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The B-mode polarization of CMB and Cosmic Neutrino Background

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It is known that in contrast with the E-mode polarization, the B-mode polarization of the Cosmic Microwave Background cannot be generated by the Compton scattering in the case of scalar mode of metric perturbation. However it is possible to generate the B-mode by the Compton scattering in the case of tensor mode of metric perturbation. For this reason, the ratio of tensor to scalar modes of metric perturbation ($r \sim C_{Bl}/C_{El}$) is estimated by comparing the B-mode power spectrum with the E-mode at least for small l. We study the CMB polarization specially B-mode due to the weak interaction of Cosmic Neutrino Background (CNB) and CMB, in addition to the Compton scattering in both cases of scalar and tensor metric perturbations. It is shown that the power spectrum C_{Bl} of the B-mode polarization receives some contributions from scalar and tensor modes, which have effects on the value of r-parameter. We also show that the B-mode polarization power spectrum can be used as an indirect probe into the CNB.

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I. INTRODUCTION

The tensor and scalar metric perturbations are described by their power spectra $P_T = A_T(\frac{k}{k})^{n_T-1}$ and $P_s = A_T(\frac{k}{k})^{n_s-1}$, $n_{T,s}$ and $A_{T,s}$ are their spectral indices and amplitudes. The tensor-to-scalar ratio $r = P_T/P_S$ and its value are very important to verify or constrain the standard scenario of Big-Bang and inflation model. Comparing the curl-like mode (B-mode) with divergence-like (E-mode) in linearly polarized CMB power spectra is one of the most important methods measuring the r-parameter value. The detection of the B-mode signals induced by the primordial gravitational waves can be one of the main targets in many ongoing and future CMB experiments [8]. The BICEP2 collaboration detected the B-mode in the primordial cosmic microwave background (CMB) for the first time and announced $r = 0.20^{+0.07}_{-0.05}$ (68%CL) [1]. This result is not consistent with the Planck limit, $r < 0.11(98\%CL)$. Some authors speculated that the observed B-mode polarization is the result of a primordial Faraday rotation of the E-mode polarization [3, 4]. The BICEP2 data could be explained by the scalar and tensor modes from primordial magnetic fields [2]. The interaction between CMB photons and CNB Neutrinos can produce the B-mode spectrum [5] even with the scalar perturbation only. The Planck group presented discussions that the result of BICEP2 can be fully attributed to cosmic dust [6]. The recent Bicep/Keck Array observation reported upper bounds on the tensor-to-scalar ratio, $r_{0.05} < 0.09$ and $r_{0.05} < 0.07$ at (95%) C.L. by using B-modes alone and combining the B-mode results with Planck temperature analysis, respectively [7]. It is crucial to know all possible effects and interactions can affect the r-parameter value. Therefore, continuing our previous work [5], we study the B-mode spectrum by considering the CMB interacting with the CNB in both cases of scalar and tensor metric perturbations in this article.

II. THE GENERATION OF POLARIZED CMB VIA SCATTERING.

Polarized CMB photons are described by the density and number operators

$$\hat{\rho} = \frac{1}{\text{tr}(\hat{\rho})} \int \frac{d^3p}{(2\pi)^3} \rho_{ij}(p) D_{ij}(p), \tag{1}$$

where $\rho_{ij}(p)$ represents the general density-matrix in the space of polarization states with a fixed energy-momentum "p", correspondingly $D_{ij}(p)$ is the number operator whose expectation value

$$\langle D_{ij}(p) \rangle \equiv \text{tr}[\hat{\rho} D_{ij}(p)] = (2\pi)^3 \delta^3(0)(2p^0)\rho_{ij}(p). \tag{2}$$

The quantum Boltzmann equation for the time-evolution of the density matrix of the polarized CMB photons reads as,

$$\frac{d\rho_{ij}}{dt} =< [H^0, \rho_{ij}] > -\frac{1}{2}\int dt < [H^0, [H^0, \rho_{ij}]] > \tag{3}$$

where H^0 is the effective Hamiltonian. The first term in the RHS is the forward scattering amplitude and the second one represents the high-order collision terms. Equation (3) can be written as [11],

$$(2\pi)^3\delta^3(0)(2p^0)\frac{d}{dt}\rho_{ij}(p) = i\langle[H_I^0(t); D_{ij}(p)]\rangle - \frac{1}{2}\int dt\langle[H_I^o(t); [H_I^0(0); D_{ij}(p)]]\rangle, \tag{4}$$

and $H_I^0(t)$ is the first order of the interacting Hamiltonian. The LHS of this equation is known as the Liouville term dealing with the effects of gravitational perturbations around the homogeneous cosmology. Using this equation, many authors investigated the effects of the Compton scattering (electromagnetic interactions) on the anisotropy and polarization of CMB (see for example [11–13]). We have studied in [5] the effects of the scattering between CMB photons and CNB neutrinos in the case of scalar metric perturbation.

