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Quantum coherent oscillations in the early universe

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Cosmic inflation is commonly assumed to be driven by quantum fields. Quantum mechanics predicts phenomena such as quantum fluctuations and tunneling of the field. Here we show an example of a quantum interference effect which goes beyond the semi-classical treatment and which may be of relevance in the early universe. We study the quantum coherent dynamics for a tilted, periodic potential, which results in genuine quantum oscillations of the inflaton field, analogous to Bloch oscillations in condensed matter and atomic systems. The underlying quantum superpositions are typically very fragile, but may persist in the early universe giving rise to quantum interference phenomena in cosmology.

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It is commonly assumed that the universe underwent inflation, an early epoch of rapid expansion [1–4]. Inflation successfully explains a number of cosmological observations, such as the near homogeneity and isotropy of the universe. In addition, quantum fluctuations during inflation are assumed to have seeded the observed large-scale inhomogeneities in the matter distribution. There is currently much interest in identifying other observable signatures of genuine quantum effects in cosmology [5–8]. Here we treat the inflaton field quantum mechanically and study its quantum dynamics. Due to quantum interference phenomena, the dynamics can significantly differ from the semi-classical predictions, with possible consequences for observations.

In the simplest model of inflation a single scalar field is responsible for the rapid expansion of the universe. Here we briefly review the main results [9, 10]. The classical action for the scalar field Φ with a potential $V(\Phi)$ and gravity is

$$S = \int d^4x \sqrt{|g|} \left(\frac{m_P^2}{2} R - \frac{1}{2} g^{\mu\nu} \nabla_\mu \Phi \nabla_\nu \Phi - V(\Phi) \right), \quad (1)$$

where R is the Ricci-scalar, g the determinant of the metric and $m_P = (8\pi G)^{-1/2}$ is the reduced Planck-mass (we use units with $\hbar = c = 1$ throughout the manuscript). The homogeneous and isotropic solution for the gravitational field is the FRW-metric, given by

$$ds^2 = -dt^2 + a(t)^2 (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2), \quad (2)$$

where for simplicity a spatially flat universe was assumed. Defining $H = \dot{a}/a$, where the dot denotes differentiation with respect to the time coordinate t , the Einstein equa-

tions for a homogeneous field $\Phi(\mathbf{x}, t) = \Phi(t)$ yield

$$\begin{aligned} H^2 &= \frac{1}{3m_P^2} \left(\frac{1}{2} \dot{\Phi}^2 + V(\Phi) \right) \\ \dot{H} &= -\frac{1}{2m_P^2} \dot{\Phi}^2 \end{aligned} \quad (3)$$

The Klein-Gordon equation on the above metric becomes

$$\ddot{\Phi} + 3H\dot{\Phi} + V'(\Phi) = 0, \quad (4)$$

where the prime denotes differentiation with respect to Φ . The Hubble parameter H , characterizing the expansion of the universe, depends on the field Φ and its potential $V(\Phi)$. These cause the scale factor $a(t)$ to grow exponentially during most of the inflationary period. Towards the end of inflation the potential energy is reduced, the “friction” term $3H\dot{\Phi}$ in eq. (4) can become negligible and the field oscillates inside the potential $V(\Phi)$. As the expansion ends, reheating starts and the field decays into other particles. This can be approximately captured by an additional effective damping term $\Gamma\dot{\Phi}$ in the equation of motion (4).

The equations of motion are analogous to the description of a particle in a potential. Despite the use of a quantum field in the model, the description so far was completely classical. In the usual treatment, inhomogeneous perturbations to the field, $\delta\Phi(\mathbf{x}, t)$, are considered in the linearized regime and quantized. These perturbations, which are interpreted to arise due to vacuum fluctuations, become squeezed due to inflation and decay into the matter distribution we see today. But the homogeneous field contribution is usually assumed to remain classical.

