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Ian Balitsky, Vladimir Kazakov, and Evgeny Sobko
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Structure constant of twist-2 light-ray operators in Regge limit

Ian Balitsky\textsuperscript{a}, Vladimir Kazakov\textsuperscript{b}, Evgeny Sobko\textsuperscript{c}
\textsuperscript{a}Physics Dept., Old Dominion University, Norfolk VA 23529 & Theory Group, JLAB, 12000 Jefferson Ave, Newport News, VA 23606
\textsuperscript{b}LPT, École Normale Supérieure, 24, rue Lhomond 75005 Paris, France & Université Paris-VI, Place Jussieu, 75005 Paris, France
\textsuperscript{c}DESY Hamburg, Theory Group, Notkestraße 85, 22607 Hamburg, Germany

We compute the normalized structure constant of three twist-2 operators in $\mathcal{N} = 4$ SYM in the leading BFKL approximation at any $N_c$. The result is applicable to other gauge theories including QCD.

**INTRODUCTION**

The problem of high-energy behavior of amplitudes has a long story \cite{1,2}. One of the most popular approaches is to reduce the gauge theory at high energies to $2+1$ effective theory which can be solved exactly or by computer simulations. Unfortunately, despite the multitude of attempts, the Lagrangian for $2+1$ QCD at high energies is not written yet. In this context the idea to solve formally the high-energy QCD or $\mathcal{N} = 4$ SYM by calculation of anomalous dimensions and structure constants in the BFKL limit seems to be very promising.

$\mathcal{N} = 4$ SYM is a superconformal theory and its most important physical properties are encoded into the OPE characterized by the spectrum of anomalous dimensions and by the structure constants. While the former is now exactly and efficiently computable at large $N_c$ due to quantum integrability \cite{3}, the calculation of the OPE structure constants is, besides days on a fast track, especially after the ground-breaking all-loop proposal of \cite{4}.

In this note we calculate the 3-point correlator of twist-2 operators $O^j(x) = \text{tr} F_{\pm i} D^j D^{−2} F^i_+ + \text{fermions + scalars}$ in $\mathcal{N} = 4$ SYM in the BFKL limit \cite{5} when $\omega = j−1 → 0$, the ’t Hooft coupling $g^2 ≡ \frac{N c^2}{16 \pi g_0^2} → 0$ and $\frac{c^2}{g_0^2}$ fixed, for arbitrary $N_c$. The symbol ’+’ in the field-strength tensor $F_{\pm i}$ means contraction with light-ray vector $n_+$ and the summation over index ‘i’ goes over two-dimensional space orthogonal to $n_+$ and $n_−$. Since the contribution of \textit{fermions+scalars} is subleading at this limit, including the internal loops, the result is valid for the pure Yang-Mills theory as well. The case of two-point correlator was elaborated in our previous paper \cite{6} where we defined the generalized operators with complex spin as special light-ray operators \cite{7} (regularized as a narrow rectangular Wilson contour called "frame") and calculated their correlator using OPE over Wilson lines \cite{8} with a rapidity cutoff and the BFKL evolution (see Fig. 1). Here we use the same light-ray operators: one along $n_+$ direction and two along $n_−$. In this case we should use more general Balitsky-Kovchegov (BK) evolution \cite{9,10} and the leading BFKL contribution comes from the BK vertex.

**LIGHT-RAY OPERATORS AND THEIR RELATION TO LOCAL OPERATORS**

The generalization of local operator $O^j$ for the case of complex spin $j$ was constructed in \cite{6}. It has a form of light-ray operator $\tilde{S}^j$ stretched along $n_+$ direction and realizing the principal series representation of $a l(2)\mathbb{C}$ with conformal spin $J = \frac{1}{2} + i N$ which is related to Lorentz spin $j$ as $J = j + 1$. The full regularized operator reads as follows:

