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Verifiable Associated Processes from Radiative Lepton Masses with Dark Matter

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Abstract

If leptons do not couple directly to the one Higgs doublet of the standard model of particle interactions, they must still do so somehow indirectly to acquire mass, as proposed recently in several models where it happens in one loop through dark matter. We analyze the important consequences of this scenario in a specific model, including Higgs decay, muon anomalous magnetic moment, $\mu \to e\gamma$, $\mu \to eee$, and the proposed dark sector.

1 Introduction

The idea that lepton masses are induced in one loop has been around for a long time. Recently it has been proposed [1, 2, 3] that the particles in the loop are distinguished from ordinary matter by an unbroken symmetry so that the lightest neutral particle among them may be the dark matter of the Universe. As an example, consider the specific proposal of Ref. [3] for generating charged-lepton masses. This model assumes the non-Abelian discrete

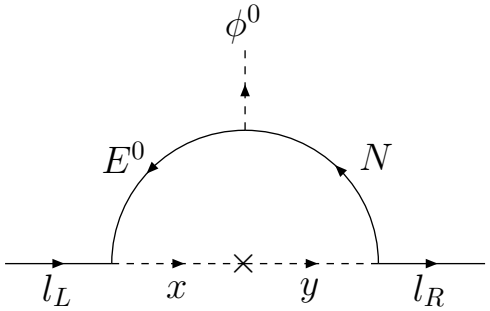

Figure 1: One-loop generation of charged-lepton mass.

symmetry A_4 under which the three families of leptons transform as

$$(\nu_i, l_i)_L \sim \underline{3}, \quad l_{iR} \sim \underline{1}, \underline{1}', \underline{1}''. \tag{1}$$

With only the one Higgs doublet (ϕ^+, ϕ^0) of the standard model (SM) transforming as $\underline{1}$, a tree-level lepton mass is forbidden. To obtain one-loop radiative lepton masses, the following new particles are added, all of which are odd under an unbroken dark Z_2 symmetry:

$$(E^0, E^-)_{L,R} \sim \underline{1}, \quad N_{L,R} \sim \underline{1}, \quad x_i^- \sim \underline{3}, \quad y_i^- \sim \underline{1}, \underline{1}', \underline{1}'', \tag{2}$$

where $(E^0, E^-), N$ are fermions and x^-, y^- are charged scalars. Note that in supersymmetry, there are also similar new particles, i.e. left and right charged sleptons and doublet Higgsinos. The soft breaking of A_4 to Z_3 lepton triality [4, 5] is encoded in the scalar off-diagonal mass-squared $x_i y_j^*$ terms. In this paper we will study the phenomenological consequences of this

proposal, including the deviation of the Higgs to charged-lepton decay from the SM, the muon anomalous magnetic moment, $\mu \to e\gamma$, $\mu \to eee$, as well as the structure of its dark sector.

2 Radiative Lepton Masses

The mass matrix linking $(\bar{N}_L, \bar{E}^0_L)$ to (N_R, E^0_R) is given by

$$\mathcal{M}_{N,E} = \begin{pmatrix} m_N & m_D \\ m_F & m_E \end{pmatrix}, \tag{3}$$

where m_N, m_E are invariant mass terms, and m_D, m_F come from the Higgs Yukawa terms $f_D \bar{N}_L E^0_R \bar{\phi}^0$, $f_F \bar{E}^0_L N_R \phi^0$ with vacuum expectation value $\langle \phi^0 \rangle = v/\sqrt{2}$. As a result, N and E^0 mix to form two Dirac fermion eigenstates

$$n_{1(L,R)} = \cos\theta_{L,R} N_{L,R} - \sin\theta_{L,R} E^0_{L,R}, \quad n_{2(L,R)} = \sin\theta_{L,R} N_{L,R} + \cos\theta_{L,R} E^0_{L,R}, \tag{4}$$

of masses $m_{1,2}$, with mixing angles

$$m_D m_E + m_F m_N = \sin\theta_L \cos\theta_L (m_1^2 - m_2^2), \tag{5}$$

$$m_D m_N + m_F m_E = \sin\theta_R \cos\theta_R (m_1^2 - m_2^2). \tag{6}$$

With the A_4 assignment of Eq. (2), and the soft breaking to Z_3 of the term $x_i y_j^*$, i.e.

$$U_\omega \begin{pmatrix} \mu_e^2 & 0 & 0 \\ 0 & \mu_\mu^2 & 0 \\ 0 & 0 & \mu_\tau^2 \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} \mu_e^2 & 0 & 0 \\ 0 & \mu_\mu^2 & 0 \\ 0 & 0 & \mu_\tau^2 \end{pmatrix}, \tag{7}$$

