\beta}^2 - \delta_d c_{\beta}^2 - \frac{(\delta_u - \delta_d)^2}{8(b + g^2 v_u v_d / 2)} s_{2\beta}^3 + \dots \quad (2.27)$$

Thus, at the minimum of the full 2 loop effective potential of MSSM, the tree-level masses of the Goldstone bosons are not zero, but can be considered to be of 1-loop order, and unlike the situation in the Standard Model they are not exactly the same, with the difference between them being effectively of 2-loop order, with an additional mass suppression when b is large, as well as a $1/\tan^3 \beta$ suppression.

III. EXPANSION OF THE 2-LOOP MSSM EFFECTIVE POTENTIAL FOR SMALL G^0, G^{\pm}

In this section we consider the leading contributions to the effective potential in an expansion in small G^0, G^{\pm} in the MSSM. In the $\overline{\text{DR}}'$ scheme the one loop order correction to the MSSM potential can be written as

$$V^{(1)}(G^0, G^{\pm}) = V^{(1)}(0, 0) + f(G^0) + 2f(G^{\pm}), \quad (3.1)$$

where the 1-loop integral function is defined as

$$f(x) = \frac{x^2}{4}(\overline{\ln} x - 3/2), \quad (3.2)$$

with

$$\overline{\ln}(x) = \ln(x/Q^2) \quad (3.3)$$

where Q is the renormalization scale. In eq. (3.1), $f(G^0) + 2f(G^{\pm})$ is the Goldstone bosons contribution and the terms independent of G^0 and G^{\pm} are

$$V^{(1)}(0, 0) = f(h^0) + f(H^0) + f(A^0) + 2f(H^{\pm}) + 2 \sum_{\tilde{f}} n_{\tilde{f}} f(\tilde{f}) - 2 \sum_{i=1}^4 f(\tilde{N}_i) - 4 \sum_{i=1}^2 f(\tilde{C}_i)$$

$$-16f(\tilde{g}) - 12f(t) - 12f(b) - 4f(\tau) + 3f(Z) + 6f(W), \quad (3.4)$$

where the sfermions are called \tilde{f} , with $n_{\tilde{f}} = 3$ for squarks and 1 for sleptons. At the two loop order, we find it convenient to expand for small G^0 and G^\pm , neglecting quadratic terms, in the form

$$V^{(2)}(G^0, G^\pm) = V^{(2)}(0, 0) + \frac{1}{2}A(G^0)\Delta_1^0 + A(G^\pm)\Delta_1^\pm + \frac{1}{2}\Omega^0 G^0 + \Omega^\pm G^\pm + \dots, \quad (3.5)$$

where Δ_1^0 , Δ_1^\pm , Ω^0 , and Ω^\pm do not depend on G^0 or G^\pm , and

$$A(x) = x(\overline{\ln}x - 1). \quad (3.6)$$

The expressions for $V^{(1)}(0, 0)$ and $V^{(2)}(0, 0)$ can be obtained by taking $G^0, G^\pm = 0$ in every expression that contributes to $V^{(1)}$ and $V^{(2)}$ in ref. [25]. We prefer to write in this way because we want to deal with the Goldstone bosons separately. The logarithmic terms $G^0\overline{\ln}G^0$ and $G^\pm\overline{\ln}G^\pm$ are included in $A(G^0)$ and $A(G^\pm)$. The ellipses represent terms in higher order of G^0 and G^\pm .

To obtain the expressions for Δ_1^0 , Δ_1^\pm , Ω^0 , and Ω^\pm , we first expand the 2-loop integral functions defined in ref. [25] that involve scalars:

$$f_{SSS}(G, x, y) = f_{SSS}(0, x, y) + P_{SS}(x, y)A(G) + R_{SS}(x, y)G + \mathcal{O}(G^2), \quad (3.7)$$

$$f_{SS}(G, x) = A(x)A(G), \quad (3.8)$$

$$f_{FFS}(x, y, G) = f_{FFS}(x, y, 0) + P_{FF}(x, y)A(G) + R_{FF}(x, y)G + \mathcal{O}(G^2), \quad (3.9)$$

$$f_{\overline{FF}S}(x, y, G) = f_{\overline{FF}S}(x, y, 0) + P_{\overline{FF}}(x, y)A(G) + R_{\overline{FF}}(x, y)G + \mathcal{O}(G^2), \quad (3.10)$$

$$f_{SSV}(G, x, y) = f_{SSV}(0, x, y) + R_{SV}(x, y)G + \mathcal{O}(G^2), \quad (3.11)$$

$$F_{VS}(x, G) = 3A(x)A(G), \quad (3.12)$$

$$F_{VVS}(x, y, G) = F_{VVS}(x, y, 0) + P_{VV}(x, y)A(G) + R_{VV}(x, y)G + \mathcal{O}(G^2). \quad (3.13)$$

For the P and R functions defined in this way, we find:

$$P_{SS}(x, y) = \frac{A(x) - A(y)}{x - y}, \quad (3.14)$$

$$P_{SS}(x, x) = 1 + A(x)/x, \quad (3.15)$$

$$P_{FF}(x, y) = -2\left[\frac{x A(x) - y A(y)}{x - y}\right], \quad (3.16)$$

$$P_{FF}(x, x) = -2x - 4A(x), \quad (3.17)$$

$$P_{\overline{FF}}(x, y) = -2P_{SS}(x, y), \quad (3.18)$$

$$P_{VV}(x, y) = 3P_{SS}(x, y), \quad (3.19)$$

and

$$R_{SS}(x, y) = \left\{ (x+y)^2 + 2A(x)A(y) - 2xA(x) - 2yA(y) + (x+y)I(0, x, y) \right\} / (x-y)^2 \quad (3.20)$$

$$R_{SS}(x, x) = -3 - 2A(x)/x - A(x)^2/2x^2 \quad (3.21)$$

$$R_{FF}(x, y) = - \left[(x+y) \left\{ 2A(x)A(y) - 2xA(x) - 2yA(y) + (x+y)^2 \right\} + 2(x^2 + y^2)I(0, x, y) \right] / (x-y)^2 \quad (3.22)$$

$$R_{FF}(x, x) = 8x + 2A(x) + 2A(x)^2/x \quad (3.23)$$

$$R_{\overline{FF}}(x, y) = -2R_{SS}(x, y) \quad (3.24)$$

$$R_{VV}(x, y) = \frac{1}{4xy(x-y)^2} \left[3A(x)A(y) \{ x^2 + y^2 + 6xy \} - 24xy \{ xA(x) + yA(y) \} + 14xy(x^2 + y^2) + 20x^2y^2 - 3(x-y)^2 \{ xI(0, 0, x) + yI(0, 0, y) \} + 3(x+y)^3 I(0, x, y) \right] \quad (3.25)$$

$$R_{VV}(0, x) = \frac{11}{4} + \frac{3}{x}I(0, 0, x) - \frac{9A(x)}{2x} \quad (3.26)$$

$$R_{SV}(x, y) = \frac{1}{y} \left\{ 3(x+y)I(0, x, y) - 3xI(0, 0, x) + 3A(x)A(y) + 2xy + y^2 \right\} \quad (3.27)$$

$$R_{SV}(x, 0) = -x + 6A(x) \quad (3.28)$$

Expressions for $I(0, x, y)$ and $I(0, 0, x)$ in the notation of the present paper in terms of logarithms and dilogarithms can be found in equation (2.26)-(2.28) of [25]. The expansion of these functions in terms of small G^0 and G^\pm also can be obtained from eqs. (2.29)-(2.31) of the same reference.

[Although they are not needed for the MSSM as discussed in this paper, for the $\overline{\text{MS}}$ scheme, we find instead for the expansions of the relevant functions defined in eqs. (4.17) and (4.18) of ref. [25] the results:

$$f_{VS}(x, G) = 3A(x)A(G) + 2xA(G), \quad (3.29)$$

$$f_{VVS}(x, y, G) = f_{VVS}(x, y, 0) + p_{VV}(x, y)A(G) + r_{VV}(x, y)G + \mathcal{O}(G^2), \quad (3.30)$$

where

$$p_{VV}(x, y) = P_{VV}(x, y) + 2, \quad (3.31)$$

$$r_{VV}(x, y) = R_{VV}(x, y) - 1. \quad (3.32)$$

These could be useful for example in non-supersymmetric two-Higgs doublet models. The other functions do not differ between the $\overline{\text{MS}}$ and $\overline{\text{DR}}$ schemes.]

Hence, one can write the expressions for Δ_1^0 , Δ_1^\pm , Ω^0 , and Ω^\pm in terms of the functions defined above. For the MSSM, we find:

$$\begin{aligned}
\Delta_1^0 = & (\lambda_{G^0 A^0 h^0})^2 P_{SS}(A^0, h^0) + (\lambda_{G^0 A^0 H^0})^2 P_{SS}(A^0, H^0) + (\lambda_{G^0 G^0 h^0})^2 P_{SS}(0, h^0) \\
& + (\lambda_{G^0 G^0 H^0})^2 P_{SS}(0, H^0) + 2|\lambda_{G^0 G^+ H^-}|^2 P_{SS}(0, H^+) \\
& + \sum_{\tilde{f}, \tilde{f}'} n_{\tilde{f}} |\lambda_{G^0 \tilde{f} \tilde{f}'}|^2 P_{SS}(\tilde{f}, \tilde{f}') + \frac{1}{2} \lambda_{G^0 G^0 h^0 h^0} A(h^0) + \frac{1}{2} \lambda_{G^0 G^0 H^0 H^0} A(H^0) \\
& + \frac{1}{2} \lambda_{G^0 G^0 A^0 A^0} A(A^0) + \lambda_{G^0 G^0 H^+ H^-} A(H^+) + \sum_{\tilde{f}} n_{\tilde{f}} \lambda_{G^0 G^0 \tilde{f} \tilde{f}'} A(\tilde{f}) \\
& + 6|Y_{\tilde{t}\tilde{t}G^0}|^2 P_{FF}(t, t) + 6t(Y_{\tilde{t}\tilde{t}G^0})^2 P_{\overline{FF}}(t, t) \\
& + 6|Y_{\tilde{b}\tilde{b}G^0}|^2 P_{FF}(b, b) + 6b(Y_{\tilde{b}\tilde{b}G^0})^2 P_{\overline{FF}}(b, b) \\
& + 2|Y_{\tau\tau G^0}|^2 P_{FF}(\tau, \tau) + 2\tau(Y_{\tau\tau G^0})^2 P_{\overline{FF}}(\tau, \tau) \\
& + \sum_{i,j=1}^2 \left\{ 2|Y_{\tilde{C}_i^+ \tilde{C}_j^- G^0}|^2 P_{FF}(\tilde{C}_i, \tilde{C}_j) + 2\sqrt{\tilde{C}_i \tilde{C}_j} \text{Re}[Y_{\tilde{C}_i^+ \tilde{C}_j^- G^0} Y_{\tilde{C}_j^+ \tilde{C}_i^- G^0}] P_{\overline{FF}}(\tilde{C}_i, \tilde{C}_j) \right\} \\
& + \sum_{i,j=1}^4 \left\{ |Y_{\tilde{N}_i \tilde{N}_j G^0}|^2 P_{FF}(\tilde{N}_i, \tilde{N}_j) + \sqrt{\tilde{N}_i \tilde{N}_j} \text{Re}[(Y_{\tilde{N}_i \tilde{N}_j G^0})^2] P_{\overline{FF}}(\tilde{N}_i, \tilde{N}_j) \right\} \\
& + \frac{3g^2}{2} A(W) + \frac{3(g^2 + g'^2)}{4} A(Z)
\end{aligned} \tag{3.33}$$

