Classifying the bosonic quartic couplings

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Mapping the genuine bosonic quartic couplings

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The larger center–of–mass energy of the Large Hadron Collider Run 2 opens up the possibility of a more detailed study of the quartic vertices of the electroweak gauge bosons. Our goal in this work is to classify all operators possessing quartic interactions among the electroweak gauge bosons that do not exhibit triple gauge–boson vertices associated to them. We obtain all relevant operators in the non-linear and linear realizations of the $SU(2)_L \otimes U(1)_Y$ gauge symmetry.

I. INTRODUCTION

The recent discovery of a Higgs-like boson by the ATLAS and CMS collaborations [1] brings us a step closer to a full check of the standard model (SM). The SM has been subject to a large number of precision tests during the past decades [2] without any putative indication of deviations from its predictions for the particle couplings, which in the case of fermion-gauge interactions have been tested to close to per mil accuracy. Since the SM is a gauge theory based on the $SU(2)_L \otimes U(1)_Y$ group it fixes completely the structure of the trilinear (TGC) and quartic (QGC) electroweak gauge–boson couplings. Therefore, it is important to establish whether these couplings indeed are in agreement with the SM predictions.

Precise knowledge of the gauge–boson self–interactions not only can serve to further establish the SM in the case of agreement with its predictions, but also any observed deviation can indicate the existence of new physics. For instance, new heavy bosons can generate a tree level contribution to four gauge–boson couplings while its effect in the triple–gauge vertex would only appear at one one–loop [3], and consequently be suppressed with respect to the quartic one. Moreover, the comparison of deviations in TGC and QGC [4] can be used to determine whether the $SU(2)_L \otimes U(1)_L$ is linearly [5–13] or nonlinearly [14–16] realized in low energy effective theory of the electroweak breaking sector.

Presently the trilinear gauge-boson couplings are known to agree with the SM within a few percent [17–19]. On the other hand, there are sparse direct data on anomalous QGC and, for a long time, the most stringent bound on QGC stemmed from their indirect effects to the Z physics via their one–loop contributions to the oblique corrections [20–23]; a situation that is starting to change [24–27] . The LEP collaborations directly probed $W^+W^-\gamma\gamma$ and $ZZ\gamma\gamma$ interactions in the reactions $e^+e^- \to W^+W^-\gamma$ [28] and $Z\gamma\gamma$ [29]. At the Tevatron, the D0 collaboration studied the $W^+W^-\gamma\gamma$ vertex in diffractive events exhibiting dielectron and missing energy [30]. At the LHC, the ATLAS and CMS collaborations studied the production of $V\gamma\gamma$ with $V = Z$ or $W^\pm$ to constrain the $VV\gamma\gamma$ QGC [24]. Moreover, the ATLAS collaboration analyzed the W^+W^- and $ZW^\pm$ pairs via vector boson fusion to bound the QGC among four massive electroweak vector bosons [25] while the CMS studied the $Z\gamma jj$, $W^\pm\gamma jj$ and $W^\pm W^\pm jj$ productions to probe QGC [26]. In addition to these inclusive processes the CMS collaboration also probed the $W^+W^-\gamma\gamma$ vertex through the exclusive $\gamma\gamma \to W^+W^-$ production [27].

The direct study of QGC requires either the production of three gauge bosons or the pair production of gauge bosons in vector boson fusion [23]. Therefore, the LHC Run 2 opens the possibility of testing systematically anomalous QGC due to the large center–of–mass energy. Here we focus on genuine QGC, that is, QGC that do not have any TGC associated to them since the best bounds on the Wilson coefficients in the latter case are obtained from direct study of TGC. In a scenario where the $SU(2)_L \otimes U(1)_Y$ is realized linearly the lowest order QGC are given by dimension-eight operators [21]. On the other hand, if the gauge symmetry is implemented nonlinearly the lowest order QGC appear at $\mathcal{O}(p^4)$ [4, 16].

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In the previous phenomenological [20–23, 31–38] and experimental [24–30] analyses of QGC just a partial list of effective operators has been considered. Our goal in this work is to classify all genuine QGC in the non-linear and linear realizations of the gauge symmetry including up to two derivatives acting on the gauge boson fields. This will facilitate the translation of bounds between different notations.

The outline of the paper is as follows. We start by listing the most general set of Lorentz structures which can be involved in quartic gauge boson vertices containing up to two derivatives acting on the gauge fields in Sec. II. In Sec. III we present the most general effective Lagrangian which generates QGC in scenarios in which the observed Higgs-like particle is indeed a fundamental state belonging to an $SU(2)_L$ doublet and for which the gauge symmetry is linearly realized. In these scenarios QGC appear at dimension-8 independently on the number of derivatives, and we find a total of 10 independent operators and derived the relations between the coefficients of the generated Lorentz structures that this implies. Section IV contains the most general effective Lagrangian which gives rise to those QGC in scenarios with a dynamical light Higgs for which the electroweak symmetry realization is chiral. In this case QGC with no derivatives and two derivatives appear at $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ respectively and we find the same number of independent chiral operators as Lorentz structures. Finally in Sec. V we summarize our conclusions. The article is complemented with a set of appendices containing the most lengthy expressions as well as some technical details.

II. GENERAL LORENTZ STRUCTURES

Without loss of generality, initially we construct the possible Lorentz structures that are invariant under $U(1)_{\rm em}$ and which contain four vector bosons. We group the vertices in terms of the number of derivatives that they contain, and we restrict ourselves to vertices exhibiting zero or two derivatives, or, equivalently, to operators with mass dimension up to six.

A. Vertices with zero derivatives

Since the $U(1)_{\rm em}$ invariance requires that the photon field appears only as part of the electromagnetic field strength, the zero-derivative vertices do not contain photons, *i.e.* the QGC exhibit only massive electroweak gauge bosons. These structures are:

$$
\begin{aligned}
\mathcal{Q}^{\partial=0}_{WW,1} &= W^{+\mu} W^-_\mu W^{+\nu} W^-_\nu \quad , \quad \mathcal{Q}^{\partial=0}_{WW,2} = W^{+\mu} W^{-\nu} W^+_\mu W^-_\nu \; , \\
\mathcal{Q}^{\partial=0}_{WZ,1} &= W^{+\mu} W^-_\mu Z^\nu Z_\nu \quad\quad , \quad \mathcal{Q}^{\partial=0}_{WZ,2} = W^{+\mu} W^{-\nu} Z_\mu Z_\nu \; , \\
\mathcal{Q}^{\partial=0}_{ZZ} &= Z^\mu Z_\mu Z^\nu Z_\nu \; .
\end{aligned}
\tag{1}
$$

The first four structures in Eq. (1) modify the SM quartic couplings $W^+W^-W^+W^-$ and W^+W^-ZZ, while the last one leads to a QGC not present in the SM. The effective Lagrangian containing these five structures has the general form:

$$
\mathcal{L}^{\partial=0}_Q = \sum_{i=1}^2 c_i^{0,WW} \mathcal{Q}^{\partial=0}_{WW,i} + \sum_{i=1}^2 c_i^{0,WZ} \mathcal{Q}^{\partial=0}_{WZ,i} + c^{0,ZZ} \mathcal{Q}^{\partial=0}_{ZZ} \; .
\tag{2}
$$

B. Vertices with 2 derivatives

The quartic vertices containing two derivatives can be classified according to the number of photons in the vertex as $U(1)_{\rm em}$ requires that for each photon field at least one derivative must appear.

1. Vertices with 2 derivatives and two photons

The vertices with two derivatives and containing two photons are constructed using two photon field strengths plus two $W^\pm$ or Z fields. In this case there are only four possible Lorentz structures:

$$
\begin{aligned}
\mathcal{Q}^{\partial=2}_{\gamma W,1} &= F_{\mu\nu} F^{\mu\nu} W^{+\alpha} W^-_\alpha \quad , \quad \mathcal{Q}^{\partial=2}_{\gamma W,2} = F_{\mu\nu} F^{\mu\alpha} W^{+\nu} W^-_\alpha \; , \\
\mathcal{Q}^{\partial=2}_{\gamma Z,1} &= F_{\mu\nu} F^{\mu\nu} Z^\alpha Z_\alpha \quad\quad , \quad \mathcal{Q}^{\partial=2}_{\gamma Z,2} = F_{\mu\nu} F^{\mu\alpha} Z^\nu Z_\alpha \; .
\end{aligned}
\tag{3}
$$

2. Vertices containing 2 derivatives and a single photon

In this group of QGC, one derivative appears in the photon field strength while the other derivative can appear in the anti-symmetric combination $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ with $V_\mu = Z_\mu$ or $W_\mu^\pm$ or in the symmetric combinations

$$X_{\mu\nu}^\pm = \partial_\mu W_\nu^\pm + \partial_\nu W_\mu^\pm \;, \qquad Y_{\mu\nu} = \partial_\mu Z_\nu + \partial_\nu Z_\mu \;. \tag{4}$$

In this case one can construct nine Lorentz structures corresponding to anomalous $\gamma Z W^+ W^-$ interactions:

$$
\begin{aligned}
\mathcal{Q}_{\gamma ZW,1}^{\partial=2} &= F_{\mu\nu} Z^{\mu\nu} W^{+\alpha} W_\alpha^- & , \quad \mathcal{Q}_{\gamma ZW,2}^{\partial=2} &= F_{\mu\nu} Z^{\mu\alpha} (W^{+\nu} W_\alpha^- + W^{-\nu} W_\alpha^+) \;\;, \\
\mathcal{Q}_{\gamma ZW,3}^{\partial=2} &= F^{\mu\nu}(W_{\mu\nu}^+ W_\alpha^- Z^\alpha + W_{\mu\nu}^- W_\alpha^+ Z^\alpha) & , \quad \mathcal{Q}_{\gamma ZW,4}^{\partial=2} &= F^{\mu\nu}(W_{\mu\alpha}^+ W_\nu^- Z^\alpha + W_{\mu\alpha}^- W_\nu^+ Z^\alpha) \;, \\
\mathcal{Q}_{\gamma ZW,5}^{\partial=2} &= F^{\mu\nu}(W_{\mu\alpha}^+ W^{-\alpha} Z_\nu + W_{\mu\alpha}^- W^{+\alpha} Z_\nu) & , \quad \mathcal{Q}_{\gamma ZW,6}^{\partial=2} &= F_{\mu\nu} Y^{\mu\alpha}(W^{+\nu} W_\alpha^- + W^{-\nu} W_\alpha^+) \;\;, \\
\mathcal{Q}_{\gamma ZW,7}^{\partial=2} &= F_{\mu\nu} Z_\alpha (X^{+\mu\alpha} W^{-\nu} + X^{-\mu\alpha} W^{+\nu}) & , \quad \mathcal{Q}_{\gamma ZW,8}^{\partial=2} &= F_{\mu\nu} Z^\nu (X^{+\mu\alpha} W_\alpha^- + X^{-\mu\alpha} W_\alpha^+) \;\;, \\
\mathcal{Q}_{\gamma ZW,9}^{\partial=2} &= F_{\mu\nu} Z^\mu (X_\alpha^{+\alpha} W^{-\nu} + X_\alpha^{-\alpha} W^{+\nu}) & , &
\end{aligned}
\tag{5}
$$

while there are three Lorentz structures associated to the coupling of a single photon to three Z's

$$\mathcal{Q}_{\gamma ZZ,1}^{\partial=2} = F_{\mu\nu} Z^{\mu\nu} Z^\alpha Z_\alpha \;, \quad \mathcal{Q}_{\gamma ZZ,2}^{\partial=2} = F_{\mu\nu} Z^{\mu\alpha} Z^\nu Z_\alpha \;, \quad \mathcal{Q}_{\gamma ZZ,3}^{\partial=2} = F_{\mu\nu} Y^{\mu\alpha} Z^\nu Z_\alpha \;. \tag{6}$$

