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Is there really a $W \to \tau\nu$ puzzle?

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According to the Particle Data Group, the measurements of $\mathcal{B}(W^+ \to \tau^+\nu_\tau)$ and $\mathcal{B}(W^+ \to \ell^+\nu_\ell)$ ($\ell = e, \mu$) disagree with one another at the 2.3σ level. In this paper, we search for a new-physics (NP) explanation of this $W \to \tau\nu$ puzzle. We consider two NP scenarios: (i) the W mixes with a W' boson that couples preferentially to the third generation, (ii) $\tau_{L,R}$ and $\nu_{\tau L}$ mix with isospin-triplet leptons. Unfortunately, once other experimental constraints are taken into account, neither scenario can explain the above experimental result. Our conclusion is that the $W \to \tau\nu$ puzzle is almost certainly just a statistical fluctuation.

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I. INTRODUCTION

At present, there are a few measurements that are in potential disagreement with the predictions of the standard model (SM) of particle physics. One hint of lepton non-universality involves the leptonic decays of the W. According to the Particle Data Group [1], we have

$$\begin{aligned} \mathcal{B}(W^+ \to e^+\nu_e) &= (10.71 \pm 0.16)\% \ , \\ \mathcal{B}(W^+ \to \mu^+\nu_\mu) &= (10.63 \pm 0.15)\% \ , \\ \mathcal{B}(W^+ \to \tau^+\nu_\tau) &= (11.38 \pm 0.21)\% \ , \end{aligned} \tag{1}$$

yielding

$$\frac{2\mathcal{B}(W^+ \to \tau^+\nu_\tau)}{\mathcal{B}(W^+ \to e^+\nu_e) + \mathcal{B}(W^+ \to \mu^+\nu_\mu)} = 1.067 \pm 0.029 \ . \tag{2}$$

The SM prediction for this ratio is 0.999 to a very good approximation, so there is a difference at the level of 2.3σ. We refer to this as the "$W \to \tau\nu$ puzzle." Of course, this could simply be a statistical fluctuation. But could it in fact be due to the presence of new physics (NP)?

In the past, the only theoretical studies that attempted to directly address the $W \to \tau\nu$ puzzle involved models with two Higgs doublets. Specifically, it was suggested that the excess in $W \to \tau\nu$ events is due to contamination by a light charged Higgs, with mass m_W, decaying via $H^+ \to \tau^+\nu_\tau$ [2]. However, recently the data of the four LEP collaborations was combined and a search for pair-produced charged Higgs bosons was performed [3]. No significant excess of $\tau^+\nu_\tau$ final states was observed compared to the SM background, so that a lower limit can be set on the mass of the charged Higgs as a function of the $H^+ \to \tau^+\nu_\tau$ branching ratio. While the LEP study does not completely rule out two-Higgs-doublet models with the most general couplings, it does severely restrict the available parameter space [4].

An alternative explanation of the puzzle is that the W-τ-ν_τ coupling is itself increased. This possibility was considered in Ref. [6] using an effective field theory (EFT) approach. Here the NP effects are encapsulated by including higher-dimensional operators, each with its own arbitrary Wilson coefficient. The authors study the effect of different flavor symmetries; they conclude that it is difficult to resolve the W-τ-ν_τ puzzle in this framework, mainly due to the constraints arising from Z and τ decays. However, we note that this analysis is not the most general – the question of neutrino masses has not been considered. In the EFT, neutrino mass operators arise at dimension five. The authors of Ref. [6] write, "The only gauge-invariant operator of dimension five violates lepton number, and thus it can be safely neglected under the assumption that the violation of that symmetry occurs at scales much higher than

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$\Lambda \sim 1$ TeV." But this assumption is not necessarily true. Indeed, in the present paper we consider several models giving rise to lepton-number violation at a scale of $O(1)$ TeV (see Sec. III), and generating non-zero neutrino masses. These are correlated with the contribution to the W-τ-ν_τ coupling. The connection between neutrino masses and the W-τ-ν_τ coupling is ignored in the EFT approach, but is taken into account here.

A larger W-τ-ν_τ coupling can also improve some other discrepancies with the SM. Below we discuss several other measurements that are sensitive to the W-τ-ν_τ coupling. It should be noted that, while Eq. (1) is a direct measurement of the W-τ-ν_τ coupling, these other measurements are indirect probes of this coupling, and there may be other new-physics contributions to these decays [5].

Consider first the decay $B^+ \to \tau^+\nu_\tau$. Its branching ratio is [1]

$$\mathcal{B}(B^+ \to \tau^+\nu_\tau) = (1.14 \pm 0.27) \times 10^{-4} \ , \tag{3}$$

while the SM prediction is [7]

$$\begin{aligned}\mathcal{B}(B^+ \to \tau^+\nu_\tau) &= \tau_{B^+} G_F^2 m_\tau^2 f_B^2 |V_{ub}|^2 \frac{m_B}{8\pi}\left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 \\ &= (0.81 \pm 0.08) \times 10^{-4} \ .\end{aligned} \tag{4}$$

Here the FLAG average $f_B = (190.5 \pm 4.2)$ MeV [8] and the CKMfitter result $|V_{ub}| = (3.55 \pm 0.16) \times 10^{-3}$ [9] have been used. From the above numbers, we see that there is a small (1.5σ) disagreement between the measurement and the SM prediction. It is stressed in Ref. [7] that the size of the disagreement depends on the value taken for $|V_{ub}|$, and there is a long-standing discrepancy between the determinations of $|V_{ub}|$ from inclusive $B \to X_u \ell^+\nu$ and exclusive $\bar{B} \to M\ell\bar{\nu}$ decays [1]. Indeed, if the inclusive value for $|V_{ub}|$ is used, the disagreement disappears. Still, if the SM prediction of Eq. (4) holds, the agreement with experiment can be improved if the W-τ-ν_τ coupling is increased.

Another example, similar to the above process, involves $D_s^+ \to \tau^+\nu_\tau$ and $D_s^+ \to \ell^+\nu_\ell$ ($\ell = e, \mu$) decays. Experimentally, it is found that [1]

$$R_{D_s} \equiv \frac{\mathcal{B}(D_s^+ \to \tau^+\nu_\tau)}{\mathcal{B}(D_s^+ \to \ell^+\nu_\ell)} = 10.0 \pm 0.6 \ . \tag{5}$$

In the SM, this ratio is predicted to be

$$R_{D_s} = \frac{m_\tau^2}{m_\mu^2} \frac{(1 - m_\tau^2/m_{D_s^+}^2)^2}{(1 - m_\mu^2/m_{D_s^+}^2)^2} = 9.742 \pm 0.013 \ . \tag{6}$$

Due to the large experimental error on R_{D_s}, there is no discrepancy with the SM, but, at the 3σ level, a 10% increase in the W-τ-ν_τ coupling is allowed. This measurement is thus consistent with that of Eq. (1).

τ decays would obviously be affected by a change in the W-τ-ν_τ coupling. Consider first $\tau^- \to e^-\nu_\tau\bar{\nu}_e$. Here the SM predicts [1]

$$R_\tau \equiv \frac{\mathcal{B}(\tau^- \to e^-\nu_\tau\bar{\nu}_e)}{\mathcal{B}(\mu^- \to e^-\nu_\mu\bar{\nu}_e)} = \frac{\tau_\tau}{\tau_\mu}\left(\frac{m_\tau}{m_\mu}\right)^5 \ = \ (17.77 \pm 0.03)\% \ , \tag{7}$$

where τ_i represents the mean lifetime of particle i. The experimental value for the above ratio is

$$R_\tau \approx (17.83 \pm 0.04)\% \ , \tag{8}$$

assuming the branching ratio for the μ decay is $\approx 100\%$ [1]. This τ decay channel therefore allows very little (less than a percent) change in the W-τ-ν_τ coupling.

