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Warped Seesaw is Physically Inverted

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Abstract

Warped extra dimensions can address both the Planck-weak and flavor hierarchies of the Standard Model (SM). In this paper we discuss the SM neutrino mass generation in a scenario in which a SM singlet bulk fermion — coupled to the Higgs and the lepton doublet near the IR brane — is given a *Majorana* mass of order the Planck scale on the UV brane. Despite the resemblance to a type I seesaw mechanism, a careful investigation based on the *mass* basis for the singlet 4D modes reveals a very different picture. Namely, the SM neutrino masses are generated dominantly by the exchange of the *TeV*-scale mass eigenstates of the singlet, that are pseudo-Dirac and have a sizable Higgs-induced mixing with the SM doublet neutrino: remarkably, in warped 5D models the anticipated type I seesaw morphs into a *natural* realization of the so-called "inverse" seesaw. This understanding uncovers an intriguing and direct link between neutrino mass generation (and possibly leptogenesis) and *TeV*-scale physics. We also perform estimates using the dual CFT picture of our framework, which back-up our 5D calculation.

1 Motivation and summary

The Randall-Sundrum (RS1) model [1] with a warped extra dimension [in particular, five-dimensional (5D) anti de-Sitter space (AdS)], coupled with an appropriate mechanism [2] to stabilize the size of the extra dimension, provides an attractive solution to the Planck-weak hierarchy problem of the Standard Model (SM). The basic idea is that localizing the SM Higgs boson near the IR brane results in scale of its vacuum expectation value (VEV) being warped-down to the $\sim$ TeV scale relative to that of 4D graviton (i.e., the Planck scale) which is localized near the UV brane. By the correspondence between AdS space and conformal field theories (CFTs) in lower space-time dimension [3], this idea is dual to a purely 4D theory, where the SM Higgs boson is a composite of some new strong dynamics [4].

In addition, the warped framework with the SM fermions arising as zero-modes of fermion fields propagating in the extra dimension can also account for the *charged* fermion mass and mixing angle (flavor) hierarchies of the SM as follows [6, 7, 8]. The effective 4D Yukawa couplings are dictated by the *overlap* of fermion zero-mode profiles with the Higgs boson, the latter being localized near/on the TeV/IR brane. The crux of this idea is that small changes in the *five*-dimensional (5D) mass parameters can result in large variations in the (extra-dimensional) profiles of the fermion zero modes at the TeV brane, thus (easily) generating the desired hierarchies in these Yukawa couplings, i.e., the SM fermion masses. It is interesting that such a scenario for SM fermions is dual to SM fermions being partially composite also [9], to degrees determined by scaling dimensions of the fermionic operators to which they couple (this scaling dimension is dual to the 5D mass parameter). The point then is that the coupling to Higgs is dictated by the amount of composite admixture in SM fermions, which can be hierarchical even with small differences in the scaling dimensions of the fermionic operators, provided there is a large energy range for the associated renormalization group evolution (RGE). Of course, 5D fermions necessitate 5D gauge fields [5].

In such a "bulk" SM in warped extra dimension (see also [10]), there are also Kaluza-Klein (KK) excitations of SM particles, which have masses starting at and quantized in units of roughly TeV scale and profiles which are peaked near the TeV brane. These new particles inherently contribute to various types of precision tests of the SM. Thus, there are indirect constraints on the KK mass scale in this model; the worry being that KK scale much larger than $\sim$ TeV will jeopardize the solution to the Planck-weak hierarchy problem. Those from electroweak tests can be controlled by suitable custodial symmetries [11], allowing a few TeV KK scale [12]. As far as flavor violation is concerned, there is a built-in suppression of such effects in this framework, roughly an analog of Glashow-Iliopoulos-Maiani (GIM) mechanism in the SM [7, 8, 13]. Still, KK scale above $\sim$ 10 TeV might be required (modulo the option of fine-tuning of flavor parameters) in order to be consistent with flavor precision data [14].

Of course, this situation can be mitigated by use of appropriate flavor symmetries [15] such that a few TeV KK mass scale can be once again allowed[1]. For a review of the framework and its phenomenology (and more references), see, for example, [16].

In this paper, we study the SM *neutrino* masses in this framework: clearly there are two options to begin with, namely, Dirac or Majorana type mass. For Majorana neutrinos, an incarnation of the standard type I seesaw mechanism [17] has been incorporated in the warped extra dimensional framework [18, 19, 20]: we will focus only on this model in this paper.[2] In this model, SM singlet neutrinos (denoted generically by N) are added in the bulk to the above framework of SM-charged fermions, aka the "right-handed" (RH) neutrino in the 4D case, even though it gives massive 4D modes with *both* chiralities in the 5D version (a fact which will turn out to be crucial in our work). This singlet neutrino field has a coupling to lepton doublet and Higgs on (or near) the IR brane, from which the singlet neutrino 5D field acquires a Dirac mass term with the doublet (or LH) neutrino field once EW symmetry breaking (EWSB) occurs, i.e., Higgs develops a VEV (just like for charged SM fermions). However, the *difference* from charged fermion case is that we assume that lepton-number is broken only on the UV brane (i.e., it is still a good symmetry in the bulk and on the TeV brane). This choice essentially manifests itself as a Majorana mass term for the UV brane-localized value of the bulk singlet neutrino field. (Obviously, no such mass terms are allowed for the charged fermions.)

Note that adding a Majorana mass term (or lepton-number violation) only on the UV brane is technically natural by 5D locality. It is also quite generic in scenarios where the bulk EW gauge group is extended to $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ in order to satisfy bounds from EW precision tests [11]. Here $SU(2)_R \times U(1)_{B-L}$ is spontaneously broken down to $U(1)_Y$ (hypercharge of the SM) on the Planck brane, either by boundary conditions or Planckian VEV of a localized scalar (this is equivalent to the former case in the large VEV limit), whereas $SU(2)_L \times U(1)_Y \to U(1)_{\rm EM}$ occurs by the Higgs VEV localized near the IR brane. In this setup N will be typically charged under $SU(2)_R \times U(1)_{B-L}$[3] while remaining neutral under the SM gauge group. Such a choice of the bulk gauge symmetry (and breaking) implies that a Majorana mass term for N, which would break $SU(2)_R \times U(1)_{B-L}$, is only allowed on Planck brane, i.e., it is forbidden in the bulk and on TeV brane.

We contextualize our contribution by first recapitulating the approaches used in previous studies. It turns out that most of the earlier studies of this model [18, 20] were performed

[1] In addition, there are lower bounds on the KK scale from absence of any signal of *direct* production of these KK particles at the LHC, but those from run 1 are still below the few TeV limit that we get from precision tests.

[2] For other scenarios (for either Dirac or Majorana case) see, for example, references [6, 21, 24]. We will comment on models with a bulk Majorana mass for the singlet at the end of this section.

[3] In fact, in the canonical case, this SM singlet simply corresponds to the $SU(2)_R$ doublet partner of the charged RH lepton, i.e., it is not added "by hand", rather its presence is required by the bulk gauge symmetry.

employing the "usual" (i.e., similarly to the *charged*-SM fermions) KK modes of the SM *singlet* field as the basis, where the above-mentioned Planck brane localized Majorana mass term is treated as a (not necessarily small) "perturbation" or at the least an "add-on": we will call this simply the "KK" basis. [4]

In more detail, in these earlier papers the KK decomposition for *singlet* field [5] is performed *neglecting* the Majorana mass on UV brane, giving zero (chiral) and massive, Dirac (KK) modes, just like for doublet lepton and, in general, SM charged fermion fields. Afterwards, turning on the Planck brane localized Majorana mass term results in the would-be zero-mode acquiring a large Majorana mass. Furthermore, it leads to mixing (via Majorana mass terms) *among* the would-be zero *and* (already massive) KK modes so that clearly the would-be zero modes and KK modes are *not* the mass eigenstates. Finally, including EWSB leads to mass terms between the SM neutrino and the entire tower of singlet modes; integrating out the latter then generates a mass for the SM neutrino, which is thus purely Majorana in nature, deriving from the above-mentioned Majorana mass terms for the singlet modes.

The *advantages* of the KK basis are its familiarity (from the numerous studies of *charged* fermion masses, where of course such Majorana mass terms are absent). As we will detail in what follows, it is perhaps the quickest/easiest way to obtain the SM neutrino mass formula in the 5D model. Indeed, the exchange of non-zero KK singlet modes with Dirac mass terms quantized in units of TeV-scale gives *negligible* contribution to the SM neutrino mass (*inspite* of these modes having Majorana mass terms also): almost all of this effect then comes instead from the would-be *zero*-mode (i.e., *no* Dirac mass term), with a super-large Majorana mass term. This "anatomy" of the SM neutrino mass gives it the appearance of *type I high-scale* seesaw.

In addition, the "intermediate" seesaw scale which is typically needed in type I high-scale seesaw models for obtaining the right SM neutrino mass can be naturally realized in the 5D model, i.e., even with input parameters being Planckian, via a natural choice of 5D mass of the singlet. In contrast, in 4D models such a seesaw scale often has to be introduced as a "new" scale.

In this paper, we *re*-consider the model using the *mass* basis (instead of the above KK one) for the singlet 4D modes, neglecting the mass mixing with doublet due to Higgs VEV.

[4]An exception is reference [19], which employed the *full* mass basis, i.e., for all modes (entire tower) of neutrinos (i.e., diagonalizing *also* the effect of doublet and singlet mixing due to EWSB, which we neglect here to begin with, rather it can be genuinely treated as a insertion/perturbation). However, this study focussed only on mass of the *lightest* (i.e., mostly SM) neutrino state, i.e., it did *not* (at least explicitly) work out the spectrum of heavier states. Hence, the "inner workings" of the SM neutrino mass, whose exchange is responsible for its generation, is not clear from such an analysis.

[5]At leading order in Higgs VEV, the *doublet* lepton *KK* modes will play no role in the generation of the SM neutrino mass, no matter which basis we use. So, we will only keep the doublet *zero*-mode, i.e., (approximately) the SM doublet lepton, from now on.

The reason is that this is the basis necessary to analyze processes involving *on*-shell singlet neutrinos, such as direct collider signals of singlet neutrino states and leptogenesis [25].

What we find is that the character of the seesaw is "changed" when the mass basis is employed! Namely, even though the SM neutrino mass is obtained exchanging the mass eigenstates of the singlet (similarly to exchanging would-be zero and KK modes), we show that

- the *TeV*-scale *mass* eigenstates of the singlet actually give a *significant* contribution to the SM neutrino mass (the end result being of course the same as in KK basis); in fact, this is the *dominant* effect for the natural versions of the model.

Given *also* their *un*suppressed Yukawa couplings to Higgs and the SM neutrino (following from their profile leaning towards TeV brane, where Higgs is also localized), at first sight, it seems somewhat counter-intuitive that the SM neutrino mass comes out very small: indeed, this is due to these modes being mostly Dirac, i.e., with a highly suppressed Majorana mass term.

A similar mechanism in *four* dimensions goes by the name "inverse" seesaw [22], i.e., where the very small SM neutrino mass arises from exchange of (possibly TeV-mass) singlet mode which is pseudo Dirac and has sizable EWSB mass term with the SM neutrino. Thus, we discover that, in mass basis, the dynamical picture of a seemingly high-scale type-I seesaw model in warped 5D is that of an "inverse" see-saw. Actually, it is crucial that the Majorana mass term for these TeV-mass modes in the 5D model is *naturally* small, as opposed to generic 4D inverse seesaw models, where such a smallness can be rather an ad-hoc assumption.

Phenomenologically, we then see that – for the purpose of leptogenesis or probing *directly* the mechanism of the SM neutrino mass generation in this 5D model by producing the responsible singlet states at the LHC/future colliders – the center of attention becomes TeV-mass singlet modes, as in the usual/4D inverse seesaw models.

Furthermore, the CFT interpretation of this seesaw model has not been discussed in the literature thus far, even though the charged SM fermion case has been thoroughly studied in this way, providing physical intuition to the problem. Such a dual CFT description of warped seesaw for neutrino masses will be similarly extremely useful, offering an alternative picture for SM neutrino mass generation. In fact, we find that

- the CFT viewpoint allows us to quickly unveil the true nature of the seesaw mechanism and clarifies the naturalness of the small Majorana component of the TeV-scale mass eigenstates.

We end this section with a comment on scenarios in which the singlet is given a bulk Majorana mass. A major difference compared to the models we analyze in this paper is that

Basis → Features ↓	KK (would-be mass modes *neglecting* UV brane Majorana mass term)	mass (for singlet only, i.e., neglecting Higgs VEV)	CFT [N_R (external) and composites (with 2 sectors *mixing*)]
Advantage/Use	familiar from charged fermion analysis; easy to obtain m_ν	needed for *on*-shell production (LHC and/or leptogenesis)	elucidates seesaw structure easy to obtain m_ν "bridge" between mass and KK bases
Nature of seesaw (details below)	Type I (high-scale) (for *both* $c_N < -1/2$ and $> -1/2$)	(Dominantly) inverse for $c_N > -1/2$ "Combination" for $c_N < -1/2$	(Significantly) inverse (for *both* $\left[\mathcal{O}_N\right] > 5/2$ and $< 5/2$)
fraction of (net) m_ν from $\sim$ TeV-scale modes	0 (from each *Dirac* mode)	≈ 1 (~ 1) for $c_N > (<) - 1/2$ (from pseudo-Dirac *pair*)	~ 1 (for *both* $\left[\mathcal{O}_N\right] > 5/2$ and $< 5/2$) (from each Dirac composite)
heavy (Majorana) mode Mass for $c_N > -1/2$ Mass for $c_N < -1/2$ fraction of (net) m_ν	would-be zero-mode, *not* mass eigenstate $M_N^{\rm UV} \times \left(\frac{\rm TeV}{M_{\rm Pl}}\right)^{2\,c_N+1}$ $M_N^{\rm UV}$ 1	"special/single" mode $M_N^{\rm UV} \times \left(\frac{M_N^{\rm UV}}{M_{Pl}}\right)^{\frac{1}{-2\,c_N}-1}$ $M_N^{\rm UV}$ $\ll 1$ for $c_N > -1/2$ $\gg 1$ ("cancels" $\gg 1$ below) for $c_N < -1/2$	external N_R $M_N^{\rm bare}\left(\frac{\mu}{M_{\rm Pl}}\right)^{5-2\left[\mathcal{O}_N\right]}$ $M_N^{\rm bare}$ 0
fraction of (net) m_ν from *sum* of intermediate modes	0	$\ll 1$ for $c_N > -1/2$ $\gg 1$ for $c_N < -1/2$	unknown (for *both* $c_N < -1/2$ and $> -1/2$)

Table 1: A comparison of the three bases used for studying this model. Note that the bulk mass for singlet field in the 5D model (c_N) is dual (in the CFT picure) to $\left(2 - \left[\mathcal{O}_N\right]\right)$, where $\left[\mathcal{O}_N\right]$ is the scaling dimension of the singlet operator in the CFT basis. Whereas, the Majorana mass on the Planck brane in the 5D model ($M_N^{\rm UV}$) corresponds to the bare mass for the external singlet ($M_N^{\rm bare}$) in the CFT interpretation.

in the former case a sizable bulk mass would significantly distort the spectrum of the KK modes and produce a tower of Majorana states, as opposed to pseudo-Dirac. Unfortunately, this is not a phenomenologically viable option because the SM neutrinos would acquire a large mass as well. A realistic model can be obtained taking a very tiny bulk Majorana mass, which corresponds to making a tuning roughly of order the UV/IR hierarchy. Then one can safely treat the bulk Majorana mass as a perturbation of the KK basis, whose leading effect is the generation of small Majorana mass splitting and lepton-number violating couplings for the 4D modes of the singlet. From a dual CFT perspective, this is equivalent to assume that there exists a tiny violation of the lepton number within the large N dynamics. We thus see that models with a bulk Majorana mass reproduce the SM neutrino masses precisely as in the 4D inverse seesaw mechanism, and still at the price of tuning. On the other hand, 5D scenarios with an UV-localized Majorana mass offer a theoretically compelling justification for the smallness of the SM neutrino masses.

Here is the outline for the rest of this paper. We begin with a review of the above 5D model, setting-up our notation in section 2. In order to set the stage for our new analysis, it is necessary to first give a more extensive review of the various related results from earlier literature, namely, that of the KK basis calculation done earlier. We do this in section 3. We then move onto *our* findings.

Our mass basis calculation of the SM neutrino mass is given in section 4; this is a somewhat tedious procedure and so we begin (subsection 4.1) with a qualitative *summary* of the subsequent results, followed by setting-up the mass basis in subsection 4.2. The main results are summarized in subsection 4.3. In table 1 we give a snapshot of the features in each of the three bases mentioned above (i.e., KK, mass and CFT). Each entry will be clarified below. The full details of 5D calculation are relegated to Appendix A.

