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An Analysis of $\mathsf{B-L=-2}$ Operators from Matter-Higgs Interactions in a Class of Supersymmetric $\mathsf{SO(10)}$ Models

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Abstract

Recently interest in GUT baryogenesis has been resurrected due to the observation that B-violating dimension seven operators that arise in grand unified theories that also violate $\mathsf{B-L}$ produce baryon asymmetry that cannot be wiped out by sphaleron processes. While a general analysis of such higher dimensional operators from a bottom up approach exists in the literature, a full analysis of them derived from grand unification does not exist. In this work we present a complete analysis of $\mathsf{B-L}=-2$ operators within a realistic $\mathsf{SO(10)}$ grand unification where the doublet-triplet splitting is automatic via a missing partner mechanism. Specifically we compute all allowed dimension five, dimension seven and dimension nine operators arising from matter-Higgs interactions. The relative strength of all the allowed $\mathsf{B-L}=-2$ operators is given. Such interactions are useful in the study of neutrino masses, baryogenesis, proton decay and $n-\bar{n}$ oscillations within a common realistic grand unification framework.

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1 Introduction

Analyses of higher dimensional operators within an effective field theory framework to explore physics beyond the Standard Model has a long history [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23]. Included in such effective theories are $\mathsf{B}-\mathsf{L}$ violating operators. Such operators already appear in the study of see saw neutrino masses and in the study of $n-\bar{n}$ oscillation and have gained further interest recently in the context of GUT scale baryogenesis [24, 25, 26]. Thus while baryogenesis arising from baryon and lepton number violating but $\mathsf{B}-\mathsf{L}$ preserving interactions from GUT models is wiped out by sphaleron interactions which violate $\mathsf{B}+\mathsf{L}$ and preserve $\mathsf{B}-\mathsf{L}$, this is not the case for $\mathsf{B}-\mathsf{L}$ violating interactions. The simplest GUT model $\mathsf{SU(5)}$ with renormalizable interactions and R parity conservation has only $\mathsf{B}-\mathsf{L}$ preserving interactions and is not a desirable model for GUT scale baryogenesis. However, $\mathsf{SO(10)}$ models [27, 28] can generate $\mathsf{B}-\mathsf{L}$ violating interactions. While a significant amount of work has been done recently in the study of baryogenesis within $\mathsf{SO(10)}$ using $\mathsf{B}-\mathsf{L}$ violating interactions [25, 26], to our knowledge there is as yet no complete analysis of $\mathsf{B}-\mathsf{L}$ violating interactions that arise in $\mathsf{SO(10)}$. In this work we give a full and rigorous analysis of such interactions and compute dimension 5, 7 and 9 $\mathsf{B}-\mathsf{L}=-2$ interactions within a class of $\mathsf{SO(10)}$ models where the $\mathsf{B}-\mathsf{L}$ violating operators arise from matter-Higgs interactions. The model we consider has a natural doublet-triplet splitting within the framework of a missing partner mechanism [29, 30, 31, 32] (for a recent application of $\mathsf{SO(10)}$ missing partner model see [33]). While our analysis is done within a specific model, it is likely to be applicable to a broader class of models where the light spectrum is that of MSSM.

The outline of the rest of the paper is as follows: In Sec.(2), we give the details of the $\mathsf{SO(10)}$ model. Here, we also discuss the spontaneous breaking of the $\mathsf{SO(10)}$ GUT symmetry down to the symmetry of the Standard Model gauge group, i.e., the symmetry $\mathsf{SU(3)_C}\times\mathsf{SU(2)_L}\times\mathsf{U(1)_Y}$ using $\mathsf{126}+\overline{\mathsf{126}}+\mathsf{210}$ multiplets. The GUT sector of this model is the same as discussed in [34] . The symmetry breaking in these models was investigated in [35] where a cubic equation for spontaneous symmetry breaking was obtained. The models of [34] had in addition 10-plets of Higgs and were later extended to include $\mathsf{120}$-plet of Higgs [36]. Further applications of such models were made in a number of works [37, 38, 39]. While the GUT sector of the model considered here is identical to the previous works the model overall is different for the following reasons: In the usual GUT models to which the works listed above belong, the choice of the Higgs content is arbitrary. For example, in the previous models one can add any number of additional Higgs fields, such as one or more $\mathsf{10}$-plets and $\mathsf{120}$-plets as there is no principle that restricts it. In the missing partner model the Higgs sector is strictly constrained and in $\mathsf{SO(10)}$ only few examples are known [31, 32]. Specifically the GUT sector must be enchored either in $\mathsf{126}+\overline{\mathsf{126}}$ or $\mathsf{560}+\overline{\mathsf{560}}$. Even more stringent is the constraint on the light Higgs sector, i.e., the sector which provides a component to the light Higgs doublet. Thus once an anchor in the GUT sector is assumed the light sector cannot be chosen in an arbitrary fashion as is possible in the usual GUT models. This is needed to satisfy two constraints: first to ensure that the light sector has an excess number of Higgs doublets by one over the heavy Higgs sector while there is an exact match of the Higgs triplets/anti-triplets between the light and the heavy sectors. This ensures that all Higgs doublets will become heavy except one and all Higgs triplets will become heavy because of mixing between the light and the heavy sectors. Second, that all the exotic fields in the light Higgs sector will become heavy as a result of mixing with the heavy fields. These constrains are strong enough to eliminate a large number of models except the ones listed in [31, 32]. For example, for the case when the GUT sector is assumed to be $560+\overline{560}$, which breaks the GUT symmetry down to $\mathsf{SU(3)_C}\times\mathsf{SU(2)_L}\times\mathsf{U(1)_Y}$, the light sector must consist only of the fields $2\times\mathsf{10}+\mathsf{320}$. Further, no masses are allowed for the light sector and the mass generation is allowed only via mixings with the heavy sector. This automatically requires several couplings to vanish. The model we consider here requires us to choose in an unambiguous manner a light sector which is $2\times\mathsf{10}+\mathsf{120}$. The doublet-triplet splitting has been an Achilles heel of grand unification and the missing partner mechanism is one of the ways the problem can be redressed. Thus $\mathsf{SO(10)}$ model which contains many desirable features coupled with the missing partner mechanism provides a natural framework for grand unification.

In Sec.(3), we give details of the doublet-triplet splitting and determine the linear combination of the fields in the $2\times\mathsf{10}+\mathsf{120}$ plet of Higgs that produce a pair of light Higgs doublets. In Sec.(4), we give analysis of the $\mathsf{B}-\mathsf{L}=-2$ operators arising from matter-Higgs interactions. A discussion of results is given in Sec.(5). Conclusions are given in Sec.(6). Further details of the analysis are given in several appendices. In Appendix A, we define the notation and give the decomposition of the $\mathsf{SO(10)}$ multiplets in terms of the $\mathsf{SU(5)}$ multiplets. Appendix B contains the reduction of $\mathsf{24},\mathsf{45},\mathsf{50},\mathsf{75}$ plets of $\mathsf{SU(5)}$ fields in terms of component states with $\mathsf{SU(3)_C}\times\mathsf{SU(2)_L}\times\mathsf{U(1)_Y}$ quantum numbers. These fields enter in the spontaneous breaking of GUT and electroweak symmetry. In Appendix C, we give additional details of the GUT symmetry breaking. In Appendix D, we give a further discussion of the following sets of $\mathsf{SO(10)}$ Higgs couplings in $SU(5)\times U(1)$ decomposition: $\mathsf{10}\cdot\mathsf{126}\cdot\mathsf{210}$, $\mathsf{10}\cdot\overline{\mathsf{126}}\cdot\mathsf{210}$, $\mathsf{120}\cdot\mathsf{126}\cdot\mathsf{210}$ and $\mathsf{120}\cdot\overline{\mathsf{126}}\cdot\mathsf{210}$. These couplings enter in the doublet-triplet splitting. The analysis of these couplings is based on the oscillator mechanism [40, 41] using techniques developed in [42, 43, 44, 45, 46, 47, 48]. An analysis of such couplings in Pati-Salam subgroups was given in earlier works of [37, 49] using techniques of [49]. Finally in Appendix F we exhibit some of the coefficients of $\mathsf{B}-\mathsf{L}=-2$ operators. In this work we do not discuss $\mathsf{B}-\mathsf{L}$ violating interactions that arise from four-point Higgs interactions. There are a large number of such interactions and they include couplings of the type $(\mathsf{126}\times\mathsf{126})_r\cdot(X\times Y)_r$ and $(\mathsf{126}\times\overline{\mathsf{126}})_r\cdot(X\times Y)_r$ where $X,Y=\mathsf{10},\ \mathsf{45},\ \mathsf{54},\ \mathsf{120},\ \mathsf{126},\ \overline{\mathsf{126}},\ \mathsf{210}$. Many of these operators are discussed in [25]. A full analysis of the $\mathsf{B}-\mathsf{L}$ violating interactions from this set is outside the scope of this work and requires a separate analysis.