$$\tilde{S}^{j+1}(x_{1\perp}) = \tilde{S}_{gl}^{j+1}(x_{1\perp}) + \frac{i}{2}(j-1)\tilde{S}_{j}^{j+1}(x_{1\perp}) - \frac{1}{2}(j-1)\tilde{S}_{sc}^{j+1}(x_{1\perp}), \quad (1)$$

where for example, the regularized gluon operator is:

$$\tilde{S}_{gl}^{j+1}(x_{1\perp}) = \lim_{|x_{1\perp}| → 0} |x_{1\perp}|^{-\gamma_j} \tilde{S}_{gl}^{j+1}(x_{1\perp}, x_{3\perp}),$$

$$S_{gl}^{j+1}(x_{1\perp}, x_{3\perp}) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} d x_{11} - d x_{31} \frac{\text{tr} F_+}{x_{11} x_{31}} F_{\pm i}(x_{31}),$$

and $x_1 = (x_{1-0}, x_{1\perp})$, $x_3 = (x_{3-0}, x_{3\perp})$. The anomalous dimension $\gamma_j$ corresponds to operator $\tilde{S}^{j+1}(x_{1\perp})$. Here we introduced the notation $[1,3]_i$ for rectangular Wilson contour with coordinates $x_1, x_3$ of two diagonally opposite corners, as in Fig. 1. In the case of even integer Lorentz spin $j$ it can be rewritten as an integral of
local operator $O^j(x)$ with dimension $\Delta(j)$ along a light ray direction $n_+$:

$$S^{j+1}(x_\perp)_{|j\in \text{Even}} \sim \int_{-\infty}^{\infty} dx_- O^j(x)$$

(2)

In this case the correlator of two light-ray operators stretched along $n_+$ and $n_-$ vectors, normalized as $(n_+ n_-) = 1$, is just the double integral of two-point correlator of local operators w.r.t. light-ray directions $n_+$:

$$<S^{j_1+1}(x_\perp) S^{j_2+1}(y_\perp) > = \frac{\delta(j_1 - j_2) b_{j_1}}{|x-y|^{n_{\perp}}}$$

(3)

In this note, we calculate the correlator of three light-ray operators, restricting ourselves to a particular simple kinematics: one light-ray operator is stretched along $n_+$ light-ray direction and two other — along $n_-$. The correlator of 3 light-ray operators can be obtained by integrating the correlator of 3 local operators along these light-rays. The tensor structures of such local correlators are known from general group-theoretical considerations [11], up to a few structure constants depending on the coupling and symmetry charges. The main problem which we are addressing here is the calculation of these non-trivial constants. Remarkably, if the coordinates of all 3 light-rays in the transverse space are restricted to the same line all these structures collapse into a single one [13], with a single overall structure constant which we are going to compute. Note that after a conformal transformation the three points in the transverse space take arbitrary positions.

However, the configuration with two collinear light-ray operators is singular, so we first consider three different light-ray directions $n_1$, $n_2$, $n_3$ and then take the limit $n_2 \to n_3$. The result of integration along light-rays is quite simple and contains only one unknown overall constant

$$<S^{j_1+1}(x_\perp) S^{j_2+1}(y_\perp) S^{j_3+1}(z_\perp) > = C_{(n_1 \{ \Delta(j_1) \}, \{ j_i \})}$$

$$\cdot \frac{1}{|x-y|^2 |y-z|^2 |z-x|^2}$$

(4)

where we used a short-hand notation $[a_i, j, k] = \frac{1}{2}(a_i + a_j - a_k - 1)$ and $\{a_i\} \equiv \{a_1, a_2, a_3\}$. In what follows, we assume the existence of a good analytic continuation for $C_{(n_1 \{ \Delta(j_1) \}, \{ j_i \})}$ to non-integer $\{j_i\}$’s. We take the limit $n_1 = n_+$, $n_2 = n_-$, $n_3 \to n_2$ with the normalization $(n_+ n_-) = 1$. In BFKL regime $j_1 = 1 + \omega_i \to 1$ we obtain:

$$<S^{2+\omega_i}(x_\perp) S^{2+\omega_i}(y_\perp) S^{2+\omega_i}(z_\perp) >$$

$$= \lim_{n_3 \to n_2 \to n_-} \frac{\omega_2 + \omega_3 - \omega_1}{\omega_2 + \omega_3 - \omega_1} \times$$

$$\times \frac{C_{+, -}(\{ \Delta \}, \{ 1 + \omega_i \})}{|x-y|^2 |y-z|^2 |z-x|^2}$$