A second decay is $\tau^- \to \pi^-\nu_\tau$. In the SM, the branching ratio for this decay can be expressed as [10]

$$\begin{aligned}\mathcal{B}(\tau^- \to \pi^-\nu_\tau) &= \frac{G_F^2|V_{ud}|^2}{16\pi} f_\pi^2 \tau_\tau m_\tau^3 \left(1 - \frac{m_\pi^2}{m_\tau^2}\right)^2 \delta_{\tau/\pi} \\ &= (10.67 \pm 0.23)\% \ .\end{aligned} \tag{9}$$

Here $\delta_{\tau/\pi}$ represents the small radiative corrections to the decay rate; it is known very well: $\delta_{\tau/\pi} = 1.0016 \pm 0.0014$ [11]. Above we have used the FLAG average $f_\pi = (130.2 \pm 1.4)$ MeV [8] and the CKMfitter result $|V_{ud}| = (0.97425 \pm 0.00022)$

[9]. (The biggest source of the $\sim 2\%$ error in the predicted branching ratio is the lattice value for f_π which has a $\sim 1\%$ error.) The prediction in Eq. (9) should be compared with the measured value [1]

$$\mathcal{B}(\tau^- \to \pi^- \nu_\tau) = (10.91 \pm 0.07)\% \ . \tag{10}$$

In this case the predicted value has a larger error than the measured value and they are consistent with each other. Still, if we allow for a 3σ (upward) deviation from the measured value and trust the predicted central value, we find that a 2% increase in the W-τ-ν_τ coupling is allowed. Also, it must be remembered that this decay can be affected by other new-physics contributions [12].

Finally, there are the charged-current decays $\bar{B} \to D^{(*)+}\ell^-\bar{\nu}_\ell$, which have been measured by the BaBar [13], Belle [14] and LHCb [15] Collaborations. It is found that the values of the ratios $\mathcal{B}(\bar{B} \to D^{(*)+}\tau^-\bar{\nu}_\tau)/\mathcal{B}(\bar{B} \to D^{(*)+}\ell^-\bar{\nu}_\ell)$ ($\ell = e, \mu$) considerably exceed their SM predictions. The experimental results and theoretical predictions can be combined to yield [16]

$$\begin{aligned} R_D &\equiv \frac{\mathcal{B}(\bar{B} \to D^+\tau^-\bar{\nu}_\tau)_{\rm exp}/\mathcal{B}(\bar{B} \to D^+\tau^-\bar{\nu}_\tau)_{\rm SM}}{\mathcal{B}(\bar{B} \to D^+\ell^-\bar{\nu}_\ell)_{\rm exp}/\mathcal{B}(\bar{B} \to D^+\ell^-\bar{\nu}_\ell)_{\rm SM}} = 1.37 \pm 0.18 \ , \\ R_{D^*} &\equiv \frac{\mathcal{B}(\bar{B} \to D^{*+}\tau^-\bar{\nu}_\tau)_{\rm exp}/\mathcal{B}(\bar{B} \to D^{*+}\tau^-\bar{\nu}_\tau)_{\rm SM}}{\mathcal{B}(\bar{B} \to D^{*+}\ell^-\bar{\nu}_\ell)_{\rm exp}/\mathcal{B}(\bar{B} \to D^{*+}\ell^-\bar{\nu}_\ell)_{\rm SM}} = 1.28 \pm 0.08 \ . \end{aligned} \tag{11}$$

The measured values of R_D and R_{D^*} represent deviations from the SM of 2.0σ and 3.8σ, respectively. This is the $R_{D^{(*)}}$ puzzle. In this case, the discrepancies with the SM are too large to be explained entirely by an increase in the W-τ-ν_τ coupling. Even so, such an increase would lead to larger theoretical predictions for $\mathcal{B}(\bar{B} \to D^{(*)+}\tau^-\bar{\nu}_\tau)$, which would reduce the disagreement with experiment.

We therefore see that an increased W-τ-ν_τ coupling can explain the $W \to \tau\nu$ puzzle, and can also improve other discrepancies with the SM. (It must also be conceded that not all measurements support the idea of an increased coupling.) The purpose of this paper is to attempt to find a NP model in which the W-τ-ν_τ coupling can be made larger.

To this end we consider two NP possibilities. In the first, we assume that a W' boson exists that couples preferentially to the third generation. The mixing of this W' with the SM W could then lead to an increased W-τ-ν_τ coupling. In the second, we allow the $\tau_{L,R}$ and $\nu_{\tau L}$ to mix with isospin-triplet leptons. Once again, this mixing could generate a larger W-τ-ν_τ coupling. Unfortunately, as we will see, once constraints from other measurements are taken into account, neither NP scenario can reproduce the measured W-τ-ν_τ coupling. Because of the difficulty in finding a reasonably simple NP explanation, we are forced to conclude that the $W \to \tau\nu$ puzzle is probably just a statistical fluctuation.

We begin in Sec. II with an evaluation of the potential for W-W' mixing to lead to an increased W-τ-ν_τ coupling once all experimental constraints are taken into account. This analysis is repeated in Sec. III for the mixing of the $\tau_{L,R}$ and $\nu_{\tau L}$ with isospin-triplet leptons. (The details of the formalism of this mixing are given in the Appendix.) We conclude in Sec. IV.

II. *W*-*W*′ MIXING

There has been another recent hint of lepton non-universality. The LHCb Collaboration measured the ratio of decay rates for $B^+ \to K^+\ell^+\ell^-$ ($\ell = e, \mu$) in the dilepton invariant mass-squared range 1 GeV$^2 \le q^2 \le 6$ GeV^2 [17], and found

$$\begin{aligned} R_K &\equiv \frac{\mathcal{B}(B^+ \to K^+\mu^+\mu^-)}{\mathcal{B}(B^+ \to K^+e^+e^-)} \\ &= 0.745^{+0.090}_{-0.074}\ ({\rm stat}) \pm 0.036\ ({\rm syst}) \ . \end{aligned} \tag{12}$$

This differs from the SM prediction of $R_K = 1 \pm O(10^{-4})$ [18] by 2.6σ. A NP explanation of this R_K puzzle was offered in Ref. [19]. Here the NP is assumed to couple preferentially to the third generation, giving rise to the operator[1]

$$G(\bar{b}'_L\gamma_\mu b'_L)(\bar{\tau}'_L\gamma^\mu\tau'_L) \ , \tag{13}$$

[1] The $(V-A)\times(V-A)$ form of this operator follows the analysis of Ref. [20]. There it is found that the only NP operator that can reproduce the experimental value of R_K is $(\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu P_L \ell)$. This is consistent with the NP explanations for the $B \to K^{(*)}\mu^+\mu^-$ angular distributions measured by LHCb [21].

where $G = O(1)/\Lambda_{NP}^2 \ll G_F$, and the primed fields are the fermion eigenstates in the gauge basis. When one transforms to the mass basis, this generates the operator $(\bar{b}_L \gamma_\mu s_L)(\bar{\mu}_L \gamma^\mu \mu_L)$ that contributes to $\bar{b} \to \bar{s}\mu^+\mu^-$. (There is also a contribution to $\bar{b} \to \bar{s}e^+e^-$, but it is much smaller.)