2 The SO(10) model

The $\mathsf{SO(10)}$ model we discuss has the following particle content [32]

$$\mathsf{126}(\Delta_{\mu\nu\rho\sigma\lambda}),\quad \overline{\mathsf{126}}(\overline{\Delta}_{\mu\nu\rho\sigma\lambda}),\quad \mathsf{210}(\Phi_{\mu\nu\rho\sigma}),\quad \mathsf{10}_1({}^1\Gamma_\mu),\quad \mathsf{10}_2({}^2\Gamma_\mu),\quad \mathsf{120}(\Sigma_{\mu\nu\lambda}). \tag{1}$$

Here the fields $\mathsf{126}+\overline{\mathsf{126}}+\mathsf{210}$ constitute the heavy sector while the fields $2\times\mathsf{10}+\mathsf{120}$ constitute the light sector. Additionally, the model contains three generations of matter fields which reside in three copies of $\mathsf{16}$-plet spinor representation of $\mathsf{SO(10)}$. The light fields consisting of $2\times\mathsf{10}+\mathsf{120}$, together with the heavy sector, generate the desired doublet-triplet splitting and make all the triplets and doublets heavy except for one pair of Higgs doublets. This will be discussed in detail in the next section. Here we discuss the breaking of the $\mathsf{SO(10)}$ GUT symmetry to the Standard Model gauge group. The spontaneous breaking of the GUT symmetry for this model was first discussed in [35]. Here we discuss it to set up the framework for the discussion in the following sections. Thus the breaking comes about as follows: The $\mathsf{126}+\overline{\mathsf{126}}$ multiplets reduces the rank of the group and the $\mathsf{210}$ plet breaks the rest of the gauge symmetry down to the Standard Model gauge group. The superpotential that breaks the GUT symmetry is

$$W_{GUT}=m_\Phi\Phi_{\mu\nu\sigma\xi}\Phi_{\mu\nu\sigma\xi}+m_\Delta\Delta_{\mu\nu\sigma\xi\zeta}\overline{\Delta}_{\mu\nu\sigma\xi\zeta}+\lambda\Phi_{\mu\nu\sigma\xi}\Phi_{\sigma\xi\rho\tau}\Phi_{\rho\tau\mu\nu}+\eta\Phi_{\mu\nu\sigma\xi}\Delta_{\mu\nu\rho\tau\zeta}\overline{\Delta}_{\sigma\xi\rho\tau\zeta}, \tag{2}$$

where m_Φ is the mass of the $\mathsf{210}$-plet field and m_Δ is the mass of the $\mathsf{126}+\overline{\mathsf{126}}$ multiplets. The fields that develop VEVs are $\mathsf{SU(3)_C}\times\mathsf{SU(2)_L}\times\mathsf{U(1)_Y}$ singlets: $\mathbf{S}_{1_{126}}$, $\mathbf{S}_{1_{\overline{126}}}$, $\mathbf{S}_{1_{210}}$, $\mathbf{S}_{24_{210}}$, $\mathbf{S}_{75_{210}}$. Here for example $\mathbf{S}_{24_{210}}$ means that the Standard Model singlet is in the $\mathsf{24}$ plet of $\mathsf{SU(5)}$ contained in the 210 plet of $\mathsf{SO(10)}$ (See Appendix B for further details). The superpotential, when expanded in terms of these Standard Model singlets, gives (see Appendix C for details)

$$\begin{aligned}
W_{GUT} \quad = \quad & m_\Phi\left(\frac{3}{4}\mathbf{S}^2_{75_{210}}+\frac{5}{12}\mathbf{S}^2_{24_{210}}+\frac{3}{80}\mathbf{S}^2_{1_{210}}+\cdots\right)+m_\Delta\left(\frac{15}{2}\mathbf{S}_{1_{126}}\mathbf{S}_{1_{\overline{126}}}+\cdots\right)\\
&+\lambda\left(\frac{1}{18}\mathbf{S}^3_{75_{210}}-\frac{1}{18}\mathbf{S}^2_{75_{210}}\mathbf{S}_{24_{210}}+\frac{25}{864}\mathbf{S}_{75_{210}}\mathbf{S}^2_{24_{210}}+\frac{1}{40}\mathbf{S}^2_{75_{210}}\mathbf{S}_{1_{210}}\right.\\
&\qquad\left.-\frac{35}{3888}\mathbf{S}^3_{24_{210}}-\frac{1}{192}\mathbf{S}^2_{24_{210}}\mathbf{S}_{1_{210}}-\frac{3}{3200}\mathbf{S}^3_{1_{210}}+\cdots\right)\\
&+\eta\left(-\frac{3}{16}\mathbf{S}_{1_{210}}\mathbf{S}_{1_{126}}\mathbf{S}_{1_{\overline{126}}}+\cdots\right).
\end{aligned} \tag{3}$$

Vanishing of the D-terms implies $\mathbf{S}_{1_{126}}=\mathbf{S}_{1_{\overline{126}}}$, while the F-terms yield

$$\begin{aligned}
\frac{3}{40}m_\Phi\mathbf{S}_{1_{210}}+\lambda\left(\frac{1}{40}\mathbf{S}^2_{75_{210}}-\frac{1}{192}\mathbf{S}^2_{24_{210}}-\frac{9}{3200}\mathbf{S}^2_{1_{210}}\right)-\frac{3}{16}\eta\mathbf{S}^2_{1_{126}} &= 0,\\
\frac{5}{6}m_\Phi\mathbf{S}_{24_{210}}+\lambda\left(-\frac{1}{18}\mathbf{S}^2_{75_{210}}+\frac{25}{432}\mathbf{S}_{75_{210}}\mathbf{S}_{24_{210}}-\frac{35}{1296}\mathbf{S}^2_{24_{210}}-\frac{1}{96}\mathbf{S}_{24_{210}}\mathbf{S}_{1_{210}}\right) &= 0,\\
\frac{3}{2}m_\Phi\mathbf{S}_{75_{210}}+\lambda\left(\frac{1}{6}\mathbf{S}^2_{75_{210}}-\frac{1}{9}\mathbf{S}_{75_{210}}\mathbf{S}_{24_{210}}+\frac{25}{864}\mathbf{S}^2_{24_{210}}+\frac{1}{20}\mathbf{S}_{75_{210}}\mathbf{S}_{1_{210}}\right) &= 0,\\
15m_\Delta\mathbf{S}_{1_{126}}-\eta\frac{3}{8}\mathbf{S}_{1_{210}}\mathbf{S}_{1_{126}} &= 0.
\end{aligned} \tag{4}$$

For the sake of brevity, we define

$$\mathcal{M}_\Delta\equiv\frac{m_\Delta}{\eta},\qquad \mathcal{M}_\Phi\equiv\frac{m_\Phi}{\lambda}. \tag{5}$$

Note that $\mathbf{S}_{1_{210}}$ can be immediately solved for and is given by

$$\mathbf{S}_{1_{210}}=40\mathcal{M}_\Delta. \tag{6}$$

Except for a trivial solution, $\mathbf{S}_{24_{210}}$ satisfies a cubic equation

$$\begin{aligned}
9\mathbf{S}^3_{24_{210}}+24\mathbf{S}^2_{24_{210}}\left(28\mathcal{M}_\Delta-45\mathcal{M}_\Phi\right)+64\mathbf{S}_{24_{210}}&\left(320\mathcal{M}^2_\Delta-279\mathcal{M}_\Delta\mathcal{M}_\Phi\right.\\
&\left.+972\mathcal{M}^2_\Phi\right)+13824\left(\mathcal{M}_\Delta-2\mathcal{M}_\Phi\right)\left(4\mathcal{M}_\Delta+3\mathcal{M}_\Phi\right)^2=0.
\end{aligned} \tag{7}$$

Once $\mathbf{S}_{24_{210}}$ is determined, $\mathbf{S}_{75_{210}}$ and $\mathbf{S}_{1_{126}} = \mathbf{S}_{1_{\overline{126}}}$ are given by

$$\mathbf{S}_{75_{210}} = \frac{5\left[\mathbf{S}^2_{24_{210}} + 24\mathbf{S}_{24_{210}}\left(\mathcal{M}_\Delta - 2\mathcal{M}_\Phi\right)\right]}{6\left[1\mathbf{S}_{24_{210}} + 8\left(4\mathcal{M}_\Delta + 3\mathcal{M}_\Phi\right)\right]}, \tag{8}$$

$$\begin{aligned}\mathbf{S}_{1_{126}} \cdot \mathbf{S}_{1_{\overline{126}}} = {} & \left(\frac{\lambda}{\eta}\right)\frac{1}{216\left(1\mathbf{S}_{24_{210}} + 32\mathcal{M}_\Delta + 24\mathcal{M}_\Phi\right)}\left[5\mathbf{S}^3_{24_{210}} - 32\mathbf{S}^2_{24_{210}}\left(8\mathcal{M}_\Delta + 39\mathcal{M}_\Phi\right)\right. \\ & - 1728\mathbf{S}_{24_{210}}\left(7\mathcal{M}^2_\Delta - 7\mathcal{M}_\Delta\mathcal{M}_\Phi - 6\mathcal{M}^2_\Phi\right) \\ & \left. - 13824\mathcal{M}_\Delta\left(3\mathcal{M}_\Delta - 2\mathcal{M}_\Phi\right)\left(4\mathcal{M}_\Delta + 3\mathcal{M}_\Phi\right)\right]. \end{aligned} \tag{9}$$

Thus all the VEVs, i.e., $\mathbf{S}_{1_{210}}$, $\mathbf{S}_{75_{210}}$, $\mathbf{S}_{1_{126}}$, $\mathbf{S}_{1_{\overline{126}}}$ can be determined in terms of just one VEV, i.e., $\mathbf{S}_{24_{210}}$ using the minimization conditions Eq.(4). We note that the VEVs depend on all the four parameters m_Δ, m_Φ, λ and η. Table 2 gives the numerical estimates of these Standard Model singlets. Note that in Table 2 corresponding to each set of $\mathcal{M}_\Delta$ and $\mathcal{M}_\Phi$, there exists three solutions for $\mathbf{S}_{24_{210}}$ as given by Eq.(7). We note that the cubic equation for spontaneous symmetry breaking exhibited in Eq.(7) was first obtained in the work of [35].