In section 5 we scrutinize the 5D model from a 4D CFT perspective. We finally present our conclusions in section 6, where we also discuss some directions for future work.

2 The 5D Model

We consider a slice of AdS_5 geometry described by the following metric:

$$ds^2 = \left(\frac{R}{z}\right)^2 \eta_{ab}\, dx^a dx^b, \tag{1}$$

where $\eta_{ab} = \mathrm{diag}(+,-,-,-,-)$ and $x^a = (x^\mu, z)$, with $\mu = 0,1,2,3$ and the fifth coordinate confined within the interval $R \leqslant z \leqslant R'$, where R is the AdS curvature radius. [6] At the boundary $z = R$ (R') we locate a UV (IR) brane. The SM fermions are in the bulk and, for simplicity, the SM Higgs boson is taken to be localized on the IR brane, although we think that the arguments presented here can be straightforwardly generalized, giving similar results, as long as the Higgs boson is peaked towards the IR brane.

In order to be consistent with bounds from EW precision tests, we consider a minimally extended bulk gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ with $SU(2)_R \times U(1)_{B-L}$ spontaneously broken down to $U(1)_Y$ on the UV brane. Since detailed dynamics responsible for such a spontaneous breaking is not of central interest here, we will not discuss it for brevity. However, it is worth to mention that in this framework the SM singlet neutrino is charged under $SU(2)_R \times U(1)_{B-L}$. Since the Majorana mass term for the singlet breaks this gauge symmetry it can appear only on the UV brane.

The quadratic action for SM singlet neutrino [7] in the background of Eq. (1), including a UV-localized Majorana mass ($S_{\rm UV}$), is:

$$\begin{aligned} S &= \int d^5x \sqrt{g} \left\{ \frac{i}{2} \left(\bar\Psi e^M_a \gamma^a D_M \Psi - D_M \bar\Psi e^M_a \gamma^a \Psi \right) - m_D \bar\Psi \Psi \right\} + S_{\rm UV} \qquad (2) \\ &= \int d^5x \left(\frac{R}{z}\right)^4 \left\{ -i\bar\chi\, \bar\sigma^\mu \partial_\mu \chi - i\psi\, \sigma^\mu \partial_\mu \bar\psi + \frac{1}{2}\left(\psi \overleftrightarrow{\partial_5} \chi - \bar\chi \overleftrightarrow{\partial_5} \bar\psi \right) + \frac{c_N}{z}\left(\psi\chi + \bar\chi\bar\psi \right) \right\} + S_{\rm UV}. \end{aligned}$$

In the first line the Fünfbein reads $e^a_M = (R/z)\delta^a_M$, $D_M = \partial_M + \omega_M$ with the spin connection given by $\omega_M = \left(\frac{\gamma_\mu\gamma_5}{4z}, 0\right)$. For the gamma matrices we use the conventions of [19]:

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar\sigma^\mu & 0 \end{pmatrix} \qquad \sigma^0 = -1, \quad \gamma^5 = \begin{pmatrix} i1 & 0 \\ 0 & -i1 \end{pmatrix}. \tag{3}$$

[6] As a reference it is useful to recall that much of the literature uses the equivalent line element $ds^2 = e^{-2ky}\eta_{\mu\nu}dx^\mu dx^\nu - dy^2$, with $0 \le y \le \frac{1}{k}\ln(kR')$ related to ours by $z = \frac{e^{ky}}{k}$ and $k = 1/R$.

[7] For simplicity, we describe *one* generation, but our analysis can be easily extended to more.

In the second line we explicitly wrote the action in terms of Weyl spinors:

$$\Psi = \begin{pmatrix} \chi_\alpha \\ \bar{\psi}^{\dot{\alpha}} \end{pmatrix},$$

and defined the real number $c_N \equiv m_D R$, and $\overleftrightarrow{\partial_5} \equiv \overrightarrow{\partial_5} - \overleftarrow{\partial_5}$.

The UV-localized Majorana mass term is defined as a quadratic term for ψ:

$$S_{\rm UV} = \int d^5x \left(\frac{R}{z}\right)^4 \frac{d}{2}\delta(z-R)\psi\psi + {\rm hc}, \tag{4}$$

where $d \equiv M_N^{\rm UV} R$.

We also introduce a coupling between Ψ, a Higgs $\mathcal{H}$ localized on the IR-brane at $z = R'$, and the electroweak doublet 5D field Ψ_L:

$$\delta S \;=\; -\int d^4x \int dz \left(\frac{R}{z}\right)^4 \delta(z-R')\lambda_5 \mathcal{H}\Psi_L\Psi \tag{5}$$

where λ_5 is 5D Yukawa coupling with mass dimension -1. In our notation $c_{N,L}$ denote the 5D mass parameters for RH (singlet) and LH (doublet) neutrinos (which, in turn, determine profiles for zero-modes in the extra dimension). We will follow convention that $c_L = 1/2$ ($c_N = -1/2$) is constant profile for the LH (RH) zero mode, $c_L > 1/2$ ($c_N < -1/2$) being localized close to the Planck brane. Values $c_L \gtrsim 1/2$ are expected to explain the smallness of the charged lepton masses [8]

All dimensionful parameters are taken to be $O(1)$ in units of AdS curvature scale ($k \equiv 1/R$) and in turn, the latter mass scale is set to be the 4D Planck mass scale (denoted by $M_{\rm Pl}$). In the following, by "TeV scale", we tacitly mean the scale $1/R'$ which sets size of KK masses.

3 SM neutrino mass using KK basis

In this section, we will first review previous results obtained using what we call the KK basis and present our new work in the following section. As outlined in the introduction, this KK basis is characterized by an *a-posteriori* consideration of the effects of the UV brane Majorana mass term on the modes (both zero and massive KK) which had been obtained with*out* this UV brane mass term: essentially this "addition" generates Majorana mass terms for all these modes: see, for example, reference [18].[9]

[8]There might be some leeway here, due to the profile of RH charged lepton. In any case, formulae below can be easily generalized to $c_L < 1/2$ by replacing $\sim ({\rm TeV}/M_{\rm Pl})^{c_L - 1/2}$ by $\sim \sqrt{1/2 - c_L}$.

[9]Note that in the literature, there are usages of "KK" basis with other meanings, for example, while dealing with charged fermions (i.e., no Majorana mass!), some authors denote by it the mass (i.e., physical) basis *before* taking into account EWSB (Higgs VEV), i.e., doublet and singlet modes are separate, whereas some others reserve it for the final, i.e., post-EWSB, mass basis. Once again, our KK basis for *singlet* is the one with*out* taking into account *both* Majorana mass term on Planck brane *and* mass mixing with doublet leptons via EWSB.

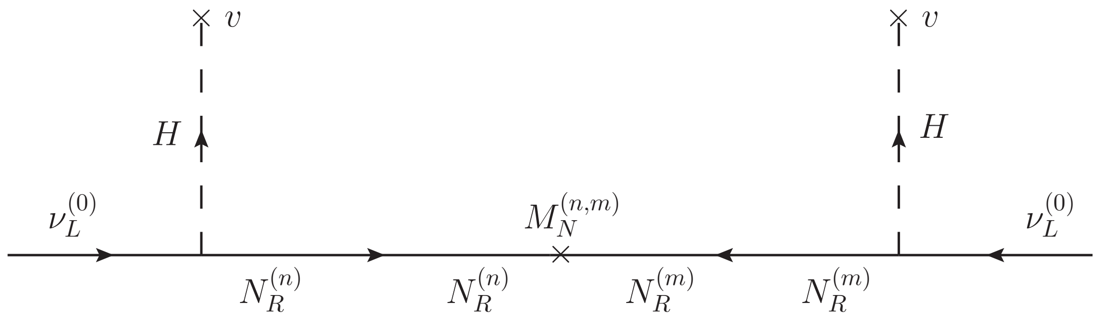

Figure 1: The (vanishing) SM neutrino mass contribution from exchange of massive/KK modes in KK basis, where $M_N^{(n,m)}$ (n, $m \neq 0$) denote Majorana mass terms.

To begin with, we provide a simple derivation – using equations of motion (EOM) – of the formula for the SM neutrino mass. The result that we are about to derive was already obtained and used in earlier works [18, 20]; rather than following the approach used in the literature we present a different one, that makes the relevant physics more transparent.

We use 4-component Dirac spinors notation, with $N_R^{(0)}$ being singlet *chiral* zero-mode, $N_R^{(n\neq0)}$ being singlet *non*-zero KK modes (Dirac i.e. have both L and R chiralities) and $\nu_L^{(0)}$ being (doublet) SM neutrino (left-handed only). We have the following mass terms

$$\mathcal{L}_{\text{mass}} = \sum_{n,m=0,1,2\ldots} \frac{1}{2} M_N^{(n,m)} \overline{\left[N^{(n)\,c}\right]_L} N_R^{(m)} + \sum_{n=1,2\ldots} m_n \overline{N_L^{(n)}} N_R^{(n)} + \sum_{m=0,1\ldots} m_D^{(0,m)} \overline{\nu_L^{(0)}} N_R^{(m)} + \text{h.c.} \tag{6}$$

where $m_D^{(0,m)}$ is the (effective) Dirac mass for the two *different* types of neutrino modes induced by the Higgs VEV. These EWSB-induced mass terms are given simply by 5D Yukawa coupling (along with Higgs VEV) multiplied by product of profiles of LH (zero) and RH (zero or KK, labelled m) neutrino modes at the IR brane. Similarly, $M_N^{(n,m)}$ are Majorana mass terms between various singlet modes, obtained by multiplying the Majorana mass term on the UV brane by relevant profiles at the UV brane. Finally, m_n are the usual Dirac masses for the non-zero KK modes. [10]

We simply use equation of motion for $N_L^{(n\neq0)}$ which implies $N_R^{(n\neq0)} = 0$, since only term in Lagrangian involving $N_L^{(n)}$ is the KK mass with $N_R^{(n)}$. Whereas, EOM for $N_R^{(0)}$ sets itself to $\nu_L^{(0)} m_D^{(0,0)} / M_N^{(0,0)}$. Plugging these expressions for $N_R^{(n)}$ ($n = 0, 1\ldots$) back into the Lagrangian

[10] In [18] the Dirac masses are denoted by D_n (our m_n). The Majorana mass terms between singlet modes, which we denoted as $M_N^{(n,m)}$, is denoted A_{nm}. Finally, the Dirac mass between LH zero mode and RH zero/KK modes, which we called $m_D^{(0,m)}$, is denoted C_{0n} in [18].

we get

$$\mathcal{L} \;\ni\; -\frac{1}{2}\frac{\left[m_D^{(0,0)}\right]^2}{M_N^{(0,0)}}\overline{\nu_L^{(0)}}\left[\nu^{(0)\,c}\right]_R \tag{7}$$

Equivalently, we can represent the use of EOM's by Feynman diagrams: see Fig. 1. In this KK basis, it is the right chirality of the KK mode which couples to both Higgs VEV at one end and has Majorana mass term on the other side. Thus, we have to pick the "$\not{p}$" piece of propagator, which does *not* contribute to the mass term (again, despite the non-zero KK modes having Majorana mass terms). [11] This argument is *not* valid for $N_R^{(0)}$, so the entire contribution comes from the would-be zero-mode.

The formula for the SM neutrino mass from the would-be zero mode exchange looks like the usual, type I seesaw, i.e.,

$$m_\nu \;\equiv\; \frac{m_D^{\mathrm{eff}\;2}}{M_N^{\mathrm{eff}}} \tag{8}$$

where $m_D^{\mathrm{eff}} = m_D^{(0,0)}$ for the case of would-be zero mode, with

$$m_D^{(0,0)} \;\approx\; \begin{cases} a_{>-1/2} Y_5\; v \left(\frac{\mathrm{TeV}}{M_{\mathrm{Pl}}}\right)^{c_L-\frac{1}{2}} & \text{for } c_N > -\frac{1}{2} \\ a_{<-1/2} Y_5\; v \left(\frac{\mathrm{TeV}}{M_{\mathrm{Pl}}}\right)^{c_L-\frac{1}{2}} \times \left(\frac{\mathrm{TeV}}{M_{\mathrm{Pl}}}\right)^{-c_N-\frac{1}{2}} & \text{for } c_N < -\frac{1}{2} \end{cases} \tag{9}$$

where the superscript $(0,0)$ on m_D indicates that this is the mass term between two zero modes, obtained by combining their profiles at the TeV brane (we assumed $c_L > 1/2$ for simplicity here). Also, $Y_5 \equiv \lambda_5/R$ denotes the Yukawa coupling of brane-localized Higgs to bulk fermions in units of AdS curvature scale (k).

Here (and in what follows), we have kept track of *parametric* effects, i.e., relegating the $O(1)$ factors to seperate formulae:

$$a_{>-1/2} \;\approx\; \sqrt{\frac{(2c_N+1)(2c_L-1)}{2}} \tag{10}$$

$$a_{<-1/2} \;\approx\; \sqrt{\frac{(-2c_N-1)(2c_L-1)}{2}} \tag{11}$$

Similarly, the effective Majorana mass in Eq. (8) is given by the Majorana mass *term* of the would-be zero mode *with itself*, $M_N^{\mathrm{eff}} = M_N^{(0,0)}$ [12], with

$$M_N^{(0,0)} \;\approx\; M_N^{\mathrm{UV}} \times \begin{cases} b_{>-1/2}\left(\frac{\mathrm{TeV}}{M_{\mathrm{Pl}}}\right)^{1+2\,c_N} & \text{for } c_N > -\frac{1}{2} \\ b_{<-1/2} & \text{for } c_N < -\frac{1}{2} \end{cases}. \tag{12}$$

[11] However, the exchange of the KK mode can correct kinetic term for SM neutrino and this, after canonically normalizing kinetic term, will induce mass correction of order $\mathcal{O}(v^4)$, which is higher-order than $\mathcal{O}(v^2)$ contribution from exchange of would-be zero-mode.

[12] We emphasize that (see also next section) these KK basis modes are *not* the mass eigenstates; in order to make this point explicit, we denote this mass term as above, instead of simply $M_N^{(0)}$, which would give the impression that it is actually a physical *mass*.

namely, size of Majorana mass term on UV brane, denoted by M_N^{UV}, multiplied by (square of) the profile of the would-be zero mode for the RH neutrino at the UV brane this time. Once again, b's above are $O(1)$ factors, given by

$$b_{>-1/2} \approx (2c_N + 1) \tag{13}$$

$$b_{<-1/2} \approx -(2c_N + 1) \tag{14}$$

Plugging the singlet would-be zero mode Majorana mass from Eq. (12) and its Dirac mass with doublet zero mode from Eq. (9) into the "master" formula in Eq. (8), we get (for *both* $c_N <$ and $> -1/2$)

$$m_\nu \approx \left(c_L - \frac{1}{2}\right) \frac{Y_5^2 v^2}{M_N^{UV}} \left(\frac{\mathrm{TeV}}{M_{\mathrm{Pl}}}\right)^{2(c_L - c_N - 1)} \tag{15}$$

As promised, deriving formula for the SM neutrino mass is a very straightforward task in KK basis!

It is remarkable that the strong dependence on c_N is similar whether we consider $c_N < -1/2$ or $c_N > -1/2$. This requires more explanation. First of all, as can be seen from Eq(9), for $c_N < -1/2$, the Dirac mass is exponentially suppressed by the fact that the profile of RH singlet would-be zero mode is peaked at UV brane and highly suppressed at IR brane. On the other hand, the Dirac mass for $c_N > -1/2$ does not show any strong sensitivity in c_N, which again comes from the fact that the profile at IR brane is unsuppressed and has very little c_N-dependence in this case. In the case of Majorana mass, however, the situation is *interestingly* reversed (see Eq(12)). Namely, it is now $c_N > -1/2$ case that acquires exponential suppression and only a mild c_N-dependence for $c_N < -1/2$ (arising from the profile on the UV brane). After combining these two effects, one can now, at least intuitively, see that in both $c_N <$ and $> -1/2$ cases the SM neutrino mass gets strong c_N dependence as explicitly shown in Eq(15). What's really remarkable is that everything works out just right such that both cases reveal exactly the same c_N-dependence. In section 5, we will come back to this point and provide another way to understand it in a somewhat less coincidental manner. The above-mentioned results in KK basis are summarized in the left column of table 1.