In Ref. [22], it was pointed out that, assuming the scale of NP is much larger than the weak scale, the operator of Eq. (13) should be made invariant under the full $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge group. One way to do this is to write the NP operator as

$$\begin{aligned}\mathcal{O}_{NP} &= G_2(\bar{Q}'_L \gamma_\mu \sigma^I Q'_L)(\bar{L}'_L \gamma^\mu \sigma^I L'_L) \\ &= G_2 \left[2(\bar{Q}'^i_L \gamma_\mu Q'^j_L)(\bar{L}'^j_L \gamma^\mu L'^i_L) - (\bar{Q}'_L \gamma_\mu Q'_L)(\bar{L}'_L \gamma^\mu L'_L) \right] ,\end{aligned} \tag{14}$$

where G_2 is $O(1)/\Lambda_{NP}^2$. Here $Q' \equiv (t', b')^T$ and $L' \equiv (\nu'_\tau, \tau')^T$. The key point is that $\mathcal{O}_{NP}$ contains both neutral-current (NC) and charged-current (CC) interactions. The NC and CC pieces can be used to respectively explain the R_K and $R_{D^{(*)}}$ puzzles. One NP model that contains the above operator involves vector leptoquarks [23]. Another assumes the addition of a set of massive vector bosons that transform as an $SU(2)_L$ triplet, and that are coupled to both quark and lepton currents [16]. It is this second NP model that is of interest for the $W \to \tau\nu$ puzzle.

In Ref. [24], a formalism was presented for adding to the SM a real spin-1 isospin-triplet V_μ^a $(a = 1, 2, 3)$ with vanishing hypercharge. It describes heavy vector particles, one charged (W') and one neutral (Z'), that couple to the SM left-handed fermionic currents. This was adapted in Ref. [16] to the specific case where the V couples principally to the third-generation fermions. The simplified Lagrangian is given by

$$\mathcal{L}_V = -\frac{1}{4} D_{[\mu} V^a_{\nu]} D^{[\mu} V^{\nu]a} + \frac{1}{2} m_V^2 V^a_\mu V^{\mu a} + i g_H V^a_\mu (H^\dagger T^a \overset{\leftrightarrow}{D}{}^\mu H) + V^a_\mu J^{\mu a} ,$$

where $T^a = \sigma^a/2$, $D_{[\mu} V^a_{\nu]} \equiv D_\mu V^a_\nu - D_\nu V^a_\mu$ with $D_\mu V^a_\nu = \partial_\mu V^a_\nu + g \epsilon^{abc} W^b_\mu V^c_\nu$, and

$$J^{\mu a} = g_q \lambda^q_{ij} \left(\bar{Q}'^i_L \gamma^\mu T^a Q'^j_L \right) + g_l \lambda^l_{ij} \left(\bar{L}'^i_L \gamma^\mu T^a L'^j_L \right) . \tag{15}$$

Here $\lambda^{q,l}_{ij}$ are Hermitian flavor matrices and $\lambda^q_{33} = \lambda^l_{33} = 1$. In Ref. [16] it was shown that tree-level Z' and W' exchange can respectively explain the R_K and $R_{D^{(*)}}$ puzzles.

It is emphasized in Ref. [24] that the V^a_μ fields in Eq. (15) are not the mass eigenstates as they mix with the W^a_μ after electroweak symmetry breaking. In particular, the physical W mass eigenstate is

$$(W^\pm)_{phys} = W^\pm \cos\theta_C + V^\pm \sin\theta_C , \tag{16}$$

where θ_C is the charged-current mixing angle. Naively, this angle could be as large as $O(m_W/m_V)$, which equals 0.08 for $m_V = 1$ TeV. In the presence of such mixing, the W-τ-ν_τ coupling is given by

$$g(\cos\theta_C + (g_l/g)\lambda^l_{33} \sin\theta_C) . \tag{17}$$

The experimental measurement of the W-τ-ν_τ coupling could therefore be reproduced if the expression in parentheses equals 1.033. Given that $\lambda^l_{33} = 1$, this could happen if, for example,

$$g_l = g \quad , \qquad \theta_C = 0.034 . \tag{18}$$

On the face of things, this appears to be possible. However, constraints from the neutral-current sector must be taken into account. In the presence of mixing, the physical Z mass eigenstate is given by

$$(Z^0)_{phys} = Z^0 \cos\theta_N + V^0 \sin\theta_N . \tag{19}$$

The key point [24] is that, for small mixing angles,

$$\theta_C \simeq \frac{M_W}{M_Z} \theta_N . \tag{20}$$

Thus, constraints on θ_N lead directly to constraints on θ_C. And θ_N can be bounded by the data on Z decays. For example, consider $Z \to \tau^+\tau^-$. The $Z \to \ell^+\ell^-$ data are [1]

$$\begin{aligned}\mathcal{B}(Z \to e^+e^-) &= (3.363 \pm 0.004)\% , \\ \mathcal{B}(Z \to \mu^+\mu^-) &= (3.366 \pm 0.007)\% , \\ \mathcal{B}(Z \to \tau^+\tau^-) &= (3.370 \pm 0.008)\% ,\end{aligned} \tag{21}$$

leading to

$$\frac{2\mathcal{B}(Z \to \tau^+\tau^-)}{\mathcal{B}(Z \to e^+e^-) + \mathcal{B}(Z \to \mu^+\mu^-)} = 1.0016 \pm 0.0027 \ . \tag{22}$$

Theoretically, we have

$$\frac{2\mathcal{B}(Z \to \tau^+\tau^-)}{\mathcal{B}(Z \to e^+e^-) + \mathcal{B}(Z \to \mu^+\mu^-)} = \frac{(a^Z_{\tau_L})^2 + (a^Z_{\tau_R})^2}{(a^Z_{\ell_L})^2 + (a^Z_{\ell_R})^2} \ , \tag{23}$$

where $a^Z_f = I_{3L} - Q_{em} \sin^2\theta_W$ is the $Zf\bar{f}$ coupling. In the SM, the $Z\ell^+\ell^-$ couplings are given by

$$a^Z_{\ell_L} = -\frac{1}{2} + \sin^2\theta_W \ , \quad a^Z_{\ell_R} = \sin^2\theta_W \ . \tag{24}$$

In the presence of Z^0-V^0 mixing, the coupling of the Z^0 to left-handed τ's is modified:

$$a^Z_{\tau_L} = \left(-\frac{1}{2} + \sin^2\theta_W\right)\cos\theta_N + (g_l/g)\cos\theta_W \lambda^l_{33} \sin\theta_N \ . \tag{25}$$

($a^Z_{\tau_R}$ is unchanged from the SM.) Taking $\lambda^l_{33} = 1$, $\sin^2\theta_W = 0.231$, and $g_l = g$, this yields

$$-0.0026 \le \theta_N \le 0.0017 \ \ (3\sigma) \ . \tag{26}$$

This corresponds to the constraint $\theta_C < 0.0015$, which rules out the solution of Eq. (18).

We note in passing that a similar result can be found by considering $Z \to \nu_\tau\bar{\nu}_\tau$ decays. In the SM,

$$\frac{\mathcal{B}(Z \to \nu_e\bar{\nu}_e)}{\mathcal{B}(Z \to e^+e^-)} = \frac{\left(\frac{1}{2}\right)^2}{\left(-\frac{1}{2} + \sin^2\theta_W\right)^2 + \left(\sin^2\theta_W\right)^2} \ , \tag{27}$$

so that, using Eq. (21),

$$\mathcal{B}(Z \to \nu_e\bar{\nu}_e) = (6.687 \pm 0.008)\% \ . \tag{28}$$

The SM therefore predicts that

$$\mathcal{B}(Z \to \text{invisible}) = 3\mathcal{B}(Z \to \nu_e\bar{\nu}_e) = (20.062 \pm 0.024)\% \ . \tag{29}$$

Experimentally, we have [1]

$$\mathcal{B}(Z \to \text{invisible}) = (20.0 \pm 0.06)\% \ . \tag{30}$$

As above, $\mathcal{B}(Z \to f\bar{f})$ is proportional to $(a^Z_{f_L})^2 + (a^Z_{f_R})^2$. In the SM, the $Z\nu_\ell\bar{\nu}_\ell$ couplings are given by

$$a^Z_{\nu_{\ell L}} = \frac{1}{2} \ , \quad a^Z_{\nu_{\ell R}} = 0 \ . \tag{31}$$