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$, and U_ω is the familiar [6] unitary matrix derivable from A_4, the charged-lepton mass matrix is given by

$$\mathcal{M}_l = U_\omega^\dagger \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \tag{8}$$

with

$$m_e = -if' f_e \mu_e^2 \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_{1e}^2)(k^2 - m_{2e}^2)} \left[\frac{m_1 \cos\theta_R \sin\theta_L}{k^2 - m_1^2} - \frac{m_2 \cos\theta_L \sin\theta_R}{k^2 - m_2^2} \right], \quad (9)$$

where f' is the $E_L^0 l_L x^*$ Yukawa coupling, f_e is the $N_R e_R y_1^*$ Yukawa coupling, and $m_{1e,2e}$ are the mass eigenvalues of the 2×2 mass-squared matrix

$$\mathcal{M}_{xy_1}^2 = \begin{pmatrix} m_x^2 & \mu_e^2 \\ \mu_e^2 & m_{y_1}^2 \end{pmatrix}, \quad (10)$$

with $\mu_e^2 = \sin\theta_e \cos\theta_e (m_{1e}^2 - m_{2e}^2)$, and similarly for m_μ and m_τ. It is clear that the residual Z_3 triality [4, 5] remains exact with $e, \mu, \tau \sim 1, \omega^2, \omega$, and the Higgs coupling matrix as well as the anomalous magnetic moment matrix are diagonal, as far as Fig. 1 is concerned. In other words, flavor is not violated in Higgs decays and $\mu \to e\gamma$ is not mediated by the new particles of Eq. (2).

3 Anomalous Higgs Yukawa Couplings

One immediate consequence of a radiative charged-lepton mass is that the Higgs Yukawa coupling $h\bar{l}l$ is no longer exactly m_l/v as in the SM. Its deviation is not suppressed by the usual one-loop factor of $16\pi^2$ and may be large enough to be observable [7]. Moreover, this deviation is finite and calculable exactly in one loop. For discussion, compare our proposal to the usual consideration of the deviation of the Higgs coupling from m_l/v from new physics in terms of higher-dimensional operators, i.e.

$$-\mathcal{L} = f_l \bar{l}_L l_R \phi^0 \left(1 + \frac{\Phi^\dagger \Phi}{\Lambda^2} \right), \quad (11)$$

where $\Lambda^2 >> v^2$. This implies $m_l = (f_l v/\sqrt{2})(1 + v^2/2\Lambda^2)$, whereas the Higgs coupling is $(f_l/\sqrt{2})(1 + 3v^2/2\Lambda^2) \simeq (m_l/v)(1 + v^2/\Lambda^2)$. However, this approach is only valid for $v^2 << \Lambda^2$, which guarantees the effect to be small. In the present case, if our result is

interpreted as an expansion in powers of v^2, then it is a sum of infinite number of terms for both m_l and the Higgs coupling, but each sum is finite. Their ratio is not necessarily small because some particles in the loop could be light, as shown below.

There are three contributions to the $h\bar{l}l$ coupling: (1) the Yukawa terms $(f_D/\sqrt{2})h\bar{N}_L E^0_R$ and $(f_F/\sqrt{2})h\bar{E}^0_L N_R$, (2) the scalar trilinear $(\lambda_x v)hx^*x$ term, and (3) the scalar trilinear $(\lambda_y v)hy^*y$ term. In the following expressions, the couplings $f_{D,F}$ do not appear explicitly because they have been expressed in terms of the fermion masses $m_{1,2}$ and angles $\theta_{L,R}$. Consider $h\bar{\tau}\tau$. The first contribution is given by

$$f^{(1)}_\tau = \frac{f' f_\tau \sin 2\theta_\tau}{32\pi^2 v}[c_R s_L T_1 + s_L s_R T_2 + c_L c_R T_3 + c_L s_R T_4], \tag{12}$$

where $x_{ij} = (\frac{m_{i\tau}}{m_j})^2$, $s_{L,R} = \sin\theta_{L,R}$, $c_{L,R} = \cos\theta_{L,R}$ and

$$\begin{aligned}
F_N(x) &= \frac{x(1+x)\ln x}{(1-x)^2} + \frac{2}{1-x}, \quad H(x) = \frac{x}{x-1}\ln x \\
T_1 &= [2m_2 s_L c_L s_R c_R - m_1(s^2_L c^2_R + c^2_L s^2_R)][F_N(x_{11}) - F_N(x_{21})], \\
T_2 &= m_2 s_L c_L (c^2_R - s^2_R)[H(x_{22}) - H(x_{12})] - m_1 s_R c_R (c^2_L - s^2_L)[H(x_{21}) - H(x_{11})], \\
T_3 &= m_1 s_L c_L (c^2_R - s^2_R)[H(x_{21}) - H(x_{11})] - m_2 s_R c_R (c^2_L - s^2_L)[H(x_{22}) - H(x_{12})], \\
T_4 &= [2m_1 c_L c_R s_L s_R - m_2(s^2_L c^2_R + c^2_L s^2_R)][F_N(x_{12}) - F_N(x_{22})].
\end{aligned} \tag{13}$$