Before moving to a study of the mass basis, we stress that in type I high-scale seesaw models (including the 5D realization above) there *appears* to be a "new hierarchy" of mass scales. This is because the (effective) seesaw scale needed is $\sim O\left(10^{12}\right)$ GeV, i.e., ~ 6 orders of magnitude smaller than Planck scale[13]. In order to achieve this in the 4D models,

[13] In other words, it is *not* enough to get small m_ν – which is accomplished by the basic seesaw mechanism for *any* high scale for singlet neutrino mass, but we need to get its right size as well, which requires seesaw scale to be high, but not as much as Planck scale!

one is usually forced to introduce *new* dynamics for this purpose, often requiring its own explanations. This is what would also happen in our model if we took $M_N^{\rm UV} \ll M_{\rm Pl}$. Importantly, in warped 5D models there is an interesting alternative. In fact, the desired seesaw scale can be obtained from Planckian-size $M_N^{\rm UV}$ *naturally*, it suffices to choose $|c_N|$ a bit smaller than $1/2$ for $M_N^{\rm eff}$ to be (much) smaller than the Planck scale. Specifically, in order to get the observed size of the SM neutrino masses, given that $c_L \sim 0.6$ is a "natural" choice[14] for reproducing charged lepton masses [i.e., $m_D^{(0,0)} \sim O(10 \text{ GeV})$][15], we can choose $c_N \sim -0.3 > -1/2$ so that for natural size of $M_N^{\rm UV}$ [namely $\sim O(M_{\rm Pl})$], we get $M_N^{\rm eff} \sim O\left(10^{12}\right)$ GeV, giving us $m_\nu \sim O(0.1)$ eV as required.

4 SM neutrino mass using mass basis

The reader must be warned that the KK basis is *not* even remotely close to the mass basis. Indeed, the Majorana mass term for low-lying (TeV-scale) KK modes can be much larger than KK (Dirac) mass itself:

$$M_N^{(1,1)} \quad \sim \quad M_N^{\rm UV} \times \begin{cases} \left(c_N + \frac{1}{2}\right)^2 \left(\frac{\rm TeV}{M_{\rm Pl}}\right)^{-2\,c_N - 1}, \text{ for } c_N < -1/2 \\ \left(c_N + \frac{1}{2}\right)^2 \left(\frac{\rm TeV}{M_{\rm Pl}}\right)^{2\,c_N + 1}, \text{ for } c_N > -1/2 \end{cases} \tag{16}$$

where we are interested in $c_N \sim -1/2$ and $M_N^{\rm UV} \lesssim M_{\rm Pl}$ so that (typically) $M_N^{(1,1)} \gg$ TeV. This demonstrates that the Majorana mass terms can*not* really be treated as a "perturbation" (i.e., that it should be included from the beginning).

We therefore decide to analyze the warped seesaw model using *mass* basis *directly*. Such a step is necessary for the study of direct production of singlet neutrino states at colliders, similarly for the consideration of their effects in the early universe (relevant perhaps for leptogenesis). Namely, we include the effect of the Majorana mass on the Planck brane a *priori* such that all modes are (from the start) Majorana[16]. The two approaches must of course agree on the final result. Nonetheless, we will see that this change of basis has some "surprises" in store for us that will elucidate the nature of the seesaw mechanism itself! An

[14] i.e., it can account for charged lepton mass hierarchies *and* suppress flavor violation with*out* any significant structure in the 5D Yukawa couplings, in addition to being safer from EW precision tests than $c_L < 1/2$.

[15] Note that this (i.e., neutrino) Dirac mass is only suppressed by *one* factor of doublet lepton profile, cf. charged lepton mass involving two such factors; that is why we can take $O(10 \text{ GeV})$ as Dirac mass term for neutrino, instead of $\sim O(\text{GeV})$ for charged lepton, say, τ, mass.

[16] Strictly speaking and as mentioned earlier, EWSB will actually further mix the singlet modes in this "mass" basis with doublet modes, but *that* effect can be genuinely treated as a perturbation, just like it is often done for charged SM fermions: we will neglect it – at this stage – for simplicity and so continue to call it the mass basis, again for the singlet modes *by themselves*. Of course, these EWSB-induced mass terms between the singlet modes and the doublet *zero*-mode (i.e., the SM neutrino) are crucial later, i.e., in generating mass for the SM neutrino.

intuitive understanding of our results immediately follows from the CFT interpretation in section 5.

4.1 Summary

We first give highlights of the mass basis analysis, before entering quantitative details in the next subsection.

It turns out that basically all the singlet mass eigenstates (except one) are "pseudo-Dirac", i.e., form pairs with (roughly) the "original" Dirac-like mass, but with very *small* mass splitting within each pair, induced by the Majorana mass term on the UV brane. This spectrum comes with a regular spacing between these pairs, given by $\sim$ TeV (the usual KK scale): in other words, each $\sim$ TeV interval (starting at $\sim$ TeV itself) in mass has 2 almost degenerate Majorana modes. In addition to the mass spectrum, we need to know the *couplings* to Higgs (and doublet lepton) of these singlet modes; they turn out to be *sizable*, given the localization of these mass eigenstates near TeV brane. These two properties (which are *qualitatively* similar for both $c_N <$ and $> -1/2$) can then be combined as done above in the KK basis in order to get the SM neutrino mass.

We find that using the **mass basis** points to a strikingly different underlying mechanism of the generation of SM neutrino mass, giving the same end result for the SM neutrino mass itself. First of all, in the mass basis, the contribution of $\sim TeV$ mass singlet states to the SM neutrino mass is similar in size (for both $c_N <$ and $> -1/2$) to the final result. Thus, even though it "started out" trying to be type I, the *same* 5D model (again, in the mass basis) is reminiscent of the so-called "inverse" seesaw mechanism in the context of (purely) 4D models [22]. Namely, *both* this 5D model and the 4D models in [22] (and follow-ups) are characterized by SM neutrino mass originating from exchange of a singlet mode(s) with very *small* Majorana mass term combined with its couplings to Higgs *not* being small! In other words, the mechanism for the generation of SM neutrino mass might be "closer at hand" than would have been anticipated in the KK basis: for example,

- the TeV mass singlet states, whose exchange generates the SM neutrino mass, can potentially be probed at the LHC (or future colliders).

Furthermore,

- for leptogenesis, the focus might be on the decay of these TeV singlet states, which does *not* require the universe to be reheated to temperatures (much) above a TeV, thus avoiding the issue of the (too slow) phase transition of the high temperature scenario. [17]

[17] It is known [23] that the transition from such a high-temperature phase (i.e., $\gg$ TeV) to the usual warped model below temperature of $\sim$ TeV might proceed *too* slowly, which might then become a bottleneck in implementing a standard (i.e., *high*-scale) leptogenesis scenario.

Overall, we thus see that the mass basis picture leads to a dramatic *shift* in the expected phenomenology. Indeed, from the KK basis one might erroneously be drawn to conclude that the physics which generates the SM Majorana neutrino mass can*not* be probed *directly* at the LHC (or foreseeable colliders), and that leptogenesis *would* require the universe to be reheated to temperatures (much) above a TeV, which might then pose a problem in these scenarios (as mentioned above). Our results show that none of this is true.

Note that reference [24] actually added an extra (i.e., beyond the N discussed above) singlet in the bulk to this model in order to implement inverse seesaw in 5D (which is the way it is done in usual, 4D models), but our claim here is that there is no "need" to do so. [18]

Next, we mention finer points about the mass basis analysis. For example, consider the "fate" (in the mass basis) of the would-be zero mode of the KK basis. We can show that there is indeed one mode which is *un*paired: it seems to not conform to the "one *pair*-per-TeV bin" rule. Hence, it is termed a "special" mode, with what one might therefore call a "purely" Majorana mass. It is somewhat tempting to "identify" it with the would-be zero mode of the KK basis discussed earlier. However, we find that this "mapping" is not quite accurate. After a careful calculation, we discover that

- (i) for $c_N > -1/2$, the special mode in the mass basis is *not* at the would-be zero mode mass, but instead is parametrically *higher* (while still being smaller than the Majorana mass term on the UV brane), with a coupling to the Higgs which is similar to would-be zero-mode however. Thus, its contribution to the SM neutrino mass is *negligible*. Similarly, we can show that the effect of the (much) heavier than $\sim$ TeV *paired* modes is small, i.e., sum over these mass eigenstates from bottom-up is convergent. Hence, we can indeed say that the SM neutrino mass is *dominantly* of inverse seesaw nature, i.e., it basically arises from exchange of $\sim$ TeV mass eigenstates mentioned above[19].

  (ii) $c_N < -1/2$: the special mode *is* in fact (roughly) at the would-be zero-mode mass. Nevertheless its coupling to Higgs is actually *un*suppressed, giving *too* large a contribution to the SM neutrino mass. However, we show that this contribution is similar in size to the effect of the other, i.e., *higher* than $\sim$ TeV, paired modes (i.e., this sum is now *not* dominated by the low-lying modes, cf. $c_N > -1/2$ case above).

[18] In more detail, in 4D inverse seesaw model, we consider *two* Weyl spinor singlets, which form a pseudo-Dirac state. Reference [24] attempted to mimic this in the 5D model by incorporating two (chiral) zero-modes, i.e., one from each of two (singlet) bulk fields. However, we see that such a "proliferation" of bulk singlets is actually not necessary since a *single* bulk field does have *two* chiralities at the *non*-zero mode level: we find that these form the required pseudo-Dirac state.

[19] Again, it *is* more than one pair of modes which contribute here, i.e., involving more like a "tower", albeit rapidly convergent, of inverse seesaws, but this is a minor variation with respect to the usual 4D model of this type.

We therefore *conjecture* that these two contributions (again, those of the single/special mode and the heavy, paired ones, with each of them being *too* large) cancels against one another, leaving behind that of the $\sim$ TeV modes mentioned above (which on its own is the "correct" size); in this sense, we have sort of a "hybrid" of inverse and type I seesaws here.

Finally, as far as the curious feature about dependence on c_N of the final SM neutrino mass is concerned, we can boil it down to

- the dependence on c_N of the Majorana mass *splitting* between the two ($\sim$ TeV) mass eigenstates in each pair being similar for $c_N > -1/2$ *and* $< -1/2$ (as mentioned above, this splitting is essentially what generates the bottomline SM neutrino mass for *both* ranges of c_N).

The picture arising from our mass basis calculation is summarized in the middle column of table 1.

4.2 Setting-up the calculation

We now show derivation of the above claims. Once again, in this approach, we take into account the Majorana mass term on the UV brane from the get-go so that *all* singlet modes are strictly speaking *Majorana*. The calculation is rather straightforward, even if tedious: see Appendix A for details. It turns out that these Majorana mass modes can be divided into two types: light modes and special modes. The low-lying (TeV-mass) modes come *in pairs* of pseudo-Dirac particles (a Weyl spinor with mass m and another of mass $\sim -m$) and similar couplings to the SM Higgs and SM doublet neutrino. We will denote the two modes within each pair (and values of their masses and couplings) by the subscripts $\pm$, respectively. Of course, we have an infinite tower of such modes, counted by $n = 1, 2, \cdots$, so each n actually stands for two, "$\pm$", modes. In addition, at a mass scale much larger than $\sim$ TeV (essentially dictated by Majorana mass term on UV brane, but appropriately modulated by profiles), we find an *un*paired/single mode, which we dub "special".

The single/special, Majorana mode (mass M_N^{special}, coupling y^{special} with Higgs and doublet neutrino zero-mode) gives the usual type I seesaw contribution to the SM neutrino mass

$$m_\nu^{\text{special}} = \frac{\left(v\, y^{\text{special}}\right)^2}{M_N^{\text{special}}}$$

as in Fig. 2 [with $(m + \Delta m) \to M_N^{\text{special}}$] , where $v\, y^{\text{special}}$ is the Dirac mass with doublet neutrino zero-mode as usual.

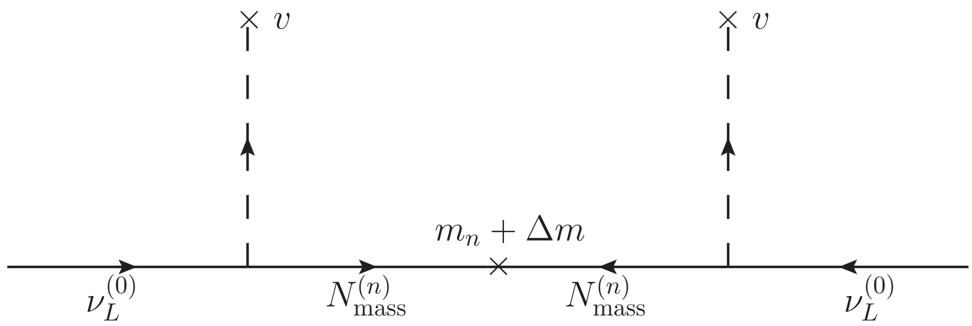

Figure 2: The SM neutrino mass from exchange of one singlet mode in mass basis, labelled $N_{\text{mass}}^{(n)}$ and of mass $(m_n + \Delta m)$.

Each mode of a pair of Majorana modes (mass $m_{n\,\pm}$, *magnitude* of coupling $y_{n\,\pm}$) gives a contribution to the SM neutrino mass which is similar to the above. However, given the near-degeneracy with*in* each pair, it is convenient to consider their *combined* effect:

$$\begin{aligned} m_\nu^{\text{pair}} &= v^2 \left(\frac{y_{n\,+}^2}{m_{n\,+}} - \frac{y_{n\,-}^2}{m_{n\,-}} \right) \\ &\approx \frac{y_n^2 v^2}{m_n} \left(2\frac{\Delta y}{y_n} - \frac{\Delta m}{m_n} \right) \end{aligned} \tag{17}$$

again, as in Fig. 2.[20] Here $\Delta y = y_{n\,+} - y_{n\,-}$ and $\Delta m = m_{n\,+} - m_{n\,-}$.

The procedure then is to determine the masses and couplings from a detailed 5D calculation, plug these into above formulae, and finally sum over the pairs of Majorana modes.

4.3 Results

In this section, we will simply summarize the results of the above outlined procedure, referring the reader to the appendix A for the actual calculation. As already mentioned in the summary above, each of the two cases $c_N >$ and $< -1/2$ has to be treated on its own.

(i) $c_N > -1/2$

We begin with the case of $c_N > -1/2$, which is the phenomenologically *viable* option, i.e., can give the known size of the SM neutrino masses with natural choices of the bulk parameters.

The *special* mode

[20]Equivalently, we can treat the small Majorana splitting (Δm) as a "mass insertion" in getting to the 2nd term of the above result.

The first surprising element is that the *mass* of special mode [for a derivation, see appendix A.2[21]] is *parametrically different* than the Majorana mass of the would-be zero mode in the KK basis: namely, we find that

$$M_N^{\text{special}} \approx f_{>-1/2} M_N^{\text{UV}} \times \left(\frac{M_N^{\text{UV}}}{M_{Pl}} \right)^{-\frac{1}{2}\frac{1}{c_N}-1} \tag{18}$$

with the $O(1)$ factor given by

$$f_{>-1/2} \approx 2 \left(\frac{-\pi \tan(c_N \pi)}{\Gamma^2(-c_N + 1/2)} \right)^{\frac{1}{2c_N}} \tag{19}$$

i.e., it is smaller than the input of M_N^{UV} (given that $c_N > -1/2$, the exponent is *positive* and we assume $M_N^{UV} \lesssim M_{\text{Pl}}$ here), but it is *larger* than the would-be zero mode mass in 1st line of Eq. (12). On the other hand, the *coupling* of special mode to the SM Higgs *is* (roughly) *similar* to that of the would-be zero-mode (apart from the absence of the $\sqrt{1/2 + c_N}$ factor [which anyway is $\sim O(1)$]), i.e., the EWSB-induced Dirac mass with the SM doublet neutrino, m_D^{eff}, is approximately[22]:

$$m_D^{\text{eff,special}} \sim m_D^{(0,0)} \text{ [where } m_D^{(0,0)} \text{ is 1st line of Eq. (9)].} \tag{20}$$

Thus it is clear that special mode's contribution to SM neutrino mass is *too* small to reproduce Eq. (15).

***Low*-lying modes**

It is the TeV-mass physical modes which shoulder the responsibility of generating the SM neutrino mass. Their Yukawa coupling to the Higgs and the SM lepton doublet is suppressed only by the latter's profile at the TeV brane, given that these singlet profiles are peaked near the TeV brane, i.e., m_D^{eff} is again similar to $m_D^{(0,0)}$ in 1st line of Eq. (9).

Naively, one might then expect a *too* large SM neutrino mass from exchange of these modes, given the $\sim$ TeV mass for these modes. However, the crucial point is that the fraction of (primordially) "Majorana natured"-mass is *naturally* very *small*. From the explicit 5D mass basis calculation we find that the mass and coupling splitting are given by (see appendix

[21]Following [18], M_N^{UV} in units of M_{Pl} is denoted by d in appendix A also.

[22]The reason for this similarity is, in turn, that of the profiles, i.e., they are both leaning towards the IR brane. Although it might not be needed (given the expectation based on these profiles), for an actual derivation of this coupling, see appendix A.3.