Here we consider the quantum dynamics of the homogeneous inflaton field. To this end, we consider the equation of motion (4) for the quantized homogeneous field $\hat{\Phi}$ and study the resulting quantum dynamics. We consider standard quantization with $[\hat{\Phi}(\mathbf{x}), \hat{\pi}(\mathbf{y})] = i\hbar\delta(\mathbf{x} - \mathbf{y})$, where $\hat{\pi}$ is the conjugate momentum field operator. In

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the usual, semi-classical treatment, the field has always a specific well-defined value, say Φ_1 . In this treatment, quantum tunnelling can transform the field from one well-defined value to another, $\Phi_1 \rightarrow \Phi_2$ [11]. The system can probabilistically “penetrate” a classical barrier, but it materializes with a classically well defined value afterwards. However, quantum mechanically the field can be in a superposition of different amplitudes, $|\Phi_1\rangle + |\Phi_2\rangle$, which is not captured by a semi-classical treatment. Until a measurement is performed, or decoherence occurs, superpositions of different amplitudes are physically valid solutions. Usually, such states are fragile and would quickly decohere, as for example in condensed matter systems. Within the single-field inflationary model, however, one can expect quantum coherence to be preserved, as there is no decoherence channel within the model. Assuming that additional deleterious effects destroy the quantum coherence after some time t^* , the field can evolve into quantum superpositions for times $t < t^*$. For a quantum state, one can only assign probabilities to each specific value of the field, but none of the possible superposed outcomes are yet classically realized.

In our discussion the quantized nature of gravity is neglected. In other words the gravitational degrees of freedom remain fully classical and we interpret $H = \langle H \rangle$ as an effectively classical variable. Clearly such a treatment is incomplete in describing the full evolution during all of inflation and the entanglement with gravitational degrees of freedom is not accounted for. To incorporate the latter, a quantum theory of gravity would be necessary, which is outside the scope of this work.

At first, we discuss the dynamics without the “damping” term $3H\dot{\Phi}$, to get insight into the quantum behavior of the system. We will later include this term, which is necessary for the slow-roll behavior. In this regime the quantum dynamics of the field is governed by

$$\ddot{\Phi} + V'(\Phi) = 0. \quad (5)$$

Rather than studying the decomposition of the field into individual modes, we consider the equation of motion for the entire field. This resembles the quantum evolution of a system in first quantization. Note that the dynamics of quantum systems in the presence of potentials has been extensively studied in low-energy quantum theory [12]. Here we can directly apply some results to the field operator $\hat{\Phi}$, since the equation of motion is mathematically analogous to the Heisenberg equation of motion for the position operator in non-relativistic quantum theory.

In particular, we consider the model by Abbott [13] in which the potential is of the form

$$V(\hat{\Phi}) = V_0 \cos\left(\frac{2\pi\hat{\Phi}}{f}\right) + \epsilon \frac{\hat{\Phi}}{f}, \quad (6)$$

as also depicted in Fig. 1. We assume this potential describes the dynamics in the regime that we are interested

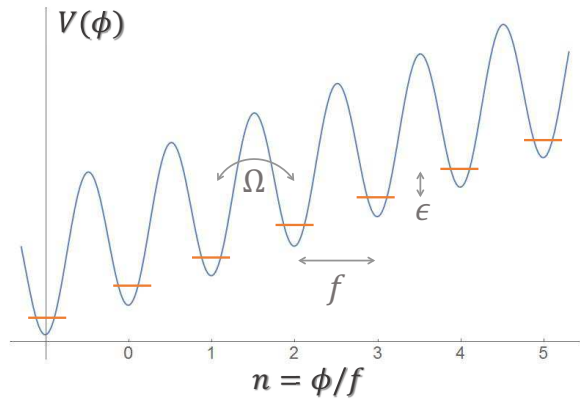


FIG. 1. The assumed potential for the homogeneous field Φ , which consists of a periodic part and an additional tilt (see eq. (6)). The minima (sites) are labeled by n , the orange lines represent the local ground states. The field can tunnel from one site to another, governed by the rate density Ω . Due to the tilt in the potential, the quantum dynamics results in coherent, periodic oscillations over many sites.

in, which may also be the case at some intermediate time during inflation. This model has also been studied in the context of a cyclic universe [14]. The quantum dynamics for this type of potential is very different from the classical counterpart: quantum systems undergo coherent, periodic oscillations. The eigenstates for this potential are not localized inside a potential well; rather they are delocalized over several minima. Thus, if the system is initially inside a well, it will not have a well-defined energy but will be in a superposition of different energy eigenstates. The system will evolve in time and spread out (this being also the physical reason for quantum tunneling). In other words, the states localized in the minima are not ground states or true vacuum states, but will slowly delocalize. Importantly, the delocalization is also altered by the linear contribution to the potential: the “tilt” is responsible for coherent oscillations of the system, also known as Bloch oscillations [15, 16]. These oscillations can extend over many minima and are quantum mechanical in nature, as they arise due to quantum interference. Such oscillations have been observed in various experiments [17–19].