3 Light and heavy Higgs fields after spontaneous breaking of the GUT symmetry

As mentioned in Sec.(2), the doublet-triplet splitting arises as a consequence of mixing between the light sector consisting of $2 \times \mathsf{10} + \mathsf{120}$ plets of Higgs fields and the heavy sector consisting of $\mathsf{126} + \overline{\mathsf{126}} + \mathsf{210}$ of Higgs fields. The interactions mixing the light and the heavy fields are given in Appendix D. As discussed in [32] the light sector contains four pairs of Higgs doublets and four pairs of Higgs triplets/anti-triplets while the heavy sector consists of three pairs of Higgs doublets and four pairs of Higgs triplets/antitriplets leaving only one pair of light Higgs doublet. This light Higgs doublet pair is, in general, a linear combination of the Higgs doublets in the $\mathsf{10}$-plets, in the $\mathsf{120}$-plet and in the $\overline{\mathsf{126}}$-plet of Higgs fields and there is no component of it in the heavy Higgs sector which breaks the GUT symmetry. Actually it turns out that the light Higgs field in the present model is a linear combination only of the Higgs doublets that arise from the $\mathsf{10}_1 + \mathsf{10}_2$ plets and from the $\mathsf{120}$ plet of $\mathsf{SO(10)}$ Higgs fields. In this section, we discuss the details of the analysis to determine the exact combination of these doublets in the residual light doublet pair. The superpotential that enters in the doublet-triplet splitting is given by

$$W_{DT} = \mathrm{A}\ {}^1\Gamma_\mu\Delta_{\mu\nu\sigma\xi\zeta}\Phi_{\mu\sigma\xi\zeta} + \mathrm{B_r}\ {}^r\Gamma_\mu\overline{\Delta}_{\mu\nu\sigma\xi\zeta}\Phi_{\mu\sigma\xi\zeta} + \mathrm{C}\ \Sigma_{\mu\nu\sigma}\Delta_{\nu\sigma\xi\zeta\rho}\Phi_{\mu\xi\zeta\rho} + \overline{\mathrm{C}}\ \Sigma_{\mu\nu\sigma}\overline{\Delta}_{\nu\sigma\xi\zeta\rho}\Phi_{\mu\xi\zeta\rho}, \tag{10}$$

where $\mathrm{r} = 1, 2$. Next, we exhibit the Higgs doublet pairs (D) consisting of up- and down-type Higgs and Higgs triplet/anti-triplet pairs (T) that participate in the missing partner mechanism. Note that we could have added the following set of terms to W_{DT} allowed by the gauge invariance of the theory

$$\Delta W_{DT} = m_{rs}\ \mathsf{10}^{(r)} \cdot \mathsf{10}^{(s)} + \lambda_\Sigma\ \mathsf{120} \cdot \mathsf{120} + \lambda^{(r)}_{10\cdot 120\cdot 210}\ \mathsf{10}^{(r)} \cdot \mathsf{120} \cdot \mathsf{210} + \lambda_{120^2\cdot 210}\ \mathsf{120} \cdot \mathsf{120} \cdot \mathsf{210} \tag{11}$$

The missing partner mechanism requires

$$m_{rs} = 0,\ \lambda_\Sigma = 0,\ \lambda^{(r)}_{10\cdot 120\cdot 210} = 0,\ \lambda_{120^2\cdot 210} = 0. \tag{12}$$

which represents a significant reduction of parameters.

1. Pairs of D and T in the Heavy Sector

$$\begin{aligned} \mathsf{126} + \overline{\mathsf{126}} \supset {} & 2\mathrm{D}\left\{\left({}^{(5_{\overline{126}})}\mathsf{D}^a, {}^{(\overline{5}_{126})}\mathsf{D}_a\right); \left({}^{(45_{126})}\mathsf{D}^a, {}^{(\overline{45}_{\overline{126}})}\mathsf{D}_a\right)\right\} + 3\mathrm{T}\left\{\left({}^{(5_{\overline{126}})}\mathsf{T}^\alpha, {}^{(\overline{5}_{126})}\mathsf{T}_\alpha\right);\right. \\ & \left.\left({}^{(45_{126})}\mathsf{T}^\alpha, {}^{(\overline{45}_{\overline{126}})}\mathsf{T}_\alpha\right); \left({}^{(50_{\overline{126}})}\mathsf{T}^\alpha, {}^{(\overline{50}_{126})}\mathsf{T}_\alpha\right)\right\} \\ \mathsf{210} \supset {} & 1\mathrm{D}\left({}^{(5_{210})}\mathsf{D}^a, {}^{(\overline{5}_{210})}\mathsf{D}_a\right) + 1\mathrm{T}\left({}^{(5_{210})}\mathsf{T}^\alpha, {}^{(\overline{5}_{210})}\mathsf{T}_\alpha\right) \end{aligned}$$

2. Pairs of D and T in the Light Sector

$$2 \times \mathsf{10} \supset 2\mathrm{D}\left\{\left({}^{(5_{10_1})}\mathsf{D}^a, {}^{(\overline{5}_{10_1})}\mathsf{D}_a\right); \left({}^{(5_{10_2})}\mathsf{D}^a, {}^{(\overline{5}_{10_2})}\mathsf{D}_a\right)\right\} + 2\mathrm{T}\left\{\left({}^{(5_{10_1})}\mathsf{T}^\alpha, {}^{(\overline{5}_{10_1})}\mathsf{T}_\alpha\right);\right.$$

$\mathsf{SU(2)_L}$ Doublet (Up-Type)	$\mathsf{SU(3)_C}$ Triplet	$\mathsf{SU(2)_L}$ Doublet (Down-Type)	$\mathsf{SU(3)_C}$ Anti-Triplet
${}^{(5_{10_1})}\mathsf{D}^a$	${}^{(5_{10_1})}\mathsf{T}^\alpha$	${}^{(\overline{5}_{10_1})}\mathsf{D}_a$	${}^{(\overline{5}_{10_1})}\mathsf{T}_\alpha$
${}^{(5_{10_2})}\mathsf{D}^a$	${}^{(5_{10_2})}\mathsf{T}^\alpha$	${}^{(\overline{5}_{10_2})}\mathsf{D}_a$	${}^{(\overline{5}_{10_2})}\mathsf{T}_\alpha$
${}^{(5_{120})}\mathsf{D}^a$	${}^{(5_{120})}\mathsf{T}^\alpha$	${}^{(\overline{5}_{120})}\mathsf{D}_a$	${}^{(\overline{5}_{120})}\mathsf{T}_\alpha$
${}^{(5_{\overline{126}})}\mathsf{D}^a$	${}^{(5_{\overline{126}})}\mathsf{T}^\alpha$	${}^{(\overline{5}_{126})}\mathsf{D}_a$	${}^{(\overline{5}_{126})}\mathsf{T}_\alpha$
${}^{(5_{210})}\mathsf{D}^a$	${}^{(5_{210})}\mathsf{T}^\alpha$	${}^{(\overline{5}_{210})}\mathsf{D}_a$	${}^{(\overline{5}_{210})}\mathsf{T}_\alpha$
${}^{(45_{120})}\mathsf{D}^a$	${}^{(45_{120})}\mathsf{T}^\alpha$	${}^{(\overline{45}_{120})}\mathsf{D}_a$	${}^{(\overline{45}_{120})}\mathsf{T}_\alpha$
${}^{(45_{126})}\mathsf{D}^a$	${}^{(45_{126})}\mathsf{T}^\alpha$	${}^{(\overline{45}_{\overline{126}})}\mathsf{D}_a$	${}^{(\overline{45}_{\overline{126}})}\mathsf{T}^\alpha$
–	${}^{(50_{\overline{126}})}\mathsf{T}^\alpha$	–	${}^{(\overline{50}_{126})}\mathsf{T}_\alpha$

Table 1: Symbolic representation of up–type and down–type Higgs doublets, and Higgs triplet and anti-triplet pairs in the $\mathsf{SO(10)}$ missing partner model discussed in this work.

$$\begin{aligned}&\left({}^{(5_{10_2})}\mathsf{T}^\alpha, {}^{(\overline{5}_{10_2})}\mathsf{T}_\alpha\right)\Big\}\\ \mathsf{120}\quad\supset\quad & 2\mathrm{D}\left\{\left({}^{(5_{120})}\mathsf{D}^a, {}^{(\overline{5}_{120})}\mathsf{D}_a\right); \left({}^{(45_{120})}\mathsf{D}^a, {}^{(\overline{45}_{120})}\mathsf{D}_a\right)\right\} + 2\mathrm{T}\left\{\left({}^{(5_{120})}\mathsf{T}^\alpha, {}^{(\overline{5}_{120})}\mathsf{T}_\alpha\right);\right.\\ &\left.\left({}^{(45_{120})}\mathsf{T}^\alpha, {}^{(\overline{45}_{120})}\mathsf{T}_\alpha\right)\right\}\end{aligned}$$

3. Residual Set of Light modes: 1D

Here, for example, ${}^{(45_{120})}\mathsf{D}^a$ means that the doublet is in the $\mathsf{45}$ plet of $\mathsf{SU(5)}$ contained in the 120 plet of $\mathsf{SO(10)}$. Here $\alpha = 1, 2, 3$ and $a = 4, 5$ represent $\mathsf{SU(3)}$ color and $\mathsf{SU(2)}$ weak indices, respectively. The result of the above analysis is summarized in Table 1. The mass terms for the $\mathsf{SU(2)}$ doublets in the superpotential can be written as

$$\left({}^{(5_{10_1})}\mathsf{D}^a, {}^{(5_{10_2})}\mathsf{D}^a, {}^{(5_{120})}\mathsf{D}^a, {}^{(5_{\overline{126}})}\mathsf{D}^a, {}^{(5_{210})}\mathsf{D}^a, {}^{(45_{120})}\mathsf{D}^a, {}^{(45_{126})}\mathsf{D}^a\right) M_d \begin{pmatrix} {}^{(\overline{5}_{10_1})}\mathsf{D}_a \\ {}^{(\overline{5}_{10_2})}\mathsf{D}_a \\ {}^{(\overline{5}_{120})}\mathsf{D}_a \\ {}^{(\overline{5}_{126})}\mathsf{D}_a \\ {}^{(\overline{5}_{210})}\mathsf{D}_a \\ {}^{(\overline{45}_{120})}\mathsf{D}_a \\ {}^{(\overline{45}_{\overline{126}})}\mathsf{D}_a \end{pmatrix}, \tag{13}$$

where the doublet mass matrix M_d receives contributions from eq. (2) and Eq.(10) and is given by

$$
M_d = \begin{pmatrix}
0 & 0 & 0 & \mathsf{d}_2 & \mathsf{d}_1 & 0 & \left(\frac{b_1}{a}\right)\mathsf{d}_3 \\
0 & 0 & 0 & 0 & 0 & 0 & \left(\frac{b_2}{a}\right)\mathsf{d}_3 \\
0 & 0 & 0 & \mathsf{d}_5 & \mathsf{d}_4 & 0 & \left(\frac{\overline{c}}{c}\right)\mathsf{d}_6 \\
\left(\frac{b_1}{a}\right)\mathsf{d}_2 & \left(\frac{b_2}{a}\right)\mathsf{d}_2 & \left(\frac{\overline{c}}{c}\right)\mathsf{d}_5 & \mathsf{d}_9 & \mathsf{d}_{11} & \left(\frac{\overline{c}}{c}\right)\mathsf{d}_7 & 0 \\
\left(\frac{b_1}{a}\right)\left(\frac{\mathbf{S}_{1_{\overline{126}}}}{\mathbf{S}_{1_{126}}}\right)\mathsf{d}_1 & \left(\frac{b_2}{a}\right)\left(\frac{\mathbf{S}_{1_{\overline{126}}}}{\mathbf{S}_{1_{126}}}\right)\mathsf{d}_1 & \left(\frac{\overline{c}}{c}\right)\left(\frac{\mathbf{S}_{1_{\overline{126}}}}{\mathbf{S}_{1_{126}}}\right)\mathsf{d}_4 & \left(\frac{\mathbf{S}_{1_{\overline{126}}}}{\mathbf{S}_{1_{126}}}\right)\mathsf{d}_{11} & \mathsf{d}_{10} & 0 & 0 \\
0 & 0 & 0 & \mathsf{d}_7 & 0 & 0 & \left(\frac{\overline{c}}{c}\right)\mathsf{d}_8 \\
\mathsf{d}_3 & 0 & \mathsf{d}_6 & 0 & 0 & \mathsf{d}_8 & \mathsf{d}_{12}
\end{pmatrix}, \tag{14}
$$