The second contribution is given by

$$f^{(2)}_\tau = \frac{\lambda_x v f' f_\tau \sin 2\theta_\tau s_L c_L}{32\pi^2 m_1 m_2}[c^2_\tau T'_1 + s^2_\tau T'_2], \tag{14}$$

where $c_\tau = \cos\theta_\tau$, $s_\tau = \sin\theta_\tau$ and

$$\begin{aligned}
F(x,y) &= \frac{1}{x-y}\left[\frac{x}{x-1}\ln x - \frac{y}{y-1}\ln y\right] \quad x \neq y, \quad F(x,x) = \frac{1}{x-1} - \frac{\ln x}{(x-1)^2}, \\
T'_1 &= m_2[F(x_{11},x_{11}) - F(x_{11},x_{21})] - m_1[F(x_{12},x_{12}) - F(x_{12},x_{22})], \\
T'_2 &= m_2[F(x_{11},x_{21}) - F(x_{21},x_{21})] - m_1[F(x_{12},x_{22}) - F(x_{22},x_{22})].
\end{aligned} \tag{15}$$

The third contribution is given by

$$f_\tau^{(3)} = \frac{\lambda_y v f' f_\tau \sin 2\theta_\tau s_L c_L}{32\pi^2 m_1 m_2}[s_\tau^2 T_1' + c_\tau^2 T_2']. \tag{16}$$

Combining all three contributions and using Eq. (9) for the tau mass, the effective Higgs Yukawa coupling $\tilde{f}_\tau$ is given by

$$\begin{aligned}\frac{\tilde{f}_\tau v}{m_\tau} &= \frac{[f_\tau^{(1)} + f_\tau^{(2)} + f_\tau^{(3)}]v}{m_\tau} \\ &= \frac{c_R s_L T_1 + s_L s_R T_2 + c_L c_R T_3 + c_L s_R T_4 + \frac{v^2 s_L c_L}{m_1 m_2}[(\lambda_x c_\tau^2 + \lambda_y s_\tau^2)T_1' + (\lambda_x s_\tau^2 + \lambda_y c_\tau^2)T_2']}{s_L c_R m_1[H(x_{21}) - H(x_{11})] + s_R c_L m_2[H(x_{12}) - H(x_{22})]}.\end{aligned} \tag{17}$$