To study the effect in detail, we can directly use the developed techniques in low-energy quantum theory [20–23]. It is convenient to express the full dynamics of the system with potential (6) in the Hamiltonian formulation, where the dynamics of $\hat{\Phi}$, eq. (5), is generated by an effective Hamiltonian $\hat{H} = \int d^3x \hat{\mathcal{H}}$ with the Hamiltonian density

$$\hat{\mathcal{H}} = -\frac{\Omega}{4} \sum_{n=-\infty}^{\infty} (\hat{a}_n \hat{a}_{n+1}^\dagger + \hat{a}_{n+1} \hat{a}_n^\dagger) + \epsilon \sum_{n=-\infty}^{\infty} n \hat{a}_n^\dagger \hat{a}_n. \quad (7)$$

Here, the operator \hat{a}_n^\dagger (\hat{a}_n) creates (annihilates) a field excitation in the n -th local potential well minimum (see

Fig. 1), and Ω is the hopping rate per unit volume between neighboring sites. In terms of these operators, the field is $\hat{\Phi} = f \sum_n n \hat{a}_n^\dagger \hat{a}_n$. The use of the above Hamiltonian assumes only nearest-neighbor hopping, and neglects any higher energy levels inside the minima (only the local false vacuum states are considered).

In the Heisenberg picture, the operator $\hat{\Phi}(t)$ evolves in time according to eq. (5), which is generated by the above effective Hamiltonian via $\dot{\hat{\Phi}}(t) = i [\hat{H}, \hat{\Phi}]$. The same Hamiltonian can also be used to study the dynamics in the Schroedinger picture [24, 25], i.e. the dynamics of the state, or wave functional, is given by $|\Psi(t)\rangle = \hat{U} |\Psi(0)\rangle = e^{-i\hat{H}t} |\Psi(0)\rangle$. The Schroedinger picture can be instructive to gain insight into the quantum processes that arise.

Classically, the system would be accelerated by the linear term in eq. (6), but if trapped in a potential minimum, it would remain bound. Semi-classically, the system can tunnel to neighboring sites, governed by the hopping rate $\omega = \int d^3x \Omega$ and the potential tilt. The full quantum dynamics given by eq. (7) predicts a coherent quantum phenomenon: Bloch oscillations of the system at a frequency $\omega_B = \int d^3x \epsilon$. These oscillations can be described in terms of the eigenstates of the Hamiltonian (7), the Wannier-Stark states [26] $|\psi_m\rangle$ with eigenenergies $E_m = m\omega_B$ with integer m . In terms of the localized states $|\Phi_n\rangle = |n\rangle$ corresponding to site n , these are given by

$$|\psi_m\rangle = \sum_n J_{n-m} \left(\frac{\omega}{2\omega_B} \right) |n\rangle, \quad (8)$$

where J_k are Bessel functions of the first kind. Using the summation property of the Bessel function, $\sum_n J_n(x) J_{n+k}(x) e^{2iky} = i^k J_k(2x \sin y) e^{-ikx/2}$, one finds the propagator from site k to n to be

$$\begin{aligned} \langle n | \hat{U} | k \rangle &= \sum_j e^{-iE_j t} \langle n | \psi_j \rangle \langle \psi_j | k \rangle \\ &= J_{n-k} \left(\frac{\omega}{\omega_B} \sin \left(\frac{\omega_B t}{2} \right) \right) e^{i\frac{1}{2}(n+k)\omega_B t} e^{-ikx/2}. \end{aligned} \quad (9)$$

Thus, if the system is initially localized at some site (or in other words, is in a false vacuum) $|\Psi(0)\rangle = |n=0\rangle$, the time evolution is

$$|\Psi(t)\rangle = \sum_n i^n J_n \left(\frac{\omega}{\omega_B} \sin \left(\frac{\omega_B t}{2} \right) \right) e^{-\frac{i}{2}\omega_B t} |n\rangle. \quad (10)$$

The system coherently spreads periodically over many sites, with the Bloch frequency ω_B . The largest oscillation amplitude of the field is on the order of $L \approx f\omega/\omega_B$. After a time $T_B = 2\pi/\omega_B$, the state is given by its initial value, $|\Psi(T_B)\rangle = |\Psi(0)\rangle$, see Fig. 2. At each instant of time, one can only assign a probability to a certain field value, given by $|\langle n | \Psi(t) \rangle|^2$. The state remains delocalized until a measurement or decoherence occurs.