We adopt the Stokes parameters I, Q, U and V to describe the polarizations of CMB photons,

$$\hat{\rho} = \begin{pmatrix} I+Q & U-iV \\ U+iV & I-Q \end{pmatrix}, \tag{5}$$

where the parameter I is total intensity, Q and U are intensities of linear polarizations, whereas the parameter V indicates net circular polarization or the difference between left- and right-circular polarizations intensities. The time evaluation of stocks parameters for linear polarization with collision terms on the right side of Boltzmann equation (4), include the Compton scattering and photon-neutrino interaction [5],

$$\frac{d}{dt}(Q \pm iU) \approx C_{e\gamma}^{\pm} \mp i\dot{\kappa}_{\pm}(Q \pm iU), \tag{6}$$

where RHS of the first term $C_{e\gamma}^{\pm}$ comes from the effects of Compton scattering (see for example [11–13]), whereas the second term comes from the photon-neutrino interaction and $\dot{\kappa}_{\pm}$ is given by

$$\dot{\kappa}_{\pm} = -\frac{\sqrt{2}}{6\pi k^0}\alpha\,G^F\int d\mathbf{q}f_\nu(q) \times \varepsilon_{\mu\nu\rho\sigma}\,\epsilon_2^\mu\epsilon_1^\nu k^\rho q^\sigma$$

$$= \frac{\sqrt{2}}{6\pi k^0}\alpha\,G^F\int d\mathbf{q}f_\nu(q) \times \left[q^0\vec{k}.(\epsilon_1\times\epsilon_2) + k^0\vec{q}.(\epsilon_1\times\epsilon_2)\right] \tag{7}$$

$$= \frac{\sqrt{2}}{6\pi}\alpha\,G^F\left[\frac{n_\nu}{2} + \int d\mathbf{q}f_\nu(q)\vec{q}.(\epsilon_1\times\epsilon_2)\right], \tag{8}$$

which is expressed in terms of the CNB neutrino number density n_ν and bulk velocity

$$q^i = \frac{1}{n_\nu} \int d\mathbf{q} q^i f_\nu. \tag{9}$$

If we apply above equation for Cosmic Neutrino Background (CNB), the second term on the right is much smaller than the first one due to the smallness of the CNB neutrino bulk velocity. For the rest of paper, we consider $\dot{\kappa}_\pm \simeq \frac{\sqrt{2}}{12\pi} \alpha G^F n_\nu$. Note that the time evolution of CMB circular polarization V- Stokes parameter was investigated in Ref. [10], and the total density I does not vary in time.

Another point which must be mentioned is the fact that in the standard model, there is one species of neutrinos, the left-handed Dirac neutrino field, whereas the left-handed Majorana neutrino field has two species for it being a self-conjugated field. As a result, in the Majorana neutrino case, the interacting Hamiltonian H_I^0 has an additional "conjugated" part, compared with the interacting Hamiltonian H_I^0 in the Dirac neutrino case. These two parts have the same contribution to $\frac{d\rho_{ij}}{dt}$, so that $\dot{\kappa}_\pm|_M$ in case of Majorana neutrino is twice larger than one in the Dirac neutrino case $\dot{\kappa}_\pm|_M \simeq 2\dot{\kappa}_\pm$, for more detail see [10]. For the rest, the subscript "M" indicates Majorana neutrino, while we don't use any subscript for Dirac one

III.   POWER SPECTRUM OF SCALAR MODES

As shown in [11, 12], the Compton scattering in the presence of scalar perturbation can be a source for the E-mode of the CMB linear polarization. The B-mode one can only be generated by the Compton scattering in the presence of tensor perturbation. However, the B-mode polarization of CMB in scalar perturbation can be generated by the CMB-CNB interaction together with the Compton scattering [5], and we generalize these results to the case of tensor perturbations in the following. The scalar and tensor perturbations of metric are indicated by S and T superscript respectively. In general, for a given mode of metric perturbation $\mathbf{K}$, we can choose the coordinate system where $\mathbf{K} \parallel \hat{\mathbf{z}}$ and $(\hat{\mathbf{e}}_1, \hat{\mathbf{e}}_2) = (\hat{\mathbf{e}}_\theta, \hat{\mathbf{e}}_\phi)$. Also for each plane wave, we describe the scattering as the transport through a plane parallel medium [14, 15]. In the case of the scalar perturbation, the Boltzmann equation (6) is given as [5],

$$\frac{d}{d\eta}\Delta_I^{(S)} + iK\mu\Delta_I^{(S)} + 4[\dot{\psi} - iK\mu\phi] = \dot{\tau}[-\Delta_I^{(S)} + \Delta_{I0}^{(S)} + i\mu v_b + \frac{1}{2}P_2(\mu)\Pi] \tag{10}$$