$$\begin{aligned}
\Omega^0 = & (\lambda_{G^0 A^0 h^0})^2 R_{SS}(A^0, h^0) + (\lambda_{G^0 A^0 H^0})^2 R_{SS}(A^0, H^0) + (\lambda_{G^0 G^0 h^0})^2 R_{SS}(0, h^0) \\
& + (\lambda_{G^0 G^0 H^0})^2 R_{SS}(0, H^0) + 2|\lambda_{G^0 G^+ H^-}|^2 R_{SS}(0, H^+) + \sum_{\tilde{f}, \tilde{f}'} n_{\tilde{f}} |\lambda_{G^0 \tilde{f} \tilde{f}'}|^2 R_{SS}(\tilde{f}, \tilde{f}') \\
& + 6|Y_{\tilde{t}\tilde{t}G^0}|^2 R_{FF}(t, t) + 6t(Y_{\tilde{t}\tilde{t}G^0})^2 R_{\overline{FF}}(t, t) \\
& + 6|Y_{\tilde{b}\tilde{b}G^0}|^2 R_{FF}(b, b) + 6b(Y_{\tilde{b}\tilde{b}G^0})^2 R_{\overline{FF}}(b, b) \\
& + 2|Y_{\tau\tau G^0}|^2 R_{FF}(\tau, \tau) + 2\tau(Y_{\tau\tau G^0})^2 R_{\overline{FF}}(\tau, \tau) \\
& + \sum_{i,j=1}^2 \left\{ 2|Y_{\tilde{C}_i^+ \tilde{C}_j^- G^0}|^2 R_{FF}(\tilde{C}_i, \tilde{C}_j) + 2\sqrt{\tilde{C}_i \tilde{C}_j} \text{Re}[Y_{\tilde{C}_i^+ \tilde{C}_j^- G^0} Y_{\tilde{C}_j^+ \tilde{C}_i^- G^0}] R_{\overline{FF}}(\tilde{C}_i, \tilde{C}_j) \right\} \\
& + \sum_{i,j=1}^4 \left\{ |Y_{\tilde{N}_i \tilde{N}_j G^0}|^2 R_{FF}(\tilde{N}_i, \tilde{N}_j) + \sqrt{\tilde{N}_i \tilde{N}_j} \text{Re}[(Y_{\tilde{N}_i \tilde{N}_j G^0})^2] R_{\overline{FF}}(\tilde{N}_i, \tilde{N}_j) \right\} \\
& + \frac{g^2 + g'^2}{4} \left\{ (c_\alpha c_{\beta_0} + s_\alpha s_{\beta_0})^2 R_{SV}(H^0, Z) + (s_\alpha c_{\beta_0} - c_\alpha s_{\beta_0})^2 R_{SV}(h^0, Z) \right\} \\
& + \frac{g^2}{2} \left\{ (c_{\beta_0} c_{\beta_\pm} + s_{\beta_0} s_{\beta_\pm})^2 R_{SV}(0, W) + (s_{\beta_0} c_{\beta_\pm} - c_{\beta_0} s_{\beta_\pm})^2 R_{SV}(H^\pm, W) \right\}
\end{aligned} \tag{3.34}$$

$$\begin{aligned}
\Delta_1^\pm = & |\lambda_{h^0 G^+ H^-}|^2 P_{SS}(h^0, H^+) + |\lambda_{A^0 G^+ H^-}|^2 P_{SS}(A^0, H^+) + |\lambda_{H^0 G^+ H^-}|^2 P_{SS}(H^0, H^+) \\
& + |\lambda_{h^0 G^+ G^-}|^2 P_{SS}(0, h^0) + |\lambda_{H^0 G^+ G^-}|^2 P_{SS}(0, H^0) + |\lambda_{G^0 G^+ H^-}|^2 P_{SS}(0, H^+) \\
& + \sum_{\tilde{f}, \tilde{f}'} n_{\tilde{f}} |\lambda_{G^+ \tilde{f} \tilde{f}^*}|^2 P_{SS}(\tilde{f}, \tilde{f}') + \lambda_{G^+ H^+ G^- H^-} A(H^+) + \frac{1}{2} \lambda_{H^0 H^0 G^+ G^-} A(H^0) \\
& + \frac{1}{2} \lambda_{h^0 h^0 G^+ G^-} A(h^0) + \frac{1}{2} \lambda_{A^0 A^0 G^+ G^-} A(A^0) + \sum_{\tilde{f}} n_{\tilde{f}} \lambda_{G^+ G^- \tilde{f} \tilde{f}^*} A(\tilde{f}) \\
& + 3 \left\{ |Y_{\tilde{t} b G^+}|^2 + |Y_{\tilde{b} t G^-}|^2 \right\} P_{FF}(t, b) + 6 Y_{\tilde{t} b G^+} Y_{\tilde{b} t G^-} \sqrt{tb} P_{\overline{FF}}(t, b) \\
& + |Y_{\tilde{\nu}_\tau G^-}|^2 P_{FF}(0, \tau) + \sum_{i=1}^2 \sum_{j=1}^4 \left[\left\{ |Y_{\tilde{C}_i^+ \tilde{N}_j G^-}|^2 + |Y_{\tilde{C}_i^- \tilde{N}_j G^+}|^2 \right\} P_{FF}(\tilde{C}_i, \tilde{N}_j) \right. \\
& \left. + 2 \text{Re}[Y_{\tilde{C}_i^+ \tilde{N}_j G^-} Y_{\tilde{C}_i^- \tilde{N}_j G^+}] \sqrt{\tilde{C}_i \tilde{N}_j} P_{\overline{FF}}(\tilde{C}_i, \tilde{N}_j) \right] \\
& + \frac{3g^2}{2} A(W) + \frac{3(g^2 - g'^2)^2}{4(g^2 + g'^2)} A(Z) \\
& + \frac{g^2 g'^2}{2(g^2 + g'^2)} (c_{\beta_\pm} v_d + s_{\beta_\pm} v_u)^2 \left\{ g^2 P_{VV}(0, W) + g'^2 P_{VV}(W, Z) \right\}
\end{aligned} \tag{3.35}$$

$$\begin{aligned}
\Omega^\pm = & |\lambda_{h^0 G^+ H^-}|^2 R_{SS}(h^0, H^+) + |\lambda_{A^0 G^+ H^-}|^2 R_{SS}(A^0, H^+) + |\lambda_{H^0 G^+ H^-}|^2 R_{SS}(H^0, H^+) \\
& + |\lambda_{h^0 G^+ G^-}|^2 R_{SS}(0, h^0) + |\lambda_{H^0 G^+ G^-}|^2 R_{SS}(0, H^0) + |\lambda_{G^0 G^+ H^-}|^2 R_{SS}(0, H^+) \\
& + \sum_{\tilde{f}, \tilde{f}'} n_{\tilde{f}} |\lambda_{G^+ \tilde{f} \tilde{f}^*}|^2 R_{SS}(\tilde{f}, \tilde{f}') + 3 \left\{ |Y_{\tilde{t} b G^+}|^2 + |Y_{\tilde{b} t G^-}|^2 \right\} R_{FF}(t, b) \\
& + 6 Y_{\tilde{t} b G^+} Y_{\tilde{b} t G^-} \sqrt{tb} R_{\overline{FF}}(t, b) + |Y_{\tilde{\nu}_\tau G^-}|^2 R_{FF}(0, \tau) \\
& + \sum_{i=1}^2 \sum_{j=1}^4 \left[\left\{ |Y_{\tilde{C}_i^+ \tilde{N}_j G^-}|^2 + |Y_{\tilde{C}_i^- \tilde{N}_j G^+}|^2 \right\} R_{FF}(\tilde{C}_i, \tilde{N}_j) \right. \\
& \left. + 2 \text{Re}[Y_{\tilde{C}_i^+ \tilde{N}_j G^-} Y_{\tilde{C}_i^- \tilde{N}_j G^+}] \sqrt{\tilde{C}_i \tilde{N}_j} R_{\overline{FF}}(\tilde{C}_i, \tilde{N}_j) \right] + \frac{(g^2 - g'^2)^2}{4(g^2 + g'^2)} R_{SV}(0, Z) \\
& + \frac{g^2}{4} \left\{ (c_\alpha c_{\beta_\pm} + s_\alpha s_{\beta_\pm})^2 R_{SV}(H^0, W) + (s_\alpha c_{\beta_\pm} - c_\alpha s_{\beta_\pm})^2 R_{SV}(h^0, W) \right. \\
& \left. + (s_{\beta_0} c_{\beta_\pm} - c_{\beta_0} s_{\beta_\pm})^2 R_{SV}(A^0, W) + (c_{\beta_0} c_{\beta_\pm} + s_{\beta_0} s_{\beta_\pm})^2 R_{SV}(0, W) \right\} \\
& + \frac{g^2 g'^2}{2(g^2 + g'^2)} (c_{\beta_\pm} v_d + s_{\beta_\pm} v_u)^2 \left\{ g^2 R_{VV}(0, W) + g'^2 R_{VV}(W, Z) \right\}.
\end{aligned} \tag{3.36}$$

All of the associated couplings appearing above are taken from Section II of ref. [43], using the following coefficients:

$$k_{uh^0} = k_{dH^0} = c_\alpha, \quad k_{uH^0} = -k_{dh^0} = s_\alpha, \tag{3.37}$$

$$k_{uG^0} = k_{dA^0} = i s_{\beta_0}, \quad k_{uA^0} = -k_{dG^0} = i c_{\beta_0}, \tag{3.38}$$

$$k_{uG^+} = k_{dH^+} = s_{\beta_\pm}, \quad k_{uH^+} = -k_{dG^+} = c_{\beta_\pm}. \tag{3.39}$$



FIG. 3.1: The leading contribution at fixed loop order to the effective potential as $G^0, G^\pm \rightarrow 0$ comes from vacuum diagrams with chains of $\ell - 1$ one-loop subdiagrams involving heavy particles connected by $\ell - 1$ Goldstone boson propagators.