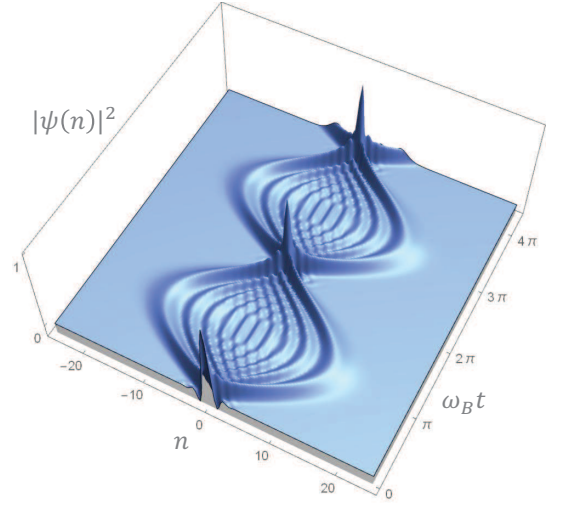


FIG. 2. Bloch oscillations of the system, for an initially localized state inside a potential well. The system spreads as a coherent superposition over many sites of the potential (labeled by n). The oscillations are periodic with frequency ω_B , which depends on the tilt of the potential. On average, the system remains in the potential well in which it started.

One can also find the solutions in the Heisenberg picture [21, 23], i.e. the time evolved field state $\hat{\Phi}(t)$. To this end, we follow the algebraic approach as developed in Ref. [23]. The displacement operator \hat{D} shifts the field from one site to the next, i.e. $\hat{D}|n\rangle = |n+1\rangle$ and $\hat{D}^\dagger|n\rangle = |n-1\rangle$, and has the properties $\hat{D}\hat{D}^\dagger = 1$, $[\hat{\Phi}, \hat{D}] = f\hat{\Phi}$. The Heisenberg equation of motion for the displacement operator gives the solution $\hat{D}(t) = \hat{D}(0)e^{i\omega_B t}$. In terms of the displacement operator, the field operator evolves in time as

$$\dot{\hat{\Phi}}(t) = \frac{if\omega}{4} \left(\hat{D}(0)e^{i\omega_B t} - \hat{D}^\dagger(0)e^{-i\omega_B t} \right) \quad (11)$$

with the solution

$$\hat{\Phi}(t) = \hat{\Phi}(0) + \frac{f\omega}{4\omega_B} \left(\hat{D}(0) (e^{i\omega_B t} - 1) + \hat{D}^\dagger(0) (e^{-i\omega_B t} - 1) \right) \quad (12)$$

The above results highlight how Bloch oscillations can arise, but they are insufficient to make a link to inflation. We relate the results to cosmological parameters via eq. (3), interpreting the expansion parameter H as a semi-classical mean value. But a semi-classical approximation is invalid when superpositions of vastly different states are involved [27], as is the case shown in Fig. 2. Instead, we therefore focus on another initial condition, in which the field has initial Gaussian spread over many sites, $|\Psi(0)\rangle = \mathcal{N} \sum_n e^{-n^2/2\sigma^2} |n\rangle$, where \mathcal{N} is the normalization factor. In this case, the relevant quantity becomes

$$\langle \dot{\hat{\Phi}}^2 \rangle = \frac{f^2\omega^2}{8} \left(1 - e^{-1/\sigma^2} \cos(2\omega_B t) \right) \approx \frac{f^2\omega^2}{4} \sin^2(\omega_B t), \quad (13)$$