(5)

where $\Delta_i = \Delta(1 + \omega_i, g^2)$ is given by BFKL spectrum (see below). We explicitly pulled out the denominator $\frac{1}{\omega_2 + \omega_3 - \omega_1}$ because it will emerge in our forthcoming calculation using the BK evolution. We interpret

$$\lim_{n_2 n_3 \to 0} \frac{\omega_2 + \omega_3 - \omega_1}{\omega_2 + \omega_3 - \omega_1}$$

as a delta function $\delta(\omega_2 + \omega_3 - \omega_1)$ reflecting the boost invariance. In addition, we keep $\omega_i$ positive through the paper.

Finally, the structure constant is normalized using the corresponding 2-point correlators:

$$C_{\omega_1, \omega_2, \omega_3} = C_{+, -}(\{ \Delta \}, \{ 1 + \omega_i \})$$

(6)

**DECOMPOSITION OVER DIPOLES AND BK EVOLUTION**

When calculating the two-point correlator [6] we used a point splitting regularization in orthogonal direction, replacing light-rays by infinitely narrow Wilson frames with inserted fields in the corners (see Fig. 1). Now, for the sake of simplicity, we carry out our calculation for pure Wilson frames, related to our operators with zero $R$-charge in the following way:

$$\partial_{x_{13\perp}} \partial_{x_{31\perp}} \int \int \frac{dx_1 dx_3}{(x_3 - x_1 - 2t^{\omega_1})} [x_1, x_3] \to$$

$$\to x_{13\perp} \to 0, \omega_1 \to 0 [x_{13\perp}]^{\gamma_1} c(g^2_{YM}, N, \omega_1) S^{x_1 + x_3}(x_{1\perp})$$

(7)

The coefficient $c(g^2_{YM}, N, \omega_1)$ (denoted below as $c(\omega_i)$) depends on the local regularization procedure and at weak coupling it behaves as $c(\omega_i) \sim g^2_{YM} \omega_i$, but its explicit form is irrelevant for us because we are going to calculate the normalized structure constant where it cancels. In general, there are a few types of leading twist-2 operators which appear in this decomposition but in the BFKL limit a single one with the smallest anomalous dimension survives. In addition, in the $\omega_i \to 0$ limit only the term built out of gauge fields alone does contribute [6].

Following the OPE method [8], the pure Wilson frames can be replaced by regularized color dipoles:

$$[x_1, x_3] \to N(1 - U^{x_1 + x_3}(x_{1\perp}, x_{3\perp}))$$

(8)

where

$$U^{x_1 + x_3}(x_{1\perp}, x_{3\perp}) = 1 - \frac{1}{N} \text{tr}(U^{x_1 + x_3})$$

(9)

$$U^{x_1 + x_3} = P \exp[ig_{YM} \int_{-\infty}^{\infty} dx_+ A^{x_1 + x_3}(x)]$$

(10)

$$A^{x_1 + x_3}(x) = \int d^4k \theta(\sigma_+ - |k_+|) e^{ikx} A_+ (k)$$

(11)

and $\sigma_+$ is a longitudinal cutoff in $n_+$ direction. Now we
can write:

\[
\langle S^{2+ω_1}(x_{1⊥},x_{3⊥})S^{2+ω_2}(y_{1⊥},y_{3⊥})S^{2+ω_3}(z_{1⊥},z_{3⊥}) \rangle =
- \mathcal{D}_⊥ \int \left. dx_{1⊥} \int dx_{3⊥} x_{3⊥}^{-2ω_3} \times \right.
\left. \int dy_{1+} \int dy_{3+} y_{3+}^{-2ω_2} \int dz_{1+} \int dz_{3+} z_{3+}^{-2ω_3} \times \right.
\left. \langle \sigma^{2+}(x_{1⊥},y_{1⊥},z_{1⊥})W^{σ_2}(y_{1⊥},y_{3⊥})W^{σ_3}(z_{1⊥},z_{3⊥}) \rangle \right)
\]