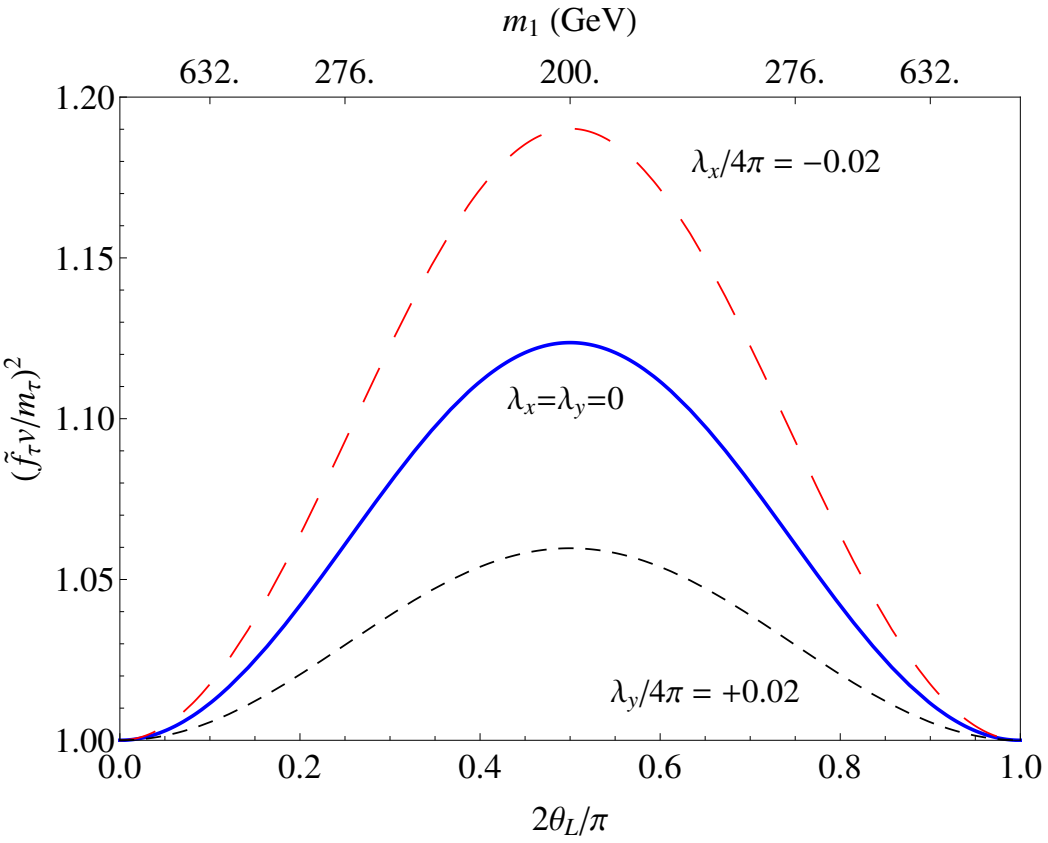

Figure 2: The ratio $(\tilde{f}_\tau v/m_\tau)^2$ plotted against θ_L with various $\lambda_{x,y}$ for the case $\theta_L = \theta_R$.

To simplify the analysis, we focus on $\theta_L = \theta_R$, in which case $f_D = f_F$. We use the relation $f_D v/\sqrt{2} = s_L c_L (m_1 - m_2) = s_L c_L m_1 (1 - m_2/m_1)$ from fermion mixing to define m_1 as a function of θ_L for a constant ratio $m_2/m_1 = 2.2$ and coupling $f_D/\sqrt{4\pi} = -0.19$. In this parameterization, the combination $s_L c_L m_1$ remains constant, and also appears in the radiative mass formula for each charged lepton. In addition, we use the value $f'/\sqrt{4\pi} = -0.6$. For the scalars in the tau sector, we choose fixed mass ratios $m_{1\tau}/m_1 = 5.7$ and $m_{2\tau}/m_1 = 1.1$. To satisfy the mass formula, we verify that the product $f_\tau \sin 2\theta_\tau$ is not too large. We have checked that the values used here also allow solutions for the muon and electron radiative masses. In Fig. 2 we plot the effective Yukawa coupling from Eq. (17) as a function of θ_L, using the values $f_\tau/\sqrt{4\pi} = -0.54$, $\theta_\tau = 0.8$ for the $\lambda_{x,y}$ curves. We see that a significant deviation from the SM prediction is possible.

4 Muon Anomalous Magnetic Moment

Another important consequence of a radiative charged-lepton mass is that the same particles which generate m_l also contribute to its anomalous magnetic moment. This differs from the usual contribution of new physics, because there is again no $16\pi^2$ suppression. There are three contributions to the anomalous magnetic moment. The main contribution is given by

$$\Delta a_\mu = \frac{m_\mu^2}{m_1 m_2} \left\{ \frac{s_L c_R m_2 [G(x_{11}) - G(x_{21})] + s_R c_L m_1 [G(x_{22}) - G(x_{12})]}{s_L c_R m_1 [H(x_{11}) - H(x_{21})] + s_R c_L m_2 [H(x_{22}) - H(x_{12})]} \right\}, \tag{18}$$

where $x_{ij} = (\frac{m_{i\mu}}{m_j})^2$ and

$$G(x) = \frac{2x \ln x}{(x-1)^3} - \frac{x+1}{(x-1)^2}. \tag{19}$$

In the simplifying case we are considering, Eq. (18) is independent of $\theta_L = \theta_R$. In Fig. 3 we plot $m_{1\mu}$ against m_1 for various ratios $m_{2\mu}/m_{1\mu}$ in order to show the values of m_1 and $m_{1,2\mu}$ which can account for the discrepancy between the experimental measurement [8] and the

SM prediction [9]

$$\Delta a_\mu = 39.35 \pm 5.21_{\rm th} \pm 6.3_{\rm exp} \times 10^{-10} \tag{20}$$

We have combined the experimental and theoretical uncertainties in quadrature, which corresponds to the curved limits of the shaded regions. The lower limit of 200 GeV for m_1 corresponds to $\theta_L = \pi/4$.