and

$$
\begin{aligned}
a &\equiv \frac{\imath}{5!}\mathrm{A}; \quad b_{1,2} \equiv \frac{\imath}{5!}\mathrm{B}_{1,2}; \quad c \equiv \frac{\imath}{5!}\mathrm{C}; \quad \overline{c} \equiv \frac{\imath}{5!}\overline{\mathrm{C}}, \\
\mathsf{d}_1 &\equiv \frac{a}{2\sqrt{5}}\mathbf{S}_{1_{126}}, \\
\mathsf{d}_2 &\equiv -a\left[\frac{\sqrt{3}}{10}\mathbf{S}_{1_{210}} + \frac{\sqrt{3}}{20}\mathbf{S}_{24_{210}}\right], \\
\mathsf{d}_3 &\equiv a\left[-\frac{1}{4\sqrt{6}}\mathbf{S}_{24_{210}} + \frac{1}{4\sqrt{15}}\mathbf{S}_{75_{210}}\right], \\
\mathsf{d}_4 &\equiv -\frac{c}{\sqrt{30}}\mathbf{S}_{1_{126}}, \\
\mathsf{d}_5 &\equiv c\left[-\frac{1}{10\sqrt{2}}\mathbf{S}_{1_{210}} + \frac{3}{40\sqrt{2}}\mathbf{S}_{24_{210}}\right], \\
\mathsf{d}_6 &\equiv -c\left[\frac{1}{48}\mathbf{S}_{24_{210}} + \frac{1}{12\sqrt{10}}\mathbf{S}_{75_{210}}\right], \\
\mathsf{d}_7 &\equiv c\left[\frac{1}{48\sqrt{3}}\mathbf{S}_{24_{210}} + \frac{1}{12\sqrt{30}}\mathbf{S}_{75_{210}}\right], \\
\mathsf{d}_8 &\equiv -c\left[\frac{1}{20\sqrt{6}}\mathbf{S}_{1_{210}} + \frac{1}{240\sqrt{6}}\mathbf{S}_{24_{210}} + \frac{1}{12\sqrt{15}}\mathbf{S}_{75_{210}}\right], \\
\mathsf{d}_9 &\equiv 2m_\Delta - \eta\left[\frac{2}{5\sqrt{15}}\mathbf{S}_{1_{210}} + \frac{3}{20}\sqrt{\frac{3}{5}}\mathbf{S}_{24_{210}}\right], \\
\mathsf{d}_{10} &\equiv 2m_\Phi - \lambda\left[\frac{3}{10\sqrt{2}}\mathbf{S}_{1_{210}} + \frac{1}{2}\sqrt{\frac{3}{5}}\mathbf{S}_{24_{210}}\right], \\
\mathsf{d}_{11} &\equiv \frac{1}{5}\eta\mathbf{S}_{1_{126}}, \\
\mathsf{d}_{12} &\equiv m_\Delta + \eta\left[-\frac{1}{6\sqrt{15}}\mathbf{S}_{24_{210}} + \frac{1}{15\sqrt{6}}\mathbf{S}_{75_{210}}\right].
\end{aligned} \tag{15}
$$

Substituting the values of $\mathbf{S}_{\mathbf{1_{210}}}$, $\mathbf{S}_{75_{210}}$, $\mathbf{S}_{1_{126}}$, $\mathbf{S}_{1_{\overline{126}}}$ in terms of just one VEV, i.e., $\mathbf{S}_{24_{210}}$, all the matrix elements of M_d can be determined in terms of a single VEV, i.e., $\mathbf{S}_{24_{210}}$. An illustrative example of the numerical sizes of $\mathbf{S}_{\mathbf{1_{210}}}$, $\mathbf{S}_{24_{210}}$, $\mathbf{S}_{75_{210}}$, $\mathbf{S}_{1_{126}}$ is given in Table 2. The matrix M_d is non-symmetric and is diagonalized by two 7×7 unitary matrices U_d and V_d:

$$
{U_d}^\dagger M_d V_d = \mathrm{diag}\left(0, \mathsf{m}_{d_2}, \mathsf{m}_{d_3}, \cdots, \mathsf{m}_{d_7}\right). \tag{16}
$$

The columns of the matrices U_d and V_d are the eigenvectors of matrices ${M_d}^\dagger M_d$ and $M_d {M_d}^\dagger$ respectively,

$$
U_d^\dagger[M_d^\dagger M_d]U_d = \mathrm{diag}\left(0, \mathsf{m}_{d_2}^2, \mathsf{m}_{d_3}^2, \cdots, \mathsf{m}_{d_7}^2\right) = V_d^\dagger[M_d M_d^\dagger]V_d. \tag{17}
$$

The mass eigenstates of the Higgs doublet fields are expressed in terms of the primitive Higgs doublet fields through

$\mathcal{M}_\Delta$ (GeV)	$\mathcal{M}_\Phi$ (GeV)	$\mathbf{S}_{1_{210}}$ (GeV)	$\mathbf{S}_{24_{210}}$ (GeV)	$\mathbf{S}_{75_{210}}$ (GeV)	$\mathbf{S}_{1_{126}}$ (GeV)
10^{15}	10^{15}	4×10^{16}	1.10×10^{16}	-1.78×10^{15}	$\imath 1.28\times 10^{16}$
		4×10^{16}	$(1.72-\imath 8.08)\times 10^{16}$	$(-2.94-\imath 4.20)\times 10^{16}$	$(1.38+\imath 2.75)\times 10^{16}$
		4×10^{16}	$(1.72+\imath 8.08)\times 10^{16}$	$(-2.94+\imath 4.20)\times 10^{16}$	$(1.38-\imath 2.75)\times 10^{16}$
6.67×10^{15}	6.67×10^{15}	2.67×10^{17}	7.36×10^{16}	-1.19×10^{16}	$\imath 8.57\times 10^{16}$
		2.67×10^{17}	$(1.14-\imath 5.39)\times 10^{17}$	$(-1.96-\imath 2.80)\times 10^{17}$	$(9.23+\imath 18.4)\times 10^{16}$
		2.67×10^{17}	$(1.14+\imath 5.39)\times 10^{17}$	$(-1.96+\imath 2.80)\times 10^{17}$	$(9.23-\imath 18.4)\times 10^{16}$
1.25×10^{16}	1.25×10^{16}	5×10^{17}	1.38×10^{17}	-2.22×10^{16}	$\imath 1.61\times 10^{17}$
		5×10^{17}	$(2.14-\imath 10.10)\times 10^{17}$	$(-3.67-\imath 5.24)\times 10^{17}$	$(1.72+\imath 3.44)\times 10^{17}$
		5×10^{17}	$(2.14+\imath 10.10)\times 10^{17}$	$(-3.67+\imath 5.24)\times 10^{17}$	$(1.72-\imath 3.44)\times 10^{17}$
6.67×10^{15}	2×10^{16}	2.67×10^{17}	1.66×10^{17}	-1.02×10^{17}	$\imath 6.51\times 10^{16}$
		2.67×10^{17}	$(8.68-\imath 12.49)\times 10^{17}$	$(-1.84-\imath 7.71)\times 10^{17}$	$(1.65+\imath 3.17)\times 10^{17}$
		2.67×10^{17}	$(8.68+\imath 12.49)\times 10^{17}$	$(-1.84+\imath 7.71)\times 10^{17}$	$(1.65-\imath 3.17)\times 10^{17}$
2×10^{16}	6.67×10^{15}	8×10^{17}	-1.15×10^{17}	-6.24×10^{15}	2.41×10^{17}
		8×10^{17}	$(-2.89-\imath 8.96)\times 10^{17}$	$(-5.69-\imath 3.87)\times 10^{17}$	$(2.87+\imath 5.36)\times 10^{17}$
		8×10^{17}	$(-2.89+\imath 8.96)\times 10^{17}$	$(-5.69+\imath 3.87)\times 10^{17}$	$(2.87-\imath 5.36)\times 10^{17}$

Table 2: A Numerical estimate of the VEVs of the Standard Model singlets in $\mathsf{210}$, $\mathsf{126}$ and $\overline{\mathsf{126}}$-plets arising in the spontaneous breaking of the $\mathsf{SO(10)}$ GUT gauge symmetry under the assumption $\lambda=\eta\sim 1$ and $\mathbf{S}_{1_{126}}=\mathbf{S}_{1_{\overline{126}}}$.

$$\begin{pmatrix} {}^{1}\mathsf{D}'_a \\ {}^{2}\mathsf{D}'_a \\ {}^{3}\mathsf{D}'_a \\ {}^{4}\mathsf{D}'_a \\ {}^{5}\mathsf{D}'_a \\ {}^{6}\mathsf{D}'_a \\ {}^{7}\mathsf{D}'_a \end{pmatrix} = {V_d}^{\dagger} \begin{pmatrix} {}^{(\overline{5}_{10_1})}\mathsf{D}_a \\ {}^{(\overline{5}_{10_2})}\mathsf{D}_a \\ {}^{(\overline{5}_{120})}\mathsf{D}_a \\ {}^{(\overline{5}_{126})}\mathsf{D}_a \\ {}^{(\overline{5}_{210})}\mathsf{D}_a \\ {}^{(\overline{45}_{120})}\mathsf{D}_a \\ {}^{(\overline{45}_{\overline{126}})}\mathsf{D}_a \end{pmatrix} ; \quad \begin{pmatrix} {}^{1}\mathsf{D}'^a \\ {}^{2}\mathsf{D}'^a \\ {}^{3}\mathsf{D}'^a \\ {}^{4}\mathsf{D}'^a \\ {}^{5}\mathsf{D}'^a \\ {}^{6}\mathsf{D}'^a \\ {}^{7}\mathsf{D}'^a \end{pmatrix} = {U_d}^{\dagger} \begin{pmatrix} {}^{(5_{10_1})}\mathsf{D}^a \\ {}^{(5_{10_2})}\mathsf{D}^a \\ {}^{(5_{120})}\mathsf{D}^a \\ {}^{(5_{\overline{126}})}\mathsf{D}^a \\ {}^{(5_{210})}\mathsf{D}^a \\ {}^{(45_{120})}\mathsf{D}^a \\ {}^{(45_{126})}\mathsf{D}^a \end{pmatrix}. \tag{18}$$