A.2)

$$\begin{aligned}
\frac{\Delta m}{m_n} &\approx h_{>-1/2}\frac{\text{TeV}}{m_n}\frac{1}{M_N^{\text{UV}}/M_{Pl}}\left(\frac{m_n}{M_{\text{Pl}}}\right)^{-2\,c_N} \quad \textit{irr}\text{espective of } c_N \\
&\approx h_{>-1/2}\frac{1}{M_N^{\text{UV}}/M_{Pl}}\left(\text{TeV}/M_{Pl}\right)^{-2\,c_N}, \text{ for } m_n \sim \text{TeV} \qquad (21)\\
\frac{\Delta y}{y_n} &= -c_N\frac{\Delta m}{m_n} \qquad (22)
\end{aligned}$$

where the leading order mass m_n and coupling y_n are given by

$$\begin{aligned}
m_n &\approx \left(n+\frac{1}{2}(1-c_N)\right)\pi \quad (\text{TeV}) \qquad (23)\\
y_n &\approx Y_5\sqrt{2c_L-1}\left(\frac{\text{TeV}}{M_{\text{Pl}}}\right)^{c_L-1/2}. \qquad (24)
\end{aligned}$$

(assuming $c_L > 1/2$ as before). The $\mathcal{O}(1)$ factor $h_{>-1/2}$ is given by

$$h_{>-1/2} \approx \frac{4^{c_N}\pi}{\Gamma^2(-c_N+1/2)}. \qquad (25)$$

As is discussed in detail in section A.2, the above formula for the $\mathcal{O}(1)$ factor [and similarly Eqs. (23) and (24)] is valid for any low-lying modes with not so small n and more precise expression that holds even for the first few modes can be found there.

Notice that the mass (and similarly coupling) splitting is clearly $\ll 1$, as long as $c_N < 0$ and $M_N^{\text{UV}} \lesssim M_{\text{Pl}}$, i.e., for a (very) *wide* range of parameter space. (We would like to again emphasize here that the above estimate for Majorana mass splitting holds both for $c_N >$ and $< -1/2$.) It should be clear from Eq. (21) and Eq. (22) that the contribution from the mass splitting to the SM neutrino mass is similar in size to that due to coupling splitting.

Plugging Eqs. (21), (22), (24) and (23) into the general formula in Eq. (17) and summing over such modes, we find that SM neutrino mass formula becomes

$$m_\nu \approx \; h_{>-1/2}(2c_N+1)\sum_n \frac{\text{TeV}}{M_N^{\text{UV}}/M_{Pl}}\frac{(y_n v)^2}{m_n^2}\left(\frac{m_n}{M_{\text{Pl}}}\right)^{-2\,c_N} \qquad (26)$$

Approximating m_n by $\sim n$ TeV, we can see that this sum goes as $\sim \left(n_{\text{max}}^{-2\,c_N-1}-1\right)$, where n_{max} ($\gg 1$) denotes a *naive* cut-off on the sum approaching from $n=1$. Thus this sum is convergent for $c_N > -1/2$, which implies that it is dominated by the *lightest*, i.e., $\sim$ TeV mass modes (this argument is valid only for $c_N > -1/2$). This is one of our main results. As far as the quantitative aspect is concerned, as indicated earlier, the expressions for masses and couplings given above are a very good approximation for low-lying modes with not so small n. However, since, as we just learnt, the contribution from the first few modes is significant, a more careful treatment is needed to get a more reliable final result. We do this in the appendix A, and, as can be seen in section A.4, the final answer for SM neutrino mass by performing numerical sum with improved $\mathcal{O}(1)$ factor shows excellent agreement with the result obtained in the KK basis.

Having established the above quantitative result, we now turn our attention to its qualitative features. For this purpose, it is clear that we can simply focus on the contribution from the lightest TeV mode. By setting $m_n \sim$ TeV in Eq. (26) and noticing that the Dirac mass $y_n v$ is approximately $m_D^{(0,0)}$ [compare Eq. (24) with Eq. (9)], we get for $c_N > -1/2$

$$m_\nu \quad \sim \quad \frac{1}{M_N^{\rm UV}/M_{Pl}} \frac{\left[m_D^{(0,0)}\right]^2}{\rm TeV} \left({\rm TeV}/M_{Pl}\right)^{-2\,c_N} . \tag{27}$$

Clearly it has the same form as Eq. (8), where the "effective" Majorana mass in this case can be *defined* by

$$M_N^{\rm eff} \quad \sim \quad M_N^{\rm UV} \left({\rm TeV}/M_{Pl}\right)^{1+2\,c_N} \tag{28}$$

which is *identical* to the would-be zero mode mass in the KK basis [see 1st line of Eq. (12)]. Thus, it is easy to see that we reproduce the KK basis result already at this estimate-level. However, it is important to realize that there is no "special" physics at $M_N^{\rm eff}$ in the *mass* basis, this scale is just an "illusion".

Modes *near* special mode

Based on the sum over low-lying modes being convergent, combined with the special mode (by itself, i.e., unpaired) giving too small an effect, we can anticipate that the modes near special mode will have a very small contribution to the SM neutrino mass. Indeed a dedicated analysis of the mass *and* coupling splittings of these modes confirms this expectation. Similarly, we can estimate that the modes *much* above the special one also contribute negligibly.

(ii) $c_N < -1/2$

Finally, for the sake of completeness we also briefly comment on the case $c_N < -1/2$, even though does not give the observed size of neutrino masses for natural values of the bulk parameters.

Special mode

Here, a similar analysis [for a derivation, see appendix A.2] shows that the special mode (in the mass basis) *is* indeed at the mass of the would-be zero-mode:

$$M_N^{\rm special} \quad \approx f_{<-1/2} \quad M_N^{\rm UV} \text{ for } c_N < -\frac{1}{2} \tag{29}$$

with the $\mathcal{O}(1)$ factor given by

$$f_{<-1/2} \approx -\left(2c_N + 1\right). \tag{30}$$

but there is more to it than meets the eye! Namely, it is not just the mass, but also the *coupling* to the Higgs is a player in this game of the generation of SM neutrino mass. It turns out that the "analogy" between the special mode of mass basis and the would-be zero mode of KK basis, based on similarity in their masses, does *not* extend to their coupling to the Higgs: from the *detailed* 5D calculation (see appendix A.3), we find that the coupling of the special mode to the Higgs is *not* suppressed by the factor of the would-be zero mode profile at the TeV brane simply because the special mode is peaked near the *TeV* brane (instead of near Planck brane for the would-be zero mode). So, this is a rather *un*expected result: see section 5 for some "understanding" of it in the CFT basis. Thus, we have

$$\begin{aligned} m_D^{\text{special,single}} &\sim v \left(\frac{\text{TeV}}{M_{\text{Pl}}} \right)^{c_L - \frac{1}{2}} \\ &\sim \left(\frac{M_{\text{Pl}}}{\text{TeV}} \right)^{-c_N - \frac{1}{2}} m_D^{(0,0)} \text{ (2nd factor is second line of Eq. (9))} \\ &\gg m_D^{(0,0)} \end{aligned} \tag{31}$$

(where we have labelled it "single" – in addition to special – since it is after all an *unpaired* mode: further reasons will be made clear later). In other words, it is actually similar to the Dirac mass term (with the SM doublet neutrino) of the would-be zero mode in the KK basis for the *other* value of $c_N (> -1/2)$ [see *1st* line of Eq. (9), even though we have $c_N < -1/2$ in this case]. Equivalently, it is (roughly) same as the coupling of the *non*-special or KK modes, *ir*respective of c_N: again, the point is that *all* these modes are peaked near the TeV brane. Substituting Eqs. (29) and (31) as the effective masses into Eq. (8), we see that

$$\begin{aligned} m_\nu^{\text{special,single}} &\sim \frac{v^2}{M_N^{UV}} \left(\frac{\text{TeV}}{M_{\text{Pl}}} \right)^{2\left(c_L - \frac{1}{2}\right)} \\ &\sim m_\nu \text{ [of Eq. (15)]} \times \left(\frac{M_{\text{Pl}}}{\text{TeV}} \right)^{-2\, c_N - 1} \end{aligned} \tag{32}$$

i.e., the contribution of the special mode by itself is too *large* compared to the KK basis result of Eq. (15).

Nonetheless, there is no reason to "worry" here, since only after summing *all* mass eigenstates would the result for the SM neutrino mass agree with that obtained using the KK basis. So, we now proceed to considering the contribution of the other modes carefully.

***Low*-lying modes**

Let us start with the low-lying modes, i.e., much below the special (single) one. We can show that the Majorana mass (and similarly coupling) splitting for these non-special modes – for the case $c_N < -1/2$ being considered here – is also given by Eq. (21) that we used for

$c_N > -1/2$ earlier (see appendix A.2 and A.3). Also, the Dirac mass with the SM doublet neutrino for these modes is similar to that of the special mode in Eq. (31): equivalently, to that for the low-lying modes for the case $c_N > -1/2$ (again, this is expected based on all these profiles being peaked near the TeV brane). Thus, we see that the *lowest* TeV-scale modes (*no* sum yet!) give a contribution to the SM neutrino mass that is similar in form to that discussed above for $c_N > -1/2$. In other words, it is clear that, even for $c_N < -1/2$, the first few mass eigenstates (by themselves) contribute to the SM neutrino mass at order unity.

However, *un*like for $c_N > -1/2$ that we studied earlier, for the case of $c_N < -1/2$, as we include more and more low-lying modes, the *sum* seems to actually "diverge" from this *bottom-up* viewpoint: this is easy to see from the 2nd line of Eq. (26), where sum is $\sim \left(n_{\max}^{-2\,c_N - 1} - 1 \right) \sim n_{\max}^{-2\,c_N - 1}$ for the case of $c_N < -1/2$. Obviously, these modes then also give *too* large contribution to the SM neutrino mass:

$$m_\nu^{\mathrm{non-special}} \sim n_{\max}^{-2\,c_N - 1} \times m_\nu \text{ [of Eq. (15)]} \tag{33}$$

We can thus naturally hope that the above sum might (up to the contribution of lightest modes) cancel against the special (single) mode contribution [Eq. (32)] – *both* being overly large. In order to check this possibility, let us estimate the above sum of modes by cutting it off at (roughly) mass of the special mode itself, i.e., set $n_{\max} \sim M_N^{\mathrm{UV}}/\mathrm{TeV}$: this might be a reasonable way to proceed, since we do expect properties of modes to change as we make the transition across the special mode mass. This assumption gives

$$\begin{aligned} m_\nu^{\mathrm{non-special}} &\sim \left(\frac{M_N^{\mathrm{UV}}}{\mathrm{TeV}} \right)^{-2\,c_N - 1} \times m_\nu \text{ [of Eq. (15)]} \\ &\sim m_\nu^{\mathrm{special,single}} \times \left(\frac{M_N^{\mathrm{UV}}}{M_{\mathrm{Pl}}} \right)^{-2\,c_N - 1} \end{aligned} \tag{34}$$

where in 2nd line above, we have used Eq. (32). So, even though the collective effect of the light modes is much *larger* than the "right" answer, m_ν, it is still parametrically much smaller than the special (single) mode contribution. [23] Another crucial contribution must come from somewhere else.

Modes *near* special mode

What remains to be considered for the resolution of the above "discrepancy" is to take into account a "threshold" effect at the scale of the special mode, i.e., include the contribution to the SM neutrino mass from the *paired* modes *near* the special one. Indeed, we find

[23] Note that we are assuming $M_N^{\mathrm{UV}} \ll M_{\mathrm{Pl}}$ here, although the hierarchy here need only be an order of magnitude or so for the 5D mass basis results (for the special mode) to be valid.

that the modes just above and below the special mode are also "special" (even if *paired*) in the sense that the naive extrapolation for their properties from the formulae for low-lying modes is simply invalid. For example, 1st line of Eq. (21) would give mass splitting $\sim \left(M_N^{\rm UV}/M_{\rm Pl}\right)^{-2\,c_N-1} \times$ TeV, i.e., $\ll$ TeV, by setting $m_n \sim M_N^{\rm UV}$, but actually we find that it is $\sim$ TeV (see appendix A.2 and A.3). And, the Dirac mass with the SM doublet neutrino for these modes (at the leading order) is similar to that of the special, single mode, i.e., Eq. (31) (again, as dictated by all these profiles being peaked near the TeV brane). Thus, for each such pair, the contribution to the SM neutrino mass from the mass splitting *by itself* (i.e., setting couplings to be exactly *degenerate*: we will return to the splitting in couplings momentarily!) is

$$
\begin{aligned}
m_\nu^{\rm special,one-pair}\ (\text{mass splitting } \textit{only}) \quad &\sim \quad v^2 \left(\frac{\rm TeV}{M_{\rm Pl}}\right)^{2\left(c_L-\frac{1}{2}\right)} \frac{\Delta M_{\rm special}}{M_{\rm special}^2} \\
&\sim \quad v^2 \left(\frac{\rm TeV}{M_{\rm Pl}}\right)^{2\left(c_L-\frac{1}{2}\right)} \frac{\rm TeV}{M_N^{\rm UV\,2}}
\end{aligned} \tag{35}
$$

Now, the number of such special, *paired* modes is approximately given by (see appendix A.2)

$$
\eta_{\rm special,paired} \quad \sim \quad \left(\frac{M_{\rm Pl}}{\rm TeV}\right)\left(\frac{M_N^{\rm UV}}{M_{\rm Pl}}\right)^{-2\,c_N} \tag{36}
$$

Upon summing Eq. (35) over these special modes, we then get

$$
m_\nu^{\rm special,all-pairs}\ (\text{mass splitting } \textit{only}) \quad \sim \quad \frac{v^2}{M_N^{\rm UV}} \left(\frac{\rm TeV}{M_{\rm Pl}}\right)^{2\left(c_L-\frac{1}{2}\right)} \left(\frac{M_N^{\rm UV}}{M_{\rm Pl}}\right)^{-2\,c_N-1} \tag{37}
$$

i.e., same size as the sum over *non*-special modes (cut-off as above), see Eqs. (34) and (32), so that this is still not enough to cancel the excessive contribution of the special, single mode.

However, what "saves the day" is that the effect of the *coupling* splitting for these paired-special modes is actually larger, i.e., dominates over the mass splitting. In detail, the *relative* splitting in coupling (and hence in Dirac mass term with the SM doublet neutrino) is given by (see appendix A.3)

$$
\delta^{\rm special}_{\rm coupling} \quad \sim \quad \left(\frac{\rm TeV}{M_{\rm Pl}}\right)\left(\frac{M_N^{\rm UV}}{M_{\rm Pl}}\right)^{2\,c_N} \tag{38}
$$

so that contribution to the SM neutrino mass from this effect for *each* pair is:

$$
\begin{aligned}
m_\nu^{\rm special,one-pair}\ (\text{coupling splitting}) \quad &\sim \quad v^2 \left(\frac{\rm TeV}{M_{\rm Pl}}\right)^{2\left(c_L-\frac{1}{2}\right)} \frac{\delta^{\rm special}_{\rm coupling}}{M_N^{\rm UV}} \\
&\sim \quad v^2 \left(\frac{\rm TeV}{M_{\rm Pl}}\right)^{2\left(c_L-\frac{1}{2}\right)} \frac{\rm TeV}{M_{\rm Pl}^2} \left(\frac{M_N^{\rm UV}}{M_{\rm Pl}}\right)^{2\,c_N-1}
\end{aligned} \tag{39}
$$

clearly larger than the mass splitting effect of Eq. (35). And, summing over special mode pairs, gives (we multiply the previous result by $\eta_{\text{special,paired}}$):

$$m_\nu^{\text{special,all-pairs}} \text{ (coupling splitting)} \sim \frac{v^2}{M_N^{\text{UV}}} \left(\frac{\text{TeV}}{M_{\text{Pl}}}\right)^{2\left(c_L - \frac{1}{2}\right)}. \tag{40}$$

which is indeed larger than sum of non-special modes (cut-off at special mode mass) in Eq. (34). Importantly, the above collective effect is parametrically *comparable* to that of the special mode by itself in Eq. (32). So the two "special" contributions – single and paired (again, with mass $\sim M_N^{\text{UV}}$) – *can* cancel each other to a large extent!

We thus conjecture that this is precisely what happens: it is the sum over *all* modes – special (paired and single) and ordinary below it – which can reproduce the KK basis result for $c_N < -1/2$.