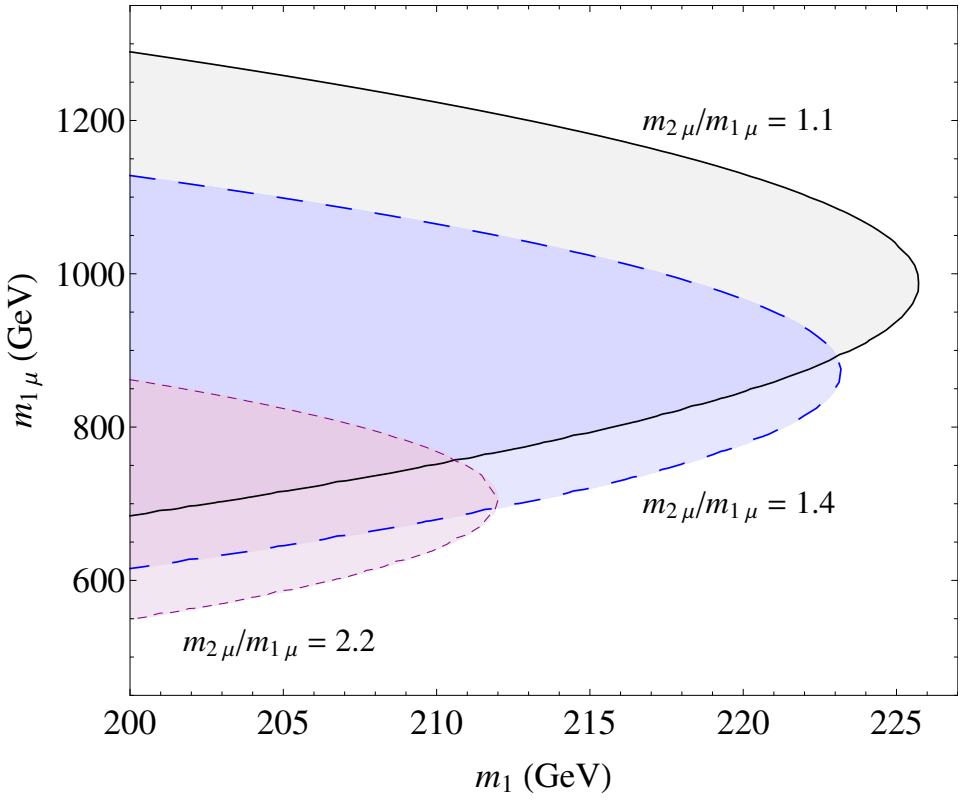

Figure 3: Values of m_1 and $m_{1,2\mu}$ which can explain Δa_μ for the case $\theta_L = \theta_R$.

The subdominant contributions to Δa_μ from f'^2 , and f_μ^2 are negative as expected , i.e.

$$\begin{aligned}(\Delta a_\mu)' = \frac{-m_\mu^2}{32\pi^2}\Big\{ \quad & f'^2\Big[\tfrac{s_L^2}{m_1^2}\Big(c_\mu^2 J(x_{11}) + s_\mu^2 J(x_{21})\Big) + \tfrac{c_L^2}{m_2^2}\Big(c_\mu^2 J(x_{12}) + s_\mu^2 J(x_{22})\Big)\Big] \\ & + f_\mu^2\Big[\tfrac{c_R^2}{m_1^2}\Big(s_\mu^2 J(x_{11}) + c_\mu^2 J(x_{21})\Big) + \tfrac{s_R^2}{m_2^2}\Big(s_\mu^2 J(x_{12}) + c_\mu^2 J(x_{22})\Big)\Big]\Big\},\end{aligned} \tag{21}$$

where

$$J(x) = \frac{x \ln x}{(x-1)^4} + \frac{x^2 - 5x - 2}{6(x-1)^3}. \tag{22}$$

The third contribution is from s exchange which will be introduced in the next section and is given by

$$(\Delta a_\mu)'' = \sum_{i=1}^{3} \frac{-f^2 |U_{\mu i}|^2 m_\mu^2}{16\pi^2 m_E^2} G_\gamma(x_i), \tag{23}$$

where $x_i = \frac{m_{s_i}^2}{m_E^2}$ and

$$G_\gamma(x) = \frac{2x^3 + 3x^2 - 6x^2 \ln x - 6x + 1}{6(x-1)^4} < \frac{1}{6}. \tag{24}$$

The mass of E^- has a lower limit of $m_E \simeq 300$ GeV, which is numerically equivalent to $G_F m_E^2 \simeq 1$ used in the following section, due to our parameterization for the fermion mixing of N and E^0. Hence $(\Delta a_\mu)''$ is less than $10^{-10} f^2$, which for $f < 1$ is below the present experimental sensitivity of 10^{-9} and thus can be neglected.

5 Rare Lepton Decays

Whereas Z_3 lepton triality is exact in Fig. 1, the corresponding diagram for neutrino mass breaks it, as shown below. The new particles are three real scalars $s_{1,2,3} \sim \underline{3}$ under A_4.