We identify the light Higgs doublet pair to be $\left({}^{1}\mathsf{D}'_a, {}^{1}\mathsf{D}'^a\right)\equiv(\mathbf{H_d}_a, \mathbf{H_u}^a)$ while all the remaining mass eigenstates of the Higgs doublets are superheavy. Thus, the inverse transformation of Eq.(18) gives

$$\begin{aligned} &{}^{(\overline{5}_{10_1})}\mathsf{D}_a = V_{d_{11}}\mathbf{H_d}_a+\cdots, \quad {}^{(\overline{5}_{10_2})}\mathsf{D}_a = V_{d_{21}}\mathbf{H_d}_a+\cdots, \quad {}^{(\overline{5}_{120})}\mathsf{D}_a = V_{d_{31}}\mathbf{H_d}_a+\cdots, \\ &{}^{(\overline{5}_{126})}\mathsf{D}_a = V_{d_{41}}\mathbf{H_d}_a+\cdots, \quad {}^{(\overline{5}_{210})}\mathsf{D}_a = V_{d_{51}}\mathbf{H_d}_a+\cdots, \quad {}^{(\overline{45}_{120})}\mathsf{D}_a = V_{d_{61}}\mathbf{H_d}_a+\cdots, \\ &{}^{(\overline{45}_{\overline{126}})}\mathsf{D}_a = V_{d_{71}}\mathbf{H_d}_a+\cdots, \end{aligned} \tag{19}$$

and

$$\begin{aligned} &{}^{(5_{10_1})}\mathsf{D}^a = U_{d_{11}}\mathbf{H_u}^a+\cdots, \quad {}^{(5_{10_2})}\mathsf{D}^a = U_{d_{21}}\mathbf{H_u}^a+\cdots, \quad {}^{(5_{120})}\mathsf{D}^a = U_{d_{31}}\mathbf{H_u}^a+\cdots, \\ &{}^{(5_{\overline{126}})}\mathsf{D}^a = U_{d_{41}}\mathbf{H_u}^a+\cdots, \quad {}^{(5_{210})}\mathsf{D}^a = U_{d_{51}}\mathbf{H_u}^a+\cdots, \quad {}^{(45_{120})}\mathsf{D}^a = U_{d_{61}}\mathbf{H_u}^a+\cdots, \\ &{}^{(45_{126})}\mathsf{D}^a = U_{d_{71}}\mathbf{H_u}^a+\cdots. \end{aligned} \tag{20}$$

where $+\cdots$ stand for heavy Higgs doublet fields. The heavy Higgs doublets are to be integrated out and do not appear in the effective low energy theory. The matrix elements $U_{d_{k1}}$ and $V_{d_{k1}}$ are functions of $\mathbf{S}_{24_{210}}$ except for $k=4,\ 5,\ 7$ for which $U_{d_{k1}}=0=V_{d_{k1}}$. The numerical values of $U_{d_{k1}}$ and $V_{d_{k1}}$, $k=1,\ 2,\ 3,\ 6$ are given in Table 3 and Table 4. Again note that in Tables 3 and 4 corresponding to each set of $\mathcal{M}_\Delta$ and $\mathcal{M}_\Phi$, there exists three solutions for $U_{d_{k1}}$ and $V_{d_{k1}}$. This is simply because $\mathbf{S}_{24_{210}}$ satisfies a cubic equation Eq.(7).

In summary, all the eigenvalues of the doublet Higgs mass matrix given by Eq.(14) are superheavy except one pair which is massless and corresponds to the electroweak Higgs doublets of MSSM. The zero eigenmode is determined by transformation matrix elements $U_{d_{k1}}$ and $V_{d_{k1}}$ where $k=1,\cdots,7$.

$\mathcal{M}_\Delta$ (GeV)	$\mathcal{M}_\Phi$ (GeV)	$U_{d_{11}}$	$U_{d_{21}}$	$U_{d_{31}}$	$U_{d_{61}}$
10^{15}	10^{15}	$0.388 + \imath 0.273$	$-0.394 - \imath 0.277$	$-0.00732 - \imath 0.00516$	$-0.602 - \imath 0.424$
		$0.133 - \imath 0.0527$	$-0.203 + \imath 0.174$	$-0.0847 + \imath 0.149$	$-0.821 - \imath 0.453$
		$0.133 + \imath 0.0527$	$-0.203 - \imath 0.174$	$-0.0847 - \imath 0.149$	$-0.821 + \imath 0.453$
6.67×10^{15}	6.67×10^{15}	$-0.341 - \imath 0.330$	$0.347 + \imath 0.335$	$0.00644 + \imath 0.00622$	$0.530 + \imath 0.512$
		$0.0126 - \imath 0.143$	$0.0654 + \imath 0.259$	$0.0954 + \imath 0.142$	$-0.772 + \imath 0.532$
		$0.0126 + \imath 0.143$	$0.0654 - \imath 0.259$	$0.0954 - \imath 0.142$	$-0.772 - \imath 0.532$
1.25×10^{16}	1.25×10^{16}	$-0.474 + \imath 0.0142$	$0.482 - \imath 0.0144$	$0.00895 - \imath 0.000268$	$0.736 - \imath 0.0220$
		$0.132 + \imath 0.0571$	$-0.267 - \imath 0.0200$	$-0.165 + \imath 0.0455$	$-0.260 - \imath 0.900$
		$0.132 - \imath 0.0571$	$-0.267 + \imath 0.0200$	$-0.165 - \imath 0.0455$	$-0.260 + \imath 0.900$
6.67×10^{15}	2×10^{16}	$-0.125 + \imath 0.0773$	$0.130 - \imath 0.0804$	$-0.00623 - \imath 0.00386$	$0.831 - \imath 0.515$
		$-0.0806 - \imath 0.197$	$0.131 + \imath 0.436$	$0.0612 + \imath 0.293$	$-0.576 + \imath 0.571$
		$-0.0806 + \imath 0.197$	$0.131 - \imath 0.436$	$0.0612 - \imath 0.293$	$-0.576 - \imath 0.571$
2×10^{16}	6.67×10^{15}	-0.631	0.634	0.00396	-0.448
		$0.0792 + \imath 0.0411$	$-0.0542 + \imath 0.0561$	$0.0306 + \imath 0.119$	$-0.947 - 0.273$
		$0.0792 - \imath 0.0411$	$-0.0542 - \imath 0.0561$	$0.0306 - \imath 0.119$	$-0.947 + 0.273$

Table 3: A Numerical estimate of the elements of the zero mode eigenvectors using the analysis of Table 2 and under the additional assumption $a = b_{1,2} = c = \overline{c} \sim 1$.

The mass terms for the $\mathsf{SU(3)}$ color triplets in the superpotential can be written as

$$\left({}^{(5_{10_1})}\mathsf{T}^\alpha, {}^{(5_{10_2})}\mathsf{T}^\alpha, {}^{(5_{120})}\mathsf{T}^\alpha, {}^{(5_{\overline{126}})}\mathsf{T}^\alpha, {}^{(5_{210})}\mathsf{T}^\alpha, {}^{(45_{120})}\mathsf{T}^\alpha, {}^{(45_{126})}\mathsf{T}^\alpha, {}^{(50_{\overline{126}})}\mathsf{T}^\alpha\right) M_t \begin{pmatrix} {}^{(\overline{5}_{10_1})}\mathsf{T}_\alpha \\ {}^{(\overline{5}_{10_2})}\mathsf{T}_\alpha \\ {}^{(\overline{5}_{120})}\mathsf{T}_\alpha \\ {}^{(\overline{5}_{126})}\mathsf{T}_\alpha \\ {}^{(\overline{5}_{210})}\mathsf{T}_\alpha \\ {}^{(\overline{45}_{120})}\mathsf{T}_\alpha \\ {}^{(\overline{45}_{\overline{126}})}\mathsf{T}_\alpha \\ {}^{(\overline{50}_{126})}\mathsf{T}_\alpha \end{pmatrix}, \tag{21}$$

where the triplet mass matrix M_t receives contributions from eq. (2) and eq.(10) and is given by

$$M_t = \begin{pmatrix} 0 & 0 & 0 & \mathsf{t}_2 & \mathsf{t}_1 & 0 & \left(\frac{b_1}{a}\right)\mathsf{t}_3 & \mathsf{t}_9 \\ 0 & 0 & 0 & 0 & 0 & 0 & \left(\frac{b_2}{a}\right)\mathsf{t}_3 & 0 \\ 0 & 0 & 0 & \mathsf{t}_5 & \mathsf{t}_4 & 0 & \left(\frac{\overline{c}}{c}\right)\mathsf{t}_6 & 0 \\ \left(\frac{b_1}{a}\right)\mathsf{t}_2 & \left(\frac{b_2}{a}\right)\mathsf{t}_2 & \left(\frac{\overline{c}}{c}\right)\mathsf{t}_5 & \mathsf{t}_{11} & \mathsf{t}_{13} & \left(\frac{\overline{c}}{c}\right)\mathsf{t}_7 & 0 & \mathsf{t}_{16} \\ \left(\frac{b_1}{a}\right)\left(\frac{\mathbf{S}_{1_{\overline{126}}}}{\mathbf{S}_{1_{126}}}\right)\mathsf{t}_1 & \left(\frac{b_2}{a}\right)\left(\frac{\mathbf{S}_{1_{\overline{126}}}}{\mathbf{S}_{1_{126}}}\right)\mathsf{t}_1 & \left(\frac{\overline{c}}{c}\right)\left(\frac{\mathbf{S}_{1_{\overline{126}}}}{\mathbf{S}_{1_{126}}}\right)\mathsf{t}_4 & \left(\frac{\mathbf{S}_{1_{\overline{126}}}}{\mathbf{S}_{1_{126}}}\right)\mathsf{t}_{13} & \mathsf{t}_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathsf{t}_7 & 0 & 0 & \left(\frac{\overline{c}}{c}\right)\mathsf{t}_8 & \mathsf{t}_{10} \\ \mathsf{t}_3 & 0 & \mathsf{t}_6 & 0 & 0 & \mathsf{t}_8 & \mathsf{t}_{14} & 0 \\ \left(\frac{b_1}{a}\right)\mathsf{t}_9 & \left(\frac{b_2}{a}\right)\mathsf{t}_9 & 0 & \mathsf{t}_{16} & 0 & \left(\frac{\overline{c}}{c}\right)\mathsf{t}_{10} & 0 & \mathsf{t}_{15} \end{pmatrix}. \tag{22}$$