where \( \mathcal{D}_⊥ \) is a normalizing function and the generating function in LO approximation:

\[
\langle \sigma(x_{1⊥}, x_{3⊥})W^{σ_2}(y_{1⊥}, y_{3⊥})W^{σ_3}(z_{1⊥}, z_{3⊥}) \rangle = K_{BK} \ast \sigma^{σ_2}(z_{1⊥}, z_{2⊥})
\]

where \( K_{BK} \) is an integral operator having the following form in LO approximation:

\[
K_{LOBK} \ast \sigma(z_{1⊥}, z_{2⊥}) = \frac{2g^2}{\pi} \int d^2 z_2 \frac{z_2^{12}}{z_{1⊥}z_{2⊥}^2} \left[ \sigma(z_{1⊥}, z_{2⊥}) + \sigma(z_{2⊥}, z_{1⊥}) - \sigma(z_{1⊥}, z_{2⊥}) \right]
\]

Evolution of \( U^{V_2} \) goes from \( Y_1 \) to an intermediate \( Y_0 \) w.r.t. the linear part of (13), and then the BK vertex acts at \( Y_0 \) and generates two dipoles which can be contracted with \( V^{V_2} \) and \( W^{V_3} \). Schematically, it can be written as:

\[
\int dY_0 (U^{V_1} \rightarrow U^{V_0} \otimes \text{(BK vertex at } Y_0 \text{)} \otimes \left( \frac{U^{V_0}V^{V_2}}{U^{V_0}W^{V_3}} \right))
\]

The linear BFKL evolution of \( U^{V_2} \) from \( Y_1 \) to \( Y_0 \) gives:

\[
U^{V_2}(x_{1⊥}, x_{3⊥}) = \int dv \int d^2 x_0 \frac{v_1^2}{\pi^2} E_{v_1}(x_{10}, x_{30}) e^{N(v_1)Y_{10}},
\]

\[
\frac{1}{\pi^2} \int d^2 γd^2δ \frac{1}{|γ - β|^2} E_{v_1}(γ - x_0, β - x_0)U^{Y_0}(γ, β),
\]

where we denoted \( Y_{ij} \equiv Y_i - Y_j \) and we introduced the function \( E_{v_1}(z_{10}, z_{20}) = \left( \frac{1}{\pi z_{10} z_{20} } \right)^{1/2+iw} \) which projects dipoles on the eigenstates of the BFKL operator with the eigenvalues \( R(ν) = 4g^2(ψ(1) - ψ(1/2 + iν) - ψ(1/2 - iν)) \). We take here only the sector \( n = 0 \), where \( n \) is the discrete quantum number of \( SL(2, C) \) because it gives the leading contribution.

The non-linear part of BK evolution (13) is described by the following renorm group equation:

\[
\frac{∂}{∂Y} U^{Y}(γ, β) \big|_{Y=Y_0} =
\frac{2g^2}{π} \int d^2α \frac{|γ - β|^2}{|α|^2|γ - α|^2} U^{Y_0}(γ, α)U^{Y_0}(α, β)
\]

Finally, we contract the two emerging dipoles \( U^{Y_0}(γ, α) \) and \( U^{Y_0}(α, β) \) with \( V^{V_2}(y_{1⊥}, y_{3⊥}) \) and \( W^{V_3}(z_{1⊥}, z_{3⊥}) \). Thus for the planar contribution we get:

\[
\langle U^{Y_0}(x_{1⊥}, x_{3⊥})V^{V_2}(y_{1⊥}, y_{3⊥})W^{V_3}(z_{1⊥}, z_{3⊥}) \rangle_{pl} =
\frac{2g^2}{π} \int dY_0 \int dv_1 \int d^2 x_0 \frac{v_1^2}{π^2} E_{v_1}(x_{10}, x_{30}) e^{N(v_1)Y_{10}} \times \frac{1}{π^2} \int \frac{d^2α d^2β |γ - β|^2}{|α|^2|γ - α|^2} E_{v_1}(γ, x_0, β, x_0).
\]