At higher loop orders, the singularities in the effective potential as $G^0, G^\pm \rightarrow 0$ are derived from diagrams consisting of chains of $\ell - 1$ one-loop subdiagrams connected by $\ell - 1$ Goldstone boson propagators, as shown in figure 3.1. In general, the grey blobs in the figure represent 1-particle irreducible subdiagrams, but the leading contribution as $G^0, G^\pm \rightarrow 0$ at any fixed loop order ℓ comes when these are 1-loop subdiagrams. (Beyond the leading order as $G^0, G^\pm \rightarrow 0$ at a fixed loop order, one must include other diagrams.) The calculation of this class of diagrams, treating the gray blobs as constant squared-mass insertions, then reduces down to a 1-loop integration, as described in refs. [34, 35]. For G^0, G^\pm much less than the squared-mass scale of the blobs, the contributions to V_{eff} from these classes of diagrams can be written as

$$\frac{1}{16\pi^2} \sum_{n=0}^{\infty} \frac{1}{n!} \left[(\Delta^0)^n f^{(n)}(G^0) + 2(\Delta^\pm)^n f^{(n)}(G^\pm) \right] + \dots \quad (3.40)$$

where $n = \ell - 1$ with ℓ denoting the loop order, and $f^{(n)}(G)$ is the n th derivative, with

$$f^{(1)}(G) = A(G)/2, \quad (3.41)$$

$$f^{(2)}(G) = \overline{\text{In}}(G)/2 \quad (3.42)$$

$$f^{(n)}(G) = \frac{1}{2}(-1)^{n-1}(n-3)! G^{2-n} \quad (\text{for } n \geq 3), \quad (3.43)$$

and the Δ 's result from the integrations over heavy 1-particle irreducible subdiagrams. The charged and neutral Goldstone bosons G^0 and G^\pm have distinct loop expansions for these subdiagram quantities:

$$\Delta^0 = \frac{1}{16\pi^2} \Delta_1^0 + \frac{1}{(16\pi^2)^2} \Delta_2^0 + \dots \quad (3.44)$$

$$\Delta^\pm = \frac{1}{16\pi^2} \Delta_1^\pm + \frac{1}{(16\pi^2)^2} \Delta_2^\pm + \dots \quad (3.45)$$

In the following, we consider only the leading terms in small G^0 and G^\pm at each loop order,

hence the 2-loop contributions Δ_2^0 and Δ_2^\pm and higher orders can be neglected. The contributions Δ_1^0 and Δ_1^\pm are given above, as they can be read off of the known 2-loop results [the $n = 1$ term in eq. (3.40)]. From these, we can predict the leading logarithmic singularities in the 3-loop effective potential (before resummation) as $G^0, G^\pm \rightarrow 0$, corresponding to the $n = 2$ term in eq. (3.40):

$$V^{(3)} = \frac{1}{4}(\Delta_1^0)^2 \overline{\ln}(G^0) + \frac{1}{2}(\Delta_1^\pm)^2 \overline{\ln}(G^\pm) + \dots \quad (3.46)$$

where the ellipses means terms finite as $G^0, G^\pm \rightarrow 0$. The $\overline{\ln}(G^0)$ and $\overline{\ln}(G^\pm)$ terms here can be eliminated, along with the leading 2-loop order terms proportional to $G^0 \overline{\ln}(G^0)$ and $G^\pm \overline{\ln}(G^\pm)$, by the resummation described below.

IV. RESUMMATION OF LEADING GOLDSTONE CONTRIBUTIONS IN MSSM

One can now sum the contributions to V_{eff} indicated in eq. (3.40) to all loop orders, with the result

$$\frac{1}{16\pi^2} f(G^0 + \Delta^0) + \frac{2}{16\pi^2} f(G^\pm + \Delta^\pm) + \dots \quad (4.1)$$

We have checked that at the minimum of the effective potential, $G^0 + \frac{1}{16\pi^2} \Delta_1^0 = 0$ and $G^\pm + \frac{1}{16\pi^2} \Delta_1^\pm = 0$, up to terms of 2-loop order, so that eq. (4.1) is 0 and has vanishing first derivatives there, up to terms of 3-loop order. Therefore, if the effective potential V_{eff} has been obtained at loop order ℓ , then the corresponding resummed effective potential can be expressed as

$$\begin{aligned} \hat{V}_{\text{eff}} = & V_{\text{eff}} + \frac{1}{16\pi^2} \left[f(G^0 + \Delta^0) - \sum_{n=0}^{\ell-1} \frac{(\Delta^0)^n}{n!} f^{(n)}(G^0) \right] \\ & + \frac{2}{16\pi^2} \left[f(G^\pm + \Delta^\pm) - \sum_{n=0}^{\ell-1} \frac{(\Delta^\pm)^n}{n!} f^{(n)}(G^\pm) \right]. \end{aligned} \quad (4.2)$$

After expanding this equation, there are no terms involving $G^0 \overline{\ln} G^0$ and $G^\pm \overline{\ln} G^\pm$ at 2-loop order. The contributions of the different terms involving the Goldstone bosons in the 2-loop contribution were given in the previous section. From these, we find that the resummed

MSSM effective potential through 2-loop order can be written from eq. (4.2) as

$$\begin{aligned}\hat{V}_{\text{eff}} = & V^{(0)} + \frac{1}{16\pi^2} \left[V^{(1)}(0,0) + f(G^0 + \Delta^0) + 2f(G^\pm + \Delta^\pm) \right] \\ & + \frac{1}{(16\pi^2)^2} \left[V^{(2)}(0,0) + \frac{1}{2}\Omega^0 G^0 + \Omega^\pm G^\pm \right],\end{aligned}\quad (4.3)$$

where 2-loop order terms of order G^2 have been neglected, as they cannot affect the minimization conditions at 2-loop order. In summary, one replaces the 1-loop Goldstone contributions by functions with arguments shifted by the Δ 's, and sets the Goldstone boson contributions at 2-loop order to 0, with additional 2-loop terms linear in G^0 and G^\pm (but with no logarithms of them). The last terms are necessary for the minimization conditions described in the next section.

V. MINIMIZATION CONDITIONS FOR THE RESUMMED MSSM EFFECTIVE POTENTIAL

A. Minimization conditions with Goldstone boson resummation

In this section, we consider the minimization condition of the resummed effective potential, obtained by requiring the vanishing of the derivatives with respect to v_u and v_d of \hat{V}_{eff} in eq. (4.3). We note first that the 1-loop Goldstone terms have no effect, because at the minimum of \hat{V}_{eff} ,

$$f'(G^0 + \Delta^0) = 0, \quad f'(G^\pm + \Delta^\pm) = 0, \quad (5.1)$$

up to terms of 3-loop order, due to the vanishing of the arguments as noted above. The derivatives of $V^{(1)}(0,0)$ and $V^{(2)}(0,0)$ can be obtained from the expressions in ref. [25, 26]. The remaining contribution comes from the terms proportional to $\Omega^0 G^0$ and $\Omega^\pm G^\pm$ in eq. (4.3). In these terms, if the derivatives do not act on the Goldstone boson squared masses, then the result will be proportional to G^0 or G^\pm , and thus is of order 3-loop order, and can be consistently neglected. We therefore only need the derivatives of G^0 and G^\pm with respect to v_u and v_d , and keeping only the terms independent of δ_u and δ_d when expanded in terms of them. For these derivatives, we find:

$$\frac{1}{2v_u} \frac{\partial G^0}{\partial v_u} = \frac{1}{2v_u} \frac{\partial G^\pm}{\partial v_u} = -\frac{1}{2v_d} \frac{\partial G^0}{\partial v_d} = -\frac{1}{2v_d} \frac{\partial G^\pm}{\partial v_d} = -\frac{1}{4}(g^2 + g'^2)c_{2\beta}. \quad (5.2)$$

Hence, we find that the minimization conditions can be written as eqs. (2.19)-(2.20) with:

$$\delta_u = \frac{1}{16\pi^2} \hat{\Delta}_u^{(1)} + \frac{1}{(16\pi^2)^2} \hat{\Delta}_u^{(2)}, \quad (5.3)$$

$$\delta_d = \frac{1}{16\pi^2} \hat{\Delta}_d^{(1)} + \frac{1}{(16\pi^2)^2} \hat{\Delta}_d^{(2)}, \quad (5.4)$$

where

$$\hat{\Delta}_u^{(1)} = \frac{1}{2v_u} \frac{\partial}{\partial v_u} V^{(1)}(0, 0), \quad (5.5)$$

$$\hat{\Delta}_d^{(1)} = \frac{1}{2v_d} \frac{\partial}{\partial v_d} V^{(1)}(0, 0), \quad (5.6)$$

$$\hat{\Delta}_u^{(2)} = \frac{1}{2v_u} \frac{\partial}{\partial v_u} V^{(2)}(0, 0) - \frac{1}{8} (g^2 + g'^2)_{c_{2\beta}} (\Omega^0 + 2\Omega^\pm), \quad (5.7)$$

$$\hat{\Delta}_d^{(2)} = \frac{1}{2v_d} \frac{\partial}{\partial v_d} V^{(2)}(0, 0) + \frac{1}{8} (g^2 + g'^2)_{c_{2\beta}} (\Omega^0 + 2\Omega^\pm). \quad (5.8)$$

In words, this means that one can simply minimize the two-loop effective potential with all Goldstone boson squared masses replaced by 0, provided that one then includes extra terms in the 2-loop order part of the minimization condition that are proportional to the quantities Ω^0 and Ω^\pm provided in the previous section.

Explicitly, we find

$$\begin{aligned} \hat{\Delta}_u^{(1)} = & \frac{1}{2v_u} \left[\frac{1}{2} A(h^0) \frac{\partial h^0}{\partial v_u} + \frac{1}{2} A(H^0) \frac{\partial H^0}{\partial v_u} + \frac{1}{2} A(A^0) \frac{\partial A^0}{\partial v_u} + A(H^\pm) \frac{\partial H^\pm}{\partial v_u} \right. \\ & + \sum_{\tilde{f}} n_{\tilde{f}} A(\tilde{f}) \frac{\partial \tilde{f}}{\partial v_u} - \sum_{i=1}^4 A(\tilde{N}_i) \frac{\partial \tilde{N}_i}{\partial v_u} - 2 \sum_{i=1}^2 A(\tilde{C}_i) \frac{\partial \tilde{C}_i}{\partial v_u} - 6A(t) \frac{\partial t}{\partial v_u} \\ & \left. + \frac{3}{2} A(Z) \frac{\partial Z}{\partial v_u} + 3A(W) \frac{\partial W}{\partial v_u} \right], \end{aligned} \quad (5.9)$$

$$\begin{aligned} \hat{\Delta}_d^{(1)} = & \frac{1}{2v_d} \left[\frac{1}{2} A(h^0) \frac{\partial h^0}{\partial v_d} + \frac{1}{2} A(H^0) \frac{\partial H^0}{\partial v_d} + \frac{1}{2} A(A^0) \frac{\partial A^0}{\partial v_d} + A(H^\pm) \frac{\partial H^\pm}{\partial v_d} \right. \\ & + \sum_{\tilde{f}} n_{\tilde{f}} A(\tilde{f}) \frac{\partial \tilde{f}}{\partial v_d} - \sum_{i=1}^4 A(\tilde{N}_i) \frac{\partial \tilde{N}_i}{\partial v_d} - 2 \sum_{i=1}^2 A(\tilde{C}_i) \frac{\partial \tilde{C}_i}{\partial v_d} - 6A(b) \frac{\partial b}{\partial v_d} \\ & \left. - 2A(\tau) \frac{\partial \tau}{\partial v_d} + \frac{3}{2} A(Z) \frac{\partial Z}{\partial v_d} + 3A(W) \frac{\partial W}{\partial v_d} \right]. \end{aligned} \quad (5.10)$$

while the 2-loop contributions are straightforward to evaluate using eqs. (5.7) and (5.8) but rather lengthy. In general, the partial derivatives of mixing angles and squared masses, needed for finding the derivatives and thus the minimization conditions for effective poten-

tials, can be derived in the following manner. Consider diagonal squared mass matrices given by

$$M_D^2 = U M^2 U^\dagger \quad (5.11)$$

where M^2 is a gauge-eigenstate squared mass matrix and U is a unitary matrix. The derivatives of the diagonal entries of M_D^2 , which are the squared mass eigenvalues, with respect to any parameter x on which they depend, can be found by doing

$$\frac{\partial}{\partial x} (M_D^2)_{ii} = \left[U \frac{\partial M^2}{\partial x} U^\dagger \right]_{ii}, \quad (5.12)$$

with no sum on the repeated index i . In order to calculate the derivatives of the two-loop effective potential one will also need the derivatives of the mixing angles found in the unitary matrices denoted U . Those can be found by

$$\frac{\partial}{\partial x} U = A U, \quad (5.13)$$

where the matrix A has elements

$$A_{ij} = \begin{cases} \left[U \frac{\partial M^2}{\partial x} U^\dagger \right]_{ij} / [(M_D^2)_{ii} - (M_D^2)_{jj}], & (i \neq j), \\ 0 & (i = j), \end{cases} \quad (5.14)$$

with again no summation on repeated indices. One needs derivatives with respect to both the VEVs, $x = v_u, v_d$.