3. Vertices with 2 derivatives without photons

The $W^+ W^- Z Z$ quartic interactions exhibiting two derivatives are summarized by 29 distinct Lorentz structures:

$$
\begin{aligned}
\mathcal{Q}_{WZ,1}^{\partial=2} &= W^{+\mu\nu} W_{\mu\nu}^- Z^\alpha Z_\alpha & , \quad \mathcal{Q}_{WZ,2}^{\partial=2} &= W^{+\mu\nu} W_{\mu\alpha}^- Z^\alpha Z_\nu & , \\
\mathcal{Q}_{WZ,3}^{\partial=2} &= W^{+\mu\nu} Z_{\mu\nu} Z^\alpha W_\alpha^- + \text{h.c.} & , \quad \mathcal{Q}_{WZ,4}^{\partial=2} &= W^{+\mu\nu} Z_{\mu\alpha} Z_\nu W^{-\alpha} + \text{h.c.} & , \\
\mathcal{Q}_{WZ,5}^{\partial=2} &= W^{+\mu\nu} Z_{\mu\alpha} Z^\alpha W_\nu^- + \text{h.c.} & , \quad \mathcal{Q}_{WZ,6}^{\partial=2} &= Z^{\mu\nu} Z_{\mu\nu} W_\alpha^+ W^{-\alpha} & , \\
\mathcal{Q}_{WZ,7}^{\partial=2} &= Z^{\mu\nu} Z_{\mu\alpha} W_\nu^+ W^{-\alpha} & , \quad \mathcal{Q}_{WZ,8}^{\partial=2} &= X^{+\mu\nu} X_{\mu\nu}^- Z^\alpha Z_\alpha & , \\
\mathcal{Q}_{WZ,9}^{\partial=2} &= X^{+\mu\nu} X_{\mu\alpha}^- Z^\alpha Z_\nu & , \quad \mathcal{Q}_{WZ,10}^{\partial=2} &= X_\mu^{+\mu} X_\nu^{-\nu} Z^\alpha Z_\alpha & , \\
\mathcal{Q}_{WZ,11}^{\partial=2} &= X_\mu^{+\mu} X^{-\nu\alpha} Z_\alpha Z_\nu + \text{h.c.} & , \quad \mathcal{Q}_{WZ,12}^{\partial=2} &= X^{+\mu\nu} Y_{\mu\nu} Z^\alpha W_\alpha^- + \text{h.c.} & , \\
\mathcal{Q}_{WZ,13}^{\partial=2} &= X^{+\mu\nu} Y_{\mu\alpha} Z^\alpha W_\nu^- + \text{h.c.} & , \quad \mathcal{Q}_{WZ,14}^{\partial=2} &= X^{+\mu\nu} Y_{\mu\alpha} Z_\nu W^{-\alpha} + \text{h.c.} & , \\
\mathcal{Q}_{WZ,15}^{\partial=2} &= X_\mu^{+\mu} Y_\nu^\nu Z_\alpha W^{-\alpha} + \text{h.c.} & , \quad \mathcal{Q}_{WZ,16}^{\partial=2} &= X_\mu^{+\mu} Y^{\nu\alpha} Z_\nu W_\alpha^- + \text{h.c.} & , \\
\mathcal{Q}_{WZ,17}^{\partial=2} &= X_{\mu\alpha}^+ Y_\nu^\nu Z^\mu W^{-\alpha} + \text{h.c.} & , \quad \mathcal{Q}_{WZ,18}^{\partial=2} &= Y^{\mu\nu} Y_{\mu\nu} W_\alpha^+ W^{-\alpha} & , \\
\mathcal{Q}_{WZ,19}^{\partial=2} &= Y^{\mu\nu} Y_{\mu\alpha} W_\nu^+ W^{-\alpha} & , \quad \mathcal{Q}_{WZ,20}^{\partial=2} &= Y_\mu^\mu Y_\nu^\nu W_\alpha^+ W^{-\alpha} & , \\
\mathcal{Q}_{WZ,21}^{\partial=2} &= Y_\mu^\mu Y^{\nu\alpha} W_\nu^+ W_\alpha^- & , \quad \mathcal{Q}_{WZ,22}^{\partial=2} &= W^{+\mu\nu} X_{\mu\alpha}^- Z^\alpha Z_\nu + \text{h.c.} & , \\
\mathcal{Q}_{WZ,23}^{\partial=2} &= W^{+\mu\nu} Y_{\mu\alpha} Z_\nu W^{-\alpha} + \text{h.c.} & , \quad \mathcal{Q}_{WZ,24}^{\partial=2} &= W^{+\mu\nu} Y_{\mu\alpha} Z^\alpha W_\nu^- + \text{h.c.} & , \\
\mathcal{Q}_{WZ,25}^{\partial=2} &= W^{+\nu\alpha} Y_\mu^\mu Z_\nu W_\alpha^- + \text{h.c.} & , \quad \mathcal{Q}_{WZ,26}^{\partial=2} &= X^{+\mu\nu} Z_{\mu\alpha} Z_\nu W^{-\alpha} + \text{h.c.} & , \\
\mathcal{Q}_{WZ,27}^{\partial=2} &= X^{+\mu\nu} Z_{\mu\alpha} Z^\alpha W_\nu^- + \text{h.c.} & , \quad \mathcal{Q}_{WZ,28}^{\partial=2} &= X_\mu^{+\mu} Z_{\nu\alpha} Z^\nu W^{-\alpha} + \text{h.c.} & , \\
\mathcal{Q}_{WZ,29}^{\partial=2} &= Y^{\mu\nu} Z_{\mu\alpha} W_\nu^+ W^{-\alpha} + \text{h.c.} & , &
\end{aligned}
\tag{7}
$$

where h.c. stands for the hermitian conjugate. Correspondingly, there are 18 $W^+W^-W^+W^-$ effective vertices containing two derivatives that are given by:

$$
\begin{aligned}
\mathcal{Q}^{\partial=2}_{WW,1} &= W^{+\mu\nu}W^-_{\mu\nu}W^{+\alpha}W^-_{\alpha} & , & \quad \mathcal{Q}^{\partial=2}_{WW,2} = W^{+\mu\nu}W^-_{\mu\alpha}W^{+\alpha}W^-_{\nu} & , \\
\mathcal{Q}^{\partial=2}_{WW,3} &= W^{+\mu\nu}W^-_{\mu\alpha}W^+_{\nu}W^{-\alpha} & , & \quad \mathcal{Q}^{\partial=2}_{WW,4} = W^{+\mu\nu}W^+_{\mu\nu}W^{-\alpha}W^-_{\alpha} + \text{h.c.} & , \\
\mathcal{Q}^{\partial=2}_{WW,5} &= W^{+\mu\nu}W^+_{\mu\alpha}W^{-\alpha}W^-_{\nu} + \text{h.c.} & , & \quad \mathcal{Q}^{\partial=2}_{WW,6} = X^{+\mu\nu}X^-_{\mu\nu}W^{+\alpha}W^-_{\alpha} & , \\
\mathcal{Q}^{\partial=2}_{WW,7} &= X^{+\mu\nu}X^-_{\mu\alpha}W^{+\alpha}W^-_{\nu} & , & \quad \mathcal{Q}^{\partial=2}_{WW,8} = X^{+\mu\nu}X^-_{\mu\alpha}W^{-\alpha}W^+_{\nu} & , \\
\mathcal{Q}^{\partial=2}_{WW,9} &= X^{+\mu}_{\mu}X^{-\nu}_{\nu}W^{+\alpha}W^-_{\alpha} & , & \quad \mathcal{Q}^{\partial=2}_{WW,10} = X^{+\mu}_{\mu}X^{-\nu\alpha}W^+_{\nu}W^-_{\alpha} + \text{h.c.} & , \\
\mathcal{Q}^{\partial=2}_{WW,11} &= X^{+\mu\nu}X^+_{\mu\nu}W^{-\alpha}W^-_{\alpha} + \text{h.c.} & , & \quad \mathcal{Q}^{\partial=2}_{WW,12} = X^{+\mu\nu}X^+_{\mu\alpha}W^{-\alpha}W^-_{\nu} + \text{h.c.} & , \\
\mathcal{Q}^{\partial=2}_{WW,13} &= X^{+\mu}_{\mu}X^{+\nu}_{\nu}W^{-\alpha}W^-_{\alpha} + \text{h.c.} & , & \quad \mathcal{Q}^{\partial=2}_{WW,14} = X^{+\mu}_{\mu}X^{+\nu\alpha}W^-_{\nu}W^-_{\alpha} + \text{h.c.} & , \\
\mathcal{Q}^{\partial=2}_{WW,15} &= W^{+\mu\nu}X^-_{\mu\alpha}W^{+\alpha}W^-_{\nu} + \text{h.c.} & , & \quad \mathcal{Q}^{\partial=2}_{WW,16} = W^{+\mu\nu}X^-_{\mu\alpha}W^{-\alpha}W^+_{\nu} + \text{h.c.} & , \\
\mathcal{Q}^{\partial=2}_{WW,17} &= X^{+\mu}_{\mu}W^-_{\nu\alpha}W^{+\nu}W^{-\alpha} + \text{h.c.} & , & \quad \mathcal{Q}^{\partial=2}_{WW,18} = X^{+\mu\nu}W^+_{\mu\alpha}W^{-\alpha}W^-_{\nu} + \text{h.c.} & .
\end{aligned}
\tag{8}
$$

Finally, 7 different Lorentz structures are required to describe $ZZZZ$ vertices with two derivatives:

$$
\begin{aligned}
\mathcal{Q}^{\partial=2}_{ZZ,1} &= Z^{\mu\nu}Z_{\mu\nu}Z^{\alpha}Z_{\alpha} \;,\; & \mathcal{Q}^{\partial=2}_{ZZ,2} Z^{\mu\nu}Z_{\mu\alpha}Z^{\alpha}Z_{\nu} & \;, \\
\mathcal{Q}^{\partial=2}_{ZZ,3} &= Y^{\mu\nu}Y_{\mu\nu}Z^{\alpha}Z_{\alpha} \;,\; & \mathcal{Q}^{\partial=2}_{ZZ,4} = Y^{\mu\nu}Y_{\mu\alpha}Z^{\alpha}Z_{\nu} & \;, \\
\mathcal{Q}^{\partial=2}_{ZZ,5} &= Y^{\mu}_{\mu}Y^{\nu}_{\nu}Z^{\alpha}Z_{\alpha} \;,\; & \mathcal{Q}^{\partial=2}_{ZZ,6} = Y^{\mu}_{\mu}Y^{\nu\alpha}Z_{\alpha}Z_{\nu} & \;, \\
\mathcal{Q}^{\partial=2}_{ZZ,7} &= Z^{\mu\nu}Y_{\mu\alpha}Z^{\alpha}Z_{\nu} \;. &
\end{aligned}
\tag{9}
$$

Altogether any effective Lagrangian possessing two derivatives and any four gauge bosons can be written as a combination of the seventy Lorentz structures above as:

$$
\begin{aligned}
\mathcal{L}^{\partial=2}_Q &= \sum_{i=1}^{2} c^{2,\gamma W}_i \mathcal{Q}^{\partial=2}_{\gamma W,i} + \sum_{i=1}^{2} c^{2,\gamma Z}_i \mathcal{Q}^{\partial=2}_{\gamma Z,i} + \sum_{i=1}^{9} c^{2,\gamma ZW}_i \mathcal{Q}^{\partial=2}_{\gamma ZW,i} + \sum_{i=1}^{3} c^{2,\gamma ZZ}_i \mathcal{Q}^{\partial=2}_{\gamma ZZ,i} \\
&\quad + \sum_{i=1}^{29} c^{2,WZ}_i \mathcal{Q}^{\partial=2}_{WZ,i} + \sum_{i=1}^{18} c^{2,WW}_i \mathcal{Q}^{\partial=2}_{WW,i} + \sum_{i=1}^{7} c^{2,ZZ}_i \mathcal{O}^{\partial=2}_{ZZ,i} \;.
\end{aligned}
\tag{10}
$$

It is interesting to notice that the quartic vertices in Eqs. (5), (6), (7), (8), and (9) containing $X^\pm_{\mu\nu}$ or $Y_{\mu\nu}$ have not been considered before in the literature.

In summary, we have found five Lorentz structures without derivatives and seventy with two derivatives. Next we are going to build the lowest dimension electroweak gauge invariant Lagrangian which can lead to genuine quartic gauge boson vertices and map their coefficients to those of these Lorentz structures.

III. GENUINE QGC IN MODELS WITH AN ELEMENTARY HIGGS: THE LINEAR LAGRANGIAN

Assuming that the new state observed in 2012 is indeed the SM Higgs boson and belongs to a light electroweak scalar doublet, we can construct an effective theory where the $SU(2)_L \otimes U(1)_Y$ gauge symmetry is linearly realized [5–13] that can be expressed as

$$
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \sum_{n=5}^{\infty} \frac{f_n}{\Lambda^{n-4}} \mathcal{O}_n \;,
\tag{11}
$$

where the dimension–n operators $\mathcal{O}_n$ involve SM fields with couplings f_n and where Λ is a characteristic scale.

The basic blocks for constructing the effective Lagrangian leading to genuine QGC with the gauge symmetry realized linearly and their transformations are:

$$\Phi \ , \quad \text{that transforms as} \quad \Phi' = U\Phi \tag{12}$$

$$D_\mu\Phi \ , \quad \text{that transforms as} \quad D'_\mu\Phi' = UD_\mu\Phi \tag{13}$$

$$\widehat{W}_{\mu\nu} \equiv \sum_j W^j_{\mu\nu} \frac{\sigma^j}{2} \ , \quad \text{that transforms as} \quad \widehat{W}'_{\mu\nu} = U\widehat{W}_{\mu\nu}U^\dagger \tag{14}$$

$$B_{\mu\nu} \ , \quad \text{that transforms as} \quad B'_{\mu\nu} = B_{\mu\nu} \tag{15}$$

where Φ stands for the Higgs doublet, $W^i_{\mu\nu}$ is the $SU(2)_L$ field strength and $B_{\mu\nu}$ is the $U(1)_Y$ one. Here we denote an arbitrary gauge transformation by U. According to our conventions the covariant derivative is given by $D_\mu\Phi = (\partial_\mu + igW^j_\mu\frac{\sigma^j}{2} + ig'B_\mu\frac{1}{2})\Phi$ and σ^j stand for the Pauli matrices.

Notice that the covariant derivative of Φ, as well as, the field strength tensors contain terms with at least one weak gauge boson when we substitute Φ by its vacuum expectation value, v. Therefore the lowest dimension operators that leads to genuine quartic interactions are dimension eight[1]. They can be classified in three groups:

- terms that contain four covariant derivatives of the Higgs field;

- terms exhibiting two Higgs covariant derivatives and two field strength tensors;

- terms presenting four field strength tensors.

Here we focus on operators containing up to two derivatives, therefore, we will not analyze the last class, however, we present these operators in the Appendix B for the sake of completeness.