In the presence of Z^0-V^0 mixing, we have

$$a^Z_{\nu_{\tau L}} = \frac{1}{2}\cos\theta_N + (g_l/g)\cos\theta_W \lambda^l_{33} \sin\theta_N \quad , \qquad a^Z_{\nu_{\ell R}} = 0 \ , \tag{32}$$

with

$$\begin{aligned} \frac{\mathcal{B}(Z \to \text{invisible})_{NP}}{\mathcal{B}(Z \to \text{invisible})_{\text{SM}}} &= \frac{(a^Z_{\nu_{\tau L}})^2 + 2\left(\frac{1}{2}\right)^2}{3\left(\frac{1}{2}\right)^2} \\ &= \frac{20.0 \pm 0.06}{20.061 \pm 0.014} = 0.997 \pm .003 \ . \end{aligned} \tag{33}$$

Taking $\lambda^l_{33} = 1$, $\sin^2\theta_W = 0.231$, and $g_l = g$, we obtain

$$-0.0093 \le \theta_N \le 0.0046 \ \ (3\sigma) \ . \tag{34}$$

This corresponds to the constraint $\theta_C < 0.004$. This is weaker than that from $Z \to \tau^+\tau^-$, but it still rules out the solution of Eq. (18).

We therefore conclude that the $W \to \tau\nu$ puzzle cannot be explained by W-W' mixing.

III. MIXING WITH ISOSPIN-TRIPLET LEPTONS

In this section we consider the mixing of $\tau_{L,R}$ and $\nu_{\tau L}$ with isospin-triplet leptons. Such exotic leptons were examined in Ref. [25], and were allowed to mix with all three flavors of SM leptons. This then generates flavor-changing neutral-current processes (FCNCs) such as $\mu \to e\gamma$, $\tau \to \mu\mu\mu$, etc. Ref. [25] focused specifically on FCNCs, as well as on the phenomenology of the exotic leptons.

In the present paper, the isospin-triplet leptons are allowed to mix with only one flavor of SM leptons, τ and ν_τ, so that FCNCs are not generated. Thus, only flavor-conserving processes (such as $W \to \tau\nu$) are affected. Now, if the new leptons with which τ_L and $\nu_{\tau L}$ mix were singlets under $SU(2)_L$, the W-τ-ν coupling would be reduced (by the cosine of the mixing angle) [26]. However, as we show below, if the exotic leptons are isospin triplets, this coupling can be increased.

We consider two types of isospin triplets:

$$L_{L,R} \equiv \begin{pmatrix} L^+ \\ L^0 \\ L^- \end{pmatrix}_{L,R} \quad , \quad L'_{L,R} \equiv \begin{pmatrix} L'^0 \\ L'^- \\ L'^{--} \end{pmatrix}_{L,R} \quad . \tag{35}$$

L has hypercharge $Y = 0$ and is Majorana; L' has hypercharge $Y = -2$ and is Dirac. Both are vector fermions, i.e., their L and R chiralities are both isospin triplets. As shown in the Appendix [Eq. (60)], since $L^{(\prime)}$ is an isotriplet, the charged-current interactions between $L^{(\prime)0}$ and $L^{(\prime)-}$ take the form

$$g \left[\bar{L}^{(\prime)0} \gamma^\mu W_\mu^+ L^{(\prime)-} + \bar{L}^{(\prime)-} \gamma^\mu W_\mu^- L^{(\prime)0} \right] \ . \tag{36}$$

Compare this to the SM charged-current interaction terms:

$$\frac{1}{\sqrt{2}} g \left[\bar{\nu}_\tau \gamma^\mu W_\mu^+ \gamma_L \tau^- + \bar{\tau}^- \gamma^\mu W_\mu^- \gamma_L \nu_\tau \right] \ . \tag{37}$$

It is the different coefficients – $1/\sqrt{2}$ for isospin doublets, 1 for isospin triplets – that has the potential to produce an increased W-τ-ν coupling.

The basic idea is as follows. The SM fermions are

$$E_L \equiv \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \quad , \quad \tau_R^- \ . \tag{38}$$

Both L_L and L'_L have components with $Q_{em} = -1$ ($L_L^{(\prime)-}$) and $Q_{em} = 0$ ($L_L^{(\prime)0}$). Suppose the τ_L^- and $\nu_{\tau L}$ mix with these. We then have

$$(\tau_L^-)_{phys} = \tau_L^- \cos\theta_L^\tau + L_L^{(\prime)-} \sin\theta_L^\tau \ , \tag{39}$$

$$(\nu_{\tau L})_{phys} = \nu_{\tau L} \cos\theta_L^\nu + L_L^{(\prime)0} \sin\theta_L^\nu \ . \tag{40}$$

In the presence of mixing, the charged current between the physical τ_L^- and $\nu_{\tau L}$ has two pieces: τ_L^--$\nu_{\tau L}$ (isospin doublet) and $L_L^{(\prime)-}$-$L_L^{(\prime)0}$ (isospin triplet). The strength of the W-τ-ν coupling therefore changes:

$$\left.\frac{1}{\sqrt{2}}\right|_{\rm SM} \to \frac{1}{\sqrt{2}} (\cos\theta_L^\tau \cos\theta_L^\nu + \sqrt{2} \sin\theta_L^\tau \sin\theta_L^\nu) \ . \tag{41}$$

As was the case with W-W' mixing, the experimental measurement of the W-τ-ν_τ coupling could be reproduced if the expression in parentheses equals 1.033. This could happen if, for example,

$$\theta_L^\tau = \theta_L^\nu \quad , \quad \sin\theta_L^\tau = 0.28 \ . \tag{42}$$

One immediate question is: Theoretically, can such large mixing be obtained? In the case of mixing with isotriplet leptons, the answer is yes. Because both E_L and the SM Higgs are doublets under $SU(2)_L$, and because $\mathbf{2} \otimes \mathbf{2} = \mathbf{1} \oplus \mathbf{3}$, one can write dimension-four operators that involve E_L, H and $L^{(\prime)}$. When the Higgs acquires a vacuum expectation value (VEV) $v/\sqrt{2}$, it generates a mass term mixing E_L and $L^{(\prime)}$. This mass term m is naturally of $O(v)$. Assuming the exotic leptons have masses $M \simeq O(1 \text{ TeV})$, the mixing angle will be $O(m/M)$. This is in the right ballpark of the above angle.

On the other hand, this would not work if $\tau_{L,R}$ and $\nu_{\tau L}$ mix with exotic leptons of higher isospin. In this case, higher-dimension operators involving more than one Higgs field are required. These are suppressed by powers of the NP mass scale, and so the mixing angles will be correspondingly reduced.

We therefore see that the mixing of $\tau_{L,R}$ and $\nu_{\tau L}$ with exotic isotriplet leptons has the potential to explain the $W \to \tau\nu$ puzzle. But this raises further questions. Is such mixing consistent with other experimental constraints? If not, are there any mixing scenarios, even fine-tuned, in which this can be made to work? To investigate these questions, we consider four different models involving the mixing of $\tau_{L,R}$ and $\nu_{\tau L}$ with isotriplet leptons:

(i) mixing with L' alone,
(ii) mixing with L alone,
(iii) mixing with L and L',
(iv) mixing with L, L' and $\nu_{\tau R}$.

For a given model to pass all the experimental tests, it must (1) give the correct value of m_ν, (2) reproduce the measured value of the W-τ-ν_τ coupling, (3) satisfy the constraints from $Z \to \tau^+\tau^-$ and $Z \to \nu_\tau\bar{\nu}_\tau$. Regarding test (1), we know that ν_τ has a tiny mass, and there are neutrino oscillations. However, as can be seen in the Appendix, the nonzero entries in the neutrino mass matrices are all $O(v)$ or $O(1 \text{ TeV})$. As such, there is no way of implementing a seesaw mechanism, which requires a mass term of $O(10^{15} \text{ GeV})$. For this reason we require only that a model contain a massless neutrino in order to pass test (1). If a possible solution to the $W \to \tau\nu$ puzzle is found, we can then try to explain neutrino masses and oscillations by allowing all three neutrinos to mix and incorporating some sort of seesaw mechanism[2]. However, for now we are content to focus on massless neutrinos.