The preceding minimization conditions do not involve G^0 or G^\pm at all, but do include the quantities b , μ , $m_{H_u}^2$, and $m_{H_d}^2$ through the mixing angles α , β_0 , β_\pm which enter the Higgs couplings to other particles and the squared masses of the other Higgs states, h^0 , H^0 , A^0 , and H^\pm . One can now choose to eliminate any two of the parameters b , μ , $m_{H_u}^2$, and $m_{H_d}^2$ using the minimization conditions, by expanding in δ_u and δ_d . (This is analogous to eliminating the negative Higgs squared mass quantity m^2 in the Standard Model case, as explained in section IV of ref. [34].) This has the practical advantage that the effective potential minimization conditions can then be solved numerically without iteration.

B. Reexpansion to eliminate $m_{H_u}^2$ and $m_{H_d}^2$

For example, working at the minimum of the effective potential, one can choose to eliminate $m_{H_u}^2$ and $m_{H_d}^2$. To do so, it is convenient to define modified tree-level Higgs squared masses:

$$\hat{A}^0 = 2b/s_{2\beta}, \quad (5.15)$$

$$\hat{H}^\pm = \hat{A}^0 + W, \quad (5.16)$$

$$\hat{H}^0, \hat{h}^0 = \frac{1}{2} \left[\hat{A}^0 + Z \pm \sqrt{(\hat{A}^0 + Z)^2 - 4\hat{A}^0 Z c_{2\beta}^2} \right], \quad (5.17)$$

in terms of which the full tree-level squared masses appearing in the formulas above can be expanded for small δ_u, δ_d :

$$A^0 = \hat{A}^0 - \delta_u c_\beta^2 - \delta_d s_\beta^2 + \dots, \quad (5.18)$$

$$H^\pm = \hat{H}^\pm - \delta_u c_\beta^2 - \delta_d s_\beta^2 + \dots, \quad (5.19)$$

$$h^0 = \hat{h}^0 - \frac{1}{2}(\delta_u + \delta_d) + (\delta_u - \delta_d) c_{2\beta} \frac{(\hat{A}^0 - Z)}{2(\hat{H}^0 - \hat{h}^0)} + \dots, \quad (5.20)$$

$$H^0 = \hat{H}^0 - \frac{1}{2}(\delta_u + \delta_d) + (\delta_d - \delta_u) c_{2\beta} \frac{(\hat{A}^0 - Z)}{2(\hat{H}^0 - \hat{h}^0)} + \dots \quad (5.21)$$

We have already seen in eqs. (2.22)-(2.25) how to write exact expressions or expansions for the mixing angles β_0 and β_\pm in terms of the angle β and the radiative corrections δ_u and δ_d . Similarly, we find that

$$\cot(2\alpha) = \cot(2\hat{\alpha}) + \frac{\delta_d - \delta_u}{2b + (g^2 + g'^2)v_u v_d}, \quad (5.22)$$

where

$$\cot(2\hat{\alpha}) = \left(\frac{\hat{A}^0 - Z}{\hat{A}^0 + Z} \right) \cot(2\beta). \quad (5.23)$$

Thus, all of the parameters of the Higgs sector, namely the squared masses h^0, H^0, A^0, H^\pm and the angles β_0, β_\pm , and α in the effective minimization condition formulas above can be expanded (in δ_u, δ_d) about the modified tree-level values $\hat{h}^0, \hat{H}^0, \hat{A}^0, \hat{H}^\pm, \hat{\alpha}$, and β , which do not depend explicitly on $m_{H_u}^2$ or $m_{H_d}^2$. After doing this expansion, the quantities involving δ_u and δ_d from the 1-loop terms can be grouped with the 2-loop terms, and higher-order terms can be neglected consistently as 3-loop order. Then solving for $m_{H_u}^2$ and $m_{H_d}^2$ at the

minimum of the effective potential can be done without iteration.

The results of the reexpansion described above can be summarized as follows. In the expressions for $\Delta_u^{(1)}$, $\Delta_d^{(1)}$, $\Delta_u^{(2)}$, and $\Delta_d^{(2)}$ found in eqs. (5.5)-(5.8) above, one makes the replacements:

$$(h^0, H^0, A^0, H^\pm) \rightarrow (\hat{h}^0, \hat{H}^0, \hat{A}^0, \hat{H}^\pm), \quad (5.24)$$

$$\alpha \rightarrow \hat{\alpha}, \quad (5.25)$$

$$\beta_0, \beta_\pm \rightarrow \beta. \quad (5.26)$$

One then should add the following extra terms to the 2-loop parts:

$$\begin{aligned} \hat{\Delta}_u^{(2)} \rightarrow & \hat{\Delta}_u^{(2)} - \frac{1}{16}(g^2 + g'^2) \left\{ 2c_{2\beta} [\hat{\Delta}_u^{(1)} c_\beta^2 + \hat{\Delta}_d^{(1)} s_\beta^2] \overline{\ln}(\hat{A}^0) \right. \\ & + [(1 + 2c_{2\hat{\alpha}}) + s_{2\hat{\alpha}} c_\beta / s_\beta] [\hat{\Delta}_u^{(1)} + \hat{\Delta}_d^{(1)} - (\hat{\Delta}_u^{(1)} - \hat{\Delta}_d^{(1)}) \frac{\hat{A}^0 - Z}{\hat{H}^0 - \hat{h}^0} c_{2\beta}] \overline{\ln}(\hat{h}^0) \\ & + [(1 - 2c_{2\hat{\alpha}}) - s_{2\hat{\alpha}} c_\beta / s_\beta] [\hat{\Delta}_u^{(1)} + \hat{\Delta}_d^{(1)} + (\hat{\Delta}_u^{(1)} - \hat{\Delta}_d^{(1)}) \frac{\hat{A}^0 - Z}{\hat{H}^0 - \hat{h}^0} c_{2\beta}] \overline{\ln}(\hat{H}^0) \left. \right\} \\ & - \frac{1}{4} [g^2(1 + 2c_\beta^2) + g'^2 c_{2\beta}] [\hat{\Delta}_u^{(1)} c_\beta^2 + \hat{\Delta}_d^{(1)} s_\beta^2] \overline{\ln}(\hat{H}^+) \\ & + \frac{1}{8} (\hat{\Delta}_u^{(1)} - \hat{\Delta}_d^{(1)}) \left\{ -(g^2 + g'^2) s_{2\beta}^2 A(\hat{A}^0) / \hat{A}^0 + [g^2 c_{2\beta} / s_\beta^2 - 2g'^2] s_{2\beta}^2 A(\hat{H}^+) / \hat{H}^+ \right. \\ & \left. + (g^2 + g'^2)(2s_{2\hat{\alpha}} - c_{2\hat{\alpha}} c_\beta / s_\beta)(s_{2\hat{\alpha}}^2 / s_{2\beta}) [A(\hat{H}^0) - A(\hat{h}^0)] / (\hat{H}^0 + \hat{h}^0) \right\}, \quad (5.27) \end{aligned}$$

$$\begin{aligned} \hat{\Delta}_d^{(2)} \rightarrow & \hat{\Delta}_d^{(2)} + \frac{1}{16}(g^2 + g'^2) \left\{ 2c_{2\beta} [\hat{\Delta}_u^{(1)} c_\beta^2 + \hat{\Delta}_d^{(1)} s_\beta^2] \overline{\ln}(\hat{A}^0) \right. \\ & - [(1 - 2c_{2\hat{\alpha}}) + s_{2\hat{\alpha}} s_\beta / c_\beta] [\hat{\Delta}_u^{(1)} + \hat{\Delta}_d^{(1)} - (\hat{\Delta}_u^{(1)} - \hat{\Delta}_d^{(1)}) \frac{\hat{A}^0 - Z}{\hat{H}^0 - \hat{h}^0} c_{2\beta}] \overline{\ln}(\hat{h}^0) \\ & - [(1 + 2c_{2\hat{\alpha}}) - s_{2\hat{\alpha}} s_\beta / c_\beta] [\hat{\Delta}_u^{(1)} + \hat{\Delta}_d^{(1)} + (\hat{\Delta}_u^{(1)} - \hat{\Delta}_d^{(1)}) \frac{\hat{A}^0 - Z}{\hat{H}^0 - \hat{h}^0} c_{2\beta}] \overline{\ln}(\hat{H}^0) \left. \right\} \\ & - \frac{1}{4} [g^2(1 + 2s_\beta^2) - g'^2 c_{2\beta}] [\hat{\Delta}_u^{(1)} c_\beta^2 + \hat{\Delta}_d^{(1)} s_\beta^2] \overline{\ln}(\hat{H}^+) \\ & + \frac{1}{8} (\hat{\Delta}_u^{(1)} - \hat{\Delta}_d^{(1)}) \left\{ (g^2 + g'^2) s_{2\beta}^2 A(\hat{A}^0) / \hat{A}^0 + [g^2 c_{2\beta} / c_\beta^2 + 2g'^2] s_{2\beta}^2 A(\hat{H}^+) / \hat{H}^+ \right. \\ & \left. - (g^2 + g'^2)(2s_{2\hat{\alpha}} + c_{2\hat{\alpha}} s_\beta / c_\beta)(s_{2\hat{\alpha}}^2 / s_{2\beta}) [A(\hat{H}^0) - A(\hat{h}^0)] / (\hat{H}^0 + \hat{h}^0) \right\}. \quad (5.28) \end{aligned}$$

Then one can solve for $m_{H_u}^2$ and $m_{H_d}^2$ realizing the minimum of the effective potential using eqs. (2.19)-(2.20), without iteration.