A.   Operators containing only $D_\mu\Phi$

There are three independent operators belong to this class[2]:

$$\mathcal{O}_{S,0} = \left[(D_\mu\Phi)^\dagger D_\nu\Phi\right] \times \left[(D^\mu\Phi)^\dagger D^\nu\Phi\right] = \frac{g^4 v^4}{16}\left[\mathcal{Q}^{\partial=0}_{WW,2} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=0}_{WZ,2} + \frac{1}{4c_w^4}\mathcal{Q}^{\partial=0}_{ZZ}\right] \ ,$$

$$\mathcal{O}_{S,1} = \left[(D_\mu\Phi)^\dagger D^\mu\Phi\right] \times \left[(D_\nu\Phi)^\dagger D^\nu\Phi\right] = \frac{g^4 v^4}{16}\left[\mathcal{Q}^{\partial=0}_{WW,1} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=0}_{WZ,1} + \frac{1}{4c_w^4}\mathcal{Q}^{\partial=0}_{ZZ}\right] \ , \tag{16}$$

$$\mathcal{O}_{S,2} = \left[(D_\mu\Phi)^\dagger D_\nu\Phi\right] \times \left[(D^\nu\Phi)^\dagger D^\mu\Phi\right] = \frac{g^4 v^4}{16}\left[\mathcal{Q}^{\partial=0}_{WW,1} + \frac{1}{c_w^2}\mathcal{Q}^{\partial=0}_{WZ,2} + \frac{1}{4c_w^4}\mathcal{Q}^{\partial=0}_{ZZ}\right] \ .$$

The corresponding effective Lagrangian can be mapped to that in Eq. (2) with coefficients related as:

$$c_1^{0,WW} = \frac{g^4 v^4}{16}\left[\frac{f_{S,1}}{\Lambda^4} + \frac{f_{S,2}}{\Lambda^4}\right] \qquad , \quad c_2^{0,WW} = \frac{g^4 v^4}{16}\frac{f_{S,0}}{\Lambda^4} \qquad ,$$

$$c_1^{0,WZ} = \frac{g^4 v^4}{16c_W^2}\frac{f_{S,1}}{\Lambda^4} \qquad , \quad c_2^{0,WZ} = \frac{g^4 v^4}{16c_w^2}\left[\frac{f_{S,0}}{\Lambda^4} + \frac{f_{S,2}}{\Lambda^4}\right] \ , \tag{17}$$

$$c^{0,ZZ} = \frac{1}{4c_w^4}\left(c_1^{0,WW} + c_2^{0,WW}\right) \ ,$$

where s_w (c_w) stand for the sine (cosine) of the weak mixing angle θ_w verifying $\tan\theta_w = g/g'$. Notice that the linear realization of the symmetry leads to correlations between the Wilson coefficients appearing in Eq. (2).

---

[1] At dimension six, quartic vertices are generated but they are always accompanied by triple gauge boson vertices with related coefficients.

[2] We are using the conventions of Ref. [21], however, the operator $\mathcal{O}_{S,2}$ has been included.

B.   Operators containing $D_\mu\Phi$ and field strength

This class possesses seven operators[3]:

$$\mathcal{O}_{M,0} = \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\right] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] \;,\quad \mathcal{O}_{M,1} = \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] \;,$$

$$\mathcal{O}_{M,2} = [B_{\mu\nu}B^{\mu\nu}] \times \left[(D_\beta\Phi)^\dagger D^\beta\Phi\right] \qquad , \quad \mathcal{O}_{M,3} = \left[B_{\mu\nu}B^{\nu\beta}\right] \times \left[(D_\beta\Phi)^\dagger D^\mu\Phi\right] \qquad ,$$

$$\mathcal{O}_{M,4} = \left[(D_\mu\Phi)^\dagger \widehat{W}_{\beta\nu}D^\mu\Phi\right] \times B^{\beta\nu} \qquad , \quad \mathcal{O}_{M,5} = \left[(D_\mu\Phi)^\dagger \widehat{W}_{\beta\nu}D^\nu\Phi\right] \times B^{\beta\mu} + \mathrm{h.c.} \;,$$

$$\mathcal{O}_{M,7} = \left[(D_\mu\Phi)^\dagger \widehat{W}_{\beta\nu}\widehat{W}^{\beta\mu}D^\nu\Phi\right] \qquad . \tag{18}$$

These seven operators involve 23 of the 70 possible Lorentz structures as explicitly given in Appendix A. In particular none of the structures with symmetric gauge boson tensors are generated. The corresponding effective Lagrangian can be written in the form of Eq. (10) where the 23 coefficients of the Lorentz structures can be expressed in terms of 7 of them. For example, these can be chosen to be $c_{1,2}^{2,WW}$, $c_{2,3,6,7}^{2,WZ}$ and $c_2^{2,\gamma W}$ with

$$
\begin{aligned}
c_1^{2,WW} &= \frac{g^2 v^2}{8}\frac{2f_{M,0}}{\Lambda^4} \;, \\
c_2^{2,WW} &= -\frac{g^2 v^2}{8}\frac{f_{M,1}}{\Lambda^4} \;, \\
c_2^{2,WZ} &= \frac{g^2 v^2}{8}\frac{1}{c_w^2}\frac{-f_{M,1}+\frac{1}{2}f_{M,7}}{\Lambda^4} \;, \\
c_3^{2,WZ} &= \frac{g^2 v^2}{8}\frac{s_w}{c_w}\frac{f_{M,4}}{\Lambda^4} \;, \\
c_6^{2,WZ} &= \frac{g^2 v^2}{8}\left[c_w^2\frac{f_{M,0}}{\Lambda^4} + 2s_w^2\frac{f_{M,2}}{\Lambda^4} - s_w c_w\frac{f_{M,4}}{\Lambda^4}\right] \;, \\
c_7^{2,WZ} &= \frac{g^2 v^2}{8}\left[-c_w^2\frac{f_{M,1}}{\Lambda^4} - s_w^2\frac{f_{M,3}}{\Lambda^4} - 2s_w c_w\frac{f_{M,5}}{\Lambda^4} + \frac{1}{2}c_w^2\frac{f_{M,7}}{\Lambda^4}\right] \;, \\
c_2^{2,\gamma W} &= \frac{g^2 v^2}{8}\left[-s_w^2\frac{f_{M,1}}{\Lambda^4} - c_w^2\frac{f_{M,3}}{\Lambda^4} + 2s_w c_w\frac{f_{M,5}}{\Lambda^4} + \frac{1}{2}s_w^2\frac{f_{M,7}}{\Lambda^4}\right] \;,
\end{aligned}
\tag{19}
$$

and the remaining 16 coefficients satisfy the following relations

$$
\begin{aligned}
c_1^{2,WZ} &= \tfrac{1}{2c_w^2}c_1^{2,WW} & , \quad c_1^{2,\gamma Z} &= \tfrac{1}{2c_w^2}c_1^{2,\gamma W} - c_3^{2,WZ} & , \\
c_1^{2,ZZ} &= \tfrac{1}{2c_w^2}c_6^{2,WZ} + c_3^{2,WZ} & , \quad c_3^{2,\gamma ZW} &= -\tfrac{c_w}{s_w}c_3^{2,WZ} & , \\
c_1^{2,\gamma ZW} &= 2s_w c_w\left[c_1^{2,WW} - c_6^{2,WZ} - c_1^{2,\gamma W} + \tfrac{c_w^2-s_w^2}{2s_w^2}c_3^{2,WZ}\right] & , \quad c_1^{2,\gamma ZZ} &= \tfrac{1}{2c_w^2}c_1^{2,\gamma ZW} - \tfrac{c_w^2-s_w^2}{s_w c_w}c_3^{2,WZ} & , \\
c_1^{2,\gamma W} &= \tfrac{1}{2s_w^2}\left[2c_w^2(c_3^{2,WZ} + c_6^{2,WZ}) - (c_w^2 - s_w^2)c_1^{2,WW}\right] & , \quad c_3^{2,WW} &= 2c_w^2 c_2^{2,WZ} - c_2^{2,WW} & , \\
c_2^{2,ZZ} &= \tfrac{1}{2c_w^2}\left[(c_w^2 - s_w^2)(2c_w^2 c_2^{2,WZ} - c_7^{2,WZ}) + 2s_w^2 c_2^{2,\gamma W}\right] & , \quad c_2^{2,\gamma Z} &= -c_2^{2,ZZ} + \tfrac{1}{c_w^2}(c_7^{2,WZ} + c_2^{2,\gamma W}) & , \\
c_5^{2,WZ} &= \tfrac{1}{2}\left[-c_2^{2,WZ} - c_7^{2,WZ} + \tfrac{s_w^2}{c_w^2}c_2^{2,\gamma W} + 2c_2^{2,WW}\right] & , \quad c_4^{2,WZ} &= c_5^{2,WZ} + 2(c_w^2 c_2^{2,WZ} - c_2^{2,WW}) & , \\
c_2^{2,\gamma ZW} &= \tfrac{1}{2s_w c_w}\left[c_w^2(c_2^{2,WZ} - c_7^{2,WZ}) - s_w^2 c_2^{2,\gamma W}\right] & , \quad c_2^{2,\gamma ZZ} &= \tfrac{1}{c_w^2}c_2^{2,\gamma ZW} - \tfrac{c_w^2-s_w^2}{c_w s_w}(c_5^{2,WZ} + c_4^{2,WZ}) & , \\
c_{4,5}^{2,\gamma ZW} &= -\tfrac{c_w}{2s_w}(c_5^{2,WZ} + c_4^{2,WZ}) \pm \tfrac{s_w}{c_w}(c_w^2 c_2^{2WZ} - c_2^{2,WW}) & . &
\end{aligned}
\tag{20}
$$

---

[3] We follow the notation in Ref. [21] but we notice that in there an additional operator $\mathcal{O}_{M,6}$ was listed which we found to be redundant.

IV. GENUINE QGC IN MODELS WITH A DYNAMICAL HIGGS: THE CHIRAL LAGRANGIAN

In dynamical Higgs scenarios, the Higgs particle is a composite field which happens to be a pseudo-Nambu-Goldstone boson (PNG) of a global symmetry exact at scales Λ_{strong} that corresponds to the masses of the lightest strong resonances. Because the Higgs-like particle is a PNG, the effective Lagrangian is non-linear or "chiral": a derivative expansion [14, 15, 40, 41] with a global $SU(2)_L \otimes SU(2)_R$ symmetry broken to the diagonal $SU(2)_c$. The effective low-energy chiral Lagrangian is entirely written in terms of the SM fermions and gauge bosons and of the physical Higgs h. In this scenario, the basic building block at low energies is a dimensionless unitary matrix transforming as a bi-doublet of the global symmetry:

$$\mathbf{U}(x) = e^{i\sigma_a \pi^a(x)/v} , \qquad \mathbf{U}(x) \to L\,\mathbf{U}(x)R^\dagger , \tag{21}$$

where L, R denote $SU(2)_{L,R}$ global transformations, respectively and π^a are the goldstone bosons. Its covariant derivative reads

$$\mathbf{D}_\mu \mathbf{U}(x) \equiv \partial_\mu \mathbf{U}(x) + ig\frac{\sigma^j}{2}W_\mu^i(x)\mathbf{U}(x) - \frac{ig'}{2}B_\mu(x)\mathbf{U}(x)\sigma_3 . \tag{22}$$

We define the vector chiral field and its covariant derivative as

$$V_\mu \equiv (\mathbf{D}_\mu \mathbf{U})\,\mathbf{U}^\dagger , \tag{23}$$
$$D_\mu V_\alpha = \partial_\mu V_\alpha + ig[W_\mu, V_\alpha] , \tag{24}$$

and the scalar chiral field $T \equiv \mathbf{U}\sigma_3\mathbf{U}^\dagger$. These three objects transform in the adjoint of $SU(2)_L$. Moreover, the Higgs field h is a singlet under the global symmetry.

In our framework, we consider genuine QGC that appear at $\mathcal{O}(p^4)$ and $\mathcal{O}(p^6)$ and are invariant under CP. The CP transformation properties of our building blocks can be easily obtained once we know that [15]

$$\begin{aligned} CPB_\mu(\vec{x},t)(CP)^{-1} = -B_\mu(-\vec{x},t) &, \quad CPW_\mu^i(\vec{x},t)(CP)^{-1} = \sigma^2 W_\mu^i(-\vec{x},t)\sigma^2 , \\ CPT(\vec{x},t)(CP)^{-1} = -\sigma^2 T(-\vec{x},t)\sigma^2 &, \quad CPV_\mu(\vec{x},t)(CP)^{-1} = \sigma^2 V_\mu(-\vec{x},t)\sigma^2 . \end{aligned} \tag{25}$$

Our choice of phases are such that $D_\mu U$ has a well-defined transformation under CP. From the above equation we can learn that the CP conserving QGC are the ones exhibiting an even number of T's and B_μ's.

The building blocks that we use to construct genuine CP conserving QGC can be classified according the mass dimension of the operator (D) [15]. Here we list all building block operators needed to construct up to $\mathcal{O}(p^6)$ quartic operators, as well as their expressions in the unitary gauge. There is just one operator with mass dimension one

$$\text{Tr}\,[TV_\mu] = i\frac{g}{c_w}Z_\mu . \tag{26}$$

On the other hand, there are five $D = 2$ building blocks, however, only 4 of them appear in CP invariant quartic operators:

$$B_{\mu\nu} = c_w F_{\mu\nu} - s_w Z_{\mu\nu} , \tag{27}$$
$$\text{Tr}[TD_{\mu\nu}] = i\frac{g}{c_w}Y_{\mu\nu} , \tag{28}$$
$$\text{Tr}[V_\mu V_\nu] = -\frac{g^2}{2}\left(\frac{1}{c_w^2}Z_\mu Z_\nu + W_\mu^+ W_\nu^- + W_\mu^- W_\nu^+\right) , \tag{29}$$
$$\text{Tr}[T\widehat{W}_{\mu\nu}] = c_w Z_{\mu\nu} + s_w F_{\mu\nu} , \tag{30}$$

where we define the symmetric combination $D_{\mu\nu} \equiv D_\mu V_\nu + D_\nu V_\mu$.