Models (i)-(iv) are analyzed in detail in the Appendix. In all cases, we first examine the mass matrix of the neutral leptons, to see if the model passes test (1), i.e., if it predicts a tiny mass or $m = 0$ for ν_τ. We find that models (ii) and (iii) fail this test. However, models (i) and (iv) do contain ν_τ with $m = 0$. For these models, we express $(\nu_{\tau L})_{phys}$ and $(\tau^-_{L,R})_{phys}$ in terms of the gauge eigenstates. This allows us to move on to tests (2) and (3).

Consider first model (i). Here the τ^-_L mixes with L'^-_L and the $\nu_{\tau L}$ mixes with L'^0_L as in Eqs. (39) and (40). However, according to Eqs. (67) and (68) in the Appendix, the mixing angles obey

$$\sin\theta^\tau_L \simeq \frac{m'_2}{\sqrt{2}M'} \quad , \quad \sin\theta^\nu_L \simeq -\frac{m'_2}{M'} \ . \tag{43}$$

That is, they are of opposite sign[3]. Since the correction to the W-τ-ν_τ coupling is proportional to $\sin\theta^\tau_L \sin\theta^\nu_L$, mixing actually has the effect of *decreasing* the coupling. Model (i) thus fails test (2).

For completeness, how does model (i) fare with test (3)? First, consider $Z \to \tau^+\tau^-$ [see Eqs. (21)-(24)]. In the presence of mixing, the I_{3L} of the physical τ^-_L is

$$\langle(\tau^-_L)_{phys}|T_3|(\tau^-_L)_{phys}\rangle = -\frac{1}{2}\cos^2\theta^\tau_L + 0\cdot\sin^2\theta^\tau_L = -\frac{1}{2}\left(1-\sin^2\theta^\tau_L\right) \ . \tag{44}$$

(Even with mixing, $(\tau^-_R)_{phys}$ still has $I_{3L} = 0$, as in the SM.) This implies [see Eqs. (22)-(24)]

$$\frac{2\mathcal{B}(Z\to\tau^+\tau^-)}{\mathcal{B}(Z\to e^+e^-)+\mathcal{B}(Z\to\mu^+\mu^-)} = \frac{(-\frac{1}{2}\left(1-\sin^2\theta^\tau_L\right)+\sin^2\theta_W)^2+(\sin^2\theta_W)^2}{(-\frac{1}{2}+\sin^2\theta_W)^2+(\sin^2\theta_W)^2} = 1.0016 \pm 0.0027 \ , \tag{45}$$

leading to

$$|\sin\theta^\tau_L| \le 0.055 \ \ (3\sigma) \ . \tag{46}$$

Second, consider $Z \to \nu_\tau\bar{\nu}_\tau$ [see Eqs. (27)-(33)]. In the presence of mixing, the I_{3L} of the physical $\nu_{\tau L}$ is

$$\langle(\nu_{\tau L})_{phys}|T_3|(\nu_{\tau L})_{phys}\rangle = \frac{1}{2}\cos^2\theta^\nu_L + 1\cdot\sin^2\theta^\nu_L = \frac{1}{2}\left(1+\sin^2\theta^\nu_L\right) \ . \tag{47}$$

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[2] In principle, the W-τ-ν_τ puzzle could be connected to the induced non-unitarity of the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix that describes the mixing between massive neutrino species. In Ref. [27], a global fit to precision data from varying energy ranges was performed in the Minimal Unitarity Violation scheme. It was shown that the data does indeed prefer a small amount of non-unitarity in leptonic mixing. However, the best-fit point was found to slightly worsen the W-τ-ν_τ puzzle. In the present work, we ignore neutrino mixing, and hence do not consider the complications arising from a non-unitary PMNS matrix.

[3] This sign difference is due to the opposite signs of the m'_2 entries in the neutral- and charged-lepton mass matrices [Eqs. (65) and (66)]. And this is in turn due to fact that the 12 and 22 elements of the 2×2 representation of L' [Eq. (64)], which contribute to the mass matrices, are of opposite sign.

This modified I_{3L} will affect the $Z\nu_\tau\bar{\nu}_\tau$ coupling, so that [see Eq. (33)]

$$\frac{\mathcal{B}(Z \to \text{invisible})_{NP}}{\mathcal{B}(Z \to \text{invisible})_{\text{SM}}} = \frac{\left(\frac{1}{2}(1+\sin^2\theta_L^\nu)\right)^2 + 2\left(\frac{1}{2}\right)^2}{3\left(\frac{1}{2}\right)^2} = 0.997 \pm .003 \ . \tag{48}$$

This implies that

$$|\sin\theta_L^\nu| \le 0.099 \ \ (3\sigma) \ . \tag{49}$$

Combining Eqs. (46) and (49), we have

$$\cos\theta_L^\tau \cos\theta_L^\nu + \sqrt{2}\sin\theta_L^\tau \sin\theta_L^\nu < 1.002 \ . \tag{50}$$

Following Eq. (41), it was noted that this quantity should equal 1.033 to explain $W \to \tau\nu$ puzzle. This is clearly not satisfied by the above equation. Thus, even if the sign difference of Eq. (43) had not been present, model (i) could not have passed test (2). Conversely, taking values for the mixing angles large enough to explain $W \to \tau\nu$ puzzle would have resulted in failing test (3). The bottom line is that model (i) does not pass the experimental tests.

We now turn to model (iv). Here the mixing is much more complicated. The expressions for $(\nu_{\tau L})_{phys}$ and $(\tau^-_{L,R})_{phys}$ are given in Eqs. (72) and (75), and are repeated for convenience below:

$$\begin{aligned} (\tau_L^-)_{phys} &= a_{L\tau}\tau_L^- + c_{L\tau}L_L^- + d_{L\tau}L_L'^- \ , \\ (\nu_{\tau L})_{phys} &= a_\nu \nu_{\tau L} + b_\nu \nu_{\tau R}^c + c_\nu L_R^{0\ c} + d_\nu L_L'^0 + e_\nu L_R'^{0c} \ . \end{aligned} \tag{51}$$

Expressions for the coefficients are given in Eqs. (73) and (77), in terms of the various mass parameters that appear in the relevant mixing matrices. In order to obtain a massless ν_τ, we require

$$\frac{m_D^2}{m_S^2} = \eta \frac{m_2^2}{2M^2} \ , \tag{52}$$

where $\eta = -M/m_S$.

We begin by considering effects of this mixing on the neutral-current sector. In the presence of mixing, the I_{3L} of the physical τ_L^- is

$$\begin{aligned} \langle(\tau_L^-)_{phys}|T_3|(\tau_L^-)_{phys}\rangle &= -\frac{1}{2} + \frac{1}{2}(1-(a_L^\tau)^2) - (c_L^\tau)^2 \ , \\ &\approx -\frac{1}{2}\left[1 + \frac{m_2^2}{M^2} - \frac{{m_2'}^2}{2M'^2}\right] \ , \end{aligned} \tag{53}$$

while that of the physical $\nu_{\tau L}$ is

$$\begin{aligned} \langle(\nu_{\tau L})_{phys}|T_3|(\nu_{\tau L})_{phys}\rangle &= \frac{1}{2}\left[(a_L^\nu)^2 + 2\left((d_L^\nu)^2 - (e_L^\nu)^2\right)\right] \\ &\approx \frac{1}{2}\left[1 - (1+\eta)\frac{m_2^2}{2M^2} + \frac{{m_2'}^2}{M'^2}\right] \ . \end{aligned} \tag{54}$$

In both cases above, we have expanded the expressions for the coefficients, neglecting m_τ and keeping terms to leading order in the mixing parameters m_2^2/M^2 and $m_2'^2/M'^2$. Now, we saw in the study of model (i) that the constraints from the decays $Z \to \tau^+\tau^-$ and $Z \to \nu_\tau\bar{\nu}_\tau$ are quite severe. To evade these constraints, the mass parameters in model (iv) must be such that the values of I_{3L} of both $(\tau_L^-)_{phys}$ and $(\nu_{\tau L})_{phys}$ are unchanged from their SM values, i.e., $I_{3L} = -1/2$ $((\tau_L^-)_{phys})$ and $I_{3L} = 1/2$ $((\nu_{\tau L})_{phys})$. Eqs. (53) and (54) then imply that $2m_2^2/M^2 = m_2'^2/M'^2$ and $\eta = 3$.