Modes (much) *above* special mode

For the sake of completeness, especially given the "divergence" in the bottom-up approach, we should carefully estimate the effect from modes (much) above special one: we indeed find this to be convergent and negligible. In more detail, an analysis similar to that performed for modes below special one shows that the mass *splitting* in each pair for $M_{\text{Pl}} \gg m_n \gg M_N^{\text{UV}}$ is given by

$$\Delta m \text{ for } m_n \gg M_N^{\text{UV}} \sim \text{TeV} \left(\frac{M_N^{\text{UV}}}{M_{\text{Pl}}}\right) \left(\frac{m_n}{M_{\text{Pl}}}\right)^{-2\,c_N - 2} \tag{41}$$

whereas the Dirac mass term with the SM doublet neutrino is similar to the other mass eigenstates, i.e., Eq. (31). So, the contribution of each such *pair* to the SM neutrino mass is given by (the coupling splitting contributes similarly)

$$m_\nu^{\text{pair}} \sim \left(m_D^{\text{special,single}}\right)^2 \left(\frac{\text{TeV}}{m_n^2}\right) \left(\frac{M_N^{\text{UV}}}{M_{\text{Pl}}}\right) \left(\frac{m_n}{M_{\text{Pl}}}\right)^{-2\,c_N - 2} \tag{42}$$

Thus, we see that the sum over these modes (setting $m_n \sim n \times \text{TeV}$ as usual) is convergent (as long as $c_N > -3/2$). Their total contribution is much smaller than the (summed) contribution of the *low*-lying modes [see Eq. (34)] by $\sim \text{TeV}/M_N^{\text{UV}}$.

5 CFT interpretation

Let us start by reminding the reader the CFT interpretation of bulk *charged* SM fermions. In this case a massless chiral external fermion (often called "elementary") is coupled (at the UV cut-off) to a CFT fermionic operator: the scaling dimension of this operator (and hence the size of this coupling in the IR, up on RGE from UV cut-off) is related to the 5D mass parameter. The mass eigenstates, which correspond to the zero and KK modes of the 5D model, are actually *admixtures* of the external fermion and composite fermions interpolated by the CFT operator.

For the case of the singlet neutrino at hand, there is an additional feature: the external fermion (denoted by N_R) has a Majorana mass term whose size can be close to the UV cut-off. Denoting by $\mathcal{O}_N$ the CFT operator to which N_R couples, the UV Lagrangian contains

$$\mathcal{L} = \mathcal{L}_{\text{CFT}} + \lambda \overline{N_R} \mathcal{O}_N + \frac{1}{2} M_N^{\text{bare}} N_R^2 \tag{43}$$

where we are using the convention that the *engineering* dimension of $\mathcal{O}_N$ is $5/2$ so that the coupling λ is dimension*less*. We take the natural size of bare Majorana mass $M_N^{\text{bare}} \lesssim M_{\text{Pl}}$. The composite operator $\mathcal{O}_N$ actually interpolates *left*-handed composite fermionic states. These composites form *Dirac* states, with masses being quantized in units of $\sim$ TeV and with their RH partners originating from a *different* operator (which will not concern us here). Due to the above coupling, there is mixing between N_R and CFT composites so that the basis defined by the external N_R and the CFT composites is not quite the mass basis of the 5D model that we discussed above, neither is it the KK basis of 5D model. We dub it "CFT" basis. This provides yet another angle on the seesaw mechanism, allowing us to obtain quick estimates as we discuss below.

(i) $\left[\mathcal{O}_N\right] < 5/2$ or $c_N > -1/2$

The coupling $\overline{N_R} \mathcal{O}_N$ is *relevant* when the scaling dimension of operator, denoted by $\left[\mathcal{O}_N\right]$, is less than $5/2$. In this scenario, the (CFT $+N_R$) theory flows to a new fixed point and we assume it is reached rather rapidly, just below the UV cut-off $\sim M_{\text{Pl}}$. At the fixed point, N_R effectively has a scaling dimension of $\left(4 - \left[\mathcal{O}_N\right]\right)$ so that the net coupling $\overline{N_R} \mathcal{O}_N$ has a scaling dimension of 4, as appropriate for a fixed point behaviour [9].

Mass of N_R

The mass *term* for N_R can be significantly renormalized (actually reduced) compared to its bare value. The RG running is dominantly dictated by anomalous dimension of the operator N_R^2 and we find

$$M_N\left(\mu\right) \quad \sim \quad M_N^{\text{bare}} \left(\frac{\mu}{M_{\text{Pl}}}\right)^{5-2\left[\mathcal{O}_N\right]}, \text{ for } \left[\mathcal{O}_N\right] < 5/2 \tag{44}$$

where we assumed the large-N limit [24] in taking scaling dimension of N_R^2 field to be twice that of N_R (and we have set the engineering dimension of N_R to be $3/2$).

[24] Here, "N" denotes (roughly) the number of fundamental degrees of freedom in the CFT, which is not to be confused with the singlet fermion *field* N!

It is natural to assume that the "physical mass" for N_R (denoted by $M_N^{\rm phy}$) is given by the value of μ where the renormalized mass term becomes comparable to μ itself ,

$$M_N^{\rm phy} \quad \sim \quad M_N^{\rm bare} \left(\frac{M_N^{\rm phy}}{M_{\rm Pl}} \right)^{5-2\left[\mathcal{O}_N\right]} . \tag{45}$$

Solving for $M_N^{\rm phy}$ gives

$$M_N^{\rm phy} \quad \sim \quad M_N^{\rm bare} \left(\frac{M_N^{\rm bare}}{M_{\rm Pl}} \right)^{\frac{1}{2\left[\mathcal{O}_N\right]-4}-1} . \tag{46}$$

Note that the exponent on RHS in equation just above is indeed > 0 for $\left[\mathcal{O}_N\right] < 5/2$ so that $M_N^{\rm phy} < M_N^{\rm bare}$. Of course, N_R mixes with CFT states (that is why we used quotes while calling $M_N^{\rm phy}$ a mass), but it is clear that there will be a resultant mass eigenstate with significant admixture of N_R, which thus has a mass roughly given by the renormalized N_R mass term.

When matching to the 5D results, we use the standard AdS/CFT "dictionary": first, we can relate $\left[\mathcal{O}_N\right]$ to the 5D mass of N, namely, $\left[\mathcal{O}_N\right] = 2 - c_N$. Thus, it is $c_N > -1/2$ which corresponds to the relevant $\overline{N_R}\mathcal{O}_N$ coupling assumed above. And, $M_N^{\rm bare}$ in the CFT picture is dual to the Majorana mass term on the UV brane, M_N^{UV}. Plugging in the *parameters* into Eq. (46), we recover the mass of the special mode in Eq. (18).

***Low*-lying modes**

Effectively integrating out N_R at the scale $M_N^{\rm phy}$ gives rise to the composite operator $\mathcal{O}_N^2$, thus feeding lepton-number violation into the CFT sector:

$$\begin{aligned} \Delta\mathcal{L}_{CFT} \quad &\sim \quad \lambda \overline{N_R}\mathcal{O}_N + \frac{1}{2} M_N^{\rm phy} N_R^2 \\ &\rightarrow \quad \frac{\lambda^2}{M_N^{\rm phy}} \mathcal{O}_N^2, \text{ renormalized at } M_N^{\rm phy} \end{aligned} \tag{47}$$

where $\Delta\mathcal{L}_{CFT}$ denotes perturbation to the CFT Lagrangian. RG evolving this to the $\sim$ TeV scale (as before, we use $\left[\mathcal{O}_N^2\right] = 2 \times \left[\mathcal{O}_N\right]$, similarly for the engineering dimensions), where composite Higgs is interpolated by the *product* of $\mathcal{O}_N$ and O_L (latter being the doublet operator)[25], we get

$$
\begin{aligned}
\Delta\mathcal{L}_{CFT} &\sim \frac{\lambda^2}{M_N^{\text{phy}}}\left(\frac{\text{TeV}}{M_N^{\text{phy}}}\right)^{2\left[\mathcal{O}_N\right]-5}\mathcal{O}_N^2, \text{ renormalized at TeV} \\
&\sim \frac{\lambda^2}{M_N^{\text{bare}}}\left(\frac{\text{TeV}}{M_{\text{Pl}}}\right)^{2\left[\mathcal{O}_N\right]-5}\mathcal{O}_N^2 \\
&\sim \frac{\lambda^2}{\text{TeV}}\left(\frac{\text{TeV}}{M_N^{\text{phy}}}\right)^{2\left(\left[\mathcal{O}_N\right]-2\right)}\mathcal{O}_N^2
\end{aligned}
\tag{48}
$$

using Eq. (46) in 2nd line above.

Based on the above RG scaling and the requirement of stability of the system, we find that there is a *lower* limit on $\left[\mathcal{O}_N\right]$ as follows. Suppose the dimensionless coefficient (λ) appearing in the Lagrangian term of 2nd line of Eq. (47) is $\sim O(1)$, i.e., it starts being a "borderline" perturbation to the CFT. However, even with this assumption about the initial condition, as can be seen from the last line of Eq. (48), in the IR[26], it will always be a *genuine* perturbation, i.e., the coefficient (in units of the corresponding RGE scale) $\ll 1$, *as long as* $\left[\mathcal{O}_N\right] > 2$ *so that* $\mathcal{O}_N^2$ *is an* ir*relevant operator*. In 5D we thus require $c_N < 0$, which is what we assumed in our calculations.[27]

SM neutrino mass

Interpreting Eq. (48) as the main source for lepton-number violation, introducing a factor of $\sim (\text{TeV}/M_{\text{Pl}})^{2\left[\mathcal{O}_L\right]-5}$ for the (square of) coupling of doublet lepton neutrino to the CFT in the IR [9] [28] and Higgs VEV for EWSB, we estimate the SM neutrino mass:

$$
m_\nu \sim \frac{v^2}{M_N^{\text{bare}}}\left(\frac{\text{TeV}}{M_{\text{Pl}}}\right)^{2\left(\left[\mathcal{O}_N\right]+\left[\mathcal{O}_L\right]-5\right)}
\tag{49}
$$

Upon translating to the 5D parameters, we again get agreement for another physical observable, namely, the SM neutrino mass in Eq. (49) is similar to the result obtained using the 5D calculation in Eq. (15).

[25] Note that had we taken Higgs field also to be in the bulk (but with profile of its VEV/SM Higgs boson to be peaked near TeV brane), then we would have a single trace, finite/low scaling dimension CFT operator, $\mathcal{O}_H$ which can also interpolate the composite Higgs. Instead, we assumed here – mostly for simplicity – that Higgs is strictly localized on the TeV brane which implies that there is no such "Higgs" operator at higher than $\sim$ TeV energies.

[26] note that, in general, TeV here should be replaced by whatever is the IR scale

[27] In other words, for the case $\left[\mathcal{O}_N\right] < 2$, we see that $\mathcal{O}_N^2$ is a relevant operator. The "problem" with this scenario is that, even if the coefficient in Eq. (47) is *smaller* than 1, it will become (again, in appropriate units) *larger* than $\sim O(1)$ at an RG scale which is (possibly much) *above* $\sim$ TeV, i.e., there is a danger that scale invariance is then broken at that scale.

[28] Recall that, as discussed in section 3, $c_L \sim 0.6$ reproduces charged lepton masses and this corresponds to $\left[\mathcal{O}_L\right] > 5/2$, i.e. irrelevant coupling.

In the CFT picture, we can also think in terms of the SM neutrino mass actually arising from *exchange* of heavy SM singlet particles. The point is that the above lepton-number violating perturbation $\mathcal{O}_N^2$ to the CFT will induce small Majorana mass terms and lepton-number violating couplings to the Higgs for the entire tower of CFT *composites*, which of course are SM singlets and Dirac. In more detail, using Eq. (48), it is rather straightforward to estimate this effect for the *lightest* TeV-scale composites. For example, the mass splitting is of order:

$$\Delta M \text{ from } \mathcal{O}_N^2 \quad \sim \quad \frac{\text{TeV}^2}{M_N^{\text{bare}}} \left(\frac{\text{TeV}}{M_{\text{Pl}}} \right)^{2\left[\mathcal{O}_N\right]-5} \tag{50}$$

After diagonalizing these mass terms it is clear that we will obtain pairs of (almost) degenerate *Majorana* modes with mass splitting as in Eq.(50), and this is what we found in the 5D mass basis calculation. Speaking more quantitatively, relating the scaling dimension of $\mathcal{O}_N$ to c_N and identifying M_N^{bare} with M_N^{UV}, we see that this Majorana mass term has the same size as in Eq. (21) of the 5D calculation.

Armed with these Majorana mass terms for the TeV-scale composites, it is rather straightforward to show that the contribution to the SM neutrino mass from the exchange of the low-lying resonances provides an order one contribution to the SM neutrino mass. Interestingly,

- the Majorana mass term is for the *left*-handed composites (again, interpolated by $\mathcal{O}_N$), whereas coupling to the Higgs is for the R chirality so that we do not encounter any propagator suppression in the exchange of TeV-scale composites (as opposed to the KK basis), see Fig. 3.

We see from Eq. (50) that $\Delta M \ll$ TeV, as long as $\left[\mathcal{O}_N\right] > 2$ (as we assumed above for stability). Also, just to make this point more explicitly, for $N_R \overline{\mathcal{O}_N}$ coupling being close to marginal (i.e., $\left[\mathcal{O}_N\right] \sim 5/2$)[29], we get $\Delta M \sim \text{TeV}^2/M_N^{\text{bare}}$, i.e., Majorana mass term for CFT composites is *naturally* suppressed because it sort of manifests a "seesaw", with $\sim$ TeV in numerator being (roughly) Dirac mass term between N_R and (TeV-scale) CFT composite and M_N^{bare} being Majorana mass for N_R which is heavy and integrated out: of course, the "difference" from usual seesaw for SM neutrino mass is that here CFT composite also has a *Dirac* mass $\sim$ TeV (with another composite).

In addition, it is worth mentioning that the Majorana mass term which is needed for obtaining SM neutrino mass [i.e., $\sim O(0.1)$ eV] from exchange of these TeV-mass modes is actually $\sim$ keV, i.e., several orders of magnitude *larger* than simply $\sim \text{TeV}^2/M_{\text{Pl}} \sim$ meV that we would have gotten for the $N_R - \mathcal{O}_N$ coupling being marginal (as indicated above) and $M_N^{\text{bare}} \sim M_{\text{Pl}}$. Yet, here we have an interesting option:

[29] deviating from marginality does not really (at least qualitatively) change the point which follows

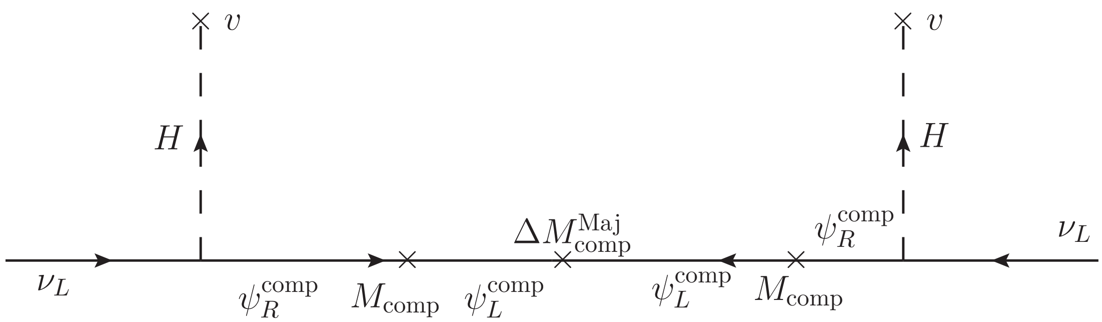

Figure 3: The SM neutrino mass generated by exchange of one composite state in the CFT basis, labelled $\psi^{\rm comp}$ with Dirac mass $M_{\rm comp}$ and Majorana mass term $\Delta M_{\rm comp}^{\rm Maj}$. The chirality structure is to be contrasted to that in Fig. 1 for the KK basis.

- for $[\mathcal{O}_N] \lesssim 5/2$, i.e., a *slightly* relevant coupling of N_R to CFT operator, naturally gives the requisite size of Majorana mass term for TeV-mass Dirac composites [as seen from Eq. (50)]: the crucial point being that a small deviation from marginality for the above coupling is "enhanced" by RGE over the large energy range.

Finally, we have seen that the TeV scale composites provide an important contribution to the SM neutrino mass. On the other hand, while N_R is *crucial* in introducing the seed of lepton-number violation in the CFT via $\mathcal{O}_N^2$, N_R itself does *not* directly couple to the Higgs. So, we learn that

- there is no *additional* contribution to the SM neutrino mass from N_R exchange per se, even though N_R has a Majorana mass: what is missing is the coupling to the Higgs.

(ii) $\left[\mathcal{O}_N\right] > 5/2$ or $c_N < -1/2$

The CFT picture for $c_N < -1/2$ should then be easy to go through; to begin with, the usual translation dictionary implies $\left[\mathcal{O}_N\right] > 5/2$ so that coupling $\overline{N_R}\mathcal{O}_N$ is now *ir*relevant. Thus, it is clear that the mass *term* for N_R is roughly the size of the Majorana mass term at the UV cut-off itself, i.e., there is negligible renormalization for it. Moreover, as before, we can argue that in spite of the mixing of N_R with CFT composites there will be an "N_R state" whose physical mass is not significantly modified relative to the N_R mass term above, i.e.,

$$M_N^{\rm phy} \quad \sim \quad M_N^{\rm bare}, \text{ for } \left[\mathcal{O}_N\right] > 5/2 \tag{51}$$

which is of course in agreement with the 5D single-special mode mass [see Eq. (29)] for this case.