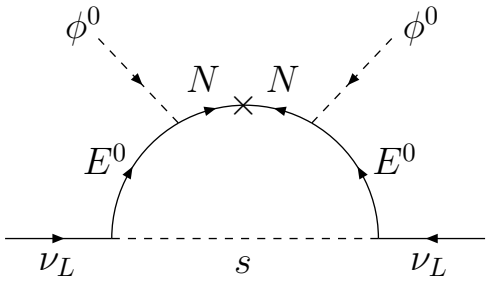

Figure 4: One-loop generation of neutrino mass.

To connect the loop, Majorana mass terms $(m_L/2)N_L N_L$ and $(m_R/2)N_R N_R$ are assumed. Since both E and N may be defined to carry lepton number, these new terms violate lepton number softly and may be naturally small. Using the Yukawa interaction $fs\bar{E}_R^0 \nu_L$, the

one-loop Majorana neutrino mass is given by

$$\begin{aligned} m_\nu &= f^2 m_R \sin^2\theta_R \cos^2\theta_R (m_1^2 - m_2^2)^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2-m_s^2)} \frac{1}{(k^2-m_1^2)^2} \frac{1}{(k^2-m_2^2)^2} \\ &+ f^2 m_L m_1^2 \sin^2\theta_L \cos^2\theta_R \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2-m_s^2)} \frac{1}{(k^2-m_1^2)^2} \\ &+ f^2 m_L m_2^2 \sin^2\theta_R \cos^2\theta_L \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2-m_s^2)} \frac{1}{(k^2-m_2^2)^2} \\ &- 2 f^2 m_L m_1 m_2 \sin\theta_L \sin\theta_R \cos\theta_L \cos\theta_R \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2-m_s^2)} \frac{1}{(k^2-m_1^2)} \frac{1}{(k^2-m_2^2)}. \end{aligned} \tag{25}$$

This formula holds for s as a mass eigenstate. If A_4 is unbroken, then $s_{1,2,3}$ all have the same mass and $\mathcal{M}_\nu$ is proportional to the identity matrix. However, if A_4 is softly broken by the necessarily real $s_i s_j$ mass terms, then the neutrino mass matrix is given by

$$\mathcal{M}_\nu = \mathcal{O} \begin{pmatrix} m_{\nu 1} & 0 & 0 \\ 0 & m_{\nu 2} & 0 \\ 0 & 0 & m_{\nu 3} \end{pmatrix} \mathcal{O}^T, \tag{26}$$

where $\mathcal{O}$ is an orthogonal matrix and $\mathcal{O} \neq 1$ breaks Z_3 lepton triality explicitly. Now each $m_{\nu i}$ may be complex because f, m_L, m_R may be complex, but a common unphysical phase, say for ν_1, may be rotated away, leaving just two relative Majorana phases for ν_2 and ν_3, owing to the relative phase between m_L and m_R with different $s_{1,2,3}$ masses. Hence $\mathcal{M}_\nu$ is diagonalized by $\mathcal{O}$, which is all that is required to obtain cobimaximal mixing [10], i.e. $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$, once U_ω is applied, as explained in Ref. [3].

The companion interaction to $f s \bar{E}_R^0 \nu_L$ is $f s \bar{E}_R^- l_L$, which induces the radiative process $l_i \to l_j + \gamma$. In the limit of exact Z_3 lepton triality, this amplitude is zero. Here it is proportional to $\sum_k U_{ik} U_{jk}^* F_k$ where $F_{1,2,3}$ refer to functions of $m^2_{s_{1,2,3}}$, and U_{ik} is the neutrino mixing matrix. Clearly, it is also zero if $F_1 = F_2 = F_3$. The amplitude for $\mu \to e\gamma$ is given by

$$A_{\mu e} = \frac{e f^2 m_\mu}{32\pi^2 m_E^2} \sum_i U_{ei}^* U_{\mu i} G_\gamma(x_i), \tag{27}$$

Using the most recent $\mu \to e\gamma$ bound [11], this branching fraction is constrained by

$$B = \frac{12\pi^2 |A_{\mu e}|^2}{m_\mu^2 G_F^2} < 5.7 \times 10^{-13}. \tag{28}$$

For small x_i and $x_1 \simeq x_2$,

$$|\sum_i U_{ei}^* U_{\mu i} G_\gamma(x_i)| = \frac{s_{13} c_{13}}{3\sqrt{2}} |x_3 - x_2|, \tag{29}$$

where $s_{13} = \sin\theta_{13}$, $c_{13} = \cos\theta_{13}$, and $\sin\theta_{23} = 1/\sqrt{2}$ has been assumed. Hence

$$B = \frac{\alpha s_{13}^2 c_{13}^2}{384\pi} \left(\frac{f^2 |x_3 - x_2|}{G_F m_E^2} \right)^2. \tag{30}$$

Let $G_F m_E^2 \simeq 1$, $f = 0.2$, $|x_3 - x_2| \simeq 0.05$, then $B = 5.6 \times 10^{-13}$, just below the experimental constraint.