The last two terms in (17) give the same contribution so it is enough to know the correlators of two dipoles [6]:

\[
\langle U^{Y_0}(γ, α)V^{Y_2}(y_{1⊥}, y_{3⊥})W^{Y_3}(z_{1⊥}, z_{3⊥}) \rangle_{pl} =
\frac{4g^4(1-N^2)}{N_c^2} \int d^2 y_0 \cdot \int dv_2 \frac{v_2}{1+ν^2} E_{ν_2}(γ, y_0 - α - y_0)E_{ν_2}(y_0, y_{30})
\]

and similarly for \( \langle U^{Y_0}(α, β)W^{Y_3}(z_{1⊥}, z_{3⊥}) \rangle_{pl} \). It was argued in [6] that we can make the following identification for rapidities in dipole correlators: \( Y_{02} = \ln \frac{y_{20}+y_{30}}{y_{20}-y_{30}} \), where \( Λ \) a cutoff whose precise value is irrelevant in LO. On the other hand, the difference of rapidities of the first dipole and of the BK vertex \( Y_{10} = \ln \frac{x_{30}}{x_{10}} \) corresponds to BFKL evolution. The integral over \( Y_0 = \ln \frac{y_{20}+y_{30}}{y_{20}-y_{30}} \) goes from \( Y_1 \) to max\( (Y_2, Y_3) \). If we plug (17)-(18) into (12) and do the integrals over light ray directions, i.e. over rapidities, we obtain the following planar contribution:

\[
\langle S^{2+ω_1}(x_{1⊥}, x_{3⊥})S^{2+ω_2}(y_{1⊥}, y_{3⊥})S^{2+ω_3}(z_{1⊥}, z_{3⊥}) \rangle_{pl} =
\frac{4g^4}{π^2 N_c^2 (N^2 - 1)^2} \delta(ω_1 - ω_2 - ω_3)D_⊥
\]

\[
\frac{1}{π^2} \int dv_1 \frac{v_1^2}{ω_2 + ω_3 - N(v_1)} \int \frac{dv_2}{(1+ν^2)^2} \frac{1}{ω_2 - N(ν_2)}
\]

\[
\int dv_3 \frac{v_3^2}{(1+ν^2)^2} \frac{1}{ω_3 - N(ν_3)} \int d^2 x_0 d^2 y_0 d^2 z_0 E_{v_1}(x_{10}, x_{30}) \cdot E_{ν_2}(y_0, y_{30}) E_{ν_2}(z_{10}, z_{30}) \Sigma_{pl}(ν_1, ν_2, ν_3; x_0, y_0, z_0)
\]
\[ Y_{pl}(\nu_1, \nu_2, \nu_3; x_0, y_0, z_0) = \int \frac{d^2 \alpha d^2 \beta d^2 \gamma}{|\gamma - \beta|^2 |\alpha - \beta|^2} E_{\nu_1}(\beta - x_0, \gamma - x_0) \cdot E_{\nu_2}(\alpha - y_0, \gamma - y_0) E_{\nu_3}(\alpha - z_0, \beta - z_0) \frac{\Omega(h_1, h_2, h_3)}{|x_0 - y_0|^2 |x_0 - z_0|^2 |y_0 - z_0|^2}, \]

where \( h_1 = \frac{1}{2} + i \nu_1, h_2 = \frac{1}{2} + i \nu_2, h_3 = \frac{1}{2} + i \nu_3 \) and the function \( \Omega(h_1, h_2, h_3) \) was presented in [15].