Just two of the eight $D = 3$ basic operators appear in CP invariant quartic vertices:

$$\text{Tr}[V_\mu D_{\nu\lambda}] = -\frac{g^2}{2}\left(\frac{1}{c_W^2}Z_\mu Y_{\nu\lambda} + W_\mu^+ X_{\nu\lambda}^- + W_\mu^- X_{\nu\lambda}^+\right) , \tag{31}$$
$$\text{Tr}[V_\mu \widehat{W}_{\nu\lambda}] = i\frac{g}{2}\left(\frac{1}{c_w}Z_\mu(c_w Z_{\nu\lambda} + s_w F_{\nu\lambda}) + W_\mu^+ W_{\nu\lambda}^- + W_\mu^- W_{\nu\lambda}^+\right) . \tag{32}$$

Of the eleven possible $D = 4$ operators just three of them contribute to CP conserving quartic vertices:

$$\text{Tr}[D_{\mu\nu}D_{\alpha\beta}] = -\frac{g^2}{2}\left(\frac{1}{c_w^2}Y_{\mu\nu}Y_{\alpha\beta} + X_{\mu\nu}^+X_{\alpha\beta}^- + X_{\mu\nu}^-X_{\alpha\beta}^+\right) , \tag{33}$$

$$\text{Tr}[\widehat{W}_{\mu\nu}\widehat{W}_{\alpha\beta}] = \frac{1}{2}\left[(c_wZ_{\mu\nu} + s_wF_{\mu\nu})(c_wZ_{\alpha\beta} + s_wF_{\alpha\beta}) + W_{\mu\nu}^+W_{\alpha\beta}^- + W_{\mu\nu}^-W_{\alpha\beta}^+\right] , \tag{34}$$

$$\text{Tr}[\widehat{W}_{\mu\nu}D_{\alpha\beta}] = \frac{ig}{2}\left[\frac{1}{c_w}(c_wZ_{\mu\nu} + s_wF_{\mu\nu})Y_{\alpha\beta} + W_{\mu\nu}^+X_{\alpha\beta}^- + W_{\mu\nu}^-X_{\alpha\beta}^+\right] . \tag{35}$$

It is interesting to notice that no dimension-five operador can give rise to p^6 CP-conserving QGC.

Using the $D = 1, 2, 3, 4$ building blocks, we construct all possible CP-conserving operators for genuine QGC, and then we remove those which can be related by total derivatives. For instance, the relation

$$\partial_\nu\left\{\text{Tr}[V_\mu D_\lambda V_\rho]\,\text{Tr}[V_\alpha V_\beta]\right\} = \text{Tr}[D_\nu V_\mu D_\lambda V_\rho]\,\text{Tr}[V_\alpha V_\beta] + \text{Tr}[V_\mu D_\nu D_\lambda V_\rho]\,\text{Tr}[V_\alpha V_\beta] + $$
$$\text{Tr}[V_\mu D_\lambda V_\rho]\,\text{Tr}[D_\nu V_\alpha V_\beta] + \text{Tr}[V_\mu D_\lambda V_\rho]\,\text{Tr}[V_\alpha D_\nu V_\beta] \tag{36}$$

can be used to eliminate operators that contain the building block $\text{Tr}[V_\mu D_\nu D_\lambda V_\rho]$. To further reduce the number of equivalent operators we also use the relation

$$D_\mu V_\nu - D_\nu V_\mu = ig\widehat{W}_{\mu\nu} - \frac{i}{2}g'B_{\mu\nu}T + [V_\mu, V_\nu] . \tag{37}$$

Moreover, we introduce a factor ig and ig' in each operator containing $\widehat{W}_{\mu\nu}$ and $B_{\mu\nu}$ respectively in order to have consistent global powers of coupling constants.

A. QGC at $\mathcal{O}(p^4)$

The lowest order genuine quartic operators are $\mathcal{O}(p^4)$, and there are two operators which respect the $SU(2)_c$ custodial symmetry, as well as C and P that are given by[4]

$$\mathcal{P}_6 = \text{Tr}[V^\mu V_\mu]\text{Tr}[V^\nu V_\nu]\mathcal{F}_6(h) = g^4\left[\frac{1}{4c_w^4}\mathcal{O}_{ZZ}^0 + \mathcal{O}_{WW,1}^0 + \frac{1}{c_w^2}\mathcal{O}_{WZ,1}^0\right]\mathcal{F}_6(h) ,$$
$$\mathcal{P}_{11} = \text{Tr}[V^\mu V^\nu]\text{Tr}[V_\mu V_\nu]\mathcal{F}_{11}(h) = g^4\left[\frac{1}{4c_w^4}\mathcal{O}_{ZZ}^0 + \frac{1}{2}\mathcal{O}_{WW,1}^0 + \frac{1}{2}\mathcal{O}_{WW,2}^0 + \frac{1}{c_w^2}\mathcal{O}_{WZ,2}^0\right]\mathcal{F}_{11}(h) , \tag{38}$$

and 3 additional CP conserving operators that violate $SU(2)_c$:

$$\mathcal{P}_{23} = \text{Tr}[V^\mu V_\mu](\text{Tr}[TV_\nu])^2\mathcal{F}_{23}(h) = g^4\left[\frac{1}{2c_w^4}\mathcal{O}_{ZZ}^0 + \frac{1}{c_w^2}\mathcal{O}_{WZ,1}^0\right]\mathcal{F}_{23}(h) ,$$
$$\mathcal{P}_{24} = \text{Tr}[V^\mu V^\nu]\text{Tr}[TV_\mu]\text{Tr}[TV_\nu]\mathcal{F}_{24}(h) = g^4\left[\frac{1}{2c_w^4}\mathcal{O}_{ZZ}^0 + \frac{1}{c_w^2}\mathcal{O}_{WZ,2}^0\right]\mathcal{F}_{24}(h) , \tag{39}$$
$$\mathcal{P}_{26} = (\text{Tr}[TV_\mu]\text{Tr}[TV_\nu])^2\mathcal{F}_{26}(h) = \frac{g^4}{c_w^4}\mathcal{O}_{ZZ}^0\mathcal{F}_{26}(h) .$$

$\mathcal{F}_i(h)$ are generic functions parametrizing the chiral-symmetry breaking interactions of h which can be expanded as $\mathcal{F}_i(h) \equiv 1 + 2\tilde{a}_i\frac{h}{v} + \tilde{b}_i\frac{h^2}{v^2} + \ldots$. As we are looking for operators whose lowest order vertex contains four gauge bosons, we will be only concerned by the constant term. So the most general Lagrangian at $\mathcal{O}(p^4)$ for genuine QGC is

$$\mathcal{L}_Q^{p=4} = \sum_{i=6,11,23,24,26} c_i^{p=4}\mathcal{P}_i . \tag{40}$$

From Eqs. (38) and (39) we see that the above Lagrangian leads to quartic gauge couplings which do not contain photons. We also see that there are five operators matching five independent Lorentz structures that do not exhibit derivatives. In Ref. [15] we can find the p^4 QGC assuming that there is no light Higgs-like state and this corresponds to the limit $\mathcal{F}_i \to 1$ in our framework. The correspondence between between the Wilson coefficients our notation and the one of Ref. [15] is

$$\alpha_4 = c_{11}^{p=4} , \quad \alpha_5 = c_6^{p=4} , \quad \alpha_6 = c_{24}^{p=4} , \quad \alpha_7 = c_{23}^{p=4} , \quad \alpha_{10} = c_{26}^{p=4} . \tag{41}$$

---

[4] We follow the notation of Refs. [4, 16].

B.   QGC at $\mathcal{O}(p^6)$

At order $\mathcal{O}(p^6)$, there is the emergence of genuine QGC containing photons as well as only four electroweak gauge bosons with two derivatives acting on them. As in Ref. [15], we construct the p^6 operators for QGC combining the $D = 1, 2, 3$, and 4 building blocks defined above. Without loss of generality we write the corresponding Lagrangian as

$$\mathcal{L}_{\mathcal{Q}}^{p=6} = \sum_i c_i^{p=6} \mathcal{T}_i^{p=6} \mathcal{F}_i^{p=6}(h) \tag{42}$$

where $\mathcal{T}_i^{p=6}$ are the $\mathcal{O}(p^6)$ operators constructed with the blocks defined above, and we denote by $\mathcal{F}_i^{p=6}(h)$ the corresponding arbitrary function parametrizing the h couplings. As already mentioned we will be only concerned with the first term of its expansion $\mathcal{F}_i^{p=6}(h) = 1$.

1. $(D = 1)^2 (D = 2)^2$ terms

There are twelve independent operators in this category:

$$
\begin{aligned}
\mathcal{T}_1^{p=6} &= \text{Tr}[T\mathcal{D}_{\mu\nu}]\text{Tr}[T\mathcal{D}^{\mu\nu}]\text{Tr}[TV^\alpha]\text{Tr}[TV_\alpha] & , \quad \mathcal{T}_2^{p=6} &= \text{Tr}[T\mathcal{D}_{\mu\nu}]\text{Tr}[T\mathcal{D}^{\mu\alpha}]\text{Tr}[TV^\nu]\text{Tr}[TV_\alpha] & , \\
\mathcal{T}_3^{p=6} &= \text{Tr}[T\mathcal{D}_\mu^\mu]\text{Tr}[T\mathcal{D}_\nu^\nu]\text{Tr}[TV^\alpha]\text{Tr}[TV_\alpha] & , \quad \mathcal{T}_4^{p=6} &= \text{Tr}[T\mathcal{D}_\mu^\mu]\text{Tr}[T\mathcal{D}^{\nu\alpha}]\text{Tr}[TV_\nu]\text{Tr}[TV_\alpha] & , \\
\mathcal{T}_5^{p=6} &= -g^2\,\text{Tr}[T\widehat{W}_{\mu\nu}]\text{Tr}[T\widehat{W}^{\mu\nu}]\text{Tr}[TV^\alpha]\text{Tr}[TV_\alpha] & , \quad \mathcal{T}_6^{p=6} &= -g^2\,\text{Tr}[T\widehat{W}_{\mu\nu}]\text{Tr}[T\widehat{W}^{\mu\alpha}]\text{Tr}[TV^\nu]\text{Tr}[TV_\alpha] & , \\
\mathcal{T}_7^{p=6} &= i\,g\,Tr[T\widehat{W}_{\mu\nu}]\text{Tr}[T\mathcal{D}^{\mu\alpha}]\text{Tr}[TV^\nu]\text{Tr}[TV_\alpha] & , \quad \mathcal{T}_8^{p=6} &= -g'^2\,B_{\mu\nu}B^{\mu\nu}\text{Tr}[TV^\alpha]\text{Tr}[TV_\alpha] & , \\
\mathcal{T}_9^{p=6} &= -g'^2\,B_{\mu\nu}B^{\mu\alpha}\text{Tr}[TV^\nu]\text{Tr}[TV_\alpha] & , \quad \mathcal{T}_{10}^{p=6} &= i\,g'\,B_{\mu\nu}\text{Tr}[T\mathcal{D}^{\mu\alpha}]\text{Tr}[TV^\nu]\text{Tr}[TV_\alpha] & , \\
\mathcal{T}_{11}^{p=6} &= -gg'\,B_{\mu\nu}\text{Tr}[T\widehat{W}^{\mu\nu}]\text{Tr}[TV^\alpha]\text{Tr}[TV_\alpha] & , \quad \mathcal{T}_{12}^{p=6} &= -gg'\,B_{\mu\nu}\text{Tr}[T\widehat{W}^{\mu\alpha}]\text{Tr}[TV^\nu]\text{Tr}[TV_\alpha] & .
\end{aligned}
\tag{43}
$$

Notice that all the effective Lagrangian in this class violate $SU(2)_c$ since they contain the T field. We present the relations between these operators and the Lorentz structures in Appendix C that allow us to see that all the operators in this group contain only neutral gauge bosons.

2. $(D = 1)(D = 2)(D = 3)$ terms

This group contains twenty one operators that violate $SU(2)_c$ due to the presence of T:

$$
\begin{aligned}
\mathcal{T}_{13}^{p=6} &= \text{Tr}[TV_\alpha]\text{Tr}[T\mathcal{D}_{\mu\nu}]\text{Tr}[V^\alpha\mathcal{D}^{\mu\nu}] & , \quad \mathcal{T}_{14}^{p=6} &= \text{Tr}[TV_\alpha]\text{Tr}[T\mathcal{D}_{\mu\nu}]\text{Tr}[V^\nu\mathcal{D}^{\mu\alpha}] & , \\
\mathcal{T}_{15}^{p=6} &= \text{Tr}[TV^\nu]\text{Tr}[T\mathcal{D}_{\mu\nu}]\text{Tr}[V_\alpha\mathcal{D}^{\mu\alpha}] & , \quad \mathcal{T}_{16}^{p=6} &= \text{Tr}[TV_\alpha]\text{Tr}[T\mathcal{D}_\mu^\mu]\text{Tr}[V^\alpha\mathcal{D}_\nu^\nu] & , \\
\mathcal{T}_{17}^{p=6} &= \text{Tr}[TV_\nu]\text{Tr}[T\mathcal{D}_\mu^\mu]\text{Tr}[V_\alpha\mathcal{D}^{\nu\alpha}] & , \quad \mathcal{T}_{18}^{p=6} &= \text{Tr}[TV^\nu]\text{Tr}[T\mathcal{D}_{\nu\alpha}]\text{Tr}[V^\alpha\mathcal{D}_\mu^\mu] & , \\
\mathcal{T}_{19}^{p=6} &= i\,g\,\text{Tr}[TV_\alpha]\text{Tr}[T\mathcal{D}_{\mu\nu}]\text{Tr}[V^\nu\widehat{W}^{\mu\alpha}] & , \quad \mathcal{T}_{20}^{p=6} &= i\,g\,\text{Tr}[TV^\nu]\text{Tr}[T\mathcal{D}_{\mu\nu}]\text{Tr}[V_\alpha\widehat{W}^{\mu\alpha}] & , \\
\mathcal{T}_{21}^{p=6} &= i\,g\,\text{Tr}[TV_\nu]\text{Tr}[T\mathcal{D}_\mu^\mu]\text{Tr}[V_\alpha\widehat{W}^{\nu\alpha}] & , \quad \mathcal{T}_{22}^{p=6} &= i\,g\,\text{Tr}[TV_\alpha]\text{Tr}[T\widehat{W}_{\mu\nu}]\text{Tr}[V^\nu\mathcal{D}^{\mu\alpha}] & , \\
\mathcal{T}_{23}^{p=6} &= i\,g\,\text{Tr}[TV^\nu]\text{Tr}[T\widehat{W}_{\mu\nu}]\text{Tr}[V_\alpha\mathcal{D}^{\mu\alpha}] & , \quad \mathcal{T}_{24}^{p=6} &= i\,g\,\text{Tr}[TV^\nu]\text{Tr}[T\widehat{W}_{\nu\alpha}]\text{Tr}[V^\alpha\mathcal{D}_\mu^\mu] & , \\
\mathcal{T}_{25}^{p=6} &= -g^2\,\text{Tr}[TV_\alpha]\text{Tr}[T\widehat{W}_{\mu\nu}]\text{Tr}[V^\alpha\widehat{W}^{\mu\nu}] & , \quad \mathcal{T}_{26}^{p=6} &= -g^2\,\text{Tr}[TV_\alpha]\text{Tr}[T\widehat{W}_{\mu\nu}]\text{Tr}[V^\nu\widehat{W}^{\mu\alpha}] & , \\
\mathcal{T}_{27}^{p=6} &= -g^2\,\text{Tr}[TV^\nu]\text{Tr}[T\widehat{W}_{\mu\nu}]\text{Tr}[V_\alpha\widehat{W}^{\mu\alpha}] & , \quad \mathcal{T}_{28}^{p=6} &= i\,g'\,\text{Tr}[TV_\alpha]B_{\mu\nu}\text{Tr}[V^\nu\mathcal{D}^{\mu\alpha}] & , \\
\mathcal{T}_{29}^{p=6} &= i\,g'\,\text{Tr}[TV^\nu]B_{\mu\nu}\text{Tr}[V_\alpha\mathcal{D}^{\mu\alpha}] & , \quad \mathcal{T}_{30}^{p=6} &= i\,g'\,\text{Tr}[TV^\nu]B_{\nu\alpha}\text{Tr}[V^\alpha\mathcal{D}_\mu^\mu] & , \\
\mathcal{T}_{31}^{p=6} &= -gg'\,\text{Tr}[TV_\alpha]B_{\mu\nu}\text{Tr}[V^\alpha\widehat{W}^{\mu\nu}] & , \quad \mathcal{T}_{32}^{p=6} &= -gg'\,\text{Tr}[TV_\alpha]B_{\mu\nu}\text{Tr}[V^\nu\widehat{W}^{\mu\alpha}] & , \\
\mathcal{T}_{33}^{p=6} &= -gg'\,\text{Tr}[TV^\nu]B_{\mu\nu}\text{Tr}[V_\alpha\widehat{W}^{\mu\alpha}] & .
\end{aligned}
\tag{44}
$$

From the results presented in Appendix C, we can see that the above operators give rise to W^+W^-ZZ, $ZZZZ$, γZZZ, γZW^+W^-, and $\gamma\gamma ZZ$ anomalous QGC.

3. $(D=3)^2$ terms

We find 11 operators in this class with all of them respecting the custodial symmetry.

$$\mathcal{T}_{34}^{p=6} = \text{Tr}[V_\alpha \mathcal{D}_{\mu\nu}]\text{Tr}[V^\alpha \mathcal{D}^{\mu\nu}] \quad , \quad \mathcal{T}_{35}^{p=6} = \text{Tr}[V_\alpha \mathcal{D}_{\mu\nu}]\text{Tr}[V^\nu \mathcal{D}^{\mu\alpha}] \quad ,$$

$$\mathcal{T}_{36}^{p=6} = \text{Tr}[V^\nu \mathcal{D}_{\mu\nu}]\text{Tr}[V_\alpha \mathcal{D}^{\mu\alpha}] \quad , \quad \mathcal{T}_{37}^{p=6} = \text{Tr}[V_\alpha \mathcal{D}_\mu^\mu]\text{Tr}[V^\alpha \mathcal{D}_\nu^\nu] \quad ,$$

$$\mathcal{T}_{38}^{p=6} = \text{Tr}[V_\nu \mathcal{D}_\mu^\mu]\text{Tr}[V_\alpha \mathcal{D}^{\nu\alpha}] \quad , \quad \mathcal{T}_{39}^{p=6} = i\,g\,\text{Tr}[V_\alpha \mathcal{D}_{\mu\nu}]\text{Tr}[V^\nu \widehat{W}^{\mu\alpha}] \quad ,$$

$$\mathcal{T}_{40}^{p=6} = i\,g\,\text{Tr}[V^\nu \mathcal{D}_{\mu\nu}]\text{Tr}[V_\alpha \widehat{W}^{\mu\alpha}] \quad , \quad \mathcal{T}_{41}^{p=6} = i\,g\,\text{Tr}[V_\nu \mathcal{D}_\mu^\mu]\text{Tr}[V_\alpha \widehat{W}^{\nu\alpha}] \quad ,$$

$$\mathcal{T}_{42}^{p=6} = -g^2\,\text{Tr}[V_\alpha \widehat{W}_{\mu\nu}]\text{Tr}[V^\alpha \widehat{W}^{\mu\nu}] \quad , \quad \mathcal{T}_{43}^{p=6} = -g^2\,\text{Tr}[V_\alpha \widehat{W}_{\mu\nu}]\text{Tr}[V^\nu \widehat{W}^{\mu\alpha}] \quad ,$$

$$\mathcal{T}_{44}^{p=6} = -g^2\,\text{Tr}[V^\nu \widehat{W}_{\mu\nu}]\text{Tr}[V_\alpha \widehat{W}^{\mu\alpha}] \quad . \tag{45}$$

From Appendix C we can learn that the effective Lagrangians in this class generate $W^+W^-W^+W^-$, W^+W^-ZZ and $ZZZZ$ quartic vertices, as well as, γZW^+W^-, γZZZ and $\gamma\gamma ZZ$. Due to the custodial symmetry the last three vertices are multiplied by s_w, therefore, vanishing in the custodial conserving limit $s_w \to 0$.

4. $(D=2)^3$ terms

There are 12 operators in this class

$$\mathcal{T}_{45}^{p=6} = \text{Tr}[T\mathcal{D}_{\mu\nu}]\text{Tr}[T\mathcal{D}^{\mu\nu}]\text{Tr}[V^\alpha V_\alpha] \quad , \quad \mathcal{T}_{46}^{p=6} = \text{Tr}[T\mathcal{D}_{\mu\nu}]\text{Tr}[T\mathcal{D}^{\mu\alpha}]\text{Tr}[V^\nu V_\alpha] \quad ,$$

$$\mathcal{T}_{47}^{p=6} = \text{Tr}[T\mathcal{D}_\mu^\mu]\text{Tr}[T\mathcal{D}_\nu^\nu]\text{Tr}[V^\alpha V_\alpha] \quad , \quad \mathcal{T}_{48}^{p=6} = \text{Tr}[T\mathcal{D}_\mu^\mu]\text{Tr}[T\mathcal{D}^{\nu\alpha}]\text{Tr}[V_\nu V_\alpha] \quad ,$$

$$\mathcal{T}_{49}^{p=6} = -g^2\,\text{Tr}[T\widehat{W}_{\mu\nu}]\text{Tr}[T\widehat{W}^{\mu\nu}]\text{Tr}[V^\alpha V_\alpha] \quad , \quad \mathcal{T}_{50}^{p=6} = -g^2\,\text{Tr}[T\widehat{W}_{\mu\nu}]\text{Tr}[T\widehat{W}^{\mu\alpha}]\text{Tr}[V^\nu V_\alpha] \quad ,$$

$$\mathcal{T}_{51}^{p=6} = i\,g\,\text{Tr}[T\widehat{W}_{\mu\nu}]\text{Tr}[T\mathcal{D}^{\mu\alpha}]\text{Tr}[V^\nu V_\alpha] \quad , \quad \mathcal{T}_{52}^{p=6} = -g'^2\,B_{\mu\nu}B^{\mu\nu}\text{Tr}[V^\alpha V_\alpha] \quad ,$$

$$\mathcal{T}_{53}^{p=6} = -g'^2\,B_{\mu\nu}B^{\mu\alpha}\text{Tr}[V^\nu V_\alpha] \quad , \quad \mathcal{T}_{54}^{p=6} = i\,g'\,B_{\mu\nu}\text{Tr}[T\mathcal{D}^{\mu\alpha}]\text{Tr}[V^\nu V_\alpha] \quad ,$$

$$\mathcal{T}_{55}^{p=6} = -gg'\,B_{\mu\nu}\text{Tr}[T\widehat{W}^{\mu\nu}]\text{Tr}[V^\alpha V_\alpha] \quad , \quad \mathcal{T}_{56}^{p=6} = -gg'\,B_{\mu\nu}\text{Tr}[T\widehat{W}^{\mu\alpha}]\text{Tr}[V^\nu V_\alpha] \quad . \tag{46}$$

It is interesting to notice that the operators in this group generate QGC among all electroweak gauge bosons except for $W^+W^-W^+W^-$.

5. $(D=2)(D=4)$ terms

This class contains 7 operators

$$\mathcal{T}_{57}^{p=6} = \text{Tr}[\mathcal{D}_{\mu\nu}\mathcal{D}^{\mu\nu}]\text{Tr}[V^\alpha V_\alpha] \quad , \quad \mathcal{T}_{58}^{p=6} = \text{Tr}[\mathcal{D}_{\mu\nu}\mathcal{D}^{\mu\alpha}]\text{Tr}[V^\nu V_\alpha] \quad ,$$

$$\mathcal{T}_{59}^{p=6} = \text{Tr}[\mathcal{D}_\mu^\mu \mathcal{D}_\nu^\nu]\text{Tr}[V^\alpha V_\alpha] \quad , \quad \mathcal{T}_{60}^{p=6} = \text{Tr}[\mathcal{D}_\mu^\mu \mathcal{D}^{\nu\alpha}]\text{Tr}[V_\nu V_\alpha] \quad ,$$

$$\mathcal{T}_{61}^{p=6} = -g^2\,\text{Tr}[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}]\text{Tr}[V^\alpha V_\alpha] \quad , \quad \mathcal{T}_{62}^{p=6} = -g^2\,\text{Tr}[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\alpha}]\text{Tr}[V^\nu V_\alpha] \quad ,$$

$$\mathcal{T}_{63}^{p=6} = i\,g\,\text{Tr}[\widehat{W}_{\mu\nu}\mathcal{D}^{\mu\alpha}]\text{Tr}[V^\nu V_\alpha] \quad . \tag{47}$$

These operators are $SU(2)_c$ invariant in the limit $s_w \to 0$ and this can be seen by their expression in terms of Lorentz structures presented in Appendix C. This class of Lagrangians generate the following QGC: W^+W^-ZZ, $ZZZZ$, γZW^+W^-, $\gamma\gamma W^+W^-$, γZZZ, and $\gamma\gamma ZZ$.

6. $(D=1)^2(D=4)$ terms

We find 7 operators in this class that violate the custodial symmetry:

$$
\begin{aligned}
\mathcal{T}_{64}^{p=6} &= \mathrm{Tr}[\mathcal{D}_{\mu\nu}\mathcal{D}^{\mu\nu}]\mathrm{Tr}[TV^{\alpha}]\mathrm{Tr}[TV_{\alpha}] &,\quad \mathcal{T}_{65}^{p=6} &= \mathrm{Tr}[\mathcal{D}_{\mu\nu}\mathcal{D}^{\mu\alpha}]\mathrm{Tr}[TV^{\nu}]\mathrm{Tr}[TV_{\alpha}] &, \\
\mathcal{T}_{66}^{p=6} &= \mathrm{Tr}[\mathcal{D}_{\mu}^{\mu}\mathcal{D}_{\nu}^{\nu}]\mathrm{Tr}[TV^{\alpha}]\mathrm{Tr}[TV_{\alpha}] &,\quad \mathcal{T}_{67}^{p=6} &= \mathrm{Tr}[\mathcal{D}_{\mu}^{\mu}\mathcal{D}^{\nu\alpha}]\mathrm{Tr}[TV_{\nu}]\mathrm{Tr}[TV_{\alpha}] &, \\
\mathcal{T}_{68}^{p=6} &= -g^2\,\mathrm{Tr}[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}]\mathrm{Tr}[TV^{\alpha}]\mathrm{Tr}[TV_{\alpha}] &,\quad \mathcal{T}_{69}^{p=6} &= -g^2\,\mathrm{Tr}[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\alpha}]\mathrm{Tr}[TV^{\nu}]\mathrm{Tr}[TV_{\alpha}] &, \\
\mathcal{T}_{70}^{p=6} &= i\,g\,\mathrm{Tr}[\widehat{W}_{\mu\nu}\mathcal{D}^{\mu\alpha}]\mathrm{Tr}[TV^{\nu}]\mathrm{Tr}[TV_{\alpha}] &. &
\end{aligned}
\tag{48}
$$

As we can see in Appendix C, this group of effective Lagrangians give rise to W^+W^-ZZ, $ZZZZ$, γZZZ, and $\gamma\gamma ZZ$ QGC.

Altogether we find 70 independent operators leading to genuine QGC in the chiral Lagrangian at $\mathcal{O}(p^6)$, so there are as many operators as independent Lorentz structures containing two derivatives. As mentioned above, this was also the case at $\mathcal{O}(p^4)$. This is somehow not unexpected: as is well known [39], a generic $U(1)_{\rm em}$ invariant Lagrangian, which is the only symmetry imposed in building the Lorentz structures, is also invariant under nonlinear $SU(2)_L \otimes U(1)_Y$ transformations.