We can integrate out N_R as before, except that this is now done at M_N^{bare}. Then, RG flowing from this scale to $\sim$ TeV, it is easy to see that the $c_N < -1/2$ (or $[\mathcal{O}_N] > 5/2$) case actually gives similar form for the coefficient of $\mathcal{O}_N^2$ operator as $c_N > -1/2$ (or $[\mathcal{O}_N < 5/2]$) that we discussed earlier; this happens mainly because the only assumption we made earlier for this purpose about $[\mathcal{O}_N]$ was that it is larger than 2, which is certainly the case for $c_N < -1/2$. Hence, the SM neutrino mass for $c_N < -1/2$ in the CFT picture is also given by Eq. (49) and, in turn, agrees with the 5D result in Eq. (15). Again, the SM neutrino mass originates only from CFT composites exchange [with Majorana mass terms for $\sim$ *TeV*-scale composites given as before: see Eq. (50)], since external N_R does not couple to the Higgs in this basis.

The above CFT picture discussion is summarized in the right column of table 1.

Contribution to the SM neutrino mass from special modes for $c_N < -1/2$

Using the CFT picture, can we understand the *un*expectedly large contribution to the SM neutrino mass of the *special* mode in *mass* basis found in the 5D calculation for $c_N < -1/2$? Note that this CFT basis is *not* exactly the mass basis. Thus, first of all, there is no obvious "contradiction" between N_R exchange in CFT picture *not* (directly) contributing to the SM neutrino mass with the fact that, in the mass basis, the special mode gives a large contribution to the SM neutrino mass, in turn, from its *un*suppressed[30] coupling to the Higgs. The point is that

- the special mode of the 5D model would in the CFT picture correspond to an *admixture* of N_R and CFT composites and the latter component of it does couple to another composite, i.e., the Higgs: first of all, this implies that the special mode *will* couple to the Higgs (as we found in the 5D calculation). Note that this point applies to both the choices of $[\mathcal{O}_L]$ (equivalently c_N).

Thus, the "origin" of the special mode and how it contributes to the SM neutrino mass is clear from the CFT perspective.

But, the main question still remains, namely, how come special mode's coupling to the Higgs is so *large*, given that the coupling between N_R and the CFT is *small* for the case $[\mathcal{O}_N] > 5/2$? The answer to this puzzle is the following. There is a whole tower of CFT composites (from $\sim$ TeV to M_{Pl}) with which N_R mixes. In particular there are many composites which have mass $\sim M_N^{\text{phy}}$, with spacing between successive modes being $\sim$ TeV. Therefore, even the small off-diagonal mass terms between N_R and these CFT composite states (denoted by $\delta m_{N_R-\text{CFT}}$) can result in *large* mixing *angles*.[31] This mixing – even if it

[30] as usual, apart from being possibly small due to choice of c_L or $[\mathcal{O}_L]$.

[31] Specifically, we can estimate that $\delta m_{N_R-\text{CFT}}$ (the mass mixing term) can be $\gg$ TeV (the mass spacing) so that there are actually *many* CFT composites with which N_R can mix significantly.

is close to maximal – does not really change the *physical* mass of N_R from the mass *term* for N_R. Conversely, the coupling can be modified significantly. In particular, we see that the special mode can acquire a *large* coupling to the Higgs by "piggy-back riding" on the coupling of its sizable admixture of (almost) degenerate CFT composites. Schematically, we have:

$$\text{special mode constitution} \quad \propto \quad N_R + a\psi^{\text{near}} + \epsilon\psi^{\text{far}} \tag{52}$$

where ψ^{near} denotes (collectively) the CFT composites with mass close to M_N^{bare} (ψ^{far} denoting rest of the CFT tower) and a is $\sim O(1)$ mixing angle, whereas $\epsilon \ll 1$. Thus, in the end, the special mode has $O(1)$ coupling to the Higgs.

Note that a similar argument applies to the case $c_N > -1/2$ or $\left[\mathcal{O}_N\right] < 5/2$ studied earlier. However, there the mixing mass term, i.e., $\delta m_{N_R-\text{CFT}}$, can be sizable to begin with, given that the coupling between N_R and CFT operator is *relevant*. Thus, the closeness in mass of some CFT composites with N_R has less of an *additional* impact as compared to the case $c_N < -1/2$ discussed above, i.e., the issue of "resonant" enhancement of mixing between N_R and CFT composites close to it is not so relevant here, as far as their contribution to the SM neutrino mass is concerned. Also, the special mode – being too heavy compared to would-be zero mode – does not contribute significantly to the SM neutrino mass, *even if* its coupling to the Higgs is taken to be unsuppressed[32] (and similarly for modes around it). Overall, that is why this issue of taking into account mixing between N_R and CFT composites is not really significant for $c_N > -1/2$, i.e., we do not expect to find (and indeed did not in the 5D calculation) any "surprises" here.

"Universal" dependence on c_N of the SM neutrino mass

Moreover, as should already be clear from the *separate* discussions for the two cases of $c_N(or\left[\mathcal{O}\right])$ above, the CFT picture leads to a simple "understanding" of why the dependence on c_N in the formula for SM neutrino mass obtained from 5D calculation is the same for $c_N < -1/2$ and $c_N > -1/2$ [see Eq (15)]; as have been discussed in section 3, this looked somewhat of a coincidence in the KK basis. Just to summarize then, the SM neutrino mass in the CFT picture is essentially dictated by the lepton-number violating effect in the CFT sector, i.e., the coefficient of the operator $\mathcal{O}_N^2$ renormalized at $\sim$ TeV scale[33]. In turn, this is determined by $\left[\mathcal{O}_N\right]$, the scaling dimension of $\mathcal{O}_N$ (that of $\mathcal{O}_N^2$ being twice in the large-N limit). The key observation is that, as long as $\left[\mathcal{O}_N^2\right] > 4$ (thus $\left[\mathcal{O}_N\right] > 2$) the RG flow of coefficient of $\mathcal{O}_N^2$ (down to the TeV scale) has a similar dependence on $\left[\mathcal{O}_N\right]$. This range of

[32] cf. for $c_N < -1/2$, where the (unexpectedly) large coupling to Higgs changed the game drastically!

[33] in the anatomical language, this operator first leads to Majorana mass terms for the CFT singlet composites, whose exchange then generates the SM neutrino mass.

$\left[\mathcal{O}_N\right]$ corresponds to $c_N < 0$, whether $c_N < -1/2$ or $c_N > -1/2$. Hence, we do not expect any qualitative change in the formula for the SM neutrino mass as we cross the $c_N = -1/2$ "threshold": again, while this marks the transition of the coupling $\overline{N_R}\mathcal{O}_N$ from relevant to irrelevant, it is $\left[\mathcal{O}_N^2\right]$ which matters for the bottomline SM neutrino mass and this operator stays irrelevant throughout this range of c_N.

6 Conclusions and outlook

We studied a simple warped 5D scenario that accommodates the SM neutrino masses. Namely, a SM singlet field is added in the bulk and coupled to the Higgs and lepton doublet fields on the IR brane. Furthermore, a Planck-size Majorana mass term for the bulk singlet field is turned on *only* at the UV brane. This is natural due to an extended bulk EW gauge symmetry (in turn, invoked in order to satisfy EW precision test bounds) under which the singlet *is* charged and which is broken only on the UV brane.

Such a framework has all the makings of type I high-scale seesaw. Indeed the bottomline formula for the SM neutrino mass in this model,

$$m_\nu \quad \propto \quad \frac{v^2}{M_N^{\mathrm{UV}}}, \tag{53}$$

seems to conform to the above expectation (here, M_N^{UV} is the Majorana mass term for the singlet on the UV brane). This result was derived in the earlier literature using the basis of the "usual" zero and KK modes, in which the Majorana mass term on the Planck brane is neglected in the KK decomposition and subsequently taken into account in the form of Majorana mass terms for the zero and KK modes. In that picture the SM neutrino mass arises entirely from the exchange of the would-be zero mode, that in practice has a super-large Majorana mass term of order M_N^{UV}. The latter is the scale that appears in the denominator of (53), whereas the numerator corresponds to the Dirac mass induced by the Higgs VEV, just like the usual 4D seesaw. On the other hand, the KK modes contribute negligibly (even though they also have very large Majorana mass terms).

In this paper, we focussed instead on the *mass* basis for the singlet neutrino modes (as might be required for studies involving *on*-shell production of the singlet neutrino states) and analyzed in detail neutrino mass generation via a 5D calculation. Such a change of basis actually turns out to lead to a paradigm shift. Our results show that Eq. (53) should be reinterpreted as

$$m_\nu \quad \propto \quad \left(\frac{v^2}{\mathrm{TeV}}\right)\left(\frac{\mathrm{TeV}}{M_N^{\mathrm{UV}}}\right). \tag{54}$$

Namely, it is the exchange of TeV mass singlet modes with unsuppressed coupling to Higgs which dominantly contribute to the SM neutrino mass, as indicated by the first factor above.

The smallness of the SM neutrino mass follows from these singlet modes being *mostly* Dirac with a tiny fraction of their mass being Majorana-natured (which accounts for the 2nd factor). What is remarkable is that these highly suppressed Majorana mass terms are completely natural, being themselves the result of an incarnation of a type I seesaw mechanism, albeit here it is for the Majorana mass term for TeV-scale singlet modes which already have a *Dirac* mass of a TeV! This picture realizes a *natural* version of a scenario dubbed "inverse" seesaw in the literature. The type I high-scale seesaw was merely a mirage.

We also presented the first discussion of the CFT interpretation of this warped seesaw model. The new ingredient relative to the case of the charged SM fermions is the Majorana mass for the external singlet field coupled to the CFT. Taking it into account we confirmed that one naturally ends up with an inverse seesaw mechanism. The CFT picture also clarifies the universal dependence on the 5D singlet mass parameter c_N in the neutrino mass formula (15), whose origin was somewhat obscured in the KK basis.

Importantly, our finding leads to a radical shift in the *phenomenology* of this scenario. Indeed, we realized that the physical source of a dominant part the SM neutrino mass – which are the TeV-mass singlet states – can potentially be directly probed at colliders. Similarly, leptogenesis may occur at a temperature of order the TeV from decays of these singlet modes. The attention is therefore on TeV-scale physics. [34]

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A Details of the 5D mass basis calculation

A.1 The 5D Model and KK decomposition

Varying the full action S in (2) with respect to $\bar{\chi}$ and ψ we get:

$$-i\bar{\sigma}^\mu \partial_\mu \chi - \partial_5 \bar{\psi} + \frac{c_N + 2}{z} \bar{\psi} = 0 \tag{55}$$

$$-i\sigma^\mu \partial_\mu \bar{\psi} + \partial_5 \chi + \frac{c_N - 2}{z} \chi + d \frac{R}{z} \delta(z - R) \psi = 0. \tag{56}$$

[34] We will detail these ideas in ongoing work [25].

The boundary conditions in the absence of $S_{\rm UV}$ are chosen to be Dirichlet for χ (and consequently Neumann for ψ). The UV-Majorana mass alters the boundary conditions at $z = R$.

Following [19], we slightly displace the UV-localized mass to $z = R + \epsilon$ and impose standard Dirichlet boundary conditions for χ at $z = R$. The effect of the localized mass is then encoded in a jump of the field: $\chi|_{R+\epsilon} = -d\ \psi|_{R+\epsilon}$. We can now send $\epsilon \to 0$. The corresponding jump in ψ may be found imposing the bulk equations of motion: $\partial_5\psi|_{R+\epsilon} = id\bar{\partial}\bar{\psi}|_{R+\epsilon}$.

Overall, the boundary conditions turn out to be:

$$\chi|_{R'^-} = 0, \qquad\qquad \chi|_{R^+} = -d\ \psi|_{R^+}. \tag{57}$$

For the sake of completeness, we also observe that the remaining two (redundant conditions) are $\partial_5\psi|_{R'^-} = 0$, $\partial_5\psi|_{R^+} = id\bar{\partial}\bar{\psi}|_{R^+}$.

Next, we perform a Kaluza-Klein reduction. Because the UV-localized mass breaks the $U(1)_\Psi$ number, the reduced 4D theory will be a dynamics of Majorana fermions. It is therefore convenient to decompose χ, ψ in terms of a single tower of Weyl fermions:

$$\chi(x,z) = \sum_n g_n(z)\xi_n(x), \qquad\qquad \bar{\psi}(x,z) = \sum_n f_n(z)\bar{\xi}_n(x), \tag{58}$$

where ξ_n satisfy Majorana equations of motion $-i\bar{\sigma}^\mu\partial_\mu\xi_n + m_n\bar{\xi}_n = 0$. The bulk equations of motion and the boundary conditions then become

$$\begin{aligned} f_n' + m_n g_n - \frac{c_N+2}{z} f_n &= 0 \qquad\qquad & g_n' - m_n^* f_n + \frac{c_N-2}{z} g_n &= 0, \\ g_n(R') &= 0 & g_n(R) &= -d f_n^*(R). \end{aligned} \tag{59}$$

The Dirac mass parameter c_N is real by Hermiticity of the action. In addition, by making a phase rotation of ψ we can always eliminate the phase in d. Since ψ is one component of Ψ, in order not to break 5D Lorentz invariance, we are actually performing a phase rotation of the 5D field Ψ itself. We conventionally take $d > 0$ from now on. Finally, m_n are real because they are the eigenvalues of the Hermiticitan differential operator defined by Eqs (59) in the metric determined by the kinetic term. Hermiticity also guarantees that the Kaluza-Klein expansion (58) is meaningful.

Consistently, observe that inserting (58) in (2) gives

$$S = \int d^4x \left[\int dz \sum_{n,m} \left(\frac{R}{z}\right)^4 (f_n^* f_m + g_n^* g_m)\right] \left\{-i\xi_n\bar{\partial}\bar{\xi}_m + \frac{1}{2}\left(m_n^*\xi_n\xi_m + m_n\bar{\xi}_n\bar{\xi}_m\right)\right\}, \tag{60}$$

The normalization is therefore defined by

$$\int dz \left(\frac{R}{z}\right)^4 (f_n^* f_m + g_n^* g_m) = \delta_{nm}. \tag{61}$$

For clarity we stress our convention for c_N, which we do by solving the zero mode equation for the right-handed fermion g_n, i.e. Eq(59) with $m_n = 0$. By plugging the solution into the action, one can easily see that $c_N = -1/2$ (as opposed to $1/2$) corresponds to flat, $c_N > -1/2$ a IR-localized and $c_N < -1/2$ a UV-localized profile.

We decide to carry out the Kaluza-Klein decomposition with real eigenfunctions f_n, g_n (as in [26]), in which m_n are allowed to acquire any (real) positive or negative value. [35] Before proceeding with the actual calculation of the spectrum, note that the eigenvalue problem is invariant under the following spurious symmetry:

$$(f_n, g_n, m_n, d) \to (f_n, -g_n, -m_n, -d). \tag{62}$$

This tells us that for $d = 0$ the solution consists of Dirac pairs: there exists an *independent* solution with eigenvalue $-m_n$ for any eigenfunction with mass m_n. This is no more true as soon as $d \neq 0$, and no exact pairing is observed.

The coupled system described by the bulk equations of motion can be decoupled in a straightforward way, yielding Bessel equation. The result is given by:

$$\begin{aligned} g_n(z) &= -\frac{1}{N_n}\frac{m_n}{|m_n|} z^{5/2}\left[J_{-c_N-1/2}(|m_n|z) + b_n Y_{-c_N-1/2}(|m_n|z)\right] \\ f_n(z) &= \frac{1}{N_n} z^{5/2}\left[J_{-c_N+1/2}(|m_n|z) + b_n Y_{-c_N+1/2}(|m_n|z)\right]. \end{aligned} \tag{63}$$

The coefficient b_n is constrained by the boundary conditions: [36]

$$-b_n = \frac{J_{-c_N-1/2}(|x_n|)}{Y_{-c_N-1/2}(|x_n|)} = \frac{J_{-c_N-1/2}(|x_n|/\Omega) - d\frac{x_n}{|x_n|} J_{-c_N+1/2}(|x_n|/\Omega)}{Y_{-c_N-1/2}(|x_n|/\Omega) - d\frac{x_n}{|x_n|} Y_{-c_N+1/2}(|x_n|/\Omega)}, \tag{66}$$

where $x_n = m_n R'$ and $\Omega \equiv R'/R$. This is the equation constraining the eigenvalues x_n. Defining $Z_\nu(y) \equiv J_\nu(y) + b_n Y_\nu(y)$, the normalization is determined by

$$\begin{aligned} N_n^2 &= R^4 \int_R^{R'} dz\; z\left[Z_\nu^2(|m_n|z) + Z_{\nu+1}^2(|m_n|z)\right] \\ &= \frac{R^4}{2}\left(\mathcal{I}_n(R') - \mathcal{I}_n(R)\right), \end{aligned} \tag{67}$$

[35] One may alternatively work with both real and imaginary components of the wave-functions, but with a constraint $m_n > 0$ on the eigenvalues (we believe this is the convention implicitly adopted in [18]). We checked that our results do not depend on which convention is used.