C. Reexpansion to eliminate μ and b

Alternatively, one could choose to eliminate $|\mu|^2$ and b . Then, the corresponding results for the tree-level mixing angles are:

$$\tan(2\beta_0) = \tan(2\beta) \left[1 + \frac{\delta_d - \delta_u}{m_{H_d}^2 - m_{H_u}^2 + Zc_{2\beta}} \right], \quad (5.29)$$

$$\tan(2\beta_{\pm}) = \tan(2\beta) \left[1 + \frac{\delta_d - \delta_u}{m_{H_d}^2 - m_{H_u}^2 + (Z - W)c_{2\beta}} \right], \quad (5.30)$$

$$\tan(2\alpha) = \tan(2\bar{\alpha}) + (\delta_d - \delta_u) \left[\frac{\tan(2\beta)}{m_{H_d}^2 - m_{H_u}^2 + 2Zc_{2\beta}} \right], \quad (5.31)$$

where one defines

$$\tan(2\bar{\alpha}) = \tan(2\beta) \left[\frac{m_{H_d}^2 - m_{H_u}^2}{m_{H_d}^2 - m_{H_u}^2 + 2Zc_{2\beta}} \right], \quad (5.32)$$

and one can expand the tree-level Higgs squared masses around the modified tree-level values defined by:

$$\bar{A}^0 = (m_{H_u}^2 - m_{H_d}^2)/c_{2\beta} - Z, \quad (5.33)$$

$$\bar{H}^{\pm} = (m_{H_u}^2 - m_{H_d}^2)/c_{2\beta} - Z + W, \quad (5.34)$$

$$\bar{H}^0, \bar{h}^0 = \frac{1}{2} \left[\bar{A}^0 + Z \pm \sqrt{(\bar{A}^0 + Z)^2 - 4\bar{A}^0 Z c_{2\beta}^2} \right], \quad (5.35)$$

with the results:

$$A^0 = \bar{A}^0 + p_u \delta_u + p_d \delta_d + \dots, \quad (5.36)$$

$$H^{\pm} = \bar{H}^{\pm} + p_u \delta_u + p_d \delta_d + \dots, \quad (5.37)$$

$$h^0 = \bar{h}^0 + \left[\delta_u (s_{\beta}^2 \bar{H}^0 + p_u \bar{h}^0) + \delta_d (c_{\beta}^2 \bar{H}^0 + p_d \bar{h}^0) \right] / (\bar{h}^0 - \bar{H}^0) + \dots, \quad (5.38)$$

$$H^0 = \bar{H}^0 + \left[\delta_u (s_{\beta}^2 \bar{h}^0 + p_u \bar{H}^0) + \delta_d (c_{\beta}^2 \bar{h}^0 + p_d \bar{H}^0) \right] / (\bar{H}^0 - \bar{h}^0) + \dots, \quad (5.39)$$

where

$$p_u = s_{\beta}^2 (1 + 2c_{\beta}^2) / c_{2\beta}, \quad (5.40)$$

$$p_d = -c_{\beta}^2 (1 + 2s_{\beta}^2) / c_{2\beta}. \quad (5.41)$$

Then the effective potential minimization conditions can be expanded in δ_u , δ_d about the modified tree-level values \bar{h}^0 , \bar{H}^0 , \bar{A}^0 , \bar{H}^\pm , $\bar{\alpha}$, and β , which do not depend explicitly on b or μ . After doing these expansions, the quantities involving δ_u and δ_d from the 1-loop terms can be grouped with the 2-loop terms, and higher-order terms can be neglected consistently as 3-loop order.

The reexpansion described above can be implemented as follows. In the expressions for $\Delta_u^{(1)}$, $\Delta_d^{(1)}$, $\Delta_u^{(2)}$, and $\Delta_d^{(2)}$ found in eqs. (5.5)-(5.8) above, one makes the replacements:

$$(h^0, H^0, A^0, H^\pm) \rightarrow (\bar{h}^0, \bar{H}^0, \bar{A}^0, \bar{H}^\pm), \quad (5.42)$$

$$\alpha \rightarrow \bar{\alpha}, \quad (5.43)$$

$$\beta_0, \beta_\pm \rightarrow \beta. \quad (5.44)$$

One then should add the following extra terms to the 2-loop parts:

$$\begin{aligned} \hat{\Delta}_u^{(2)} \rightarrow & \hat{\Delta}_u^{(2)} + \frac{1}{8}(g^2 + g'^2)c_{2\beta} [\hat{\Delta}_u^{(1)}p_u + \hat{\Delta}_d^{(1)}p_d] \ln(\bar{A}^0) + \frac{g^2 + g'^2}{8(\bar{H}^0 - \bar{h}^0)} \left\{ \right. \\ & - \{[1 + 2c_{2\bar{\alpha}}] + s_{2\bar{\alpha}}c_\beta/s_\beta\} [\hat{\Delta}_u^{(1)}(s_\beta^2\bar{H}^0 + p_u\bar{h}^0) + \hat{\Delta}_d^{(1)}(c_\beta^2\bar{H}^0 + p_d\bar{h}^0)] \ln(\bar{h}^0) \\ & + \{[1 - 2c_{2\bar{\alpha}}] - s_{2\bar{\alpha}}c_\beta/s_\beta\} [\hat{\Delta}_u^{(1)}(s_\beta^2\bar{h}^0 + p_u\bar{H}^0) + \hat{\Delta}_d^{(1)}(c_\beta^2\bar{h}^0 + p_d\bar{H}^0)] \ln(\bar{H}^0) \left. \right\} \\ & + \frac{1}{4} [g^2(1 + 2c_\beta^2) + g'^2c_{2\beta}] [\hat{\Delta}_u^{(1)}p_u + \hat{\Delta}_d^{(1)}p_d] \ln(\bar{H}^+) \\ & + \frac{1}{8} (\hat{\Delta}_d^{(1)} - \hat{\Delta}_u^{(1)}) \left\{ (g^2 + g'^2)s_{2\beta}^2 A(\bar{A}^0)/\bar{A}^0 + [2g'^2s_{2\beta}^2 - 4g^2c_\beta^2c_{2\beta}] A(\bar{H}^+)/\bar{H}^+ \right. \\ & \left. + (g^2 + g'^2)[2s_{2\bar{\alpha}} - c_{2\bar{\alpha}}c_\beta/s_\beta](s_{2\bar{\alpha}}c_{2\bar{\alpha}}/c_{2\beta})[A(\bar{h}^0) - A(\bar{H}^0)]/(\bar{H}^0 + \bar{h}^0) \right\}, \quad (5.45) \end{aligned}$$

$$\begin{aligned} \hat{\Delta}_d^{(2)} \rightarrow & \hat{\Delta}_d^{(2)} - \frac{1}{8}(g^2 + g'^2)c_{2\beta} [\hat{\Delta}_u^{(1)}p_u + \hat{\Delta}_d^{(1)}p_d] \ln(\bar{A}^0) + \frac{g^2 + g'^2}{8(\bar{H}^0 - \bar{h}^0)} \left\{ \right. \\ & - \{[1 - 2c_{2\bar{\alpha}}] + s_{2\bar{\alpha}}s_\beta/c_\beta\} [\hat{\Delta}_u^{(1)}(s_\beta^2\bar{H}^0 + p_u\bar{h}^0) + \hat{\Delta}_d^{(1)}(c_\beta^2\bar{H}^0 + p_d\bar{h}^0)] \ln(\bar{h}^0) \\ & + \{[1 + 2c_{2\bar{\alpha}}] - s_{2\bar{\alpha}}s_\beta/c_\beta\} [\hat{\Delta}_u^{(1)}(s_\beta^2\bar{h}^0 + p_u\bar{H}^0) + \hat{\Delta}_d^{(1)}(c_\beta^2\bar{h}^0 + p_d\bar{H}^0)] \ln(\bar{H}^0) \left. \right\} \\ & + \frac{1}{4} [g^2(1 + 2s_\beta^2) - g'^2c_{2\beta}] [\hat{\Delta}_u^{(1)}p_u + \hat{\Delta}_d^{(1)}p_d] \ln(\bar{H}^+) \\ & + \frac{1}{8} (\hat{\Delta}_u^{(1)} - \hat{\Delta}_d^{(1)}) \left\{ (g^2 + g'^2)s_{2\beta}^2 A(\bar{A}^0)/\bar{A}^0 + [2g'^2s_{2\beta}^2 + 4g^2s_\beta^2c_{2\beta}] A(\bar{H}^+)/\bar{H}^+ \right. \\ & \left. + (g^2 + g'^2)[2s_{2\bar{\alpha}} + c_{2\bar{\alpha}}s_\beta/c_\beta](s_{2\bar{\alpha}}c_{2\bar{\alpha}}/c_{2\beta})[A(\bar{h}^0) - A(\bar{H}^0)]/(\bar{H}^0 + \bar{h}^0) \right\}. \quad (5.46) \end{aligned}$$

Then one can solve for b and $|\mu|^2$ using eqs. (2.19)-(2.20), without iteration.

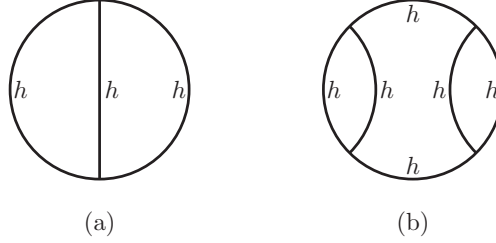


FIG. 6.1: Diagrams that give the most singular behavior as $h^0 \rightarrow 0$ for the minimization condition of the effective potential, at 2-loop order (a) and 3-loop order (b).

VI. SINGULARITIES AND SPURIOUS IMAGINARY PARTS FOR SMALL AND NEGATIVE h^0

It should be noted that there are also singularities in the effective potential for $h^0 \rightarrow 0$, and in fact these are formally more severe than the singularities coming from $G^0, G^\pm \rightarrow 0$. This can be seen, for example, from the diagrams shown in Figure 6.1, which involve only the h^0 field. The contribution of the 2-loop diagram to the effective potential is:

$$V_{\text{Fig. 6.1(a)}}^{(2)} = -\frac{1}{12}(\lambda_{h^0 h^0 h^0})^2 I(h^0, h^0, h^0) \quad (6.1)$$

$$= \frac{1}{12}(\lambda_{h^0 h^0 h^0})^2 h^0 \left[\frac{15}{2} - 3\sqrt{3}\text{Ls}_2 - 6\overline{\ln}(h^0) + \frac{3}{2}\overline{\ln}^2(h^0) \right], \quad (6.2)$$

where $\text{Ls}_2 = -\int_0^{2\pi/3} dx \ln[2 \sin(x/2)] = 0.6766277376 \dots$. This contribution is finite as $h^0 \rightarrow 0$, but derivatives of it have a squared logarithm singularity. At 3-loop order, the contribution shown in Figure 6.1 has the form

$$V_{\text{Fig. 6.1(b)}}^{(3)} = \frac{1}{16}(\lambda_{h^0 h^0 h^0})^4 \left(\frac{5}{3} - 2\zeta(3) - 2\sqrt{3}\text{Ls}_2[1 + \overline{\ln}(h^0)] - \overline{\ln}^2(h^0) + \frac{1}{3}\overline{\ln}^3(h^0) \right), \quad (6.3)$$

with a cubic logarithmic singularity even before taking derivatives, and other diagrams leading to quadratic logarithmic singularities. For contributions at L -loop order, we expect contributions with leading singularities of the form

$$V^{(L)} \propto (\lambda_{h^0 h^0 h^0})^{2L-2} \overline{\ln}^L(h^0) / (h^0)^{L-3} \quad (6.4)$$

as $h^0 \rightarrow 0$. Note that the reason these singularities are more severe than for the Goldstone case is because of the absence of triple Goldstone boson couplings. Furthermore, unlike diagrams involving Goldstone bosons, such diagrams have no larger mass scale with respect to which one can expand for small h^0 . Other diagrams involving h^0 will involve W and

Z , which have smaller physical masses than h^0 , so an expansion in small h^0 may not be appropriate. Methods for resumming non-Goldstone light boson singularities have been discussed in ref. [35]. Another way of doing a resummation is by taking advantage of the renormalization group, by simply choosing a scale Q where h^0 is positive, and not too far from the physical squared mass. As illustrated by the example in the next section, this is generally possible, and will be a sensible choice of renormalization scale from the point of view of perturbative expansions for other physical quantities. (However, note that with such a choice, the Goldstone boson squared masses could still easily be negative or 0, so that before resummation of the G^0 and G^\pm contributions the effective potential would be complex or singular at its minimum.) The reexpansions described in subsection VB or VC can also be used to eliminate the problems with $h^0 \leq 0$.