Remarkably, we can also take into account the nonplanar contribution [15, 16], thus providing the finite \( N_c \) answer for the BFKL structure constant! It appears as a single extra term \( Y_{npl} \):

\[ Y_{npl}(\nu_1, \nu_2, \nu_3; x_0, y_0, z_0) = \int \frac{d^2 \beta d^2 \gamma}{|\gamma - \beta|^2} E_{\nu_1}(\beta - x_0, \gamma - x_0) \gamma - x_0) E_{\nu_2}(\beta - y_0, \gamma - y_0) E_{\nu_3}(\beta - z_0, \gamma - z_0) = \]

\[ \Lambda(h_1, h_2, h_3) = \frac{|x_0 - y_0|^2 |x_0 - z_0|^2 |y_0 - z_0|^2}{|h_{1,2,3}|^2} \]

where \( \Lambda(h_1, h_2, h_3) \) was also presented in [15], and the full answer can be obtained from (19) by replacing \( Y_{pl} \) with \( Y \) (see in Fig. 2):

\[ Y = Y_{pl} - \frac{2\pi}{N^2} Y_{npl} \text{Re}[\psi(1)] + \psi\left(\frac{1}{2} + i \nu_1\right) - \psi\left(\frac{1}{2} + i \nu_2\right) - \psi\left(\frac{1}{2} + i \nu_3\right). \]

The integrals over \( x_0, y_0, z_0 \) are easily computable, e.g.

\[ \int d^2 x_0 E_{\nu_1}(\beta - x_0, \gamma - x_0) E_{\nu_1}^*(x_0, x_0) = \]

\[ = \left(2\tau\right)^{\frac{1}{2} + i \nu_1} F_1(\frac{1}{2} + i \nu, \frac{1}{2} + i \nu, 1 + 2 i \nu, \tau) \times \]

\[ \times F_1(\frac{1}{2} + i \nu, \frac{1}{2} + i \nu, 1 + 2 i \nu, \frac{1}{2} + i \nu^2) G(\nu) + \]

\[ + (\nu \leftrightarrow -\nu), \]

\[ G(\nu) = \frac{\nu^2}{\left(\frac{1}{2} + i \nu\right)^2 \Gamma(\frac{1}{2} + i \nu) \Gamma(1 + 2 i \nu)}, \]

where \( \tau = \frac{|x_1 - x_3| |\beta - \gamma|}{|x_1 - \beta| |x_3 - \gamma|}. \) In the limit \( x_1, x_3 \to x \) we can replace \( \frac{|x_1 - x_3| |\beta - \gamma|}{|x_1 - \beta| |x_3 - \gamma|} \to 0. \) For small \( \tau \) we can close the \( \nu_1 \) contour in the lower (upper) half-plane for first(second) term, respectively, both of them giving the same contribution. Integrals over \( \beta, \gamma \) in (19) can be reduced to \( Y_{pl} \) represented in [15] in terms of hypergeometric and Meijer G functions, and \( Y_{npl} \) in terms of \( G \) functions. Integrals over \( \nu_1 \) can be done by picking up the BFKL poles \( \omega_i = \mathfrak{R}(\nu_i^*). \)

Combining (19),(22) and (23) we come to the final expression for 3-point correlation function:

\[ \langle S^{2+\omega_1}(x_{1,1}, x_{3,1})S^{2+\omega_2}(y_{1,1}, y_{1,1})S^{2+\omega_3}(z_{1,1}, z_{3,1}) \rangle = \]

\[ = -ig^{10} \delta(\omega_1 - \omega_2 - \omega_3) \frac{c(\omega_1)c(\omega_2)c(\omega_3)}{\Omega(h_1, h_2, h_3)} H, \]

\[ \Psi(\nu_1^*, \nu_2^*, \nu_3^*) \]

\[ = \frac{2^{10}(N_c^2 - 1)^2}{\pi^2 N_c^5} \gamma_1^2(2 + \gamma_1)^4(2 + \gamma_2)^2 \times \]

\[ \times (2 + \gamma_3)^2 \frac{G(\nu_1^*) G(\nu_2^*) G(\nu_3^*)}{\tilde{N}(\nu_1^*) \tilde{N}(\nu_2^*) \tilde{N}(\nu_3^*)}, \]