Now, the W-τ-ν_τ coupling in this model is proportional to

$$\begin{aligned} K &= a_{L\tau}a_\nu + \sqrt{2}\,(c_{L\tau}c_\nu + d_{L\tau}d_\nu) \\ &= \frac{1 + \dfrac{m_2^2}{M^2 - m_\tau^2} - \dfrac{{m_2'}^2}{{M'}^2 - m_\tau^2}}{\sqrt{\left(1 + \dfrac{m_D^2}{m_S^2} + \dfrac{m_2^2}{2M^2} + \dfrac{{m_2'}^2}{M'^2}\right)\left(1 + \dfrac{m_2^2 M^2}{(M^2 - m_\tau^2)^2} + \dfrac{{m_2'}^2 {M'}^2}{2({M'}^2 - m_\tau^2)^2}\right)}} \ , \end{aligned} \tag{55}$$

$$\approx 1 - \frac{7{m_2'}^2}{4{M'}^2} + \frac{(1-\eta)}{4}\frac{m_2^2}{M^2} \ , \tag{56}$$

where we have once again neglected m_τ and kept only the leading-order terms in m_2^2/M^2 and $m_2'^2/M'^2$ in the expansion. Above, it was noted that $\eta = 3$ is required to evade the constraints from $Z \to \tau^+\tau^-$ and $Z \to \nu_\tau\bar{\nu}_\tau$. However, if $\eta \geq 1$, we have $K < 1$, so that, as was the case with model (i), mixing has the effect of reducing the W-τ-ν_τ coupling. Thus, although the model passes test (3), it now fails test (2). We therefore conclude that the $W \to \tau\nu$ puzzle cannot be explained by allowing $\tau_{L,R}$ and $\nu_{\tau L}$ to mix with isospin-triplet leptons.

IV. CONCLUSIONS

At present, there are several measurements of B decays that exhibit discrepancies with the predictions of the SM at the level of 2σ or greater. These hints of new physics have been taken very seriously – there has been a flurry of theoretical activity looking for NP explanations of the various B-decay results. Another decay that has a similar disagreement with the SM is $W \to \tau\nu$. According to the Particle Data Group, there is a 2.3σ disagreement between $\mathcal{B}(W^+ \to \tau^+\nu_\tau)$ and $\mathcal{B}(W^+ \to \ell^+\nu_\ell)$ ($\ell = e, \mu$). However, for some reason – perhaps because $\tau^- \to e^-\nu_\tau\bar{\nu}_e$ does not exhibit a similar discrepancy with the SM – little attention has been paid to this result. In the present paper, we search for a NP explanation of the $W \to \tau\nu$ puzzle.

The obvious conclusion to be drawn from the experimental measurement is that the W-τ-ν_τ coupling has been increased due to the presence of NP. Because the process is rather simple – an on-shell W decaying to $\tau\nu$ – there are only two possible ways NP can enter. Either the W mixes with a W', or the τ_L and $\nu_{\tau L}$ mix with exotic leptons. We consider both possibilities.

First, we assume that a W' boson exists that couples preferentially to the third generation. W-W' mixing could then lead to an increased W-τ-ν_τ coupling. The problem is that such a W' also comes with a neutral partner, a Z', that mixes with the SM Z. Now, the amount of W-W' and Z-Z' mixing are related. And Z-Z' mixing is strongly constrained by the experimental measurements of $Z \to \tau^+\tau^-$ and $Z \to \nu_\tau\bar{\nu}_\tau$. The upshot is that, when the constraints from Z decays are taken into account, the allowed W-W' mixing is too small to produce the necessary increase in the W-τ-ν_τ coupling.

Second, we allow $\tau_{L,R}$ and $\nu_{\tau L}$ to mix with isospin-triplet leptons. Such mixing can potentially lead to an increased W-τ-ν_τ coupling. We take two isospin-triplet leptons, one with hypercharge $Y = 0$, the other with $Y = -2$, and consider a variety of mixing scenarios. For a given scenario to succeed, it must (1) give the correct value of m_{ν_τ} ($m = 0$ is allowed), (2) reproduce the measured value of the W-τ-ν_τ coupling, and (3) satisfy the constraints from $Z \to \tau^+\tau^-$ and $Z \to \nu_\tau\bar{\nu}_\tau$. Unfortunately, all the mixing scenarios fail at least one of these tests.

Because we are unable to find a NP explanation of the $W \to \tau\nu$ puzzle, we are forced to conclude that it is almost certainly just a statistical fluctuation.

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APPENDIX

We consider two types of isospin triplets:

$$L_{L,R} \equiv \begin{pmatrix} L^+ \\ L^0 \\ L^- \end{pmatrix}_{L,R} \quad , \quad L'_{L,R} \equiv \begin{pmatrix} L'^0 \\ L'^- \\ L'^{--} \end{pmatrix}_{L,R} . \tag{57}$$

L and L' have hypercharge $Y = 0$ and $Y = -2$, respectively. Both are vector fermions, i.e., their L and R chiralities are both isospin triplets. For this reason, they may have direct mass terms. However, L is Majorana, while L' is Dirac, which means that the forms of the mass terms and mass matrices are different. The kinetic and direct mass terms for L and L' are

$$\begin{aligned} \mathcal{L}_{\rm kin} &= \frac{1}{2}\bar{L}_a i\gamma^\mu D_\mu^{ab} L_b - \frac{M}{2}(\bar{L}_a^c L_a + \bar{L}_a L_a^c) \ , \\ \mathcal{L}'_{\rm kin} &= \bar{L}'_a i\gamma^\mu D_\mu^{ab} L'_b - M'\bar{L}'_a L'_a \ , \end{aligned} \tag{58}$$

with

$$D_\mu = \partial_\mu - i\frac{g}{\sqrt{2}}(W_\mu^+ T^+ + W_\mu^- T^-) - i\frac{g}{\cos\theta_W} Z_\mu (T^3 - Q_{em}\sin^2\theta_W) - ieA_\mu Q_{em} \ . \tag{59}$$

In the above, $a, b = 1, 2, 3$ are isospin indices. Although the covariant derivative D_μ itself is representation independent, the form of the SU(2) generators $(T^\pm, T_3)$ depends on whether the fermion is an isodoublet or an isotriplet. In particular, the charged-current interactions between the isotriplets $L^{(\prime)0}$ and $L^{(\prime)-}$ are

$$g\left[\bar{L}^{(\prime)0}\gamma^\mu W_\mu^+ L^{(\prime)-} + \bar{L}^{(\prime)-}\gamma^\mu W_\mu^- L^{(\prime)0}\right] \ . \tag{60}$$

We examine models in which combinations of the following particles mix:

$$E_L \equiv \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \quad , \quad \tau_R^- \quad , \quad \nu_{\tau R} \quad , \quad L_{L,R} \quad , \quad L'_{L,R} \ . \tag{61}$$