$$\frac{d}{d\eta}(Q^{(S)} \pm iU^{(S)}) + iK\mu(Q^{(S)} \pm iU^{(S)}) = \dot{\tau}[-(Q^{(S)} \pm iU^{(S)}) - \frac{1}{2}[1 - P_2(\mu)]\Pi]$$

$$\mp i\, a(\eta)\, \dot{\kappa}_\pm\, (Q^{(S)} \pm iU^{(S)}) \tag{11}$$

$$\Pi = \Delta_{I2}^{(S)} + \Delta_{P2}^{(S)} + \Delta_{P0}^{(S)},$$

where η is the conformal time. The differential optical depth for Compton scattering is denoted by $\dot{\tau} = a n_e x_e \sigma_T$, where $a(\eta)$ is the expansion factor normalized to unity today, v_b is baryon bulk velocity, $\mu = \hat{\mathbf{K}} \cdot \hat{n}$ is the angle between the CMB photon direction $\hat{n}$ and wave number $\mathbf{K}$, n_e is the electron number density, x_e is the ionization fraction and the Thomson cross section is indicated by σ_T. The second term in R.H.S of above equation is the contribution of photon-neutrino forward scattering with $\dot{\kappa}_{\pm}$ coefficients. The source terms in these equations involve the multipole moments of temperature and polarization which are defined as $\Delta(K, \mu) = \sum_l (2l+1)(-i)^l \Delta_l(k) P_l(\mu)$, where $P_l(\mu)$ is the Legendre polynomial of order l. Temperature anisotropies Δ_I^S respect to Δ_P^S have additional sources in metric perturbations ϕ and ψ and in baryon velocity term v_b. By defining $\Delta_P^{\pm(S)} = Q^{(S)} \pm iU^{(S)}$, we rewrite Eq.(11) as following,

$$\frac{d}{d\eta}\left[\Delta_P^{\pm(S)} e^{iK\mu\eta \pm i\tilde{\kappa}(\eta,\mu) + \tilde{\tau}(\eta)}\right] = -e^{iK\mu\eta \pm i\tilde{\kappa}(\eta) + \tilde{\tau}(\eta)}\left(\frac{1}{2}\dot{\tau}[1 - P_2(\mu)]\Pi\right), \qquad (12)$$

where

$$\tilde{\kappa}(\eta, \mu) = \int_0^\eta d\eta\, a(\eta)\, \dot{\kappa}_{\pm}, \quad \tilde{\tau}(\eta) = \int_0^\eta d\eta\, \dot{\tau}. \qquad (13)$$

To obtain the value of $\Delta_P^{\pm(S)}(\hat{n})$ at present time η_0 and direction $\hat{n}$, in addition to integrate the Boltzmann equation (11) along the line of sight [16], one needs to evolve the anisotropies until the present epoch and integrate over all the Fourier modes K,

$$\Delta_P^{\pm(S)}(\hat{\mathbf{n}}) = \int d^3\mathbf{K}\xi(\mathbf{K})e^{\pm 2i\phi_{\mathbf{K},\mathbf{n}}}\Delta_{\mathbf{P}}^{\pm(\mathbf{S})}(\eta_0, \mathbf{K}, \mu), \quad \kappa(\eta) = \int_\eta^{\eta_0} d\eta\, a(\eta)\, \dot{\kappa}_{\pm}(\eta)$$

$$\Delta_P^{\pm(S)}(\eta_0, K, \mu) = \frac{3}{4}(1 - \mu^2)\int_0^{\eta_0} d\eta\, e^{ix\mu \pm i\kappa(\eta) - \tau}\, \Pi(K, \eta) \qquad (14)$$

where $x = K(\eta_0 - \eta)$, $\phi_{\mathbf{K},\mathbf{n}}$ is the angle needed to rotate the $\mathbf{K}$ and $\hat{\mathbf{n}}$ dependent basis to a fixed frame in the Sky, and $\xi(\mathbf{K})$ is a random variable used to characterize the initial amplitude of the mode $\mathbf{K}$ which follows the statistical property

$$\langle \xi^*(\mathbf{K}_1)\xi(\mathbf{K}_2)\rangle = P_S(\mathbf{K})\delta(\mathbf{K}_1 - \mathbf{K}_2) \qquad (15)$$

which $P_S(K)$ is the initial power spectrum of scalar perturbation. Using Friedmann equation in the matter dominate region, $H^2/H_0^2 = \Omega_M^0(1 + z)^3 + \Omega_\Lambda^0$, $H_0 \approx 74\,\text{km/s/Mpc}$, $\Omega_M^0 \approx 0.27, \Omega_\Lambda^0 \approx 0.73$, and $ad\eta = -dz/H(1 + z)$, as well as the conservation of total neutrino number $n_\nu = n_\nu^0(1 + z)^3$, ones can calculate

$$\kappa(z) = \int_\eta^{\eta_0} a\, d\eta\dot{\kappa}_{\pm} = \frac{\sqrt{2}}{12\pi}\alpha\, G^F n_\nu^0 \int_z^{z_{\text{lss}}} dz' \frac{(1 + z')^2}{H(z')}$$