Another possible rare decay is $\mu \to eee$, which comes from $\mu \to e(\gamma, Z) \to eee$ as well as directly through a box diagram as shown below. The amplitude for the former process with

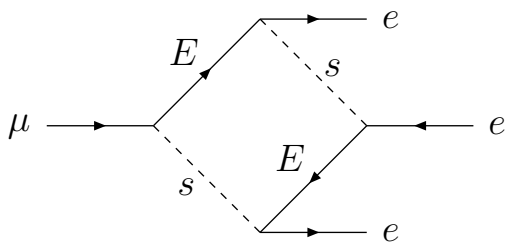

Figure 5: Box diagram for $\mu \to eee$.

a virtual photon is given by

$$\begin{aligned} i\mathcal{M}_\gamma &= \frac{-ie^2 f^2}{32\pi^2 m_E^2} \sum_{i=1}^{3} U_{ei}^* U_{\mu i} \bar{u}(p_1) \left[G_e(x_i) \left(\gamma^\alpha - \frac{q^\alpha \not{q}}{q^2} \right) P_L - i m_\mu G_\gamma(x_i) \frac{\sigma^{\alpha\beta} q_\beta}{q^2} P_R \right] u_\mu(p) \bar{u}(p_2) \gamma_\alpha v(p_3) \\ &- (p_1 \leftrightarrow p_2), \end{aligned} \tag{31}$$

where $P_{L,R} = (1 \mp \gamma_5)/2$, $q = p - p_1$ and

$$G_e(x) = \frac{7 - 36x + 45x^2 - 16x^3 + 6x^2(2x-3)\ln x}{18(x-1)^4}. \tag{32}$$

The amplitude for the process with a virtual Z boson has a similar form because $E_{L,R}$ is vector-like, but it is further suppressed by m_Z^2. The amplitude for the box diagram is given by

$$i\mathcal{M}_B = \frac{if^4[\bar{u}(p_1)\gamma_\alpha P_L u_\mu(p)\bar{u}(p_2)\gamma^\alpha P_L v(p_3) - (p_1 \leftrightarrow p_2)]}{64\pi^2 m_E^2} \sum_{i,j=1}^{3} U_{\mu i}U_{ej}^*[U_{ei}U_{ej}^* - U_{ej}U_{ei}^*]B_{ij}, \tag{33}$$

where

$$B_{ij} = \frac{B(x_i) - B(x_j)}{x_i - x_j} \quad i \neq j, \quad B_{ii} = \frac{x_i^2 - 2x_i \ln x_i - 1}{(x_i - 1)^3}, \quad B(x) = \frac{x^2 \ln x}{(x-1)^2} - \frac{1}{x-1}. \tag{34}$$

With the same specific choice of parameters as in Eq. (29) we find that the box diagram contribution is dominant. Hence the $\mu \to eee$ branching fraction is

$$B' = \frac{f^8}{2(8\pi)^4 m_E^4 G_F^2} \Big| \sum_{i,j=1}^{3} U_{\mu i}U_{ej}^*[U_{ei}U_{ej}^* - U_{ej}U_{ei}^*]B_{ij} \Big|^2. \tag{35}$$

Using the bound [14] on $\mu \to eee$ decay and for small x_i we have

$$B' = \frac{f^8}{2(8\pi)^4 m_E^4 G_F^2} \frac{\sin^2(4\theta_{13})}{8} < 1.0 \times 10^{-12}. \tag{36}$$

This constraint is easily satisfied for $G_F m_E^2 \simeq 1$, $f = 0.2$, which yields $B' = 1.35 \times 10^{-13}$.

6 Dark Matter

As for dark matter, there is a one-to-one correlation of the neutrino mass eigenstates to the $s_{1,2,3}$ mass eigenstates, the lightest of which is dark matter. Due to the presence of the A_4 symmetry, the dark matter parity of this model is also derivable from lepton parity [12]. Under lepton parity, let the new particles $(E^0, E^-), N$ be even and s, x, y be odd, then the same Lagrangian is obtained. As a result, dark parity is simply given by $(-1)^{L+2j}$, which is odd for all the new particles and even for all the SM particles. Note that the tree-level

Yukawa coupling $\bar{l}_L l_R \phi^0$ would be allowed by lepton parity alone, but is forbidden here because of the A_4 symmetry.