V. SUMMARY

In this work we have constructed the most general form of the QGC containing up to two derivatives acting on the electroweak gauge boson fields. We have shown that there are 5 independent Lorentz structures that respect the $U(1)_{em}$ symmetry and contain no derivatives while there are 70 structures exhibiting two derivatives.

We have then derived which of these QGC are generated assuming that the $SU(2)_L \otimes U(1)_Y$ gauge symmetry is linearly realized, as characteristic of scenarios with a fundamental Higgs doublet. In this case the lowest dimension that presents QGC without a TGC associated to them is eight. In this scenarios there are only three operators that contain only massive gauge bosons and no derivative acting on them; see Eq. (16). So due to the linear realization of the symmetry the Wilson coefficient of the five Lorentz structures that contain no derivatives are correlated – independently of the basis of operators used; see as example last line in Eq. (17). In the same framework, we find seven operators containing genuine QGC with two derivatives and they generate only 23 of the 70 possible Lorentz structures. So again, gauge invariance in the linear realization implies correlations among the coefficients of the different Lorentz structures, as for example those in Eq. (20).

We also classified the quartic gauge-boson interactions assuming that the $SU(2)_L \otimes U(1)_Y$ symmetry is realized nonlinearly with the global symmetry breaking $SU(2)_L \otimes SU(2)_R \to SU(2)_c$, characteristic of scenarios with a light dynamical Higgs boson. At order $\mathcal{O}(p^4)$ there are five chiral operators which generate QGC without an associated TQC. They contain only $W^\pm$ and Z and no derivatives. There are 70 independent operators at order $\mathcal{O}(p^6)$ and they contain four gauge bosons and two derivatives. This is, the chiral Lagrangian for genuine QGC contains the same number of operators as independent Lorentz structures. So no basis independent correlation can be derived between the coefficients of the Lorentz structures in this case.

At present the most sensitive searches for quartic gauge boson couplings are those involving vertices with two photons. Most of the analyses carried out by the LEP [28, 29], D0 [30] and LHC [24, 27] collaborations used the following effective Lagrangian to study the two-photon sector [32]

$$
\mathcal{L}_{\rm eff} = -\frac{\alpha_{em}\pi}{2}\frac{a_0^W}{\Lambda^2}\mathcal{Q}_{\gamma W,1}^{\partial=2} - \frac{\alpha_{em}\pi}{2}\frac{a_c^W}{\Lambda^2}\mathcal{Q}_{\gamma W,2}^{\partial=2} - \frac{\alpha_{em}\pi}{2c_w^2}\frac{a_0^Z}{\Lambda^2}\mathcal{Q}_{\gamma Z,1}^{\partial=2} - \frac{\alpha_{em}\pi}{2c_w^2}\frac{a_c^Z}{\Lambda^2}\mathcal{Q}_{\gamma Z,2}^{\partial=2} \, ,
\tag{49}
$$

where α_{em} stands for the electromagnetic fine-structure constant. In the framework of electroweak gauge invariance linearly realized, the seven operators in Eq. (18) give rise to QGC containing two photons which in the notation in Eq. (49) read:

$$
\begin{aligned}
a_0^W &= -\frac{M_W^2}{\pi\alpha_{em}}\left[s_w^2\frac{f_{M,0}}{\Lambda^2} + 2c_w^2\frac{f_{M,2}}{\Lambda^2} + s_w c_w\frac{f_{M,4}}{\Lambda^2}\right] &, \\
a_c^W &= -\frac{M_W^2}{\pi\alpha_{em}}\left[-s_w^2\frac{f_{M,1}}{\Lambda^2} - c_w^2\frac{f_{M,3}}{\Lambda^2} + 2s_w c_w\frac{f_{M,5}}{\Lambda^2} + \frac{s_w^2}{2}\frac{f_{M,7}}{\Lambda^2}\right] &, \\
a_0^Z &= -\frac{M_W^2 c_w^2}{\pi\alpha_{em}}\left[\frac{s_w^2}{2c_w^2}\frac{f_{M,0}}{\Lambda^2} + \frac{f_{M,2}}{\Lambda^2} - \frac{s_w}{2c_w}\frac{f_{M,4}}{\Lambda^2}\right] &, \\
a_c^Z &= -\frac{M_W^2 c_w^2}{\pi\alpha_{em}}\left[-\frac{s_w^2}{2c_w^2}\frac{f_{M,1}}{\Lambda^2} - \frac{1}{2}\frac{f_{M,3}}{\Lambda^2} - \frac{s_w}{c_w}\frac{f_{M,5}}{\Lambda^2} + \frac{s_w^2}{4c_w^2}\frac{f_{M,7}}{\Lambda^2}\right] &.
\end{aligned}
\tag{50}
$$

So even in the scenario with the linear realization of the gauge invariance, these four coefficients can be fully uncorrelated. Test of the presence/absence of the correlations which can point out towards an underlaying linear or chiral expansion will require the measurement with equivalent accuracy of quartic vertices involving one or zero photons, consequently requiring much more data.

At this point it is interesting to a have an idea of the LHC potential to constrain the genuine quartic couplings. For instance, Ref. [38] studied the vector boson fusion (VBF) production of W^+W^- and $W^\pm W^\pm$ pairs and obtained the prospective 95% C.L. limits on the p^4 QCG

$$-0.0045 < c_6^{p=4} < 0.0055 \quad \text{and} \quad -0.0022 < c_{11}^{p=4} < 0.0027 \tag{51}$$

for a center-of-mass energy of 14 TeV and an integrated luminosity of 300 fb^{-1}. Moreover, Ref. [31] analyzed some of the p^6 QGC containing photons ($c_{8,9,31,32,33,42,43,44,52,53,55,56,61,62}^{p=6}$) through the study of the VBF production of $\gamma\gamma$ and γe^+e^-. The typical 95% C.L. limits on the modulus of these couplings are in the range 1.2–6.3 TeV^{-2} for a center-of-mass energy of 14 TeV and an integrated luminosity of 100 fb^{-1}.

Appendix A: Relations between linear operators and Lorentz structures

In the framework where the SM gauge symmetry is realized linearly, the genuine quartic gauge couplings generated by the dimension-eight operators with two derivatives listed in Section III can be expressed in terms of the Lorentz structures defined in Section II as:

$$\mathcal{O}_{M,0} = \frac{g^2 v^2}{8} \left[2\mathcal{Q}_{WW,1}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{WZ,1}^{\partial=2} + c_w^2 \mathcal{Q}_{WZ,6}^{\partial=2} + 2s_w c_w \mathcal{Q}_{\gamma ZW,1}^{\partial=2} \right.$$
$$\left. + s_w^2 \mathcal{Q}_{\gamma W,1}^{\partial=2} + \frac{1}{2}\mathcal{Q}_{ZZ,1}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,1}^{\partial=2} + \frac{s_w^2}{2c_w^2}\mathcal{Q}_{\gamma Z,1}^{\partial=2} \right] \tag{A1}$$

$$\mathcal{O}_{M,1} = \frac{-g^2 v^2}{8} \left[\mathcal{Q}_{WW,2}^{\partial=2} + \mathcal{Q}_{WW,3}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{WZ,2}^{\partial=2} + c_w^2 \mathcal{Q}_{WZ,7}^{\partial=2} + s_w c_w \mathcal{Q}_{\gamma ZW,2}^{\partial=2} \right.$$
$$\left. + s_w^2 \mathcal{Q}_{\gamma W,2}^{\partial=2} + \frac{1}{2}\mathcal{Q}_{ZZ,2}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + \frac{s_w^2}{2c_w^2}\mathcal{Q}_{\gamma Z,2}^{\partial=2} \right] \tag{A2}$$

$$\mathcal{O}_{M,2} = \frac{g^2 v^2}{4} \left[c_w^2 \mathcal{Q}_{\gamma W,1}^{\partial=2} - 2s_w c_w \mathcal{Q}_{\gamma ZW,1}^{\partial=2} + s_w^2 \mathcal{Q}_{WZ,6}^{\partial=2} \right.$$
$$\left. + \frac{1}{2}\mathcal{Q}_{\gamma Z,1}^{\partial=2} - \frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,1}^{\partial=2} + \frac{s_w^2}{2c_w^2}\mathcal{Q}_{ZZ,1}^{\partial=2} \right] \tag{A3}$$

$$\mathcal{O}_{M,3} = \frac{-g^2 v^2}{8} \left[c_w^2 \mathcal{Q}_{\gamma W,2}^{\partial=2} - s_w c_w \mathcal{Q}_{\gamma ZW,2}^{\partial=2} + s_w^2 \mathcal{Q}_{WZ,7}^{\partial=2} \right.$$
$$\left. + \frac{1}{2}\mathcal{Q}_{\gamma Z,2}^{\partial=2} - \frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + \frac{s_w^2}{2c_w^2}\mathcal{Q}_{ZZ,2}^{\partial=2} \right] \tag{A4}$$

$$\mathcal{O}_{M,4} = \frac{g^2 v^2}{8} \left[(c_w^2 - s_w^2)\left(\mathcal{Q}_{\gamma ZW,1}^{\partial=2} - \frac{1}{2c_w^2}\mathcal{Q}_{\gamma ZZ,1}^{\partial=2} \right) - \mathcal{Q}_{\gamma ZW,3}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{WZ,3}^{\partial=2} \right.$$
$$\left. + s_w c_w \left(\mathcal{Q}_{\gamma W,1}^{\partial=2} - \mathcal{Q}_{WZ,6}^{\partial=2} - \frac{1}{2c_w^2}(\mathcal{Q}_{\gamma Z,1}^{\partial=2} - \mathcal{Q}_{ZZ,1}^{\partial=2}) \right) \right] \tag{A5}$$

$$\mathcal{O}_{M,5} = \frac{g^2 v^2}{8} \left[(c_w^2 - s_w^2)\left(\mathcal{Q}_{\gamma ZW,2}^{\partial=2} - \frac{1}{c_w^2}\mathcal{Q}_{\gamma ZZ,2}^{\partial=2} \right) \right.$$
$$\left. - \mathcal{Q}_{\gamma ZW,4}^{\partial=2} - \mathcal{Q}_{\gamma ZW,5}^{\partial=2} + \frac{s_w}{c_w}(\mathcal{Q}_{WZ,4}^{\partial=2} + \mathcal{Q}_{WZ,5}^{\partial=2}) \right.$$

$$+s_w c_w \left(2(\mathcal{Q}_{\gamma W,2}^{\partial=2} - \mathcal{Q}_{WZ,7}^{\partial=2}) - \frac{1}{c_w^2}(\mathcal{Q}_{\gamma Z,2}^{\partial=2} - \mathcal{Q}_{ZZ,2}^{\partial=2})\right)\bigg] \tag{A6}$$

$$\begin{aligned}
\mathcal{O}_{M,7} = \frac{g^2 v^2}{16} \bigg[& 2\mathcal{Q}_{WW,3}^{\partial=2} + c_w^2 \mathcal{Q}_{WZ,7}^{\partial=2} + s_w c_w \mathcal{Q}_{\gamma ZW,2}^{\partial=2} + s_w^2 \mathcal{Q}_{\gamma W,2}^{\partial=2} \\
& -(\mathcal{Q}_{WZ,5}^{\partial=2} - \mathcal{Q}_{WZ,4}^{\partial=2}) - \frac{s_w}{c_w}(\mathcal{Q}_{\gamma ZW,5}^{\partial=2} - \mathcal{Q}_{\gamma ZW,4}^{\partial=2}) \\
& +\frac{1}{2}\mathcal{Q}_{ZZ,2}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + \frac{s_w^2}{2c_w^2}\mathcal{Q}_{\gamma Z,2}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{WZ,2}^{\partial=2} \bigg]
\end{aligned} \tag{A7}$$

Appendix B: Dimension-eight operators containing four field strength tensors

There are 8 operators containing just field strength tensors that lead to genuine quartic anomalous couplings, which in the notation used in Ref. [21] are:

$$\begin{aligned}
\mathcal{O}_{T,0} &= \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\right] \times \mathrm{Tr}\left[\widehat{W}_{\alpha\beta}\widehat{W}^{\alpha\beta}\right] \quad, \quad \mathcal{O}_{T,1} = \mathrm{Tr}\left[\widehat{W}_{\alpha\nu}\widehat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\widehat{W}_{\mu\beta}\widehat{W}^{\alpha\nu}\right] \\
\mathcal{O}_{T,2} &= \mathrm{Tr}\left[\widehat{W}_{\alpha\mu}\widehat{W}^{\mu\beta}\right] \times \mathrm{Tr}\left[\widehat{W}_{\beta\nu}\widehat{W}^{\nu\alpha}\right] \quad, \quad \mathcal{O}_{T,5} = \mathrm{Tr}\left[\widehat{W}_{\mu\nu}\widehat{W}^{\mu\nu}\right] \times B_{\alpha\beta}B^{\alpha\beta} \\
\mathcal{O}_{T,6} &= \mathrm{Tr}\left[\widehat{W}_{\alpha\nu}\widehat{W}^{\mu\beta}\right] \times B_{\mu\beta}B^{\alpha\nu} \quad\quad, \quad \mathcal{O}_{T,7} = \mathrm{Tr}\left[\widehat{W}_{\alpha\mu}\widehat{W}^{\mu\beta}\right] \times B_{\beta\nu}B^{\nu\alpha} \\
\mathcal{O}_{T,8} &= B_{\mu\nu}B^{\mu\nu}B_{\alpha\beta}B^{\alpha\beta} \quad\quad\quad\quad\quad, \quad \mathcal{O}_{T,9} = B_{\alpha\mu}B^{\mu\beta}B_{\beta\nu}B^{\nu\alpha} .
\end{aligned} \tag{B1}$$