Here

$$\mathsf{t}_1 \equiv \frac{a}{2\sqrt{5}}\mathbf{S}_{1_{126}},$$

$\mathcal{M}_\Delta$ (GeV)	$\mathcal{M}_\Phi$ (GeV)	$V_{d_{11}}$	$V_{d_{21}}$	$V_{d_{31}}$	$V_{d_{61}}$
10^{15}	10^{15}	-0.00831	0.547	-0.0102	-0.837
		-0.140	$0.252 - \imath 0.0900$	-0.171	$-0.0119 + \imath 0.0.938$
		-0.140	$0.252 + \imath 0.0900$	-0.171	$-0.0119 - \imath 0.0.938$
6.67×10^{15}	6.67×10^{15}	0.00831	-0.547	0.0102	0.837
		0.140	$-0.252 + \imath 0.0900$	0.171	$0.0119 - \imath 0.938$
		0.140	$-0.252 - \imath 0.0900$	0.171	$0.0119 + \imath 0.938$
1.25×10^{16}	1.25×10^{16}	0.00831	-0.547	0.0102	0.837
		-0.140	$0.252 - \imath 0.0900$	-0.171	$-0.0119 + \imath 0.938$
		-0.140	$0.252 + \imath 0.0900$	-0.171	$-0.0119 - \imath 0.938$
6.67×10^{15}	2×10^{16}	0.00605	-0.154	0.00741	0.988
		0.243	$-0.451 - \imath 0.0385$	0.298	$0.438 - \imath 0.675$
		0.243	$-0.451 + \imath 0.0385$	0.298	$0.438 + \imath 0.675$
2×10^{16}	6.67×10^{15}	-0.00417	0.817	-0.00511	0.577
		0.100	$-0.0408 + \imath 0.0664$	0.123	$-0.500 - \imath 0.848$
		0.100	$-0.0408 - \imath 0.0664$	0.123	$-0.500 + \imath 0.848$

Table 4: A Numerical estimate of the elements of the zero mode eigenvectors using the analysis of Table 2 and under the additional assumption $a = b_{1,2} = c = \overline{c} \sim 1$.

$$
\begin{aligned}
\mathsf{t}_2 &\equiv a\left[-\frac{\sqrt{3}}{10}\mathbf{S}_{1_{210}} + \frac{1}{10\sqrt{3}}\mathbf{S}_{24_{210}}\right], \\
\mathsf{t}_3 &\equiv -a\left[\frac{1}{6\sqrt{2}}\mathbf{S}_{24_{210}} + \frac{1}{12\sqrt{5}}\mathbf{S}_{75_{210}}\right], \\
\mathsf{t}_4 &\equiv -\frac{c}{\sqrt{30}}\mathbf{S}_{1_{126}}, \\
\mathsf{t}_5 &\equiv -c\left[\frac{1}{10\sqrt{2}}\mathbf{S}_{1_{210}} + \frac{1}{20\sqrt{2}}\mathbf{S}_{24_{210}}\right], \\
\mathsf{t}_6 &\equiv -c\left[\frac{1}{24\sqrt{3}}\mathbf{S}_{24_{210}} + \frac{1}{12\sqrt{30}}\mathbf{S}_{75_{210}}\right], \\
\mathsf{t}_7 &\equiv -c\left[\frac{1}{72}\mathbf{S}_{24_{210}} + \frac{1}{36\sqrt{10}}\mathbf{S}_{75_{210}}\right], \\
\mathsf{t}_8 &\equiv -c\left[\frac{1}{20\sqrt{6}}\mathbf{S}_{1_{210}} + \frac{1}{40\sqrt{6}}\mathbf{S}_{24_{210}}\right], \\
\mathsf{t}_9 &\equiv \frac{a}{12\sqrt{5}}\mathbf{S}_{75_{210}}, \\
\mathsf{t}_{10} &\equiv c\left[-\frac{1}{60\sqrt{6}}\mathbf{S}_{24_{210}} + \frac{1}{36\sqrt{15}}\mathbf{S}_{75_{210}}\right], \\
\mathsf{t}_{11} &\equiv 2m_\Delta + \eta\left[-\frac{2}{5\sqrt{15}}\mathbf{S}_{1_{210}} + \frac{1}{10}\sqrt{\frac{3}{5}}\mathbf{S}_{24_{210}}\right], \\
\mathsf{t}_{12} &\equiv 2m_\Phi + \lambda\left[-\frac{3}{10\sqrt{2}}\mathbf{S}_{1_{210}} + \frac{1}{\sqrt{15}}\mathbf{S}_{24_{210}}\right], \\
\mathsf{t}_{13} &\equiv \frac{1}{5}\eta\mathbf{S}_{1_{126}}, \\
\mathsf{t}_{14} &\equiv m_\Delta, \\
\mathsf{t}_{15} &\equiv \frac{1}{6}m_\Delta + \eta\left[\frac{1}{60\sqrt{15}}\mathbf{S}_{1_{210}} + \frac{1}{180\sqrt{15}}\mathbf{S}_{24_{210}} + \frac{1}{60\sqrt{6}}\mathbf{S}_{75_{210}}\right], \\
\mathsf{t}_{16} &\equiv -\frac{2}{45}\eta\mathbf{S}_{75_{210}}. \qquad (23)
\end{aligned}
$$

Substitution of $\mathbf{S}_{\mathbf{1_{210}}}$, $\mathbf{S}_{75_{210}}$, $\mathbf{S}_{1_{126}}$, $\mathbf{S}_{1_{\overline{126}}}$ in terms of $\mathbf{S}_{24_{210}}$, gives all the matrix elements of the Higgs triplet mass matrix in terms of one VEV. The Higgs triplet mass matrix M_t is diagonalized by an 8×8 biunitary transformation

$$U_t{}^{\dagger} M_t V_t = \mathrm{diag}\left(\mathsf{m}_{t_1}, \mathsf{m}_{t_2}, \cdots, \mathsf{m}_{t_8}\right). \tag{24}$$

There is no zero mode in the triplet mass matrix and all the eigenvalues of this matrix are superheavy. The triplet Higgs mass spectrum is, of course, central to the study of baryon and lepton number violating dimension five operators leading to proton decay (for a review see [50, 51]), which can act as a discriminant for a variety of GUT and string models (see, e.g., [52]).

4 $\mathsf{B}-\mathsf{L}=-2$ operators from cubic matter-Higgs interactions

In this section we compute the $\mathsf{B}-\mathsf{L}=-2$ interactions arising in the model discussed in section 2. The $\mathsf{B}-\mathsf{L}$ violating interactions arise as a consequence of the singlets of $\mathsf{126}$ and $\overline{\mathsf{126}}$ gaining VEVs. In turn this VEV formation gives mass to the singlets of the $\mathsf{16}$-plets of matter. Thus the heavy fields in the model after spontaneous breaking of the GUT symmetry consist of all of the Higgs fields except for a pair of light Higgs doublets and in addition the singlet fields arising from the $\mathsf{16}$-plets of matter. From the couplings of Higgs with matter we are interested in pulling out only the parts that give $\mathsf{B}-\mathsf{L}=-2$. To obtain a low energy effective Lagrangian which contains $\mathsf{B}-\mathsf{L}=-2$ violations, we integrate on the heavy fields which can generate such interactions. In this analysis we will focus on $B-L=-2$ interactions arising from the elimination of $\mathsf{5}+\overline{\mathsf{5}}$, $\mathsf{45}+\overline{\mathsf{45}}$ fields (excluding the light modes) and the matter singlets. The elimination of $\mathsf{10}+\overline{\mathsf{10}}$ will not be considered as this requires a further overlapping analysis of Goldstones in the $SO(10)$ symmetry breaking and their absorption in the $\mathsf{10}+\overline{\mathsf{10}}$ gauge vector bosons to make them heavy in the symmetry breaking of $SO(10)$ and a full analysis of this is outside the scope of this work. Returning to the integration over $\mathsf{5}+\overline{\mathsf{5}}$, $\mathsf{45}+\overline{\mathsf{45}}$ extra care is needed in handling the integration. This is due to a mixing between the doublets and the triplets arising from $\mathsf{5}+\overline{\mathsf{5}}$ and $\mathsf{45}+\overline{\mathsf{45}}$. Further, the doublet mass matrix has a zero mode which must be extracted before integration on the heavy Higgs doublets can be performed. Similarly, integration on the $\mathsf{45}+\overline{\mathsf{45}}$ requires that we first extract out the doublet and the triplets modes before integration on them. We follow the following path in integration of the heavy fields: First we integrate on the matter singlets and then integrate on the remaining heavy Higgs fields. Another integration path is found to give the same result.