\[ \gamma_i = \gamma(1 + \omega_i) \text{ - anomalous dimension and the coefficient } \Psi(\nu_1^*, \nu_2^*, \nu_3^*). \]

\[ \text{Our final result for normalized structure constant is:} \]

\[ C_{\omega_1, \omega_2, \omega_3} = -\frac{1}{8} g_{\omega_1, \omega_2, \omega_3} f\left(\frac{g_{\omega_1, \omega_2, \omega_3}}{N_c^2} \right), \]

where \( f \) is a function which depends only on the ratios \( \{ \frac{g_{\omega_i}}{N_c^2} \}. \)

In the limit \( \frac{g_{\omega_i}}{N_c^2} \to 0 \) we get the asymptotics:

\[ \Omega(h_1^*, h_2^*, h_3^*) \to \frac{16 \pi^3}{\gamma_1 \gamma_2 \gamma_3} \frac{\gamma_1^2(\gamma_2 + \gamma_3) + \gamma_2^2(\gamma_1 + \gamma_3) + \gamma_3^2(\gamma_1 + \gamma_2)}{(1 + O(g^2/\omega_i))} + \gamma_3^2(\gamma_1 \gamma_2 \gamma_3) \]

\[ \Lambda(h_1^*, h_2^*, h_3^*) \to \frac{8 \pi^2(\gamma_1 + \gamma_2 + \gamma_3)}{\gamma_1 \gamma_2 \gamma_3}, \]

\[ \text{In this limit } \gamma_i = -\frac{g_{\omega_i}}{N_c^2} = o\left(\frac{g_{\omega_i}}{N_c^2}\right) \text{ and the main contribution to 3-point correlator (28) comes from the planar } \mathcal{O}(g^2) \text{ term} \]

\[ C_{\omega_1, \omega_2, \omega_3} = -ig^2 \frac{\sqrt{N_c^2 - 1}}{\sqrt{2\pi N_c^2}} \frac{1}{\omega_1^2 \omega_2^2 \omega_3^2} \left(\omega_1^2(\omega_2 + \omega_3) + \omega_2^2(\omega_1 + \omega_3) + \omega_3^2(\omega_1 + \omega_2) + \omega_1 \omega_2 \omega_3 \right)(1 + O(g^2)) \]

whereas the nonplanar one is \( \mathcal{O}(g^3). \) It might seem strange that the planar contribution does not start from \( \mathcal{O}(g^4) \) terms given by the leading Feynman graphs, e.g.
with 4 gluon vertices. However, in BFKL approximation we should keep $\frac{x^2}{\omega^2} \gg \omega$ [17]. In addition, when making the point-splitting regularization we have to keep $\frac{1}{\omega^2} \ln(x_{311}/(x - y))^2 \gg 1$. The limit $|x_{311}|$ has to be taken first, which makes the value $g^2 = 0$ exceptional. This order of limits leads to $O(g^2)$ behavior of (30).

**DISCUSSION**

Our result eq.(28), based on BFKL approximation is a rare example of computation of a non-BPS structure constant receiving contributions from all orders in coupling constant, including infinitely many ”wrapping” corrections. Moreover, our result is valid at any $N_c$. Since in the LO BFKL the contributions of all fields but gluons in $\mathcal{N} = 4$ SYM disappear from both the definition of operators and internal loops, the result is applicable to pure YM theory at any $N_c$, including $N_c = 3$. It would be interesting to apply our structure constants to the OPE at hard scattering in real QCD and to work out the full ”dictionary” relating them to the OPE in the 2-dimensional $SL(2,C)$ CFT – the basis of our BFKL computation. It is also not hopeless, though challenging, to compute these structure constants in the NLO approximation in $\mathcal{N} = 4$ SYM. Our present result may serve as an important, all-wrappings test for the future computations of similar quantities in the integrability approaches to planar AdS$_5$/CFT$_4$, such as [4] and the BFKL limit of quantum spectral curve [18].

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