Appendix C: Relations between $\mathcal{O}(p^6)$ Operators and Lorentz Structures

Expanding the 70 $\mathcal{O}(p^6)$ operators introduced in Sec. IV in terms of the Lorentz structures in Sec. II B we find

$$\mathcal{T}_1^{p=6} = \frac{g^4}{c_w^4}\mathcal{Q}_{ZZ,3}^{\partial=2} \tag{C1}$$

$$\mathcal{T}_2^{p=6} = \frac{g^4}{c_w^4}\mathcal{Q}_{ZZ,4}^{\partial=2} \tag{C2}$$

$$\mathcal{T}_3^{p=6} = \frac{g^4}{c_w^4}\mathcal{Q}_{ZZ,5}^{\partial=2} \tag{C3}$$

$$\mathcal{T}_4^{p=6} = \frac{g^4}{c_w^4}\mathcal{Q}_{ZZ,6}^{\partial=2} \tag{C4}$$

$$\mathcal{T}_5^{p=6} = \frac{g^4}{c_w^2}\left[c_w^2\mathcal{Q}_{ZZ,1}^{\partial=2} + 2s_w c_w\mathcal{Q}_{\gamma ZZ,1}^{\partial=2} + s_w^2\mathcal{Q}_{\gamma Z,1}^{\partial=2}\right] \tag{C5}$$

$$\mathcal{T}_6^{p=6} = \frac{g^4}{c_w^2}\left[c_w^2\mathcal{Q}_{ZZ,2}^{\partial=2} + 2s_w c_w\mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + s_w^2\mathcal{Q}_{\gamma Z,2}^{\partial=2}\right] \tag{C6}$$

$$\mathcal{T}_7^{p=6} = \frac{g^4}{c_w^3}\left[c_w\mathcal{Q}_{ZZ,7}^{\partial=2} + s_w\mathcal{Q}_{\gamma ZZ,3}^{\partial=2}\right] \tag{C7}$$

$$\mathcal{T}_8^{p=6} = \frac{g^2 g'^2}{c_w^2}\left[s_w^2\mathcal{Q}_{ZZ,1}^{\partial=2} - 2s_w c_w\mathcal{Q}_{\gamma ZZ,1}^{\partial=2} + c_w^2\mathcal{Q}_{\gamma Z,1}^{\partial=2}\right] \tag{C8}$$

$$\mathcal{T}_9^{p=6} = \frac{g^2 g'^2}{c_w^2}\left[s_w^2\mathcal{Q}_{ZZ,2}^{\partial=2} - 2s_w c_w\mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + c_w^2\mathcal{Q}_{\gamma Z,2}^{\partial=2}\right] \tag{C9}$$

$$\mathcal{T}_{10}^{p=6} = \frac{g^3 g'}{c_w^3}\left[-s_w\mathcal{Q}_{ZZ,7}^{\partial=2} + c_w\mathcal{Q}_{\gamma ZZ,3}^{\partial=2}\right] \tag{C10}$$

$$\mathcal{T}_{11}^{p=6} = \frac{g^3 g'}{c_w^2}\left[c_w s_w\left(-\mathcal{Q}_{ZZ,1}^{\partial=2} + \mathcal{Q}_{\gamma Z,1}^{\partial=2}\right) + (c_w^2 - s_w^2)\mathcal{Q}_{\gamma ZZ,1}^{\partial=2}\right] \tag{C11}$$

$$\mathcal{T}_{12}^{p=6} = \frac{g^3 g'}{c_w^2}\left[c_w s_w\left(-\mathcal{Q}_{ZZ,2}^{\partial=2} + \mathcal{Q}_{\gamma Z,2}^{\partial=2}\right) + (c_w^2 - s_w^2)\mathcal{Q}_{\gamma ZZ,2}^{\partial=2}\right] \tag{C12}$$

$$\mathcal{T}_{13}^{p=6} = \frac{g^4}{2c_w^2}\left[\mathcal{Q}_{WZ,12}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,3}^{\partial=2}\right] \tag{C13}$$

$$\mathcal{T}_{14}^{p=6} = \frac{g^4}{2c_w^2}\left[\mathcal{Q}_{WZ,14}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,4}^{\partial=2}\right] \tag{C14}$$

$$\mathcal{T}_{15}^{p=6} = \frac{g^4}{2c_w^2}\left[\mathcal{Q}_{WZ,13}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,4}^{\partial=2}\right] \tag{C15}$$

$$\mathcal{T}_{16}^{p=6} = \frac{g^4}{2c_w^2}\left[\mathcal{Q}_{WZ,15}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,5}^{\partial=2}\right] \tag{C16}$$

$$\mathcal{T}_{17}^{p=6} = \frac{g^4}{2c_w^2}\left[\mathcal{Q}_{WZ,17}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,6}^{\partial=2}\right] \tag{C17}$$

$$\mathcal{T}_{18}^{p=6} = \frac{g^4}{2c_w^2}\left[\mathcal{Q}_{WZ,16}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,6}^{\partial=2}\right] \tag{C18}$$

$$\mathcal{T}_{19}^{p=6} = \frac{g^4}{2c_w^2}\left[\mathcal{Q}_{WZ,23}^{\partial=2} + \mathcal{Q}_{ZZ,7}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,3}^{\partial=2}\right] \tag{C19}$$

$$\mathcal{T}_{20}^{p=6} = \frac{g^4}{2c_w^2}\left[\mathcal{Q}_{WZ,24}^{\partial=2} + \mathcal{Q}_{ZZ,7}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,3}^{\partial=2}\right] \tag{C20}$$

$$\mathcal{T}_{21}^{p=6} = \frac{g^4}{2c_w^2}\mathcal{Q}_{WZ,25}^{\partial=2} \tag{C21}$$

$$\mathcal{T}_{22}^{p=6} = \frac{g^4}{2c_w}\left[c_w\left(\mathcal{Q}_{WZ,26}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,7}^{\partial=2}\right) + s_w\left(\mathcal{Q}_{\gamma ZW,7}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{\gamma ZZ,3}^{\partial=2}\right)\right] \tag{C22}$$

$$\mathcal{T}_{23}^{p=6} = \frac{g^4}{2c_w}\left[c_w\left(\mathcal{Q}_{WZ,27}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,7}^{\partial=2}\right) + s_w\left(\mathcal{Q}_{\gamma ZW,8}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{\gamma ZZ,3}^{\partial=2}\right)\right] \tag{C23}$$

$$\mathcal{T}_{24}^{p=6} = \frac{g^4}{2c_w}\left[c_w\mathcal{Q}_{WZ,28}^{\partial=2} + s_w\mathcal{Q}_{\gamma ZW,9}^{\partial=2}\right] \tag{C24}$$

$$\mathcal{T}_{25}^{p=6} = \frac{g^4}{2c_w}\left[c_w\left(\mathcal{Q}_{WZ,3}^{\partial=2} + \mathcal{Q}_{ZZ,1}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,1}^{\partial=2}\right) + s_w\left(\mathcal{Q}_{\gamma ZW,3}^{\partial=2} + \mathcal{Q}_{\gamma ZZ,1}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{\gamma Z,1}^{\partial=2}\right)\right] \tag{C25}$$

$$\mathcal{T}_{26}^{p=6} = \frac{g^4}{2c_w}\left[c_w\left(\mathcal{Q}_{WZ,4}^{\partial=2} + \mathcal{Q}_{ZZ,2}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,2}^{\partial=2}\right) + s_w\left(\mathcal{Q}_{\gamma ZW,4}^{\partial=2} + \mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{\gamma Z,2}^{\partial=2}\right)\right] \tag{C26}$$

$$\mathcal{T}_{27}^{p=6} = \frac{g^4}{2c_w}\left[c_w\left(\mathcal{Q}_{WZ,5}^{\partial=2} + \mathcal{Q}_{ZZ,2}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,2}^{\partial=2}\right) + s_w\left(\mathcal{Q}_{\gamma ZW,5}^{\partial=2} + \mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{\gamma Z,2}^{\partial=2}\right)\right] \tag{C27}$$

$$\mathcal{T}_{28}^{p=6} = \frac{g^3 g'}{2c_w}\left[-s_w\left(\mathcal{Q}_{WZ,26}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,7}^{\partial=2}\right) + c_w\left(\mathcal{Q}_{\gamma ZW,7}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{\gamma ZZ,3}^{\partial=2}\right)\right] \tag{C28}$$

$$\mathcal{T}_{29}^{p=6} = \frac{g^3 g'}{2c_w}\left[-s_w\left(\mathcal{Q}_{WZ,27}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,7}^{\partial=2}\right) + c_w\left(\mathcal{Q}_{\gamma ZW,8}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{\gamma ZZ,3}^{\partial=2}\right)\right] \tag{C29}$$

$$\mathcal{T}_{30}^{p=6} = \frac{g^3 g'}{2c_w}\left[-s_w\mathcal{Q}_{WZ,28}^{\partial=2} + c_w\mathcal{Q}_{\gamma ZW,9}^{\partial=2}\right] \tag{C30}$$

$$\mathcal{T}_{31}^{p=6} = \frac{g^3 g'}{2c_w}\left[-s_w\left(\mathcal{Q}_{WZ,3}^{\partial=2} + \mathcal{Q}_{ZZ,1}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,1}^{\partial=2}\right) + c_w\left(\mathcal{Q}_{\gamma ZW,3}^{\partial=2} + \mathcal{Q}_{\gamma ZZ,1}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{\gamma Z,1}^{\partial=2}\right)\right] \tag{C31}$$

$$\mathcal{T}_{32}^{p=6} = \frac{g^3 g'}{2c_w}\left[-s_w\left(\mathcal{Q}_{WZ,4}^{\partial=2} + \mathcal{Q}_{ZZ,2}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,2}^{\partial=2}\right) + c_w\left(\mathcal{Q}_{\gamma ZW,4}^{\partial=2} + \mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{\gamma Z,2}^{\partial=2}\right)\right] \tag{C32}$$

$$\mathcal{T}_{33}^{p=6} = \frac{g^3 g'}{2c_w}\left[-s_w\left(\mathcal{Q}_{WZ,5}^{\partial=2} + \mathcal{Q}_{ZZ,2}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,2}^{\partial=2}\right) + c_w\left(\mathcal{Q}_{\gamma ZW,5}^{\partial=2} + \mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{\gamma Z,2}^{\partial=2}\right)\right] \tag{C33}$$

$$\mathcal{T}_{34}^{p=6} = \frac{g^4}{4}\left[\mathcal{Q}_{WW,11}^{\partial=2} + 2\mathcal{Q}_{WW,6}^{\partial=2} + \frac{2}{c_w^2}\mathcal{Q}_{WZ,12}^{\partial=2} + \frac{1}{c_w^4}\mathcal{Q}_{ZZ,3}^{\partial=2}\right] \tag{C34}$$

$$\mathcal{T}_{35}^{p=6} = \frac{g^4}{4}\left[\mathcal{Q}_{WW,12}^{\partial=2} + \mathcal{Q}_{WW,7}^{\partial=2} + \mathcal{Q}_{WW,8}^{\partial=2} + \frac{2}{c_w^2}\mathcal{Q}_{WZ,14}^{\partial=2} + \frac{1}{c_w^4}\mathcal{Q}_{ZZ,4}^{\partial=2}\right] \tag{C35}$$

$$\mathcal{T}_{36}^{p=6} = \frac{g^4}{4}\left[\mathcal{Q}_{WW,12}^{\partial=2} + 2\mathcal{Q}_{WW,7}^{\partial=2} + \frac{2}{c_w^2}\mathcal{Q}_{WZ,13}^{\partial=2} + \frac{1}{c_w^4}\mathcal{Q}_{ZZ,4}^{\partial=2}\right] \tag{C36}$$

$$\mathcal{T}_{37}^{p=6} = \frac{g^4}{4}\left[\mathcal{Q}_{WW,13}^{\partial=2} + 2\mathcal{Q}_{WW,9}^{\partial=2} + \frac{2}{c_w^2}\mathcal{Q}_{WZ,15}^{\partial=2} + \frac{1}{c_w^4}\mathcal{Q}_{ZZ,5}^{\partial=2}\right] \tag{C37}$$

$$\mathcal{T}_{38}^{p=6} = \frac{g^4}{4}\left[\mathcal{Q}_{WW,14}^{\partial=2} + \mathcal{Q}_{WW,10}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{WZ,16}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{WZ,17}^{\partial=2} + \frac{1}{c_w^4}\mathcal{Q}_{ZZ,6}^{\partial=2}\right] \tag{C38}$$

$$\mathcal{T}_{39}^{p=6} = \frac{g^4}{4}\left[\mathcal{Q}_{WW,18}^{\partial=2} + \mathcal{Q}_{WW,16}^{\partial=2} + \mathcal{Q}_{WZ,26}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{WZ,23}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,7}^{\partial=2} + \frac{s_w}{c_w}\left(\mathcal{Q}_{\gamma ZW,7}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{\gamma ZZ,3}^{\partial=2}\right)\right] \tag{C39}$$

$$\mathcal{T}_{40}^{p=6} = \frac{g^4}{4}\left[\mathcal{Q}_{WW,18}^{\partial=2} + \mathcal{Q}_{WW,15}^{\partial=2} + \mathcal{Q}_{WZ,27}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{WZ,24}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,7}^{\partial=2} + \frac{s_w}{c_w}\left(\mathcal{Q}_{\gamma ZW,8}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{\gamma ZZ,3}^{\partial=2}\right)\right] \tag{C40}$$

$$\mathcal{T}_{41}^{p=6} = \frac{g^4}{4}\left[-\mathcal{Q}_{WW,17}^{\partial=2} - \mathcal{Q}_{WZ,28}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{WZ,25}^{\partial=2} - \frac{s_w}{c_w}\mathcal{Q}_{\gamma ZW,9}^{\partial=2}\right] \tag{C41}$$