We begin by displaying the cubic matter-Higgs couplings which consists of $\mathsf{16}\cdot\mathsf{16}\cdot\mathsf{10}$, $\mathsf{16}\cdot\mathsf{16}\cdot\mathsf{120}$ and $\mathsf{16}\cdot\mathsf{16}\cdot\overline{\mathsf{126}}$ couplings. In $\mathsf{SU}(5)$ decomposition (for notation see Appendix A) they are given by [42]

$$\begin{aligned}
W^{(16\cdot 16\cdot 10)} &= \imath 2\sqrt{2} f^{(10_r+)}_{\acute{x}\acute{y}} \left(\mathsf{M}^{ij}_{\acute{x}}\ \mathsf{M}_{\acute{y}i}\ \mathsf{H}^{(10_r)}_{j} - \mathsf{M}_{\acute{x}}\ \mathsf{M}_{\acute{y}i}\ \mathsf{H}^{(10_r)i} + \frac{1}{8}\epsilon_{ijklm}\ \mathsf{M}^{ij}_{\acute{x}}\ \mathsf{M}^{kl}_{\acute{y}}\ \mathsf{H}^{(10_r)m}\right), && (25)\\
W^{(16\cdot 16\cdot 120)} &= \imath\frac{2}{\sqrt{3}} f^{(120-)}_{\acute{x}\acute{y}} \Big(2\mathsf{M}_{\acute{x}}\ \mathsf{M}_{\acute{y}i}\ \mathsf{H}^{(120)i} + \mathsf{M}^{ij}_{\acute{x}}\ \mathsf{M}_{\acute{y}}\ \mathsf{H}^{(120)}_{ij} + \mathsf{M}_{\acute{x}i}\ \mathsf{M}_{\acute{y}j}\ \mathsf{H}^{(120)ij}\\
&\quad -\mathsf{M}^{ij}_{\acute{x}}\ \mathsf{M}_{\acute{y}i}\ \mathsf{H}^{(120)}_{j} + \mathsf{M}_{\acute{x}i}\ \mathsf{M}^{jk}_{\acute{y}}\ \mathsf{H}^{(120)i}_{jk} - \frac{1}{4}\epsilon_{ijklm}\ \mathsf{M}^{ij}_{\acute{x}}\ \mathsf{M}^{mn}_{\acute{y}}\ \mathsf{H}^{(120)kl}_{n}\Big), && (26)\\
W^{(16\cdot 16\cdot \overline{126})} &= \imath\sqrt{\frac{2}{15}} f^{(\overline{126}+)}_{\acute{x}\acute{y}} \Big(-\sqrt{2}\mathsf{M}_{\acute{x}}\ \mathsf{M}_{\acute{y}}\ \mathsf{H}^{(\overline{126})} - \sqrt{3}\mathsf{M}_{\acute{x}}\ \mathsf{M}_{\acute{y}i}\ \mathsf{H}^{(\overline{126})i} + \mathsf{M}_{\acute{x}}\ \mathsf{M}^{ij}_{\acute{y}}\ \mathsf{H}^{(\overline{126})}_{ij}\\
&\quad -\frac{1}{8\sqrt{3}}\epsilon_{ijklm}\ \mathsf{M}^{ij}_{\acute{x}}\ \mathsf{M}^{kl}_{\acute{y}}\ \mathsf{H}^{(\overline{126})m} - \mathsf{M}_{\acute{x}i}\ \mathsf{M}_{\acute{y}j}\ \mathsf{H}^{(\overline{126})ij}_{(S)} + \mathsf{M}^{ij}_{\acute{x}}\ \mathsf{M}_{\acute{y}k}\ \mathsf{H}^{(\overline{126})k}_{ij}\\
&\quad -\frac{1}{12\sqrt{2}}\epsilon_{ijklm}\ \mathsf{M}^{lm}_{\acute{x}}\ \mathsf{M}^{rs}_{\acute{y}}\ \mathsf{H}^{(\overline{126})ijk}_{rs}\Big), && (27)
\end{aligned}$$

where the front factors $f^{(\cdot\pm)}_{\acute{x}\acute{y}}$ in Eq.(25)-Eq.(27) exhibit the symmetry and anti-symmetry in the generation indices: $f^{(\cdot\pm)}_{\acute{x}\acute{y}} = \frac{1}{2}\left(f^{(\cdot)}_{\acute{x}\acute{y}} \pm f^{(\cdot)}_{\acute{y}\acute{x}}\right)$.

Next we assume that because of spontaneous symmetry breaking the singlet field in the $\overline{\mathsf{126}}$-plet of Higgs field develops a VEV, i.e., $<\mathsf{H}^{(\overline{126})}> \ \equiv \mathbf{S}_{1_{\overline{126}}} \neq 0$, which gives mass to the singlets in the $\mathsf{16}$-plet of matter fields. Collecting the terms which contain the singlet fields of matter from Eq.(25)-Eq.(27) we have

$$\begin{aligned}
W &= \mathsf{M}_{\acute{x}}\Big\{-\imath 2\sqrt{2} f^{(10_r+)}_{\acute{x}\acute{y}}\mathsf{M}_{\acute{y}i}\ \mathsf{H}^{(10_r)i} + \imath\frac{4}{\sqrt{3}} f^{(120-)}_{\acute{x}\acute{y}}\mathsf{M}_{\acute{y}i}\ \mathsf{H}^{(120)i}\\
&\quad -\imath\sqrt{\frac{2}{5}} f^{(\overline{126}+)}_{\acute{x}\acute{y}}\mathsf{M}_{\acute{y}i}\ \mathsf{H}^{(\overline{126})i}\Big\} + \frac{1}{2}\mathsf{M}_{\acute{x}}\left\{-\imath\frac{4}{\sqrt{15}} f^{(\overline{126}+)}_{\acute{x}\acute{y}}\mathbf{S}_{1_{\overline{126}}}\right\}\mathsf{M}_{\acute{y}} && (28)
\end{aligned}$$

In the above equation the mass term for 1_{16} violates $\mathsf{B}-\mathsf{L}$. Next eliminating $\mathsf{M}_{\acute{x}}$ through $\frac{\partial W}{\partial \mathsf{M}_{\acute{x}}}=0$, we get the following 4-point matter-Higgs interactions:

$$W=\sum_{i=1}^{6} W_i, \tag{29}$$

where

$$W_1=\frac{1}{\mathbf{S}_{1_{\overline{126}}}}\imath\sqrt{15}\ \mathsf{M}_{\acute{x}i}\ \mathsf{H}^{(10_r)i}\left[f^{(10_r+)}f^{(\overline{126}+)^{-1}}f^{(10_s+)}\right]_{\acute{x}\acute{y}}\mathsf{M}_{\acute{y}j}\ \mathsf{H}^{(10_s)j}, \tag{30}$$

$$W_2=-\frac{1}{\mathbf{S}_{1_{\overline{126}}}}\imath 2\sqrt{\frac{5}{3}}\ \mathsf{M}_{\acute{x}i}\ \mathsf{H}^{(120)i}\left[f^{(120-)}f^{(\overline{126}+)^{-1}}f^{(120-)}\right]_{\acute{x}\acute{y}}\mathsf{M}_{\acute{y}j}\ \mathsf{H}^{(120)j}, \tag{31}$$

$$W_3=\frac{1}{\mathbf{S}_{1_{\overline{126}}}}\frac{\imath}{4}\sqrt{\frac{3}{5}}\ \mathsf{M}_{\acute{x}i}\ \mathsf{H}^{(\overline{126})i}\ f^{(\overline{126}+)}_{\acute{x}\acute{y}}\ \mathsf{M}_{\acute{y}j}\ \mathsf{H}^{(\overline{126})j}, \tag{32}$$

$$W_4=-\frac{1}{\mathbf{S}_{1_{\overline{126}}}}\imath 2\sqrt{10}\ \mathsf{M}_{\acute{x}i}\ \mathsf{H}^{(10_r)i}\left[f^{(10_r+)}f^{(\overline{126}+)^{-1}}f^{(120-)}\right]_{\acute{x}\acute{y}}\mathsf{M}_{\acute{y}j}\ \mathsf{H}^{(120)j}, \tag{33}$$

$$W_5=\frac{1}{\mathbf{S}_{1_{\overline{126}}}}\imath\sqrt{2}\ \mathsf{M}_{\acute{x}i}\ \mathsf{H}^{(120)i}\ f^{(120-)}_{\acute{x}\acute{y}}\ \mathsf{M}_{\acute{y}j}\ \mathsf{H}^{(\overline{126})j}, \tag{34}$$

$$W_6=\frac{1}{\mathbf{S}_{1_{\overline{126}}}}\imath\sqrt{3}\ \mathsf{M}_{\acute{x}i}\ \mathsf{H}^{(10_r)i}\ f^{(10_r+)}_{\acute{x}\acute{y}}\ \mathsf{M}_{\acute{y}j}\ \mathsf{H}^{(\overline{126})j}. \tag{35}$$

The $\mathbf{5}$-plets of Higgs can produce heavy Higgs doublets and triplets and their decays violate $\mathsf{B}-\mathsf{L}$. Further their couplings carry new sources of CP violation not subject to CKM constraints and can be large. Thus these decays can be used to produce GUT scale baryogenesis using standard techniques (see, for e.g., [24, 25, 53]). As pointed out in [24, 25] a baryon number excess produced this way in the early universe will not be washed away by sphaleron interactions at the electroweak scale. We now compute the relevant $d=5$, $d=7$ and $d=9$ operators arising from the above matter-Higgs interactions.

4.1 Operators arising from $\mathsf{M}_{\acute{x}i}\ \mathsf{H}^{(10_r)i}\ \mathsf{M}_{\acute{y}j}\ \mathsf{H}^{(10_s)j}$

Here the Higgs fields could be doublets or triplets. Thus we have three possibilities, i.e., that both the fields are doublets, both are triplets or one is a doublet and the other a triplet. Thus we write

$$\begin{aligned} W_1 &= \frac{1}{\mathbf{S}_{1_{\overline{126}}}}\imath\sqrt{15}\ \mathsf{M}_{\acute{x}i}\ \mathsf{H}^{(10_r)i}\left[f^{(10_r+)}f^{(\overline{126}+)^{-1}}f^{(10_s+)}\right]_{\acute{x}\acute{y}}\mathsf{M}_{\acute{y}j}\ \mathsf{H}^{(10_s)j} \\ &= W_1^{DD}+W_1^{TT}+W_1^{DT}, \end{aligned} \tag{36}$$

where

$$W_1^{DD} \equiv \mathcal{E}^{(rs)}_{\acute{x}\acute{y}}\ \mathsf{M}_{\acute{x}a}\ \mathsf{M}_{\acute{y}b}\ \ \mathsf{H}^{(10_r)a}\ \mathsf{H}^{(10_s)b}, \tag{37}$$

$$W_1^{TT} \equiv \mathcal{E}^{(rs)}_{\acute{x}\acute{y}}\ \mathsf{M}_{\acute{x}\alpha}\ \mathsf{M}_{\acute{y}\beta}\ \ \mathsf{H}^{(10_r)\alpha}\ \mathsf{H}^{(10_s)\beta}, \tag{38}$$

$$W_1^{DT} \equiv 2\mathcal{E}^{(rs)}_{\acute{x}\acute{y}}\ \mathsf{M}_{\acute{x}a}\ \mathsf{M}_{\acute{y}\alpha}\ \ \mathsf{H}^{(10_r)a}\ \mathsf{H}^{(10_s)\alpha}, \tag{39}$$

and where

$$\mathcal{E}^{(rs)}_{\acute{x}\acute{y}} \equiv \frac{1}{\mathbf{S}_{1_{\overline{126}}}}\left(\imath\sqrt{15}\right)\left[f^{(10_r+)}f^{(\overline{126}+)^{-1}}f^{(10_s+)}\right]_{\acute{x}\acute{y}}. \tag{40}$$

Next we obtain effective operators at low energy from each of the terms $W_1^{DD}, W_1^{TT}, W_1^{DT}$.