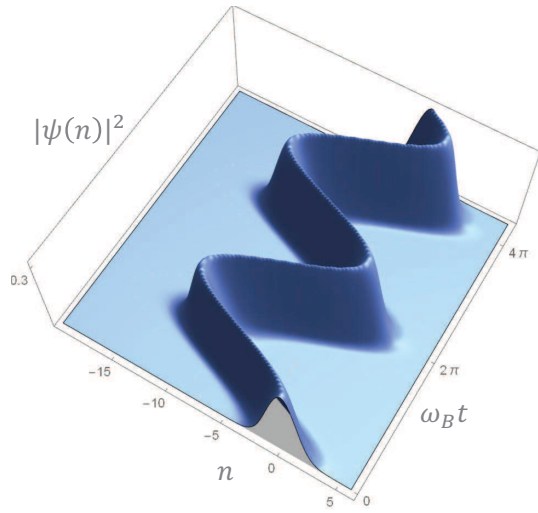


FIG. 3. Bloch oscillations for an initial Gaussian distribution over several potential wells. The system does not further delocalize, but the mean oscillates at frequency ω_B . Damping of the system causes an additional slow drift downwards, which is not shown here.

where the last approximation holds for an initially broad Gaussian. The behavior in this case is shown in Fig. 3, the system remains relatively localized such that a semi-classical value is better justified, but undergoes oscillations as a whole throughout the potential landscape. In particular, the quantity $\langle \dot{\Phi}^2 \rangle$, which is relevant here, undergoes oscillations at frequency ω_B .

In the above discussion of Bloch oscillations, the average of the field remains unaffected ($\langle \Phi \rangle = 0$), meaning that the field does not, on average, roll down the potential landscape. The quantum solution above predicts coherent oscillations, but the field comes back to its initial value after a time $T_B = 2\pi/\omega_B$ and does not roll down the potential. This is, however, required for inflationary models, such that inflation can end when the field reaches a sufficiently small potential energy. Until now, we have also neglected the additional damping term in eq. (4), $3H\dot{\Phi}$. This approximation can hold during “fast roll”, when the acceleration dominates over the friction term (for the example considered here, this constitutes the regime $H \ll \omega_B$ or $1 \gg 3H\dot{\Phi}/\ddot{\Phi} \approx f\omega/(m_P\omega_B)$, requiring that the periodicity of the field be smaller than the Planck-mass). One can expect this condition to hold at the early phase of inflation and towards the end of inflation, thus one may expect the strongest Bloch oscillations to occur then. This translates to super-horizon scales and small scales, respectively, which are not relevant for current observations.

However, a stretch of potential of the form discussed here may be encountered during the slow-roll of the field and may cause such oscillations during intermediate e-folds. This requires that the potential allows for slow roll of the field further downwards. In fact, including the

damping term $3H\dot{\Phi}$ in the dynamics of the field yields an additional slow drift of the system, which recovers the slow-roll behavior. Bloch oscillations in the presence of damping have been extensively studied [28, 29], the gradual loss of energy results in an overall slow drift given by

$$\bar{\Phi}_d = -\frac{\omega f}{2} \frac{\frac{\omega_B}{3H}}{1 + (\frac{\omega_B}{3H})^2} \approx -\frac{\omega f}{2} \cdot \frac{3H}{\omega_B} \quad (14)$$

This average drift occurs over a long time-scale (since $H \ll \omega_B$) and is superimposed on the short-time oscillations given by eq. (13). Thus slow-roll is preserved even in the presence of a potential of the form considered here. In addition to the effective damping of the field, any possible decoherence channel will destroy quantum coherence and the associated Bloch oscillations [30]. Within the single-field inflationary model, however, one would expect quantum coherence to be preserved¹. Nevertheless, decoherence will put an end to quantum phenomena at some point, at the very latest during reheating when the field decays into other fields.

We note that an overall slow drift can also occur if the potential is slightly modified to include a small periodic drive. The quantum solution for this case is given in the Appendix, which allows for the slow reduction of $\langle \hat{\Phi}(t) \rangle$ over time. The relevant term for the oscillating part in this case is

$$\langle \dot{\Phi}^2 \rangle \approx \frac{f^2 \omega^2}{4} \sin^2(\theta_B(t)), \quad (15)$$

where $\theta_B(t)$ is given below eq. (A9) and reduces to $\omega_B t$ for the case discussed above. The oscillatory part of the Hubble parameter is therefore given by

$$H_{\text{osc}}^2 = \frac{f^2 \omega^2}{24m_P^2} \sin^2(\theta_B(t)). \quad (16)$$

This term induces oscillations on top of the usual, dominant contribution of the potential, which is governed on larger time-scales by the slow drift (14).