VII. NUMERICAL EXAMPLE

The impact of the resummations described in this paper is typically numerically extremely small, at least for the minimization of the effective potential, unless one has chosen a renormalization scale where G^0 or G^\pm or h^0 vanishes exactly. To illustrate this, we consider a benchmark MSSM model with input parameters (with mass scales chosen large enough to clearly avoid all present bounds from the Large Hadron Collider, and to be roughly compatible with the h^0 physical mass near 125 GeV, with $\tan \beta$ near 25) at $Q_0 = 2000$ GeV:

$$v_u = 172.1 \text{ GeV}, \quad v_d = 6.88 \text{ GeV}, \quad (7.1)$$

$$m_{H_u}^2 = -(1500 \text{ GeV})^2, \quad m_{H_d}^2 = (2000 \text{ GeV})^2, \quad (7.2)$$

$$g = 0.6362, \quad g' = 0.3636, \quad g_3 = 1.018, \quad (7.3)$$

$$y_t = 0.785, \quad y_b = 0.296, \quad y_\tau = 0.256, \quad (7.4)$$

$$M_1 = 500 \text{ GeV}, \quad M_2 = 1000 \text{ GeV}, \quad M_3 = 2500 \text{ GeV}, \quad (7.5)$$

$$a_t = -3000 \text{ GeV}, \quad a_b = -2000 \text{ GeV}, \quad a_\tau = -1000 \text{ GeV}, \quad (7.6)$$

$$m_{Q_3}^2 = (2000 \text{ GeV})^2, \quad m_{u_3}^2 = (2100 \text{ GeV})^2, \quad m_{d_3}^2 = (2400 \text{ GeV})^2, \quad (7.7)$$

$$m_{L_3}^2 = (2200 \text{ GeV})^2, \quad m_{e_3}^2 = (2000 \text{ GeV})^2, \quad (7.8)$$

$$m_{Q_{1,2}}^2 = m_{u_{1,2}}^2 = m_{d_{1,2}}^2 = (3000 \text{ GeV})^2, \quad (7.9)$$

$$m_{L_{1,2}}^2 = (2400 \text{ GeV})^2, \quad m_{e_{1,2}}^2 = (2200 \text{ GeV})^2. \quad (7.10)$$

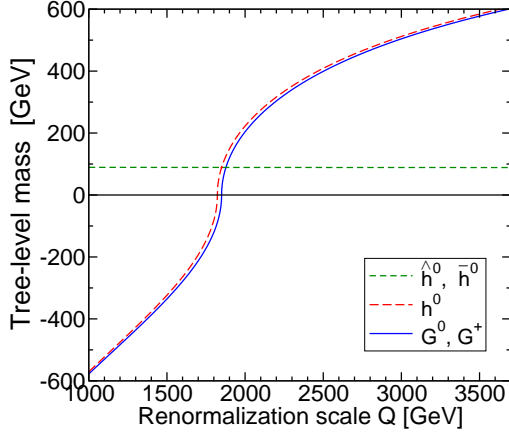


FIG. 7.1: The dependences of tree-level masses on the renormalization scale, for the Goldstone bosons G^0, G^\pm (solid blue line) and the lightest neutral Higgs bosons h^0 (long-dashed red line). The modified tree-level values \hat{h}^0 and \bar{h}^0 , defined by eqs. (5.17) and (5.35), are visually indistinguishable from each other and are nearly constant, and are shown as the short-dashed green line. In each case, $\overline{\text{sqrt}}(m^2)$ is plotted. The input parameters are defined by 2-loop renormalization group running starting from eqs. (7.1)-(7.11) at $Q_0 = 2000$ GeV.

Then, we find that the (real part) of the 2-loop MSSM effective potential as given in ref. [26] is minimized for

$$\mu = 1516.44446868 \text{ GeV}, \quad b = (522.793413744 \text{ GeV})^2. \quad (7.11)$$

Then we run the input parameters of eqs. (7.1)-(7.11) from Q_0 to a new renormalization scale Q , and require the potential to be minimized again, both using the original method of ref. [26] and then with the resummation methods of the present paper.

First, shown in Figure 7.1 are the values obtained for $\overline{\text{sqrt}}(G^0)$ and $\overline{\text{sqrt}}(G^\pm)$ and $\overline{\text{sqrt}}(h^0)$ at the minimum of the effective potential, as a function of Q , where the function

$$\overline{\text{sqrt}}(x) = x/\sqrt{|x|} \quad (7.12)$$

is used in order to plot masses while keeping information about the sign of the squared mass, while avoiding imaginary numbers. Due to the influence of very heavy squarks, these tree-level masses are seen to run very quickly. The Goldstone boson masses are visually indistinguishable from each other, and are slightly lower than the tree-level mass of h^0 . All three are negative for $Q < 1849$ GeV, and deviate very far from 0 (in the case of G^0 and G^\pm) and 125 GeV (in the case of h^0). In contrast, the modified tree-level masses \hat{h}^0 and \bar{h}^0 both remain nearly constant near 89 GeV (and are visually indistinguishable from each other on the graph). For this reason, a perturbative expansion about either one of these tree-level definitions, obtained by the re-expansions of the previous section, could be preferred at least formally. The numerical values of $\tan \beta_0$ and $\tan \beta_\pm$ at the minimum of the potential are compared to the running value of $\tan \beta \equiv v_u/v_d$ in Figure 7.2. The values of $\tan \beta_0$ and $\tan \beta_\pm$ are visually indistinguishable in the figure, but both deviate significantly from $\tan \beta$, which runs slowly from its nominal value near 25 at $Q_0 = 2000$ GeV.

Despite the large deviations of G^0 and G^\pm and h^0 from their physical values, the 2-loop

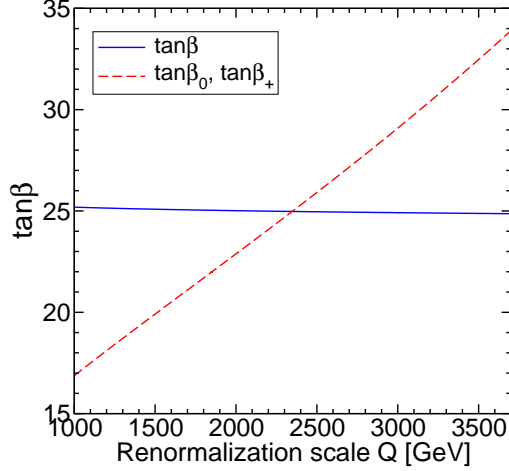


FIG. 7.2: The dependences of $\tan \beta = v_u/v_d$, the tree-level neutral pseudoscalar Higgs mixing parameter $\tan \beta_0$, and the charged Higgs mixing parameter $\tan \beta_{\pm}$, as a function of the renormalization scale Q at which the 2-loop effective potential is minimized. The input parameters are defined by running (7.1)-(7.11) starting from $Q_0 = 2000$ GeV.

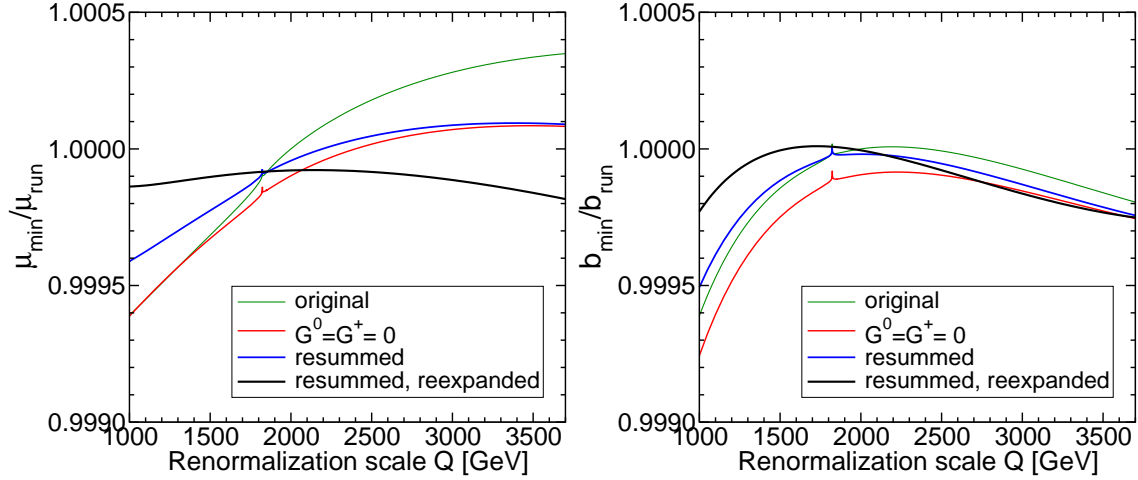


FIG. 7.3: The dependence of the ratio of $\mu_{\min}/\mu_{\text{run}}$ (left panel) and b_{\min}/b_{run} (right panel) on the renormalization scale Q . Here “run” means obtained by 2-loop renormalization group running starting from $Q_0 = 2000$ GeV with inputs from (7.1)-(7.11), while “min” means obtained by applying the effective potential minimization conditions directly at Q . The thinnest (green) lines are obtained with V_{eff} found in ref. [26]. The next thinnest (red) lines were obtained in the same way, but with $G^0 = G^{\pm} = 0$ set by hand. The thicker (blue) lines were obtained with the resummed effective potential using eqs. (5.3)-(5.8) in eqs. (2.19)-(2.20). The thickest (black) lines were obtained by further re-expanding the minimization conditions to eliminate μ and b in the radiative correction part as described in section VC.

effective potential minimization results are very stable. This is shown in Figure 7.3, which shows the ratios of the values obtained for $\mu_{\min}(Q)/\mu_{\text{run}}(Q)$ and $b_{\min}(Q)/b_{\text{run}}(Q)$, where “run” means obtained by running the MSSM 2-loop renormalization group equations [44–47] starting from Q_0 with inputs from eqs. (7.1)-(7.11), while “min” means all of the inputs are run to Q and then the effective potential minimization conditions are used to find μ and b directly at that scale. The closeness of these ratios to 1 as Q is varied is a test of the robustness of the approximations used.

Four different version of the minimization conditions are compared in Figure 7.3. First, the thinnest (green) lines show the results obtained using the real part of the original V_{eff} found in ref. [26]. By definition, the thinnest (green) curves run through 1 at $Q = Q_0 = 2000$ GeV. We note that although these curves have singularities at $G^0 = 0$ and $G^\pm = 0$, in practice these singularities are too mild to show up on the plots even for very fine grids for the data (here we used an increment of 50 MeV for Q in the vicinities of $G^0 = 0$, $G^\pm = 0$, and $h^0 = 0$). There are visible kinks near $Q = 1823$ GeV, corresponding to the scale at which h^0 crosses through 0, as discussed in the previous section. The next thinnest (red) lines show what would be obtained if one simply sets G^0 and G^\pm to 0 by hand in the effective potential before minimization. The thicker (blue) line shows the result obtained from the resummed effective potential minimization, using eqs. (5.3)-(5.8) in eqs. (2.19)-(2.20). Finally, the thickest (black) lines show the results obtained after the reexpansion of the effective potential minimization conditions to eliminate the dependence of the loop correction part on the parameters μ and b , using eqs. (5.42)-(5.46). This allows the effective potential minimization conditions to be implemented without iteration, and eliminates the possibility of kinks and singularities where G^0 , G^\pm , and h^0 run through 0. We see that in all cases the dependence on Q for each of the ratios shown in Figure 7.3 is extremely mild, well under 0.1% in all cases, despite the large magnitudes and Q dependences of the G^0 , G^\pm , and h^0 squared masses. Furthermore, the different ways of implementing the minimization conditions agree well with each other, again to better than 0.1%.