$$= \frac{\sqrt{2}}{12\pi}\alpha\, G^F n_\nu^0 \frac{2H(z')}{3\Omega_M^0 H_0^2}\bigg|_z^{z_{lss}}, \qquad (16)$$

where the present number-density of all flavor neutrinos and anti-neutrinos $n_\nu^0 = \sum(n_\nu^0 + n_{\bar\nu}^0) \approx 340\,\mathrm{cm}^{-3}$), and $\bar\kappa$ is the time average value of $\kappa(\eta)$

$$\bar\kappa = \frac{1}{\eta_0}\int_0^{\eta_0} d\eta\,\bar\kappa \simeq \frac{\sqrt{2}}{12\pi}\alpha\,G^F\,n_{\nu,0}\frac{n_{\nu,0}}{z_{lss}}\int_0^{z_{lss}} dz\frac{(1+z)^3}{H(z)} \sim 0.1, \tag{17}$$

where $z_{lss} \sim 1100$ indicates redshift at the last scattering surface, and also in the case of Majorana neutrinos

$$\bar\kappa|_M \simeq 2\times\bar\kappa \sim 0.2. \tag{18}$$

One can separate the CMB polarization in terms of the curl-free part (E-mode)

$$\Delta_E^{(S)}(\hat n) \equiv -\frac{1}{2}[\bar\eth^2\Delta_P^{+(S)}(\hat{\mathbf n}) + \eth^2\Delta_P^{-(S)}(\hat{\mathbf n})] \tag{19}$$

and divergence-free part (B-mode)

$$\Delta_B^{(S)}(\hat n) \equiv \frac{i}{2}[\bar\eth^2\Delta_P^{+(S)}(\hat{\mathbf n}) - \eth^2\Delta_P^{-(S)}(\hat{\mathbf n})] \tag{20}$$

that $\eth$ and $\bar\eth$ are spin raising and lowering operators respectively. By assuming that scalar metric perturbations are axially-symmetric around $\mathbf{K}$ so that $\eth^2 = \bar\eth^2 = \partial_\mu^2$, the E- and B-modes are given as

$$\Delta_E^{(S)}(K,\eta_0) = -\int_0^{\eta_0} d\eta g(\eta)\frac{3}{4}\Pi(K,\eta)\partial_\mu^2\left[(1-\mu^2)^2 e^{ix\mu}\cos\kappa(\eta)\right], \tag{21}$$

$$\Delta_B^{(S)}(K,\eta_0) = \int_0^{\eta_0} d\eta g(\eta)\frac{3}{4}\Pi(K,\eta)\partial_\mu^2\left[(1-\mu^2)^2 e^{ix\mu}\sin\kappa(\eta)\right], \tag{22}$$

where $g(\eta) = \dot\tau e^{-\tau}$. Note that the Compton scattering can not generate B-mode without taking into account tensor type of metric perturbations [13, 17–19], and we check this in our calculations. The polarized spectrum of CMB is then obtained by integrating over the initial power spectrum of the metric perturbation. As a result, the power spectrum for E and B modes are given by

$$C_{E,Bl}^{(S)} = \frac{1}{2l+1}\frac{(l-2)!}{(l+2)!}\int d^3K P_S(K)|\sum_m\int d\Omega Y_{lm}^*\Delta_{E,B}^{(S)}|^2. \tag{23}$$

By taking the photon-neutrino scattering into account, we showed [5] the $C_{Bl}^{(S)}$ is not zero for the scalar perturbation. The CMB polarized power spectra in multipole moments l are $C_{El}^{(S)} = \bar C_{E,l}^{(S)}(\cos^2\bar\kappa)$ and $C_{Bl}^{(S)} = \bar C_{E,l}^{(S)}(\sin^2\bar\kappa)$. $\bar C_{E,l}^{(S)}$ is the value of E-mode polarized power spectrum attributed to the Compton scattering in the case of scalar perturbation [12]:

$$\bar C_{E,l}^{(S)} = (4\pi)^2\frac{(l+2)!}{(l-2)!}\int d^3K P_S(K)|\frac{3}{4}\int_0^{\eta_0} d\eta\,g(\eta)\,\Pi(K,\eta)\frac{j_l}{x^2}|^2. \tag{24}$$

The term with $\bar\kappa$ shows that the B-mode power spectrum can be generated not only by the Compton scattering in the case of tensor mode perturbations [12], but also by the CMB-CNB interaction in the case of scalar mode perturbation.