[36] This is equivalent to the alternative solution:

$$\begin{aligned} g_n(z) &= \frac{m_n}{|m_n|} z^{5/2}\left[C_n J_{c_N+1/2}(|m_n|z) - D_n J_{-c_N-1/2}(|m_n|z)\right] \\ f_n(z) &= z^{5/2}\left[C_n J_{c_N-1/2}(|m_n|z) + D_n J_{-c_N+1/2}(|m_n|z)\right]. \end{aligned} \tag{64}$$

Indeed, using $Y_\nu(x) = \frac{J_\nu(x)\cos(\nu\pi) - J_{-\nu}(x)}{\sin(\nu\pi)}$ we get:

$$C_n = -\frac{1}{N_n}\frac{b_n}{\cos(c_N\pi)} \qquad\qquad D_n = \frac{1}{N_n}\left(1 + b_n \tan(c_N\pi)\right). \tag{65}$$

In particular, $D_n/C_n = -\cos(c_N\pi)/b_n + \sin(c_N\pi)$. The authors independently checked all results of the paper using both (64) and (63).

where $\nu = -c_N - 1/2$ and $\mathcal{I}_n(z) = z^2\left[Z_\nu^2(y) - Z_{\nu+1}(y)Z_{\nu-1}(y) + Z_{\nu+1}^2(y) - Z_{\nu+2}(y)Z_\nu(y)\right]$, $y = |m_n|z$.

A.2 Masses

We can find approximate analytic solutions for the modes satisfying $|x_n| \ll \Omega$. Using a small argument approximation of the Bessel functions for the UV boundary condition, the spectrum equation (66) is simplified to

$$-b_n = \frac{J_{-c_N-1/2}(|x_n|)}{Y_{-c_N-1/2}(|x_n|)} \approx \frac{1}{\frac{\Gamma^2(-c_N+1/2)}{\pi}\left(\frac{|x_n|}{2\Omega}\right)^{2c_N}\left[d\frac{x_n}{|x_n|} + \frac{1}{(c_N+1/2)}\left(\frac{|x_n|}{2\Omega}\right)\right] + \tan(c_N\pi)}. \quad (68)$$

To derive this expression we assumed $c_N \neq -1/2$. From now onwards we will consider $c_N < 0$. We will also assume that d is smaller than one, but much larger than the TeV-Planck hierarchy.

The ratio b_n can also be approximated for large arguments $|x_n| \gg 1$ by $b_n \approx \frac{1}{\tan\left(|x_n| + \frac{c_N}{2}\pi\right)}$. However, this approximation will break down for the first few KK modes. Because, as we will show below, these give the most important contribution to the SM neutrino mass, we keep the general expression (68) for now.

For $c_N < 0$ and $|x_n|/\Omega \ll d$ (and far from special points discussed shortly), $\tan(c_N\pi)$ can be neglected from the right-hand side of Eq. (68) and

$$-b_n = \frac{J_{-c_N-1/2}(|x_n|)}{Y_{-c_N-1/2}(|x_n|)} \approx \frac{x_n}{|x_n|}\frac{\pi}{d\Gamma^2(-c_N+1/2)}\left(\frac{|x_n|}{2\Omega}\right)^{-2c_N}. \quad (69)$$

As can be seen from $|b_n| \propto (|x_n|/\Omega)^{-2c_N} \ll 1$, the spectrum of light modes is approximately determined by $x_n = \pm x_n^0$, where x_n^0 are the zeros of $J_{-c_N-1/2}$. For n not too small, using the large argument expansion, these are approximately given by $x_n^0 \approx \left(n + \frac{1}{2}(1-c_N)\right)\pi$ with $n = 0, 1, \cdots$. Including the leading correction we get

$$\begin{aligned} x_n &= \pm x_n^0 + \delta_n \\ \delta_n &= \frac{Y_{-c_N-1/2}(|x_n^0|)}{J'_{-c_N-1/2}(|x_n^0|)}\frac{\pi}{d\Gamma^2(-c_N+1/2)}\left(\frac{|x_n^0|}{2\Omega}\right)^{-2c_N}. \end{aligned} \quad (70)$$

This result shows that the light modes are approximately Dirac pairs [37] up to a split δ_n, induced when the UV-localized Majorana mass is turned on. In other words, there are two towers of Weyl spinors, one with positive masses ("positive tower") and the other with negative masses ("negative tower"); the modes with $|x_n|/\Omega \ll d$ ("low-lying modes") form pseudo-Dirac pairs.

[37] A Dirac fermion consists of two Weyl fermions of mass $\pm m$.

In the vicinity of the zeros of the denominator of the right-hand side of (68), the function b_n is no more much smaller than one and we need a separate analysis. In this regime the mass eigenstates are identified by the fact that the denominator of the right-hand side of (68) is much smaller than one (or very close to zero):

$$\frac{d}{\pi}\Gamma^2(-c_N+1/2)\left(\frac{|x_n^{\rm special}|}{2\Omega}\right)^{2c_N}\left[\frac{x_n^{\rm special}}{|x_n^{\rm special}|}+\frac{1}{d(c_N+1/2)}\left(\frac{|x_n^{\rm special}|}{2\Omega}\right)\right]+\tan(c_N\pi)\approx 0. \quad (71)$$

As we will see shortly, the mode $x_n^{\rm special}$ that satisfies (71) is *special* in the sense that there is no analog solution of mass $\sim -x_n^{\rm special}$, that is, it is *un*paired (and so pure Majorana), unlike the usual cases where there are two modes in each TeV-bin, making up a (pseudo) Dirac pair. For this reason, we will call such mode "*single-special*" mode. Later, we will introduce "*paired-special*" modes, which, as the name indicates, consist of a pair of two Weyl fermions of mass close to the single-special and a mass splitting of order the TeV.

Now, let us discuss in detail when (71) can be satisfied. Consider first $-1/2 < c_N < 0$, for which $\tan(c_N\pi) < 0$. If $|x_n^{\rm special}| \gtrsim d\Omega$ the second term in the squared parenthesis dominates over the first term. In this case since $2c_N + 1 > 0$, $(|x_n|/\Omega)^{2c_N+1} \ll 1$ for $\forall |x_n| \ll \Omega$ and yet, for generic value of $c_N \in (-1/2, 0)$, $\tan(c_N\pi) \sim \mathcal{O}(1)$. That is, for a generic value of c_N (71) cannot be satisfied by modes below Ω. On the other hand, when $|x_n^{\rm special}| \ll d\Omega$ the first term in the squared parenthesis dominates. Because $\tan(c_N\pi) < 0$, the cancellation can occur only when the first term is positive, i.e. the solution exists only for $x_n^{\rm special} > 0$. The solution is given by:

$$\frac{x_n^{\rm special}}{2\Omega}\approx\left(\frac{-\pi\tan(c_N\pi)}{d\Gamma(-c_N+1/2)^2}\right)^{\frac{1}{2c_N}}, \qquad -\frac{1}{2}<c_N<0. \quad (72)$$

We stress out again that $x_n^{\rm special} \ll d\Omega$ and, as anticipated, there is no analog behavior at $x_n < 0$. This is how we see that the "single-special" mode is unpaired.

For $c_N \lesssim -1/2$, the second term of (71) is negative and $\tan(c_N\pi) > 0$. Again, when $|x_n^{\rm special}| \gtrsim d\Omega$ the second term in the squared parenthesis dominates. However, as in the previous case, no solution is found when $d\Omega \lesssim |x_n^{\rm special}| < \Omega$ for generic choice of $c_N < -1/2$. Similarly, for $|x_n^{\rm special}| \ll d\Omega$ the first term dominates and one would seem to find $|x_n| \sim \Omega d^{-\frac{1}{2c_N}}$; however, this value is now much larger than d, and is therefore inconsistent with the original hypothesis $|x_n^{\rm special}| \ll d\Omega$. A solution is only possible when the terms inside the squared parenthesis approximately cancel each other. This is possible only when $x_n > 0$ and thus mass of the special mode is in the positive tower (i.e. $x_n > 0$) and parametrically close to the UV-localized Majorana mass:

$$\frac{x_n^{\rm special}}{2\Omega}\sim -(c_N+1/2)d, \qquad c_N<-\frac{1}{2}. \quad (73)$$

Again, no partner at $-x_n^{\rm special}$.

In summary, with our convention $d > 0$ the single-special mode is located in the positive tower for both $c_N >$ or $< -1/2$ albeit with parametrically different mass for single-special mode. No special behavior (i.e. no singularity in the right-hand side of (68)) is present in the negative tower.

We conclude this section with a few more comments on the spectrum. We start with $-1/2 < c_N < 0$. In this case, since $|x^{\rm special}| \ll d\Omega$, the analysis leading to (70) allows us to conclude that all states with mass $|x_n| \ll |x^{\rm special}|$ are pseudo-Dirac with mass splitting of order δ_n. The denominator of (68) gets smaller as we approach the special mode in the positive tower, whereas b_n remains very small for $x_n \sim -|x^{\rm special}|$. This suffices to argue that the mass splitting for states close to the special mode is generically of order the TeV ($\delta_n \sim 1$). These pseudo-Dirac fermions have mass splitting (of order the TeV) much smaller than their mass $\sim |x^{\rm special}|$ but much larger than that of low-lying modes. We call them "paired-special" modes.

The states heavier than the special mode are again pseudo-Dirac, with a mass splitting controlled by $|b_n| \ll 1$ between $x_n^{\rm special} \ll |x_n| \ll d\Omega$.

When $c_N < -1/2$ the states with $|x_n| \ll d\Omega$ are pseudo-Dirac with mass splitting δ_n. However, since $x_n^{\rm special} \sim d\Omega$ our equation (70) breaks down before we reach the special mode; to precisely estimate the mass splitting for $|x^{\rm special}| \lesssim d\Omega$ one may perform a completely analogous analysis without dropping $\tan(c_N\pi)$. We do not quote the result for brevity. The modes at $x_n^{\rm special} \sim d\Omega$ have $b_n = \mathcal{O}(1)$ and typically a Majorana splitting of order the TeV, which is the maximal value set by the IR brane. As above, for $|x_n| \gtrsim d\Omega$ the states are pseudo-Dirac.

As we will discuss below, in order to make sense of the SM neutrino mass calculation in the case of $c_N < -1/2$ it is useful to know the number of the paired-special modes. We can address this question by determining the width of the special point (73), i.e. what condition on $\eta = x_n - x_n^{\rm special}$ follows requiring the right-hand side of (71) is allowed to be of order unity (or more precisely, of $\mathcal{O}(\tan(c_N\pi))$). This gives:

$$\eta \lesssim \tan(c_N\pi)\frac{2\pi(-1/2 - c_N)^{1-2c_N}}{\Gamma^2(-c_N + 1/2)}\Omega d^{-2c_N}. \tag{74}$$

With realistic numbers (say, $c_N = -0.7$, $d = 10^{-3}$, $\Omega \sim 10^{15}$), one finds $\eta \gg 1$ (5×10^8).

A.3 Couplings

We are interested in the couplings of ξ_n to the zero mode $L(x)$ of Ψ_L, that we identify with the Standard Model lepton doublet:

$$\Psi_L \to \Psi_L^{(0)}(z)L, \qquad \Psi_L^{(0)} = \frac{1}{\sqrt{R}}\sqrt{\frac{2c_L - 1}{1 - \Omega^{1-2c_L}}}\left(\frac{z}{R}\right)^{2-c_L}, \tag{75}$$

where $M_L = c_L/R$ is the 5D mass of Ψ_L. Introducing the canonically normalized 4D field $H = R'/R\mathcal{H}$, eq(5) becomes:

$$\delta S \;=\; -\int d^4x\; y_n H L \bar{\xi}_n, \tag{76}$$

where

$$y_n \;=\; \Omega^{-3}\lambda_5 \Psi_L^{(0)}(R') f_n(R'). \tag{77}$$

The wave function $\Psi_L^{(0)}(R')$ can be read from above. The profile of the singlet can be written as $f_n(R') = R'^{5/2} Z_{\nu+1}(|m_n|R')/N_n$, where $Z_\nu = J_\nu + b_n Y_\nu$ with $\nu = -c_N - 1/2$. We will now carefully determine $f_n(R')$ for the low-lying (pseudo-Dirac) modes $|x_n| \ll x_n^{\rm special}$. The coupling for modes around $x_n^{\rm special}$ will be analyzed subsequently.

The normalization (67) receives a contribution from $z = R'$ and one from $z = R$. To analyze the former we observe that the boundary condition for $g_n(z)$ in the IR implies $Z_\nu(|m_n|R') = 0$ (see Eq(59)). Then, from the definition (67), and using the identity $Z_{\nu+1}(|x_n|) + Z_{\nu-1}(|x_n|) = \frac{2\nu}{|x_n|} Z_\nu(|x_n|) = 0$, we get $\mathcal{I}_n(R') = R'^2[-Z_{\nu+1}Z_{\nu-1} + Z_{\nu+1}^2](|x_n|) = 2R'^2 Z_{\nu+1}^2(|x_n|)$.

In the UV the boundary condition reads $Z_\nu(|x_n|/\Omega) = d\;(x_n/|x_n|) Z_{\nu+1}(|x_n|/\Omega)$. We are interested in $\mathcal{I}_n(R)$, the UV contribution to the normalization N_n. For $|x_n| \ll |x_n^{\rm special}|$ we can use the small argument approximation of the Bessel functions. At leading order, when $c_N \neq -1/2$ (and $c_N < 1/2$), the relevant expressions are:

$$\begin{aligned}
Z_\nu(|x_n|/\Omega) &\sim \left(\frac{|x_n|}{2\Omega}\right)^{\nu} \frac{1}{\Gamma(\nu+1)}\left[1 + \mathcal{O}(\delta, |x_n|/\Omega)\right], \\
Z_{\nu-1}(|x_n|/\Omega) &\sim \left(\frac{|x_n|}{2\Omega}\right)^{\nu-1} \frac{1}{\Gamma(\nu)}\left[1 + \mathcal{O}(\delta, (|x_n|/\Omega)^3)\right], \\
Z_{\nu+2}(|x_n|/\Omega) &\sim -b_n \left(\frac{2\Omega}{|x_n|}\right)^{\nu+2} \frac{\Gamma(\nu+2)}{\pi}\left[1 + \mathcal{O}(|x_n|/\Omega)^3\right].
\end{aligned} \tag{78}$$

In order to understand whether the subleading $\mathcal{O}(\delta, |x_n|/\Omega)$ terms must be kept in our analysis we have to compare the leading order estimate of $\mathcal{I}_n(R)$ with $\mathcal{I}_n(R') \sim R'^2/|x_n|$. The leading contribution of Z_ν^2 and $Z_{\nu+1}^2$ to $\mathcal{I}_n(R)$ are suppressed by $|x_n|/\Omega$ compared to the other two and can be neglected. The dominant terms give $\mathcal{I}_n(R) \sim R^2(|x_n|/\Omega)^{2\nu-1} \sim R^2\delta_n(|x_n|/\Omega)^{-2} \sim R'^2\delta_n/|x_n|^2$, which is itself a correction of order $\delta_n/|x_n|$ of N_n. Being interested in corrections at most of order δ in the normalisation N_n, we can safely neglect $O(\delta)$ terms in (78), since they lead to $\mathcal{O}(\delta_n^2)$ corrections in N_n. A more accurate calculation gives

$$\begin{aligned}
\mathcal{I}_n(R)R^{-2} &= \left(-\frac{x_n}{|x_n|}\frac{1}{d}Z_\nu Z_{\nu-1} - Z_\nu Z_{\nu+2}\right)[1+\mathcal{O}(|x_n|/\Omega)] \\
&= -\frac{x_n}{|x_n|}\left(\frac{|x_n|}{2\Omega}\right)^{2\nu-1}\frac{2\nu+1}{d\Gamma^2(\nu+1)}[1+\mathcal{O}(\delta,|x_n|/\Omega)] \\
&= -\frac{x_n^0}{|x_n^0|}\delta_n\frac{J'_\nu(|x_n^0|)}{Y_\nu(|x_n^0|)}\left(\frac{|x_n^0|}{2\Omega}\right)^{-2}\frac{2\nu+1}{\pi}[1+\mathcal{O}(\delta)]\,.
\end{aligned} \tag{79}$$

In the second step we replaced (78) and used the definition of b_n given in (68). In the third step we neglected the correction arising from the replacement $x_n \to x_n^0$, since in our final estimate of N_n it would appear as a $\mathcal{O}(\delta^2)$ effect, which we drop.