Similar results are shown in Figure 7.4 for the determination of $m_{H_u}^2$ and $m_{H_d}^2$ from the other parameters. In this case, the thickest (black) line is obtained by reexpanding the resummed effective potential to eliminate the dependence on $m_{H_u}^2$ and $m_{H_d}^2$ in the radiative correction part of the minimization conditions, using eqs. (5.24)-(5.28), allowing them to be implemented without iteration. Again, in all cases the scale dependences are very mild, and the agreement between different methods of implementing the minimization conditions is excellent. Therefore, while conceptually important, and practically convenient, the re-summation and reexpansion does not seem to have a significant numerical effect for the minimization condition.

VIII. OUTLOOK

In this paper we have showed how to resum the Goldstone boson contributions to the MSSM effective potential and its minimization conditions. Although the numerical impact on the minimization conditions is very small compared to the results obtained by minimizing the non-resummed effective potential, or simply setting G^0 and G^\pm to 0 by hand, there is a practical benefit in that one can then reexpand the minimization conditions to implement

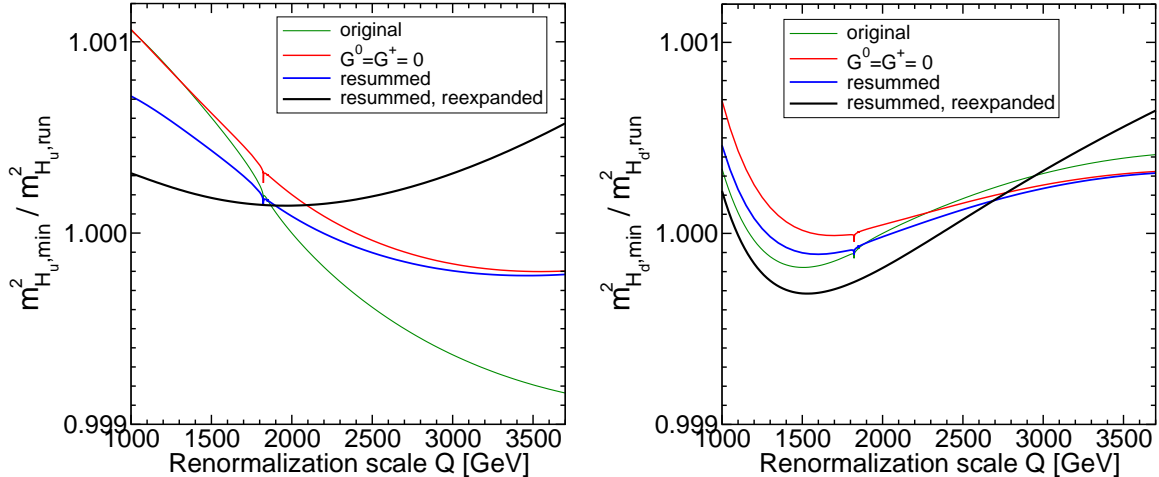


FIG. 7.4: As in figure 7.3, but for $m_{H_u}^2$ and $m_{H_d}^2$. Here, the thickest (black) line is obtained by reexpanding the resummed effective potential to eliminate the dependence on $m_{H_u}^2$ and $m_{H_d}^2$ in the radiative correction part of the minimization conditions, as described in section V B

them consistently at 2-loop order without iteration.

In addition, the resummation and reexpansions described here can be systematically applied to other calculations, for example the pole masses of the ordinary Higgs bosons. The existence of a Standard Model-like Higgs boson with mass near 125 GeV provides an opportunity to confront models with data. There has been a tremendous effort to compute the physical mass M_{h^0} using self-energy diagrammatic methods [48]–[60], the approximation based on second derivatives of the effective potential [27–33, 61–69], and effective field theory with renormalization group resummation methods [70]–[77]. (For a recent review of these approaches, see [78].) The methods described here will allow a full 2-loop self-energy diagrammatic calculation of the pole mass M_{h^0} , using modified tree-level Higgs couplings and masses that do not differ greatly from their physical values, while using VEVs that minimize the full 2-loop effective potential. (Note that the resummations described above do not attempt to address the singularities in the second derivatives of the effective potential, which are sometimes used to approximate the h^0 pole mass. Instead, the momentum dependence of the self-energy diagrams should be kept in order to find the true pole mass.) The results above can also serve as examples for other models with non-minimal Higgs sectors, such as the MSSM extended by a singlet, or non-supersymmetric two Higgs doublet models.

Acknowledgments: This work was supported in part by the National Science Foundation grant number PHY-1417028.

[1] S. R. Coleman and E. J. Weinberg, “Radiative Corrections as the Origin of Spontaneous Symmetry Breaking,” *Phys. Rev. D* **7**, 1888 (1973).

- [2] R. Jackiw, “Functional evaluation of the effective potential,” *Phys. Rev. D* **9**, 1686 (1974).
- [3] M. Sher, “Electroweak Higgs Potentials and Vacuum Stability,” *Phys. Rept.* **179**, 273 (1989), and references therein.
- [4] E. J. Weinberg and A. -q. Wu, “Understanding Complex Perturbative Effective Potentials,” *Phys. Rev. D* **36**, 2474 (1987).
- [5] C. Ford, D. R. T. Jones, P. W. Stephenson and M. B. Einhorn, “The Effective potential and the renormalization group,” *Nucl. Phys. B* **395**, 17 (1993) [hep-lat/9210033].
- [6] J. A. Casas, J. R. Espinosa and M. Quiros, “Improved Higgs mass stability bound in the standard model and implications for supersymmetry,” *Phys. Lett. B* **342**, 171 (1995) [hep-ph/9409458].
- [7] J. R. Espinosa and M. Quiros, “Improved metastability bounds on the standard model Higgs mass,” *Phys. Lett. B* **353**, 257 (1995) [hep-ph/9504241].
- [8] J. A. Casas, J. R. Espinosa and M. Quiros, “Standard model stability bounds for new physics within LHC reach,” *Phys. Lett. B* **382** (1996) 374 [hep-ph/9603227].
- [9] A. Andreassen, W. Frost and M. D. Schwartz, “Consistent Use of Effective Potentials,” *Phys. Rev. D* **91**, no. 1, 016009 (2015) [1408.0287].
- [10] G. Isidori, G. Ridolfi and A. Strumia, “On the metastability of the standard model vacuum,” *Nucl. Phys. B* **609**, 387 (2001) [hep-ph/0104016].
- [11] M. B. Einhorn and D. R. T. Jones, “The Effective potential, the renormalisation group and vacuum stability,” *JHEP* **0704**, 051 (2007) [hep-ph/0702295].
- [12] J. R. Espinosa, G. F. Giudice and A. Riotto, “Cosmological implications of the Higgs mass measurement,” *JCAP* **0805**, 002 (2008) [0710.2484].
- [13] N. Arkani-Hamed, S. Dubovsky, L. Senatore and G. Villadoro, “(No) Eternal Inflation and Precision Higgs Physics,” *JHEP* **0803**, 075 (2008) [0801.2399].
- [14] F. Bezrukov and M. Shaposhnikov, “Standard Model Higgs boson mass from inflation: Two loop analysis,” *JHEP* **0907**, 089 (2009) [0904.1537].
- [15] J. Ellis, J. R. Espinosa, G. F. Giudice, A. Hoecker and A. Riotto, “The Probable Fate of the Standard Model,” *Phys. Lett. B* **679**, 369 (2009) [0906.0954].
- [16] J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori, A. Riotto and A. Strumia, “Higgs mass implications on the stability of the electroweak vacuum,” *Phys. Lett. B* **709**, 222 (2012) [1112.3022].
- [17] S. Alekhin, A. Djouadi and S. Moch, “The top quark and Higgs boson masses and the stability of the electroweak vacuum,” *Phys. Lett. B* **716**, 214 (2012) [1207.0980].
- [18] F. Bezrukov, M. Y. Kalmykov, B. A. Kniehl and M. Shaposhnikov, “Higgs Boson Mass and New Physics,” *JHEP* **1210**, 140 (2012) [1205.2893].
- [19] G. Degrassi, S. Di Vita, J. Elias-Miro, J. R. Espinosa, G. F. Giudice, G. Isidori and A. Strumia, “Higgs mass and vacuum stability in the Standard Model at NNLO,” *JHEP* **1208**, 098 (2012) [1205.6497].
- [20] D. Buttazzo, G. Degrassi, P. P. Giardino, G. F. Giudice, F. Sala, A. Salvio and A. Strumia, “Investigating the near-criticality of the Higgs boson,” [1307.3536].
- [21] L. Di Luzio, G. Isidori and G. Ridolfi, “Stability of the electroweak ground state in the Standard Model and its extensions,” *Phys. Lett. B* **753**, 150 (2016) [1509.05028].
- [22] C. Ford, I. Jack and D.R.T. Jones, “The Standard model effective potential at two loops,” *Nucl. Phys. B* **387**, 373 (1992) [Erratum-ibid. *B* **504**, 551 (1997)] [hep-ph/0111190].
- [23] S.P. Martin, “Three-loop Standard Model effective potential at leading order in strong and top Yukawa couplings,” *Phys. Rev. D* **89**, 013003 (2014) [1310.7553].
- [24] S. P. Martin, “Four-loop Standard Model effective potential at leading order in QCD,” *Phys. Rev. D* **92**, no. 5, 054029 (2015) [1508.00912].
- [25] S.P. Martin, “Two loop effective potential for a general renormalizable theory and softly broken supersymmetry,” *Phys. Rev. D* **65**, 116003 (2002) [hep-ph/0111209].
- [26] S. P. Martin, “Two loop effective potential for the minimal supersymmetric standard model,” *Phys. Rev. D* **66**, 096001 (2002) [hep-ph/0206136].
- [27] R. Hempfling and A. H. Hoang, “Two loop radiative corrections to the upper limit of the lightest Higgs boson mass in the minimal supersymmetric model,” *Phys. Lett. B* **331**, 99 (1994) [hep-ph/9401219].
- [28] R. J. Zhang, “Two loop effective potential calculation of the lightest CP even Higgs boson mass in the MSSM,” *Phys. Lett. B* **447**, 89 (1999) [hep-ph/9808299].
- [29] J. R. Espinosa and R. J. Zhang, “MSSM lightest CP even Higgs boson mass to $O(\alpha(s)\alpha(t))$: The Effective potential approach,” *JHEP* **0003**, 026 (2000) [hep-ph/9912236].
- [30] J. R. Espinosa and I. Navarro, “Radiative corrections to the Higgs boson mass for a hierarchical