IV. POWER SPECTRUM OF TENSOR PERTURBATION MODES

The method of analysis used in previous section for the scalar perturbation can be utilized for the tensor perturbation. But the analysis is more complicate than that for the scalar perturbation because there are two independent polarized states denoted by $+$ and $\times$ for each Fourier mode. It is convenient to use two linear combinations

$$\xi^1 = (\xi^+ - i\xi^\times)/2, \quad \xi^2 = (\xi^+ + i\xi^\times)/2, \tag{25}$$

where ξ's are independent random variables and have the following statistical properties

$$\langle \xi^{1*}(\vec{K_1})\xi^1(\vec{K_2})\rangle = \langle \xi^{2*}(\vec{K_1})\xi^2(\vec{K_2})\rangle = \frac{P_h(K)}{2}\delta(\vec{K_1} - \vec{K_2}) \quad , \quad \langle \xi^{1*}(\vec{K_1})\xi^2(\vec{K_2})\rangle = 0 \tag{26}$$

where $P_h(K)$ is the primordial power spectrum of the gravitational wave. Also we define

$$\tilde{\Delta}_{P\pm}^{+(\times)}(\tau, \hat{n}, \vec{K}) = (\tilde{\Delta}_Q^{+(\times)} \pm i\tilde{\Delta}_U^{+(\times)})(\tau, \hat{n}, \vec{K}). \tag{27}$$

In the absence of CMB-CNB interaction, the polarizations generated by the gravitational wave satisfy the following Boltzman equations [20, 21]:

$$\dot{\tilde{\Delta}}_{P\pm}^{+(\times)} + ik\mu\tilde{\Delta}_{P\pm}^{+(\times)} = -\dot{\tau}[\tilde{\Delta}_{P\pm}^{+(\times)} + \psi(1 \pm i)]. \tag{28}$$

where $\dot{\tau} \equiv \frac{d\tau}{d\eta}$ and η is conformal time, the scaling factor $a(\eta_0)$ at present time η_0 is unity, and

$$\psi \equiv [\frac{1}{10}\tilde{\Delta}_{T0}^{(T)} + \frac{1}{7}\tilde{\Delta}_{T2}^{(T)} + \frac{3}{7}\tilde{\Delta}_{T4}^{(T)} - \frac{3}{5}\tilde{\Delta}_{Q0}^{\pm(T)} + \frac{6}{7}\tilde{\Delta}_{Q2}^{\pm(T)} - \frac{3}{70}\tilde{\Delta}_{Q4}^{\pm(T)}]. \tag{29}$$

In the presence of CMB-CNB interaction, the Boltzmann equations for polarizations generated by gravitational wave are modified

$$\dot{\tilde{\Delta}}_{P\pm}^{+(\times)} + ik\mu\tilde{\Delta}_{P\pm}^{+(\times)} = -\dot{\tau}[\tilde{\Delta}_{P\pm}^{+(\times)} + \psi(1 \pm i)] \mp ia(\eta)\dot{\kappa}_\pm\tilde{\Delta}_{P\pm}^{+(\times)}. \tag{30}$$

The above equation has the following solution [12]

$$\tilde{\Delta}_{P\pm}^{+(\times)}(\kappa, \eta) = \int d\eta\, e^{ix\mu \pm i\kappa \pm i\frac{\pi}{4}} S_P(k, \eta) \tag{31}$$

where $S_P(k, \eta) = \sqrt{2}\dot{\tau}(e^{-\tau})\psi$. For the tensor perturbation, the evolution equations take their simplest form after the coordinate transformation [21, 22],

$$\Delta_Q^+ = (1 + \mu^2)\cos(2\phi)\tilde{\Delta}_Q^+, \quad \Delta_Q^\times = (1 + \mu^2)\sin(2\phi)\tilde{\Delta}_Q^\times \tag{32}$$

$$\Delta_U^+ = -2\mu\sin(2\phi)\tilde{\Delta}_I^+, \qquad \Delta_U^\times = 2\mu\cos(2\phi)\tilde{\Delta}_U^\times. \tag{33}$$

Tensor perturbations can be decomposed to the general form

$$(\Delta_Q^{(T)} \pm i\Delta_U^{(T)})(\tau, \hat{n}, \vec{K}) = (\Delta_Q^+ \xi^+ + \Delta_Q^\times \xi^\times)(\tau, \hat{n}, \vec{K}) \pm i(\Delta_U^+ \xi^+ + \Delta_U^\times \xi^\times)(\tau, \hat{n}, \vec{K})$$

$$(\Delta_Q^{(T)} \pm i\Delta_U^{(T)})(\tau, \hat{n}, \vec{K}) = (1 + \mu^2)\cos 2\phi[\xi^1 + \xi^2]\tilde{\Delta}_Q^+ + (1 + \mu^2)\sin 2\phi[\xi^1 - \xi^2]i\tilde{\Delta}_Q^\times$$

$$\pm i(2\mu)(-\sin 2\phi[\xi^1 + \xi^2]\tilde{\Delta}_U^+ + (1 + \mu^2)\cos 2\phi[\xi^1 - \xi^2]i\Delta_U^\times). \quad (34)$$