The direct mass terms for L and L' are shown above [Eq. (58)]; the mass terms for E_L, τ_R and $\nu_{\tau R}$ are given by

$$-\lambda_1 \bar{E}_L H \tau_R - \lambda_D \bar{E}_L \tilde{H}^\dagger \nu_{\tau R} - \frac{m_S}{2}\nu_{\tau R}^{\bar{c}}\nu_{\tau R} + \text{h.c.} \ . \tag{62}$$

Here $H = \begin{pmatrix}\phi^+ & \phi^0\end{pmatrix}^T$ and $\tilde{H} = \begin{pmatrix}\phi^0 & -\phi^+\end{pmatrix}$. When the Higgs acquires a VEV $v/\sqrt{2}$, the τ and ν_τ obtain Dirac masses $m_1 \equiv \lambda_1 v/\sqrt{2}$ and $m_D \equiv \lambda_D v/\sqrt{2}$, respectively. The $\nu_{\tau R}$ also has a Majorana mass $m_S/2$. m_D is $O(v)$, while the size of m_S is unspecified. Mixing between E_L and $L_R^{(\prime)}$ is generated by the Yukawa terms

$$-\lambda_2 \bar{E}_L L_R \tilde{H}^\dagger - \lambda'_2 \bar{E}_L L'_R H + \text{h.c.} \ . \tag{63}$$

The mixing terms $m_2 \equiv \lambda_2 v/2$ and $m'_2 \equiv \lambda'_2 v/2$ are both $O(v)$. In the above equation, L and L' are expressed as 2×2 matrices:

$$L = \frac{1}{2}\begin{pmatrix} L^0 & \sqrt{2}L^+ \\ \sqrt{2}L^- & -L^0 \end{pmatrix} \quad , \quad L' = \frac{1}{2}\begin{pmatrix} L'^- & \sqrt{2}L'^0 \\ \sqrt{2}L'^{--} & -L'^- \end{pmatrix} \ . \tag{64}$$

Below we examine four different models: $\tau_{L,R}$ and $\nu_{\tau L}$ mixing with (i) L' alone, (ii) L alone, (iii) L and L', (iv) L, L' and $\nu_{\tau R}$. If a given model does not contain a neutrino with a tiny mass ($m = 0$ is accepted), it is excluded from further consideration. If it passes this test, we find the eigenvectors corresponding to $(\tau_{L,R})_{phys}$ and $(\nu_{\tau L})_{phys}$.

1. $\tau_{L,R}$ and $\nu_{\tau L}$ mixing with L' alone

For the neutral leptons the mass terms translate to

$$-\begin{pmatrix} \bar{\nu}_\tau & \bar{L}'^0 \end{pmatrix}_L \mathcal{M}_{\nu L'} \begin{pmatrix} \nu_{\tau L}^c \\ L_R'^0 \end{pmatrix} + \text{h.c.} \ , \quad \text{with} \quad \mathcal{M}_{\nu L'} = \begin{pmatrix} 0 & m'_2 \\ 0 & M' \end{pmatrix} \ , \tag{65}$$

while for the charged leptons the mass terms take the form

$$-\begin{pmatrix} \bar{\tau}^- & \bar{L}'^- \end{pmatrix}_L \mathcal{M}_{\tau L'} \begin{pmatrix} \tau^- \\ L'^- \end{pmatrix}_R + \text{h.c.} \ , \quad \text{with} \quad \mathcal{M}_{\tau L'} = \begin{pmatrix} m_1 & -m'_2/\sqrt{2} \\ 0 & M' \end{pmatrix} \ . \tag{66}$$

Recall that m'_2 and M' are, respectively, $O(v)$ and $O(1\ \text{TeV})$. Note that, because of gauge invariance, there is no mass term relating $L_L'^-$ and τ_R^-.

The states in Eqs. (65) and (66) are defined in the gauge basis. To transform to the mass basis one applies the unitary transformations U_L and U_R on the left-handed and right-handed states, respectively. U_L (U_R) diagonalizes $\mathcal{M}\mathcal{M}^\dagger$ $(\mathcal{M}^\dagger\mathcal{M})$. In the present case, since the mass matrices are real, the transformation matrices are orthogonal, O_L and O_R. The diagonalization of the mass matrices yields the mass eigenvalues and the decomposition of the mass eigenstates in terms of gauge eigenstates.

For the neutral leptons, this procedure is rather simple. The mass eigenvalues are $m = 0$ and $m = \sqrt{{m'_2}^2 + M'^2}$. The eigenstate that has $m = 0$ is given by Eq. (40), with

$$\sin\theta_L^\nu = -\frac{m'_2}{\sqrt{{m'_2}^2 + M'^2}} \ . \tag{67}$$

This is $O(v/M')$.

Turning to the charged-lepton mass matrix, we assume that the lighter of the two eigenvalues is the physical τ-lepton mass (m_τ). We find that the eigenstates with $m = m_\tau$ are given by Eq. (39) (and its analogue for τ_R), with

$$\begin{aligned}\sin\theta_L^\tau &= \frac{m'_2}{\sqrt{{m'_2}^2 + 2M'^2(1 - m_\tau^2/M'^2)^2}} \ , \\ \sin\theta_R^\tau &= \frac{\sqrt{2}m_1 m'_2}{\sqrt{2m_1^2{m'_2}^2 + ({m'_2}^2 + 2M'^2 - 2m_\tau^2)^2}} \ .\end{aligned} \tag{68}$$

Note that, while $\sin\theta_L^\tau = O(v/M')$, $\sin\theta_R^\tau$ is much smaller, $O(m_\tau v/M'^2)$.

2. $\tau_{L,R}$ and $\nu_{\tau L}$ mixing with L alone

For the neutral leptons, due to the Majorana nature of L, the mass terms take the form

$$-\begin{pmatrix} \bar{\nu}_{\tau L} & \overline{L_R^{0\ c}} \end{pmatrix}_L \mathcal{M}_{\nu L} \begin{pmatrix} \nu_{\tau L}^c \\ L_R^0 \end{pmatrix} + \text{h.c.} \ , \quad \text{with} \quad \mathcal{M}_{\nu L} = \begin{pmatrix} 0 & m_2/2\sqrt{2} \\ m_2/2\sqrt{2} & M/2 \end{pmatrix} \ . \tag{69}$$

The mass eigenvalues for the neutral leptons are obtained by diagonalizing $\mathcal{M}_{\nu L}$. Approximately, these are $m = -m_2^2/4M$ and $m = M/2$. However, we see immediately that this is problematic. Given that m_2 is $O(v)$ and M is $O(1\text{ TeV})$, the light neutrino mass $m_2^2/4M$ is orders of magnitude too large. We therefore conclude that the model in which $\tau_{L,R}$ and $\nu_{\tau L}$ mix with L alone is not viable.

3. $\tau_{L,R}$ and $\nu_{\tau L}$ mixing with L and L'

The mass term for the neutral leptons takes the form

$$-\begin{pmatrix} \overline{\nu_{\tau L}} & \overline{L_R^{0\ c}} & \overline{L_L'^0} & \overline{L_R'^{0c}} \end{pmatrix} \mathcal{M}_{\nu LL'} \begin{pmatrix} \nu_{\tau L}^c \\ L_R^0 \\ L_L'^{0c} \\ L_R'^0 \end{pmatrix} + \text{h.c.} \ , \quad \text{with} \quad \mathcal{M}_{\nu LL'} = \begin{pmatrix} 0 & m_2/2\sqrt{2} & 0 & m'_2/2 \\ m_2/2\sqrt{2} & M/2 & 0 & 0 \\ 0 & 0 & 0 & M'/2 \\ m'_2/2 & 0 & M'/2 & 0 \end{pmatrix} \ . \tag{70}$$

Now, the determinant of the diagonalized matrix – which is just the product of the four mass eigenvalues – is equal to the determinant of the above mass matrix. However, $\text{Det}(\mathcal{M}_{\nu LL'}) = m_2^2 {M'}^2/32$, which is nonzero. So this mass matrix does not yield an $m = 0$ eigenvalue. Furthermore, since m_2 and m'_2 are $O(v)$, while M and M' are $O(1\text{ TeV})$, there is no possibility of a seesaw mechanism. It is therefore not possible to generate a tiny mass for ν_τ, which rules out the model in which $\tau_{L,R}$ and $\nu_{\tau L}$ mix with L and L'.