- stop spectrum,” Nucl. Phys. B **615**, 82 (2001) [hep-ph/0104047].
- [31] G. Degrandi, P. Slavich and F. Zwirner, “On the neutral Higgs boson masses in the MSSM for arbitrary stop mixing,” Nucl. Phys. B **611**, 403 (2001) [hep-ph/0105096].
 - [32] A. Brignole, G. Degrandi, P. Slavich and F. Zwirner, “On the $O(\alpha(t)^2)$ two loop corrections to the neutral Higgs boson masses in the MSSM,” Nucl. Phys. B **631**, 195 (2002) [hep-ph/0112177].
 - [33] A. Brignole, G. Degrandi, P. Slavich and F. Zwirner, “On the two loop sbottom corrections to the neutral Higgs boson masses in the MSSM,” Nucl. Phys. B **643**, 79 (2002) [hep-ph/0206101].
 - [34] S. P. Martin, “Taming the Goldstone contributions to the effective potential,” Phys. Rev. D **90**, no. 1, 016013 (2014) [1406.2355].
 - [35] J. Elias-Miro, J. R. Espinosa and T. Konstandin, “Taming Infrared Divergences in the Effective Potential,” JHEP **1408**, 034 (2014) [1406.2652].
 - [36] H. H. Patel and M. J. Ramsey-Musolf, “Baryon Washout, Electroweak Phase Transition, and Perturbation Theory,” JHEP **1107**, 029 (2011) [1101.4665].
 - [37] A. Pilaftsis and D. Teresi, “Symmetry Improved 2PI Effective Action and the Infrared Divergences of the Standard Model,” J. Phys. Conf. Ser. **631**, no. 1, 012008 (2015) [1502.07986].
 - [38] A. Pilaftsis and D. Teresi, “Symmetry-Improved 2PI Approach to the Goldstone-Boson IR Problem of the SM Effective Potential,” Nucl. Phys. B **906**, 381 (2016) [1511.05347].
 - [39] S. P. Martin, “A supersymmetry primer,” [hep-ph/9709356] (version 7, 2016).
 - [40] M. B. Einhorn and D. R. T. Jones, “Scale Fixing by Dimensional Transmutation: Supersymmetric Unified Models and the Renormalization Group,” Nucl. Phys. B **211**, 29 (1983).
 - [41] B. M. Kastening, “Renormalization group improvement of the effective potential in massive ϕ^4 theory,” Phys. Lett. B **283**, 287 (1992).
 - [42] M. Bando, T. Kugo, N. Maekawa and H. Nakano, “Improving the effective potential,” Phys. Lett. B **301**, 83 (1993) [hep-ph/9210228]. “Improving the effective potential: Multimass scale case,” Prog. Theor. Phys. **90**, 405 (1993) [hep-ph/9210229].
 - [43] S. P. Martin, “Strong and Yukawa two-loop contributions to Higgs scalar boson self-energies and pole masses in supersymmetry,” Phys. Rev. D **71**, 016012 (2005) [hep-ph/0405022].
 - [44] S. P. Martin and M. T. Vaughn, “Two loop renormalization group equations for soft supersymmetry breaking couplings,” Phys. Rev. D **50**, 2282 (1994) Erratum: [Phys. Rev. D **78**, 039903 (2008)] [hep-ph/9311340].
 - [45] Y. Yamada, “Two loop renormalization group equations for soft SUSY breaking scalar interactions: Supergraph method,” Phys. Rev. D **50**, 3537 (1994) [hep-ph/9401241].
 - [46] I. Jack and D. R. T. Jones, “Soft supersymmetry breaking and finiteness,” Phys. Lett. B **333**, 372 (1994) [hep-ph/9405233].
 - [47] I. Jack, D. R. T. Jones, S. P. Martin, M. T. Vaughn and Y. Yamada, “Decoupling of the epsilon scalar mass in softly broken supersymmetry,” Phys. Rev. D **50**, 5481 (1994) [hep-ph/9407291].
 - [48] H. E. Haber and R. Hempfling, “Can the mass of the lightest Higgs boson of the minimal supersymmetric model be larger than $m(Z)$,” Phys. Rev. Lett. **66**, 1815 (1991).
 - [49] Y. Okada, M. Yamaguchi and T. Yanagida, “Upper bound of the lightest Higgs boson mass in the minimal supersymmetric standard model,” Prog. Theor. Phys. **85**, 1 (1991).
 - [50] J. R. Ellis, G. Ridolfi and F. Zwirner, “On radiative corrections to supersymmetric Higgs boson masses and their implications for LEP searches,” Phys. Lett. B **262**, 477 (1991).
 - [51] S. Heinemeyer, W. Hollik and G. Weiglein, “QCD corrections to the masses of the neutral CP-even Higgs bosons in the MSSM,” Phys. Rev. D **58**, 091701 (1998) [hep-ph/9803277]; “Precise prediction for the mass of the lightest Higgs boson in the MSSM,” Phys. Lett. B **440**, 296 (1998) [hep-ph/9807423]; S. Heinemeyer, W. Hollik and G. Weiglein, “FeynHiggs: A Program for the calculation of the masses of the neutral CP even Higgs bosons in the MSSM,” Comput. Phys. Commun. **124**, 76 (2000) [hep-ph/9812320]; “The Masses of the neutral CP - even Higgs bosons in the MSSM: Accurate analysis at the two loop level,” Eur. Phys. J. C **9**, 343 (1999) [hep-ph/9812472].
 - [52] R. V. Harlander, P. Kant, L. Mihaila and M. Steinhauser, “Higgs boson mass in supersymmetry to three loops,” Phys. Rev. Lett. **100**, 191602 (2008) [Phys. Rev. Lett. **101**, 039901 (2008)] [0803.0672].
 - [53] P. Kant, R. V. Harlander, L. Mihaila and M. Steinhauser, “Light MSSM Higgs boson mass to three-loop accuracy,” JHEP **1008**, 104 (2010) [1005.5709].
 - [54] J. S. Lee, M. Carena, J. Ellis, A. Pilaftsis and C. E. M. Wagner, “CPsuperH2.3: an Updated Tool for Phenomenology in the MSSM with Explicit CP Violation,” Comput. Phys. Commun. **184**, 1220 (2013) [1208.2212].
 - [55] T. Hahn, S. Heinemeyer, W. Hollik, H. Rzehak and G. Weiglein, “High-precision predictions

- for the light CP-even Higgs Boson Mass of the MSSM,” *Phys. Rev. Lett.* **112**, 141801 (2014) [1312.4937].
- [56] S. Borowka, T. Hahn, S. Heinemeyer, G. Heinrich and W. Hollik, “Momentum-dependent two-loop QCD corrections to the neutral Higgs-boson masses in the MSSM,” *Eur. Phys. J. C* **74**, no. 8, 2994 (2014) [1404.7074].
 - [57] W. Hollik and S. Pasetehr, “Two-loop top-Yukawa-coupling corrections to the Higgs boson masses in the complex MSSM,” *Phys. Lett. B* **733**, 144 (2014) [1401.8275]. “Higgs boson masses and mixings in the complex MSSM with two-loop top-Yukawa-coupling corrections,” *JHEP* **1410**, 171 (2014) [1409.1687].
 - [58] G. Degrassi, S. Di Vita and P. Slavich, “Two-loop QCD corrections to the MSSM Higgs masses beyond the effective-potential approximation,” *Eur. Phys. J. C* **75**, no. 2, 61 (2015) [1410.3432].
 - [59] S. Borowka, T. Hahn, S. Heinemeyer, G. Heinrich and W. Hollik, “Renormalization scheme dependence of the two-loop QCD corrections to the neutral Higgs-boson masses in the MSSM,” *Eur. Phys. J. C* **75**, no. 9, 424 (2015) [1505.03133].
 - [60] T. Hahn and S. Pasetehr, “Implementation of the $\mathcal{O}(\alpha_t^2)$ MSSM Higgs-mass corrections in *FeynHiggs*,” [1508.00562].
 - [61] M. S. Carena, M. Quiros and C. E. M. Wagner, “Effective potential methods and the Higgs mass spectrum in the MSSM,” *Nucl. Phys. B* **461**, 407 (1996) [hep-ph/9508343].
 - [62] J. R. Espinosa and R. -J. Zhang, “Complete two loop dominant corrections to the mass of the lightest CP even Higgs boson in the minimal supersymmetric standard model,” *Nucl. Phys. B* **586**, 3 (2000) [hep-ph/0003246].
 - [63] S. P. Martin, “Complete two loop effective potential approximation to the lightest Higgs scalar boson mass in supersymmetry,” *Phys. Rev. D* **67**, 095012 (2003) [hep-ph/0211366].
 - [64] A. Dedes and P. Slavich, “Two loop corrections to radiative electroweak symmetry breaking in the MSSM,” *Nucl. Phys. B* **657**, 333 (2003) [hep-ph/0212132].
 - [65] A. Dedes, G. Degrassi and P. Slavich, “On the two loop Yukawa corrections to the MSSM Higgs boson masses at large $\tan\beta$,” *Nucl. Phys. B* **672**, 144 (2003) [hep-ph/0305127].
 - [66] M. D. Goodsell, K. Nickel and F. Staub, “Two-Loop Higgs mass calculations in supersymmetric models beyond the MSSM with SARAH and SPheno,” *Eur. Phys. J. C* **75**, no. 1, 32 (2015) [1411.0675].
 - [67] H. K. Dreiner, K. Nickel and F. Staub, “On the two-loop corrections to the Higgs mass in trilinear R-parity violation,” *Phys. Lett. B* **742**, 261 (2015) [1411.3731].
 - [68] M. Goodsell, K. Nickel and F. Staub, “Generic two-loop Higgs mass calculation from a diagrammatic approach,” *Eur. Phys. J. C* **75**, no. 6, 290 (2015) [1503.03098].
 - [69] M. D. Goodsell and F. Staub, “The Higgs mass in the CP violating MSSM, NMSSM, and beyond,” [1604.05335].
 - [70] H. E. Haber and R. Hempfling, “The Renormalization group improved Higgs sector of the minimal supersymmetric model,” *Phys. Rev. D* **48**, 4280 (1993) [hep-ph/9307201].
 - [71] J. A. Casas, J. R. Espinosa, M. Quiros and A. Riotto, “The Lightest Higgs boson mass in the minimal supersymmetric standard model,” *Nucl. Phys. B* **436**, 3 (1995) [Erratum-ibid. B **439**, 466 (1995)] [hep-ph/9407389].
 - [72] M. S. Carena, J. R. Espinosa, M. Quiros and C. E. M. Wagner, “Analytical expressions for radiatively corrected Higgs masses and couplings in the MSSM,” *Phys. Lett. B* **355**, 209 (1995) [hep-ph/9504316].
 - [73] H. E. Haber, R. Hempfling and A. H. Hoang, “Approximating the radiatively corrected Higgs mass in the minimal supersymmetric model,” *Z. Phys. C* **75**, 539 (1997) [hep-ph/9609331].
 - [74] M. S. Carena, H. E. Haber, S. Heinemeyer, W. Hollik, C. E. M. Wagner and G. Weiglein, “Reconciling the two loop diagrammatic and effective field theory computations of the mass of the lightest CP - even Higgs boson in the MSSM,” *Nucl. Phys. B* **580**, 29 (2000) [hep-ph/0001002].
 - [75] S. P. Martin, “Three-loop corrections to the lightest Higgs scalar boson mass in supersymmetry,” *Phys. Rev. D* **75**, 055005 (2007) [hep-ph/0701051].
 - [76] P. Draper, G. Lee and C. E. M. Wagner, “Precise Estimates of the Higgs Mass in Heavy SUSY,” *Phys. Rev. D* **89**, 055023 (2014) [1312.5743].
 - [77] J. P. Vega and G. Villadoro, “SusyHD: Higgs mass Determination in Supersymmetry,” *JHEP* **1507**, 159 (2015) [1504.05200].
 - [78] P. Draper and H. Rzehak, “A Review of Higgs Mass Calculations in Supersymmetric Models,” *Phys. Rept.* **619**, 1 (2016) [1601.01890].