Substituting $\tilde{\Delta}_Q = \frac{1}{2}(\tilde{\Delta}_P^+ + \tilde{\Delta}_P^-)$ and $\tilde{\Delta}_U = (\frac{-i}{2})(\tilde{\Delta}_P^+ - \tilde{\Delta}_P^-)$ into Eqs. (34), we obtain

$$(\Delta_Q^{(T)} \pm i\Delta_U^{(T)})(\tau, \hat{n}, \vec{K}) = (1 + \mu^2)\xi^1 e^{2i\phi}(\tilde{\Delta}_P^{+(T)} + \tilde{\Delta}_P^{-(T)}) + (1 + \mu^2)\xi^2 e^{-2\phi}(\tilde{\Delta}_P^{+(T)} + \tilde{\Delta}_P^{-(T)})$$

$$\pm (-i)(2\mu)(\tilde{\Delta}_P^{+(T)} - \tilde{\Delta}_P^{-(T)})[-\xi^1 e^{2i\phi} - \xi^2 e^{-2i\phi}]. \quad (35)$$

Using $2\mu = \frac{1}{2}[(1 + \mu)^2 - (1 - \mu)^2]$ and $1 + \mu^2 = \frac{1}{2}[(1 + \mu)^2 + (1 - \mu)^2]$ and then similarly to the scalar perturbation, we integrate along the line of sight

$$(\Delta_Q^{(T)} + i\Delta_U^{(T)})(\tau, \hat{n}, \vec{K}) = \frac{1}{2}[(1 + \mu)^2(1 + 2i) + (1 - \mu)^2(1 - 2i)][\xi^1 e^{2i\phi} + \xi^2 e^{-2i\phi}]\tilde{\Delta}_p^+$$

$$+ \frac{1}{2}[(1 + \mu)^2(1 - 2i) + (1 - \mu)^2(1 + 2i)][\xi^1 e^{2i\phi} + \xi^2 e^{-2i\phi}]\tilde{\Delta}_p^-$$

$$(\Delta_Q^{(T)} - i\Delta_U^{(T)})(\tau, \hat{n}, \vec{K}) = \frac{1}{2}[(1 + \mu)^2(1 - 2i) + (1 - \mu)^2(1 + 2i)][\xi^1 e^{2i\phi} + \xi^2 e^{-2i\phi}]\tilde{\Delta}_p^+$$

$$+ \frac{1}{2}[(1 + \mu)^2(1 + 2i) + (1 - \mu)^2(1 - 2i)][\xi^1 e^{2i\phi} + \xi^2 e^{-2i\phi}]\tilde{\Delta}_p^-. \quad (36)$$

Using the spin raising and lowering operators twice as following,

$$\bar{\eth}^2[(1 \pm \mu)^2(1 - \mu^2)e^{ix\mu}] = [-\hat{\varepsilon}(x) \pm i\hat{\beta}(x)][(1 - \mu^2)e^{ix\mu}]$$

$$\eth^2[(1 \pm \mu)^2(1 - \mu^2)e^{ix\mu}] = [-\hat{\varepsilon}(x) \pm i\hat{\beta}(x)][(1 - \mu^2)e^{ix\mu}] \quad (37)$$

where $\hat{\varepsilon}(x) = -12 + x^2[1 - \partial_x^2] - 8x\partial_x$ and $\hat{\beta}(x) = 8x + 2x^2\partial_x$, we can separate the CMB linearly polarizations in terms of E-modes $\Delta_E^{(T)}$ and B-modes $\Delta_B^{(T)}$

$$\Delta_E^{(T)}(\hat{n}) \equiv \frac{-1}{2}[\bar{\eth}^2\Delta_P^{+(T)}(\hat{n}) + \eth^2\Delta_P^{-(T)}]$$

$$\Delta_B^{(T)}(\hat{n}) \equiv \frac{i}{2}[\bar{\eth}^2\Delta_P^{+(T)}(\hat{n}) - \eth^2\Delta_P^{-(T)}]. \quad (38)$$

As a result, we obtain

$$\Delta_E^{(T)}(\eta, \hat{n}, \vec{k}) = \sqrt{2}[(1 - \mu^2)e^{2i\varphi}\xi^1(\vec{k}) + (1 - \mu^2)e^{-2i\varphi}\xi^2(\vec{k})] \int_0^{\eta_0} d\eta e^{ix\mu} S_P^T(k, \eta)[\hat{\varepsilon}(x)\cos(\kappa(\eta) + \frac{\pi}{4})]$$

$$\Delta_B^{(T)}(\eta, \hat{n}, \vec{k}) = \sqrt{2}[(1 - \mu^2)e^{2i\varphi}\xi^1(\vec{k}) - (1 - \mu^2)e^{-2i\varphi}\xi^2(\vec{k})] \int_0^{\eta_0} d\eta e^{ix\mu} S_P^T(k, \eta)[\hat{\beta}(x)\sin(\kappa(\eta) + \frac{\pi}{4})].$$