Linking our results to observations of the cosmic microwave background anisotropies or large scale structure requires the study of inhomogeneous perturbations on top of the quantum solution for the homogeneous field. The power spectrum of such perturbations with wavenumber k is [9, 10] $P_k \propto H_k^2$, where H_k is the Hubble parameter at horizon exit. The oscillations in eq. (16) affect both, the horizon exit time and the amplitude of inhomogeneous perturbations, thus leading to a small periodic modulation of the power spectrum. Their detection could provide evidence of quantum coherent

¹ It is sometimes assumed that quantum superpositions of different field configurations are forbidden. Here we assume that they can persist in the early universe.

phenomena during inflation². The quantum coherent oscillations discussed here can appear for the specific, periodic Abbott potential [13] and small variations thereof. The potential may vary throughout the inflationary history, but if a patch of such a potential is encountered at some point, Bloch oscillations may occur. The example of Bloch oscillations may also be relevant whenever several potential minima are present. For such oscillations to occur, the boundary conditions are negligible as long as they are further away than the maximum coherent oscillation amplitude $L = f\omega/\omega_B$ [20]. Thus, for a potential with many local minima and periodicity over a finite region, similar behaviour can be expected. In string theory for example, one expects a vast potential landscape [31] where such conditions may be met. If, however, the minima are completely disordered and spread out randomly, quantum interference results in localization of the field in analogy to Anderson localization [32].

The present analysis serves as an example for a quantum interference effect that may be of relevance in cosmology. We briefly summarize the conditions under which the phenomenon can occur. The quantum mechanical nature of the homogeneous field has to be preserved, in particular, its ability to be in a quantum superposition. For this to hold it is paramount that the field does not decohere during its evolution in the periodic potential considered. In addition, we assume a semi-classical treatment of the interaction with gravity. While the field itself remains quantum mechanical and the expansion manifests itself as an effective damping parameter, the backaction of the field onto the metric is captured only semi-classically in the mean field limit. This can only hold if the superposition is not too delocalized. Finally, the discussed oscillations can occur if the potential has some periodicity at some point during inflation, and may be observable if such a stretch of potential is encountered during those e-folds that are accessible in today's experiments.

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Appendix A: Bloch oscillations

We review the solution of the quantum equation of motion (5) in the presence of a tilted, periodic potential

with periodicity f , given by the Hamiltonian

$$\hat{H} = -\frac{\omega}{4} \sum_{n=-\infty}^{\infty} (\hat{a}_n \hat{a}_{n+1}^\dagger + \hat{a}_{n+1} \hat{a}_n^\dagger) + \varepsilon(t) \sum_{n=-\infty}^{\infty} n \hat{a}_n^\dagger \hat{a}_n. \quad (\text{A1})$$

This Hamiltonian includes a time-dependent tilt and is valid in the tight-binding limit where tunnelling to only the nearest-neighboring wells is allowed. This effective Hamiltonian reproduces the equation of motion for the field when neglecting the damping due to the expansion, $\ddot{\hat{\Phi}} + V'(\hat{\Phi}) = 0$, such that the evolution of the homogeneous field is identical to a quantum particle in the presence of a potential. The Abbott-model (6) is captured when $\varepsilon(t) = \omega_B$. It is convenient to express the above Hamiltonian in terms of the operators [33] $\hat{D} = \sum \hat{a}_n \hat{a}_{n+1}^\dagger$, $\hat{D}^\dagger = \sum \hat{a}_{n+1} \hat{a}_n^\dagger$ and $\hat{\Phi} = f \sum n \hat{a}_n^\dagger \hat{a}_n$, which obey the commutation relations

$$[\hat{D}, \hat{D}^\dagger] = 0, \quad [\hat{D}, \hat{\Phi}] = -f \hat{D}, \quad [\hat{D}^\dagger, \hat{\Phi}] = f \hat{D}^\dagger. \quad (\text{A2})$$

Note that $\hat{D}|n\rangle = |n+1\rangle$ and $\hat{D}^\dagger|n\rangle = |n-1\rangle$, so the operator shifts the field to the neighboring site. In terms of these operators, the Hamiltonian becomes

$$\hat{H} = -\frac{\omega}{4} (\hat{D} + \hat{D}^\dagger) + \frac{\varepsilon(t)}{f} \hat{\Phi}. \quad (\text{A3})$$