Summing the UV and IR contributions we find

$$N_n^2 = R^4R'^2Z_{\nu+1}^2(|x_n|)\left[1-2\frac{x_n^0}{|x_n^0|}c_N\frac{\delta_n}{|x_n^0|}\left(\frac{2}{\pi|x_n^0|}\frac{J'_\nu(|x_n^0|)/Y_\nu(|x_n^0|)}{Z_{\nu+1}^2(|x_n^0|)}\right)\right]. \tag{80}$$

For later convenience we factored out $Z_{\nu+1}^2(|x_n|)$ because it automatically cancels out in the expression f_n/N_n entering y_n. This results in a $1/Z_{\nu+1}^2(|x_n|)$ factor in the δ_n correction. Despite the fact that $|x_n| = |x_n^0|(1+x^0\delta_n/|x_n^0|^2+\cdots)$, Because we content ourselves with $\mathcal{O}(\delta_n)$ effects, we can safely replace $x_n \to x_n^0$ in the squared parenthesis. On the other hand, the overall $Z_{\nu+1}^2(|x_n|)$ contributes an additional $\mathcal{O}(\delta_n)$ term to N_n, but — as anticipated — this effect cancels out from (77). More precisely, putting everything together we get:

$$y_n = \frac{\lambda_5}{R}\sqrt{\frac{2c_L-1}{1-\Omega^{1-2c_L}}}\Omega^{1/2-c_L}\mathrm{sign}(Z_{\nu+1})\left[1+\frac{x_n}{|x_n|}c_N\frac{\delta_n}{|x_n|}\left(\frac{2}{\pi|x_n^0|}\frac{1}{J_{\nu+1}^2(|x_n^0|)}\frac{J'_\nu(|x_n^0|)}{Y_\nu(|x_n^0|)}\right)\right]. \tag{81}$$

This result holds for $|x_n| \ll x_n^{\rm special}$ up to terms of order δ_n^2.

We now turn to a discussion of the couplings of the modes of mass near $x_n^{\rm special}$, which correspond to the special mode and the paired spacial modes. States in the negative tower always have $|b_n| \ll 1$ and may be analyzed in a way completely analogous to what we have done for the light modes. The result is:

$$y_n = \frac{\lambda_5}{R}\sqrt{\frac{2c_L-1}{1-\Omega^{1-2c_L}}}\Omega^{1/2-c_L}\mathrm{sign}(Z_{\nu+1})[1+\mathcal{O}(b_n)]\,. \tag{82}$$

In the positive tower the crucial difference is that b_n is unsuppressed. This implies that our estimate of the UV contribution to the normalization N_n must take this into account. In particular, (78) are no more accurate. Instead, assuming $b_n = \mathcal{O}(1)$ we find that $\mathcal{I}_n(R) \sim R^2Z_\nu Z_{\nu+2} \sim R^2(|x_n|/\Omega)^{-2\nu-2} \sim \mathcal{I}_n(R')|x_n|^{2c_N}\Omega^{-2c_N-1}$. The subleading terms are of order $(|x_n|/\Omega)^{-2c_N}$ and $(|x_n|/\Omega)$. Neglecting them, we conclude that

$$y_n = \frac{\lambda_5}{R}\sqrt{\frac{2c_L-1}{1-\Omega^{1-2c_L}}}\Omega^{1/2-c_L}\mathrm{sign}(Z_{\nu+1})\left[1+a|x_n|^{2c_N}\Omega^{-2c_N-1}\right], \tag{83}$$

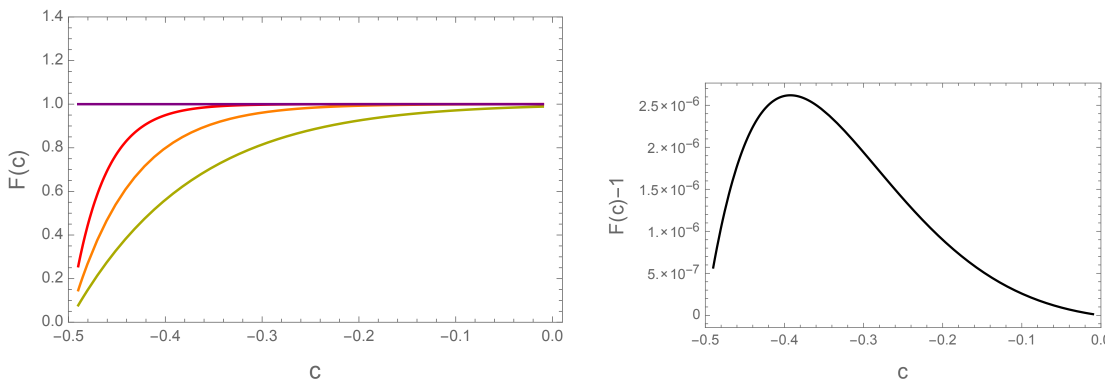

Figure 4: Function $F(c_N)$ defined in (88) for $n_{\max} = 10$ (yellow), 10^3 (orange), 10^6 (red), ∞ (purple). For comparison we also show the deviation of the purple line from 1 (right plot).

where a is some number of order one. Finally, for the special mode it is not possible to determine b_n analytically (it may well be that $|b_n| \gg 1$, so the previous derivation does not apply). Yet, for any b_n we expect

$$y_n^{\text{special}} \sim \frac{\lambda_5}{R}\sqrt{\frac{2c_L - 1}{1-\Omega^{1-2c_L}}}\Omega^{1/2-c_L}. \tag{84}$$

This estimate is correct up to a number of order unity.

A.4 SM neutrino mass for $-1/2 < c_N < 0$

The relevant part of the Lagrangian is (must change notation):

$$\mathcal{L} = \frac{m_n}{2}\bar{\xi}_n\bar{\xi}_n - y_n H L\bar{\xi}_n + \text{hc}. \tag{85}$$

Integrating out the heavy fermions ξ_n, and keeping only the leading terms in a derivative expansion gives:

$$\mathcal{L}_{\text{on-shell}} = -\frac{1}{2}(HL)^2 \sum_{m_n \lessgtr 0} \frac{y_n^2}{m_n} + \text{hc}. \tag{86}$$

Let us consider the contribution from the low-lying modes first. In this case the sum includes both the positive and negative tower up to $m_{\max} < x^{\text{special}}$. After some algebra we find:

$$\begin{aligned} \mathcal{L}_{\text{on-shell}} &= -\frac{1}{2}(HL)^2 \sum^{m_{\max}} \frac{y_n^2}{m_n} + \text{hc} \\ &= -\frac{1}{2}(HL)^2 \frac{\lambda_5^2}{dR}\left(\frac{2c_L-1}{1-\Omega^{1-2c_L}}\right)\Omega^{2+2c_N-2c_L}F(c_N) + \text{hc}, \end{aligned} \tag{87}$$

with

$$F(c_N) \equiv \frac{4^{c_N}\pi}{\Gamma^2(\nu+1)} \sum_n^{n_{\max}} \frac{1}{|x_n^0|^{2+2c_N}}\left[4c_N\left(\frac{2}{\pi|x_n^0|}\frac{1}{J_{\nu+1}^2(|x_n^0|)}\right) - 2\frac{Y_\nu(|x_n^0|)}{J_\nu'(|x_n^0|)}\right]. \tag{88}$$

In this expression, the Bessel functions are all evaluated at the zeros x_n^0 of $J_{\nu=-c_N-1/2}$. Rather than presenting the details of this computation, it is more instructive to reproduce an approximate expression valid for $n \gg 1$:

$$
\begin{aligned}
& \sum^{m_{\max}} \frac{y_n^2}{m_n} \qquad (89) \\
\to \quad & \frac{\lambda_5^2}{R^2}\left(\frac{2c_L-1}{1-\Omega^{1-2c_L}}\right)\Omega^{1-2c_L}R' \sum_{n=0}^{n_{\max}} \frac{1}{|x_n|}\left(\frac{1-2c_N\frac{\delta_n}{|x_n|}}{1+\frac{\delta_n}{|x_n|}} + \frac{1+2c_N\frac{\delta_n}{|x_n|}}{-1+\frac{\delta_n}{|x_n|}}\right) \\
= \quad & \frac{\lambda_5^2}{R^2}\left(\frac{2c_L-1}{1-\Omega^{1-2c_L}}\right)\Omega^{1-2c_L}R' \sum_{n=0}^{n_{\max}} (4c_N+2)\left(-\frac{\delta_n}{|x_n|^2}\right)\left(1+\mathcal{O}\left(\frac{\delta_n}{|x_n|}\right)\right) \\
= \quad & \frac{\lambda_5^2}{dR}\left(\frac{2c_L-1}{1-\Omega^{1-2c_L}}\right)\Omega^{2+2c_N-2c_L}\left[\frac{4^{c_N}\pi}{\Gamma^2(-c_N+1/2)}\sum_{n=0}^{n_{\max}}\frac{(4c_N+2)}{|[n+\frac{1}{2}(1-c_N)]\pi|^{2+2c_N}}\right]\left(1+\mathcal{O}\left(\frac{\delta_n}{|x_n|}\right)\right).
\end{aligned}
$$

One can verify that $F(c_N)$ consistently reduces to the quantity in the square bracket in this limit. $F(c_N)$ is a sole function of c_N. It is plotted in Figure 4 for various values of $n_{\max}$.

References

[1] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999) [arXiv:hep-ph/9905221].

[2] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83**, 4922 (1999) [arXiv:hep-ph/9907447]. J. Garriga and A. Pomarol, Phys. Lett. B **560**, 91 (2003) [hep-th/0212227].

[3] For a review, see O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, Phys. Rept. **323**, 183 (2000) [hep-th/9905111].

[4] See, for example, N. Arkani-Hamed, M. Porrati and L. Randall, JHEP **0108**, 017 (2001) [hep-th/0012148].

[5] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, Phys. Lett. B **473**, 43 (2000) [arXiv:hep-ph/9911262]; A. Pomarol, Phys. Lett. B **486**, 153 (2000) [arXiv:hep-ph/9911294];

[6] Y. Grossman and M. Neubert, Phys. Lett. B **474**, 361 (2000) [arXiv:hep-ph/9912408].

[7] T. Gherghetta and A. Pomarol, Nucl. Phys. B **586**, 141 (2000) [hep-ph/0003129].

[8] S. J. Huber and Q. Shafi, Phys. Lett. B **498**, 256 (2001) [arXiv:hep-ph/0010195]; S. J. Huber, Nucl. Phys. B **666**, 269 (2003) [hep-ph/0303183].

[9] See, for example, R. Contino and A. Pomarol, JHEP **0411**, 058 (2004) [hep-th/0406257].

[10] W. D. Goldberger and M. B. Wise, Phys. Rev. D **60**, 107505 (1999) [hep-ph/9907218]; S. Chang, J. Hisano, H. Nakano, N. Okada and M. Yamaguchi, Phys. Rev. D **62**, 084025 (2000) [hep-ph/9912498].

[11] K. Agashe, A. Delgado, M. J. May and R. Sundrum, JHEP **0308**, 050 (2003) [arXiv:hep-ph/0308036]; K. Agashe, R. Contino, L. Da Rold and A. Pomarol, Phys. Lett. B **641**, 62 (2006) [arXiv:hep-ph/0605341].

[12] M. Carena, E. Ponton, J. Santiago and C. E. M. Wagner, Nucl. Phys. B **759**, 202 (2006) [arXiv:hep-ph/0607106] and M. S. Carena, E. Pontón, J. Santiago and C. E. M. Wagner, Phys. Rev. D **76**, 035006 (2007) [arXiv:hep-ph/0701055]. For a more recent discussion/update, see, for example, C. Delaunay, O. Gedalia, S. J. Lee, G. Perez and E. Ponton, Phys. Rev. D **83**, 115003 (2011) [arXiv:1007.0243 [hep-ph]].

[13] K. Agashe, G. Perez and A. Soni, Phys. Rev. D **71**, 016002 (2005) [arXiv:hep-ph/0408134].

[14] See, for example, C. Csaki, A. Falkowski and A. Weiler, JHEP **0809**, 008 (2008) [arXiv:0804.1954 [hep-ph]]; M. Blanke, A. J. Buras, B. Duling, S. Gori and A. Weiler, JHEP **0903**, 001 (2009) [arXiv:0809.1073 [hep-ph]]; M. Bauer, S. Casagrande, U. Haisch and M. Neubert, JHEP **1009**, 017 (2010) [arXiv:0912.1625 [hep-ph]]. And, for a nice summary of estimates done in the CFT picture, see, for example, table 1 of B. Keren-Zur, P. Lodone, M. Nardecchia, D. Pappadopulo, R. Rattazzi and L. Vecchi, Nucl. Phys. B **867**, 394 (2013) [arXiv:1205.5803 [hep-ph]].

[15] See, for example, R. Barbieri, D. Buttazzo, F. Sala, D. M. Straub and A. Tesi, JHEP **1305**, 069 (2013) [arXiv:1211.5085 [hep-ph]]; D. M. Straub, JHEP **1308**, 108 (2013) [arXiv:1302.4651 [hep-ph]]; M. Konig, M. Neubert and D. M. Straub, Eur. Phys. J. C **74**, 2945 (2014) [arXiv:1403.2756 [hep-ph]].

[16] H. Davoudiasl, S. Gopalakrishna, E. Ponton and J. Santiago, "Warped 5-Dimensional Models: Phenomenological Status and Experimental Prospects," arXiv:0908.1968 [hep-ph].

[17] P. Minkowski, *Phys. Lett.* **B67** (1977) 421; T. Yanagida in *Workshop on Unified Theories, KEK Report 79-18*, p. 95, 1979. M. Gell-Mann, P. Ramond and R. Slansky, *Supergravity*, p. 315. Amsterdam: North Holland, 1979; S. L. Glashow, *1979 Cargese Summer Institute on Quarks and Leptons*, p. 687. New York: Plenum, 1980; R. N. Mohapatra and G. Senjanovic, *Phys. Rev. Lett.* **44**, 912 (1980).

[18] S. J. Huber, Q. Shafi, Phys. Lett. **B583**, 293-303 (2004). [hep-ph/0309252].

[19] C. Csaki, C. Grojean, J. Hubisz, Y. Shirman and J. Terning, Phys. Rev. D **70**, 015012 (2004) [hep-ph/0310355].

[20] G. Perez and L. Randall, JHEP **0901**, 077 (2009) [arXiv:0805.4652 [hep-ph]]; C. Csaki, C. Delaunay, C. Grojean and Y. Grossman, JHEP **0810**, 055 (2008) [arXiv:0806.0356 [hep-ph]]; M. Carena, A. D. Medina, N. R. Shah and C. E. M. Wagner, Phys. Rev. D **79**, 096010 (2009) [arXiv:0901.0609 [hep-ph]].

[21] T. Gherghetta, Phys. Rev. Lett. **92**, 161601 (2004). [hep-ph/0312392]; M. -C. Chen, Phys. Rev. **D71**, 113010 (2005). [hep-ph/0504158]; K. Agashe, T. Okui, R. Sundrum, Phys. Rev. Lett. **102**, 101801 (2009). [arXiv:0810.1277 [hep-ph]]; A. M. Iyer and S. K. Vempati, Phys. Rev. D **88**, 073005 (2013) [arXiv:1307.5773 [hep-ph]]; A. M. Iyer and S. K. Vempati, Phys. Rev. D **86**, 056005 (2012) [arXiv:1206.4383 [hep-ph]]; A. M. Iyer and S. K. Vempati, Phys. Rev. D **88**, 016005 (2013) [arXiv:1304.3558 [hep-ph]]; G. Moreau and J. I. Silva-Marcos, JHEP **0601**, 048 (2006) [hep-ph/0507145]; G. Moreau, Eur. Phys. J. C **40**, 539 (2005) [hep-ph/0407177].

[22] R. N. Mohapatra, Phys. Rev. Lett. **56**, 561 (1986);
R. N. Mohapatra and J. W. F. Valle, Phys. Rev. D **34**, 1642 (1986).

[23] See, for example, P. Creminelli, A. Nicolis and R. Rattazzi, JHEP **0203**, 051 (2002) [hep-th/0107141].

[24] C. S. Fong, R. N. Mohapatra and I. Sung, Phys. Lett. B **704**, 171 (2011) [arXiv:1107.4086 [hep-ph]].

[25] K. Agashe, P. Du, S. Hong and L. Vecchi, in preparation.

[26] L. Vecchi, Phys. Rev. D **90**, no. 2, 025017 (2014) [arXiv:1310.7862 [hep-ph]].