$$(39)$$

The E-mode power spectrum is given as

$$C_{El}^{(T)} = (4\pi)^2 \int k^2 dk P_h(k)(\int_0^{\eta_0} d\eta e^{ix\mu} S_P^{(T)}(k, \eta)[\hat{\varepsilon}(x)\frac{\cos(\kappa(\eta) + \frac{\pi}{4})}{\cos \frac{\pi}{4}}]\frac{j_l(x)}{x^2})^2, \quad (40)$$
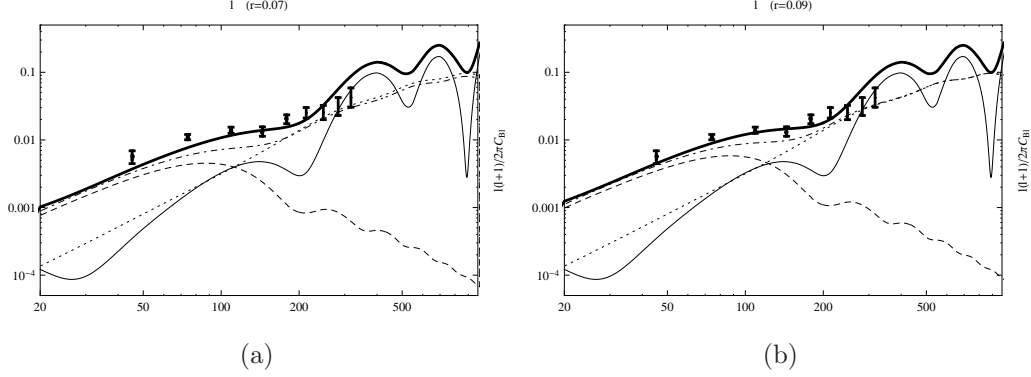
FIG. 1: The B-mode power spectrum $l(l+1)/2\pi \ \ C_{Bl}$ is plotted in terms of l and in unit $(\mu K)^2$ for the tensor to scalar ratio $r = 0.07$ (1a) and $r = 0.09$ (1b) in the case of CNB as Dirac neutrino. This file contains the LCDM power spectra that are derived from Planck (2013) parameters. Above curves indicate B-mode power spectrum in the presence of different effects such as the tensor perturbations (dashed line), lens effects (dotted line), the tensor perturbations in additional to lens effects (dotted-dashed line), Dirac CNB-CMB interaction, Compton scattering in the case of scalar perturbations (tiny solid line) and all mentioned effects (thick solid curve) and the recent observed data (Bicep/Keck Array) is added [7].

which is the same as that in the standard scenario for CMB polarizations without considering the CNB-CMB interactions, except for an additional factor $(\frac{\cos(\kappa(\eta)+\frac{\pi}{4})}{\cos\frac{\pi}{4}})^2$. This factor becomes unit if the CNB-CMB interaction ($\kappa \to 0$) is neglected and the result of standard scenario for CMB polarizations for the E-mode can be yielded. Using similar method , we obtain the power spectrum of B-mode

$$C_{Bl}^{(T)} = (4\pi)^2 \int k^2 dk P_h(k) (\int_0^{\eta_0} d\eta e^{ix\mu} S_P^{(T)}(k,\eta)[\hat{\beta}(x)\frac{\sin(\kappa(\eta)+\frac{\pi}{4})}{\sin(\frac{\pi}{4})}]\frac{j_l(x)}{x^2})^2. \tag{41}$$

Similar to the E-mode power spectrum $C_{El}^{(T)}$, the B-mode power spectrum $C_{Bl}^{(T)}$ is the same as a result of the standard scenario for CMB polarizations, except an additional factor $(\frac{\sin(\kappa(\eta)+\frac{\pi}{4})}{\sin(\frac{\pi}{4})})^2$ which becomes unit when $\kappa \to 0$. The expressions for the power spectra of E and B modes are different in κ-dependent terms, also in the $\hat{\varepsilon}$ and $\hat{\beta}$ operators, which is obtained after the angular integrals.