4. $\tau_{L,R}$ and $\nu_{\tau L}$ mixing with L, L' and $\nu_{\tau R}$

The mass terms for neutral leptons take the following form:

$$-\begin{pmatrix} \overline{\nu_{\tau L}} & \overline{\nu_{\tau R}^c} & \overline{L_R^{0\ c}} & \overline{L_L'^0} & \overline{L_R'^{0c}} \end{pmatrix} \mathcal{M}_{\nu\nu LL'} \begin{pmatrix} \nu_{\tau L}^c \\ \nu_{\tau R} \\ L_R^0 \\ L_L'^{0c} \\ L_R'^0 \end{pmatrix} + \text{h.c.} \ , \quad \text{with} \quad \mathcal{M}_{\nu\nu LL'} = \begin{pmatrix} 0 & m_D/2 & m_2/2\sqrt{2} & 0 & m'_2/2 \\ m_D/2 & m_S/2 & 0 & 0 & 0 \\ m_2/2\sqrt{2} & 0 & M/2 & 0 & 0 \\ 0 & 0 & 0 & 0 & M'/2 \\ m'_2/2 & 0 & 0 & M'/2 & 0 \end{pmatrix} \ . \tag{71}$$

m_D, m_2 and m_2' are $O(v)$, while M and M' are $O(1$ TeV$)$. However, the size of m_S is as yet undetermined.

If m_S were $O(10^{15}$ GeV$)$, it might be possible to generate a tiny mass of $O(v^2/m_S)$ for ν_τ via the seesaw mechanism. In order to establish whether this is possible, it is necessary to diagonalize $\mathcal{M}_{\nu\nu LL'}$. However, this involves solving a quintic equation, which cannot be done analytically. Still, we can get some information about the mass eigenvalues as follows. First, we know that m_D, m_2 and m_2' are less than M and M'. In the limit where these entries are neglected, the mass eigenvalues are 0, $m_S/2$, $M/2$, $M'/2$ and $-M'/2$, i.e., there are three intermediate mass eigenvalues of $O(1$ TeV$)$. When m_D, m_2 and m_2' are included, the values of these eigenvalues will be modified. However, we do not expect them to change enormously – perhaps a multiplicative factor of $10^{\pm 1}$ is possible. Second, the determinant of the diagonalized matrix – which is just the product of the five mass eigenvalues – is equal to $\mathrm{Det}(\mathcal{M}_{\nu\nu LL'}) = M'^2(2Mm_D^2 + m_2^2 m_S)/64$. If m_S is $O(10^{15}$ GeV$)$, this is $O(10^{24}$ GeV$^5)$. Given that $m_{\nu_\tau} = O(v^2/m_S) \sim 10^{-11}$ GeV, this implies that the product of the three intermediate mass eigenvalues is $O(10^{20}$ GeV$^3)$. However, as we argued above, this is many orders of magnitude larger than what is possible with $\mathcal{M}_{\nu\nu LL'}$. A seesaw mechanism can therefore not be implemented.

It is still possible to generate a neutrino mass eigenvalue $m = 0$ if $2Mm_D^2 + m_2^2 m_S = 0$. Keeping in mind the expected sizes of m_D, m_2 and M, a simple choice that satisfies this condition is $m_D^2/m_S^2 = \eta m_2^2/(2M^2)$ with $\eta = -M/m_s > 0$. This is clearly a fine-tuned solution, but we cannot overlook any possibilities. For this solution, we have

$$(\nu_{\tau L})_{phys} = a_\nu \nu_{\tau L} + b_\nu \nu_{\tau R}^c + c_\nu L_R^{0\,c} + d_\nu L_L'^0 + e_\nu L_R'^{0\,c} \ . \tag{72}$$

The coefficients are found as follows. Defining $V_\nu \equiv (a_\nu, b_\nu, c_\nu, d_\nu, e_\nu)^T$, we have $\mathcal{M}_{\nu\nu LL'} V_\nu = 0$, yielding

$$a_\nu = \frac{1}{\sqrt{1 + \frac{m_D^2}{m_S^2} + \frac{m_2^2}{2M^2} + \frac{m_2'^2}{M'^2}}} \ , \quad b_\nu = -\frac{m_D}{m_S}\, a_\nu \ , \quad c_\nu = -\frac{m_2}{\sqrt{2}M}\, a_\nu \ , \quad d_\nu = -\frac{m_2'}{M'}\, a_\nu \ , \quad e_\nu = 0 \ . \tag{73}$$

For the charged leptons, the mass terms take the following form:

$$-\begin{pmatrix} \bar{\tau}^- & \bar{L}^- & \bar{L}'^- \end{pmatrix}_L \mathcal{M}_{\tau LL'} \begin{pmatrix} \tau^- \\ L^- \\ L'^- \end{pmatrix}_R \quad , \quad \text{with} \quad \mathcal{M}_{\tau LL'} = \begin{pmatrix} m_1 & m_2 & -m_2'/\sqrt{2} \\ 0 & M & 0 \\ 0 & 0 & M' \end{pmatrix} \ . \tag{74}$$

The masses and mixings relevant for the physical left-handed (right-handed) states are found by diagonalizing $\mathcal{M}_{\tau LL'} \mathcal{M}_{\tau LL'}^T$ ($\mathcal{M}_{\tau LL'}^T \mathcal{M}_{\tau LL'}$). We have

$$\begin{aligned} (\tau_L^-)_{phys} &= a_{L\tau} \tau_L^- + c_{L\tau} L_L^- + d_{L\tau} L_L'^- \ , \\ (\tau_R^-)_{phys} &= a_{R\tau} \tau_R^- + c_{R\tau} L_R^- + d_{R\tau} L_R'^- \ . \end{aligned} \tag{75}$$

Defining $V_{L\tau} \equiv (a_{L\tau}, c_{L\tau}, d_{L\tau})^T$ and $V_{R\tau} \equiv (a_{R\tau}, c_{R\tau}, d_{R\tau})^T$, and assuming that the lightest eigenvalue for the lepton mass matrix is m_τ, the coefficients are found from

$$\mathcal{M}_{\tau LL'} \mathcal{M}_{\tau LL'}^T V_{L\tau} = m_\tau^2 V_{L\tau} \quad , \quad \mathcal{M}_{\tau LL'}^T \mathcal{M}_{\tau LL'} V_{R\tau} = m_\tau^2 V_{R\tau} \ . \tag{76}$$

This yields

$$\begin{gathered} a_{L\tau} = \frac{1}{\sqrt{1 + \frac{m_2^2 M^2}{(M^2 - m_\tau^2)^2} + \frac{m_2'^2 M'^2}{2(M'^2 - m_\tau^2)^2}}} \quad , \quad c_{L\tau} = -\frac{m_2 M}{M^2 - m_\tau^2}\, a_{L\tau} \quad , \quad d_{L\tau} = \frac{m_2' M'}{\sqrt{2}(M'^2 - m_\tau^2)}\, a_{L\tau} \ , \\ a_{R\tau} \simeq 1 \quad , \quad c_{R\tau} = O(m_\tau v/M^2) \quad , \quad d_{R\tau} = O(m_\tau v/M^2) \ . \end{gathered} \tag{77}$$

$c_{R\tau}$ and $d_{R\tau}$ are both $O(10^{-4})$. Thus, to a good approximation, there is no mixing in the right-handed sector.

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