If the Yukawa coupling f of s to leptons is small, its relic density and elastic cross section off nuclei are both controlled by the interaction $\lambda v h s^2$. As such, a recent analysis [13] claims that the resulting allowed parameter space is limited to a small region near $m_s < m_h/2$. To evade this constraint, the mechanism of Ref. [15] may be invoked. Add a complex neutral singlet scalar $\chi \sim \underline{1}'$ under A_4 with Z_2 even. The dimension-four terms of the Lagrangian are of course required to be invariant under A_4. We assume that the dimension-three terms are also invariant: χ^3, $(\chi^\dagger)^3$, $(s_1^2 + \omega^2 s_2^2 + \omega s_3^2)\chi$, and $(s_1^2 + \omega s_2^2 + \omega^2 s_3^2)\chi^\dagger$. The symmetry A_4 is broken only by the dimension-two terms: χ^2, $(\chi^\dagger)^2$, and $s_i s_j$. As a result, χ is split into χ_R and χ_I, each mixing with h radiatively. In the physical basis, the dark matter s has residual $s^2 \chi_{R,I}$ interactions which contribute to its annihilation cross section, but do not affect its scattering off nuclei through h exchange.

Let us denote the $\chi_{R,I}$ masses with $m_{R,I}$. For illustration, we assume $m_R < m_s < m_I$, and take the $\chi_I \chi_R^2$ coupling to be zero, so that the annihilations shown in Fig. 6 are controlled by the interaction terms

$$-\mathcal{L}_{int} = \frac{\lambda'}{4} s^2 \chi_R^2 + \frac{g}{2} s^2 \chi_R + \frac{g'}{3!} \chi_R^3 \tag{37}$$

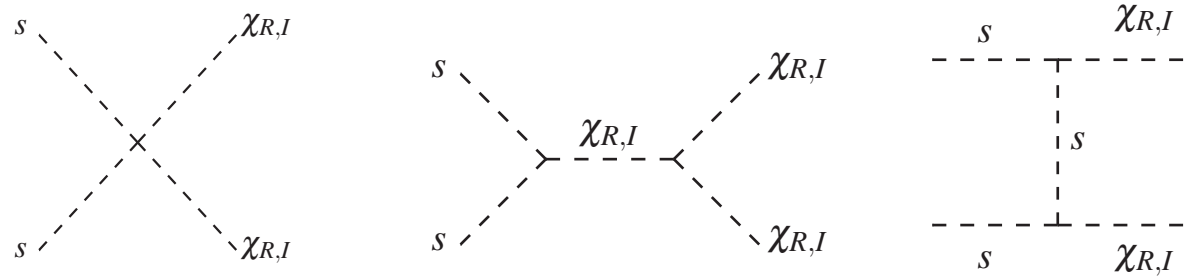

Figure 6: $s\,s$ annihilation to $\chi_{R,I}$ mass eigenstates.

As a result, the annihilation cross section times relative velocity is given by

$$\sigma \times v_{rel} = \frac{\sqrt{1-(m_R/m_s)^2}}{64\pi m_s^2}\left(\lambda' + \frac{g'g}{4m_s^2 - m_R^2} - \frac{g^2}{2m_s^2 - m_R^2}\right)^2. \tag{38}$$

Setting this equal to $2.2 \times 10^{-26}\,\mathrm{cm^3 s^{-1}}$, with $m_s = 200$ GeV and $m_R = 150$ GeV, we find

$$\lambda' + 0.073\left(\frac{\sqrt{g'g}}{100\,\mathrm{GeV}}\right)^2 - 0.174\left(\frac{g}{100\,\mathrm{GeV}}\right)^2 = 0.1514. \tag{39}$$

Note that χ_R decays to SM particles through its mixing with h. As mentioned earlier, the spin-independent elastic cross section proceeds through h exchange, with

$$\sigma_{SI} = \frac{\lambda^2 f_N^2 \mu^2 m_N^2}{\pi m_h^4 m_s^2}, \tag{40}$$

where $\mu = m_N m_s/(m_N + m_s)$ is the DM-nucleon reduced mass, $m_N = (m_p + m_n)/2 = 938.95\,\mathrm{MeV}$ is the nucleon mass, and $f_N = 0.3$ is the Higgs-nucleon coupling factor [16]. The LUX bound [17] for $m_s = 200\,\mathrm{GeV}$ is $\sigma \approx 1.5$ zb , which implies

$$\lambda < 3.3 \times 10^{-4}. \tag{41}$$

In conclusion, in the context of a specific A_4 scotogenic (dark-matter-induced) model of radiative neutrino and charged-lepton masses with the one Higgs boson of the standard model, we study finite calculable anomalous Higgs couplings with possible large deviations from the SM predictions. We show that the observed discrepancy in the muon anomalous magnetic moment may be explained by new particles in the TeV mass range, with predictions for the lepton flavor violating processes $\mu \to e\gamma$ and $\mu \to eee$. We also discuss the nature of the expected dark matter in this scenario.

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