V.   CONCLUSION.

The generation of the B-mode spectrum of CMB photons in the both cases of scalar and tensor perturbations is calculated by using Quantum Boltzmann Equation for the density matrix or
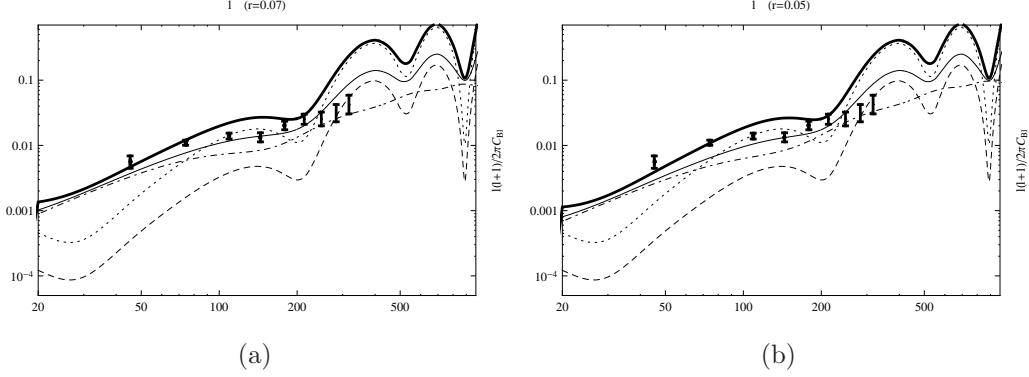
FIG. 2: The B-mode power spectrum $l(l + 1)/2\pi \ \ C_{Bl}$ is plotted in terms of l and in unit $(\mu K)^2$ for the tensor to scalar ratio $r = 0.07$ (2a) and $r = 0.05$ (2b) by considering CNB as Dirac or Majorana neutrino. This file contains the LCDM power spectra that are derived from Planck (2013) parameters. Above curves indicate B-mode power spectrum in the presence of different effects such as Dirac CNB-CMB interaction and Compton scattering in the case of scalar perturbations (dashed line), Majorana CNB-CMB interaction and Compton scattering in the presence of scalar perturbations (dotted line), the tensor perturbation in additional to lens effects (dotted-dashed line), all mentioned effects by considering CNB as Dirac neutrino (tiny solid curve) or Majorana neutrino (thick solid curve) and the recent observed data (Bicep/Keck Array) is added [7].

Stokes parameters of CMB photons, taking into account both CMB-CNB interaction and Compton scattering. As shown in Eqs.(23), (40) and (41), the power spectra of E-mode and B-mode of CMB polarizations are modified in the presence of CNB-CMB interaction by considering CNB as Dirac or Majorana neutrino. The most important point is that the B-mode is generated by the CNB-CMB interaction in the presence of scalar perturbation. The value of B-mode power spectrum in presence of tensor perturbation and CNB-CNB interaction $C_{Bl}^{(T)}$ is modified by an additional factor $(\frac{\sin(\kappa(\eta)+\frac{\pi}{4})}{\sin(\frac{\pi}{4})})^2$ which becomes unit (similar to standard scenario for generating CMB polarizations) when $\kappa \to 0$. The generated B-mode spectrum is approximately proportional to $\propto (\alpha \, G^F \, H_0^{-1} \, \bar{n}_\nu)^2$, where $\bar{n}_\nu$ is the average CNB number density from the last scattering up to present time. As discussed in Ref. [10], for the reason of neutrinos being left-handed and their gauge-couplings being parity violated, linearly polarized photons acquire their circular and magnetic-like linearly polarizations by interacting with neutrinos. It is important to mention that cosmic neutrinos can be either Dirac or Majorana types, the rate of B-mode generation $\dot{\kappa}_\pm$ for the Dirac neutrino case is about twice less than the rate for the Majorana neutrino case [10].

The B-mode power spectrum of CMB for different cases is plotted in Figs.(1) and (2). The results show that the contribution of CNB-CMB interactions (for both cases of Dirac or Majorana CNB) have caused an enhancement in C_{Bl} for $l > 300$, which could be used to show not only for the evidence for CNB, since the CNB neutrinos have not yet been directly detected, due to their weak interactions and very low energy, but also may it helps to distinguish that CNB is Dirac or Majorana neutrino.

To end this article, we would like to emphasize that the r-parameter is usually calculated by comparing B-mode and E-mode power spectrum and assuming that the observed B-mode is totally attributed to the Compton scattering in the presence of tensor perturbations $C_{Bl}^{ob} = C_{Bl}^{T}$. However our calculations show that other interactions like CNB-CMB interaction can have contribution to the B-mode power spectrum in the presence of scalar mode C_{Bl}^{S}. For this situation the observed B-mode contains a contribution from scalar perturbation in additional to tensor one, so due to this fact we have

$$r \propto C_{Bl}^{T}/C_{El}^{S} \propto (C_{Bl}^{ob} - C_{Bl}^{S})/C_{El}^{S} \tag{42}$$

As result the exact value of r-parameter is suppressed see Fig.(2). By using Eqs.(17,22,23) and above equation, ones can approximately estimate

$$r \simeq r^{*} - \sin^2 \bar{\kappa} \simeq r^{*} - \bar{\kappa}^2, \tag{43}$$

where r^{*} is the standard tensor to scalar ratio without considering CNB-CMB interaction. As above equation shows that r is approximately suppressed by $\bar{\kappa}^2$ which is about $\bar{\kappa}^2 \sim 0.01$ in the case of CNB as Dirac neutrino and $\bar{\kappa}^2|_M \sim 0.04$ in the case of CNB as Majorana neutrino.

VI. ACKNOWLEDGEMENTS

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