$$\mathcal{T}_{42}^{p=6} = \frac{g^4}{4}\left[2\mathcal{Q}_{WW,1}^{\partial=2} + \mathcal{Q}_{WW,4}^{\partial=2} + 2\mathcal{Q}_{WZ,3}^{\partial=2} + \mathcal{Q}_{ZZ,1}^{\partial=2} + 2\frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,1}^{\partial=2} + 2\frac{s_w}{c_w}\mathcal{Q}_{\gamma ZW,3}^{\partial=2} + \frac{s_w^2}{c_w^2}\mathcal{Q}_{\gamma Z,1}^{\partial=2}\right] \tag{C42}$$

$$\mathcal{T}_{43}^{p=6} = \frac{g^4}{4}\left[\mathcal{Q}_{WW,5}^{\partial=2} + 2\mathcal{Q}_{WW,3}^{\partial=2} + 2\mathcal{Q}_{WZ,4}^{\partial=2} + \mathcal{Q}_{ZZ,2}^{\partial=2} + 2\frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + 2\frac{s_w}{c_w}\mathcal{Q}_{\gamma ZW,4}^{\partial=2} + \frac{s_w^2}{c_w^2}\mathcal{Q}_{\gamma Z,2}^{\partial=2}\right] \tag{C43}$$

$$\mathcal{T}_{44}^{p=6} = \frac{g^4}{4}\left[\mathcal{Q}_{WW,5}^{\partial=2} + 2\mathcal{Q}_{WW,2}^{\partial=2} + 2\mathcal{Q}_{WZ,5}^{\partial=2} + \mathcal{Q}_{ZZ,2}^{\partial=2} + 2\frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + 2\frac{s_w}{c_w}\mathcal{Q}_{\gamma ZW,5}^{\partial=2} + \frac{s_w^2}{c_w^2}\mathcal{Q}_{\gamma Z,2}^{\partial=2}\right] \tag{C44}$$

$$\mathcal{T}_{45}^{p=6} = \frac{g^4}{2c_w^2}\left[2\mathcal{Q}_{WZ,18}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,3}^{\partial=2}\right] \tag{C45}$$

$$\mathcal{T}_{46}^{p=6} = \frac{g^4}{2c_w^2}\left[2\mathcal{Q}_{WZ,19}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,4}^{\partial=2}\right] \tag{C46}$$

$$\mathcal{T}_{47}^{p=6} = \frac{g^4}{2c_w^2}\left[2\mathcal{Q}_{WZ,20}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,5}^{\partial=2}\right] \tag{C47}$$

$$\mathcal{T}_{48}^{p=6} = \frac{g^4}{2c_w^2}\left[2\mathcal{Q}_{WZ,21}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,6}^{\partial=2}\right] \tag{C48}$$

$$\mathcal{T}_{49}^{p=6} = \frac{g^4}{2}\left[2c_w^2\mathcal{Q}_{WZ,6}^{\partial=2} + 4s_wc_w\mathcal{Q}_{\gamma ZW,1}^{\partial=2} + 2s_w^2\mathcal{Q}_{\gamma W,1}^{\partial=2} + \mathcal{Q}_{ZZ,1}^{\partial=2} + 2\frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,1}^{\partial=2} + \frac{s_w^2}{c_w^2}\mathcal{Q}_{\gamma Z,1}^{\partial=2}\right] \tag{C49}$$

$$\mathcal{T}_{50}^{p=6} = \frac{g^4}{2}\left[2c_w^2\mathcal{Q}_{WZ,7}^{\partial=2} + 2s_wc_w\mathcal{Q}_{\gamma ZW,2}^{\partial=2} + 2s_w^2\mathcal{Q}_{\gamma W,2}^{\partial=2} + \mathcal{Q}_{ZZ,2}^{\partial=2} + 2\frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + \frac{s_w^2}{c_w^2}\mathcal{Q}_{\gamma Z,2}^{\partial=2}\right] \tag{C50}$$

$$\mathcal{T}_{51}^{p=6} = \frac{g^4}{2c_w}\left[c_w\mathcal{Q}_{WZ,29}^{\partial=2} + \frac{1}{c_w}\mathcal{Q}_{ZZ,7}^{\partial=2} + s_w\mathcal{Q}_{\gamma ZW,6}^{\partial=2} + \frac{s_w}{c_w^2}\mathcal{Q}_{\gamma ZZ,3}^{\partial=2}\right] \tag{C51}$$

$$\mathcal{T}_{52}^{p=6} = \frac{g^2g'^2}{2}\left[2s_w^2\mathcal{Q}_{WZ,6}^{\partial=2} - 4s_wc_w\mathcal{Q}_{\gamma ZW,1}^{\partial=2} + 2c_w^2\mathcal{Q}_{\gamma W,1}^{\partial=2} + \frac{s_w^2}{c_w^2}\mathcal{Q}_{ZZ,1}^{\partial=2} - 2\frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,1}^{\partial=2} + \mathcal{Q}_{\gamma Z,1}^{\partial=2}\right] \tag{C52}$$

$$\mathcal{T}_{53}^{p=6} = \frac{g^2g'^2}{2}\left[2s_w^2\mathcal{Q}_{WZ,7}^{\partial=2} - 2s_wc_w\mathcal{Q}_{\gamma ZW,2}^{\partial=2} + 2c_w^2\mathcal{Q}_{\gamma W,2}^{\partial=2} + \frac{s_w^2}{c_w^2}\mathcal{Q}_{ZZ,2}^{\partial=2} - 2\frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + \mathcal{Q}_{\gamma Z,2}^{\partial=2}\right] \tag{C53}$$

$$\mathcal{T}_{54}^{p=6} = \frac{g^3g'}{2c_w}\left[-s_w\mathcal{Q}_{WZ,29}^{\partial=2} - \frac{s_w}{c_w^2}\mathcal{Q}_{ZZ,7}^{\partial=2} + c_w\mathcal{Q}_{\gamma ZW,6}^{\partial=2} + \frac{1}{c_w}\mathcal{Q}_{\gamma ZZ,3}^{\partial=2}\right] \tag{C54}$$

$$\mathcal{T}_{55}^{p=6} = \frac{g^3g'}{2}\left[2(c_w^2 - s_w^2)\mathcal{Q}_{\gamma ZW,1}^{\partial=2} + 2s_wc_w\left(\mathcal{Q}_{\gamma W,1}^{\partial=2} - \mathcal{Q}_{WZ,6}^{\partial=2}\right) + \frac{(c_w^2 - s_w^2)}{c_w^2}\mathcal{Q}_{\gamma ZZ,1}^{\partial=2} + \frac{s_w}{c_w}\left(\mathcal{Q}_{\gamma Z,1}^{\partial=2} - \mathcal{Q}_{ZZ,1}^{\partial=2}\right)\right] \tag{C55}$$

$$\mathcal{T}_{56}^{p=6} = \frac{g^3g'}{2}\left[(c_w^2 - s_w^2)\mathcal{Q}_{\gamma ZW,2}^{\partial=2} + 2s_wc_w\left(\mathcal{Q}_{\gamma W,2}^{\partial=2} - \mathcal{Q}_{WZ,7}^{\partial=2}\right) + \frac{(c_w^2 - s_w^2)}{c_w^2}\mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + \frac{s_w}{c_w}\left(\mathcal{Q}_{\gamma Z,2}^{\partial=2} - \mathcal{Q}_{ZZ,2}^{\partial=2}\right)\right] \tag{C56}$$

$$\mathcal{T}_{57}^{p=6} = \frac{g^4}{4}\left[4\mathcal{Q}_{WW,6}^{\partial=2} + \frac{2}{c_w^2}\mathcal{Q}_{WZ,8}^{\partial=2} + \frac{2}{c_w^2}\mathcal{Q}_{WZ,18}^{\partial=2} + \frac{1}{c_w^4}\mathcal{Q}_{ZZ,3}^{\partial=2}\right] \tag{C57}$$

$$\mathcal{T}_{58}^{p=6} = \frac{g^4}{4}\left[2\mathcal{Q}_{WW,7}^{\partial=2} + 2\mathcal{Q}_{WW,8}^{\partial=2} + \frac{2}{c_w^2}\mathcal{Q}_{WZ,9}^{\partial=2} + \frac{2}{c_w^2}\mathcal{Q}_{WZ,19}^{\partial=2} + \frac{1}{c_w^4}\mathcal{Q}_{ZZ,4}^{\partial=2}\right] \tag{C58}$$

$$\mathcal{T}_{59}^{p=6} = \frac{g^4}{4}\left[4\mathcal{Q}_{WW,9}^{\partial=2} + \frac{2}{c_w^2}\mathcal{Q}_{WZ,10}^{\partial=2} + \frac{2}{c_w^2}\mathcal{Q}_{WZ,20}^{\partial=2} + \frac{1}{c_w^4}\mathcal{Q}_{ZZ,5}^{\partial=2}\right] \tag{C59}$$

$$\mathcal{T}_{60}^{p=6} = \frac{g^4}{4}\left[2\mathcal{Q}_{WW,10}^{\partial=2} + \frac{2}{c_w^2}\mathcal{Q}_{WZ,11}^{\partial=2} + \frac{2}{c_w^2}\mathcal{Q}_{WZ,21}^{\partial=2} + \frac{1}{c_w^4}\mathcal{Q}_{ZZ,6}^{\partial=2}\right] \tag{C60}$$

$$\mathcal{T}_{61}^{p=6} = \frac{g^4}{4}\left[4\mathcal{Q}_{WW,1}^{\partial=2} + 2c_w^2\mathcal{Q}_{WZ,6}^{\partial=2} + 4s_w c_w\mathcal{Q}_{\gamma ZW,1}^{\partial=2} + 2s_w^2\mathcal{Q}_{\gamma W,1}^{\partial=2}\right.$$
$$\left. + \frac{2}{c_w^2}\mathcal{Q}_{WZ,1}^{\partial=2} + \mathcal{Q}_{ZZ,1}^{\partial=2} + \frac{2s_w}{c_w}\mathcal{Q}_{\gamma ZZ,1}^{\partial=2} + \frac{s_w^2}{c_w^2}\mathcal{Q}_{\gamma Z,1}^{\partial=2}\right] \tag{C61}$$

$$\mathcal{T}_{62}^{p=6} = \frac{g^4}{4}\left[2\mathcal{Q}_{WW,2}^{\partial=2} + 2\mathcal{Q}_{WW,3}^{\partial=2} + 2c_w^2\mathcal{Q}_{WZ,7}^{\partial=2} + 2s_w c_w\mathcal{Q}_{\gamma ZW,2}^{\partial=2} + 2s_w^2\mathcal{Q}_{\gamma W,2}^{\partial=2}\right.$$
$$\left. + \frac{2}{c_w^2}\mathcal{Q}_{WZ,2}^{\partial=2} + \mathcal{Q}_{ZZ,2}^{\partial=2} + \frac{2s_w}{c_w}\mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + \frac{s_w^2}{c_w^2}\mathcal{Q}_{\gamma Z,2}^{\partial=2}\right] \tag{C62}$$

$$\mathcal{T}_{63}^{p=6} = \frac{g^4}{4}\left[\mathcal{Q}_{WW,15}^{\partial=2} + \mathcal{Q}_{WW,16}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{WZ,22}^{\partial=2} + \mathcal{Q}_{WZ,29}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,7}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{\gamma ZW,6}^{\partial=2} + \frac{s_w}{c_w^3}\mathcal{Q}_{\gamma ZZ,3}^{\partial=2}\right] \tag{C63}$$

$$\mathcal{T}_{64}^{p=6} = \frac{g^4}{2c_w^2}\left[2\mathcal{Q}_{WZ,8}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,3}^{\partial=2}\right] \tag{C64}$$

$$\mathcal{T}_{65}^{p=6} = \frac{g^4}{2c_w^2}\left[2\mathcal{Q}_{WZ,9}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,4}^{\partial=2}\right] \tag{C65}$$

$$\mathcal{T}_{66}^{p=6} = \frac{g^4}{2c_w^2}\left[2\mathcal{Q}_{WZ,10}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,5}^{\partial=2}\right] \tag{C66}$$

$$\mathcal{T}_{67}^{p=6} = \frac{g^4}{2c_w^2}\left[\mathcal{Q}_{WZ,11}^{\partial=2} + \frac{1}{c_w^2}\mathcal{Q}_{ZZ,6}^{\partial=2}\right] \tag{C67}$$

$$\mathcal{T}_{68}^{p=6} = \frac{g^4}{2c_w^2}\left[2\mathcal{Q}_{WZ,1}^{\partial=2} + c_w^2\mathcal{Q}_{ZZ,1}^{\partial=2} + 2s_w c_w\mathcal{Q}_{\gamma ZZ,1}^{\partial=2} + s_w^2\mathcal{Q}_{\gamma Z,1}^{\partial=2}\right] \tag{C68}$$

$$\mathcal{T}_{69}^{p=6} = \frac{g^4}{2c_w^2}\left[2\mathcal{Q}_{WZ,2}^{\partial=2} + c_w^2\mathcal{Q}_{ZZ,2}^{\partial=2} + 2s_w c_w\mathcal{Q}_{\gamma ZZ,2}^{\partial=2} + s_w^2\mathcal{Q}_{\gamma Z,2}^{\partial=2}\right] \tag{C69}$$

$$\mathcal{T}_{70}^{p=6} = \frac{g^4}{2c_w^2}\left[\mathcal{Q}_{WZ,22}^{\partial=2} + \mathcal{Q}_{ZZ,7}^{\partial=2} + \frac{s_w}{c_w}\mathcal{Q}_{\gamma ZZ,3}^{\partial=2}\right] \tag{C70}$$

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