The Heisenberg equations of motion for $\hat{D}(t)$ is $\dot{\hat{D}}(t) = -i [\hat{D}, \hat{H}] = i\varepsilon(t)\hat{D}(t)$, which yields

$$\hat{D}(t) = \hat{D}_0 e^{i \int_0^t dt \varepsilon(t)} \text{ and } \hat{D}^\dagger(t) = \hat{D}_0^\dagger e^{-i \int_0^t dt \varepsilon(t)}, \quad (\text{A4})$$

where \hat{D}_0 is the operator at time $t = 0$. With this, we can solve the equation of motion for the field operator $\dot{\hat{\Phi}} = -i [\hat{\Phi}, \hat{H}] = i(\omega/4) (\hat{D}^\dagger(t) - \hat{D}(t))$:

$$\hat{\Phi} = \hat{\Phi}(0) + i \frac{f\omega}{4} \int_0^t ds \left(\hat{D}_0^\dagger e^{-i \int_0^s du \varepsilon(u)} - \hat{D}_0 e^{i \int_0^s du \varepsilon(u)} \right). \quad (\text{A5})$$

The integrals can be computed analytically for the case of a constant tilt with periodic driving, $\varepsilon(t) = \omega_B - \epsilon_1 \cos(\omega_1 t)$, using the property of the Bessel functions $e^{ix \sin \theta} = \sum_{k=-\infty}^{\infty} J_k(x) e^{ik\theta}$. If there exist an integer k_* such that $\omega_B - k_* \omega_1 = 0$, the solution is

$$\begin{aligned} \hat{\Phi} = & \hat{\Phi}(0) - i \frac{f\omega}{4} (\hat{D}_0^\dagger - \hat{D}_0) J_{k_*} \left(\frac{\epsilon_1}{\omega_1} \right) t \\ & - i \frac{f\omega}{4} \hat{D}_0^\dagger \sum_{k \neq k_*} J_k \left(\frac{\epsilon_1}{\omega_1} \right) \frac{2e^{-i\omega_k t/2}}{\omega_k} \sin \left(\frac{\omega_k t}{2} \right) \\ & + i \frac{f\omega}{4} \hat{D}_0 \sum_{k \neq k_*} J_k \left(\frac{\epsilon_1}{\omega_1} \right) \frac{2e^{i\omega_k t/2}}{\omega_k} \sin \left(\frac{\omega_k t}{2} \right) \end{aligned} \quad (\text{A6})$$

² Note that a different potential could lead to classical oscillations with a similar signature; discerning the quantum oscillations from classical ones requires knowledge of the potential.

where $\omega_k = \omega_B - k\omega_1$. Note that this expression reduces to eq. (12) for $\epsilon_1 = 0$, which is the typical case for Bloch oscillations. Neglecting the oscillating terms, we can write

$$\hat{\Phi} \approx \hat{\Phi}(0) - i \frac{f\omega}{4} \left(\hat{D}_0^\dagger - \hat{D}_0 \right) J_{k_*} \left(\frac{\epsilon_1}{\omega_1} \right) t \quad (\text{A7})$$

so the field evolves linearly in time in the presence of the harmonic drive. In particular, it can slowly roll down: if the parameters ϵ_1/ω_1 are close to a zero of J_{k_*} , the field can roll arbitrarily slow.

The square of the field velocity, relevant for the expansion rate, is found directly from the Heisenberg equation of motion:

$$\dot{\hat{\Phi}}^2 = \frac{f^2\omega^2}{16} \left(2 - \hat{D}_0^{\dagger 2} e^{-i2\omega_B t + i2\frac{\epsilon_1}{\omega_1} \sin(\omega_1 t)} - \hat{D}_0^2 e^{i2\omega_B t - i2\frac{\epsilon_1}{\omega_1} \sin(\omega_1 t)} \right) \quad (\text{A8})$$

For an initial Gaussian-distributed field over many sites $|\Psi(0)\rangle = \mathcal{N} \sum_n e^{-n^2/2\sigma^2} |n\rangle$, as in the main text, the expectation value becomes

$$\langle \dot{\hat{\Phi}}^2 \rangle \approx \frac{f^2\omega^2}{4} \sin^2(\theta_B(t)) \quad (\text{A9})$$

with $\theta_B(t) = \omega_B t - \frac{\epsilon_1}{\omega_1} \sin(\omega_1 t)$.

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