4.1.1 Evaluating W_1^{DD}

$$
\begin{aligned}
W_1^{DD\prime} \quad = \quad & \sum_{r,s=1}^{2} \mathcal{E}^{(rs)}_{\acute{x}\acute{y}} \; \mathsf{M}_{\acute{x}a} \; \mathsf{M}_{\acute{y}b} \left[U_{d_{r1}} \; \mathbf{H_u}^a + \sum_{M=2}^{7} U_{d_{rM}} \; \mathcal{H}^a_{\mathbf{u}M} \right] \left[U_{d_{s1}} \; \mathbf{H_u}^b + \sum_{N=2}^{7} U_{d_{sN}} \; \mathcal{H}^b_{\mathbf{u}N} \right] \\
& + \frac{\imath}{2\sqrt{2}} \epsilon_{ijkla} \; \mathsf{M}^{ij}_{\acute{x}} \; \mathsf{M}^{kl}_{\acute{y}} \sum_{N=2}^{7} \sum_{r=1}^{2} f^{(10_r+)}_{\acute{x}\acute{y}} U_{d_{rN}} \; \mathcal{H}^a_{\mathbf{u}N} \\
& + \imath 2\sqrt{2} \; \mathsf{M}^{ia}_{\acute{x}} \; \mathsf{M}_{\acute{y}i} \sum_{N=2}^{7} \sum_{r=1}^{2} f^{(10_r+)}_{\acute{x}\acute{y}} V_{d_{rN}} \; \mathcal{H}_{\mathbf{d}Na} \\
& + \frac{1}{2} \sum_{N=2}^{7} \mathsf{m}_{d_N} \; \mathcal{H}^a_{\mathbf{u}N} \; \mathcal{H}_{\mathbf{d}Na}.
\end{aligned} \tag{41}
$$

In Eq.(41), for example, $\mathcal{H}_{\mathbf{d}2a} \equiv {}^2\mathsf{D}'_a$, $\mathcal{H}_{\mathbf{d}3a} \equiv {}^3\mathsf{D}'_a, \ldots\ldots, \mathcal{H}^a_{\mathbf{u}4} \equiv {}^4\mathsf{D}'^a$, $\mathcal{H}^a_{\mathbf{u}5} \equiv {}^5\mathsf{D}'^a, .$ and that the second and third lines come from the $\mathsf{16} \cdot \mathsf{16} \cdot \mathsf{10}$ coupling. Eliminating $\mathcal{H}^a_{\mathbf{u}N}$ and $\mathcal{H}_{\mathbf{d}Na}$ through $\dfrac{\partial W_1^{DD\prime}}{\partial \mathcal{H}^a_{\mathbf{u}N}} = 0$ and $\dfrac{\partial W_1^{DD\prime}}{\partial \mathcal{H}_{\mathbf{d}Na}} = 0$, we get $\mathsf{B} - \mathsf{L} = -2$ operators with four fields consisting of two matter fields and two light Higgs fields, operators with five fields one of which is a light Higgs field and the other matter fields, and operators with six matter fields. In addition we also get $\mathsf{B} = 0$, $\mathsf{L} = 0$ operators with four matter fields. Thus we have

$$
\begin{aligned}
W_1^{DD\prime} = \quad & \sum_{r,s=1}^{2} \mathcal{E}^{(rs)}_{\acute{x}\acute{y}} U_{d_{r1}} U_{d_{s1}} \mathbf{L}_{\acute{x}a} \; \mathbf{L}_{\acute{y}b} \; \mathbf{H_u}^a \; \mathbf{H_u}^b \\
& + 16 \Big[\mathbf{L}_{\acute{u}a} \; \mathbf{L}_{\acute{v}b} \; \mathbf{Q}^{a\alpha}_{\acute{w}} \; \mathbf{D}^{\mathsf{c}}_{\acute{x}\alpha} \; \mathbf{Q}^{b\beta}_{\acute{y}} \; \mathbf{D}^{\mathsf{c}}_{\acute{z}\beta} + \epsilon^{ac} \epsilon^{bd} \; \mathbf{L}_{\acute{u}a} \; \mathbf{L}_{\acute{v}b} \; \mathbf{E}^{\mathsf{c}}_{\acute{w}} \; \mathbf{L}_{\acute{x}c} \; \mathbf{E}^{\mathsf{c}}_{\acute{y}} \; \mathbf{L}_{\acute{z}d} \\
& + \epsilon^{bc} \; \mathbf{L}_{\acute{u}a} \; \mathbf{L}_{\acute{v}b} \; \mathbf{Q}^{a\alpha}_{\acute{w}} \; \mathbf{D}^{\mathsf{c}}_{\acute{x}\alpha} \; \mathbf{E}^{\mathsf{c}}_{\acute{y}} \; \mathbf{L}_{\acute{z}c} + \epsilon^{ac} \; \mathbf{L}_{\acute{u}a} \; \mathbf{L}_{\acute{v}b} \; \mathbf{E}^{\mathsf{c}}_{\acute{w}} \; \mathbf{L}_{\acute{x}c} \; \mathbf{Q}^{b\alpha}_{\acute{y}} \; \mathbf{D}^{\mathsf{c}}_{\acute{z}\alpha} \Big] \\
& \times \sum_{p,q=1}^{2} \left(\sum_{M=2}^{7} \frac{U_{d_{pM}} \left\{ \sum_{s=1}^{2} f^{(10_s+)}_{\acute{w}\acute{x}} V_{d_{sM}} \right\}}{\mathsf{m}_{d_M}} \right) \mathcal{E}^{(pq)}_{\acute{u}\acute{v}} \left(\sum_{N=2}^{7} \frac{U_{d_{qN}} \left\{ \sum_{r=1}^{2} f^{(10_r+)}_{\acute{y}\acute{z}} V_{d_{rN}} \right\}}{\mathsf{m}_{d_N}} \right) \\
& + \imath 8\sqrt{2} \Big[\mathbf{L}_{\acute{w}a} \; \mathbf{L}_{\acute{x}b} \; \mathbf{Q}^{b\alpha}_{\acute{y}} \; \mathbf{D}^{\mathsf{c}}_{\acute{z}\alpha} \; \mathbf{H_u}^a + \epsilon^{bc} \; \mathbf{L}_{\acute{w}a} \; \mathbf{L}_{\acute{x}b} \; \mathbf{E}^{\mathsf{c}}_{\acute{y}} \; \mathbf{L}_{\acute{z}c} \; \mathbf{H_u}^a \Big] \\
& \times \sum_{N=2}^{7} \frac{\left\{ \sum_{p,q=1}^{2} U_{d_{p1}} \mathcal{E}^{(pq)}_{\acute{w}\acute{x}} U_{d_{qN}} \right\} \left\{ \sum_{s=1}^{2} f^{(10_s+)}_{\acute{y}\acute{z}} V_{d_{sN}} \right\}}{\mathsf{m}_{d_N}} \\
& - 16 \Big[\mathbf{E}^{\mathsf{c}}_{\acute{w}} \; \mathbf{L}_{\acute{x}a} \; \mathbf{U}^{\mathsf{c}}_{\acute{y}\alpha} \; \mathbf{Q}^{a\alpha}_{\acute{z}} + \epsilon_{ab} \; \mathbf{Q}^{a\alpha}_{\acute{w}} \; \mathbf{D}^{\mathsf{c}}_{\acute{x}\alpha} \; \mathbf{U}^{\mathsf{c}}_{\acute{y}\beta} \; \mathbf{Q}^{b\beta}_{\acute{z}} \Big] \\
& \times \sum_{N=2}^{7} \frac{\left\{ \sum_{r=1}^{2} f^{(10_r+)}_{\acute{w}\acute{x}} U_{d_{rN}} \right\} \left\{ \sum_{s=1}^{2} f^{(10_s+)}_{\acute{y}\acute{z}} V_{d_{sN}} \right\}}{\mathsf{m}_{d_N}}.
\end{aligned} \tag{42}
$$

4.1.2 Evaluating W_1^{TT}

For the case when both the Higgs fields are triplet, they are both superheavy and their elimination leads to the following set of terms

$$
\begin{aligned}
W_1^{TT\prime} = \quad & 16 \Big[\epsilon^{\alpha\gamma\delta} \epsilon^{\beta\mu\nu} \; \mathbf{D}^{\mathsf{c}}_{\acute{u}\alpha} \; \mathbf{D}^{\mathsf{c}}_{\acute{v}\beta} \; \mathbf{U}^{\mathsf{c}}_{\acute{w}\gamma} \; \mathbf{D}^{\mathsf{c}}_{\acute{x}\delta} \; \mathbf{U}^{\mathsf{c}}_{\acute{y}\mu} \; \mathbf{D}^{\mathsf{c}}_{\acute{z}\nu} + \mathbf{D}^{\mathsf{c}}_{\acute{u}\alpha} \; \mathbf{D}^{\mathsf{c}}_{\acute{v}\beta} \; \mathbf{Q}^{a\alpha}_{\acute{w}} \; \mathbf{L}_{\acute{x}a} \; \mathbf{Q}^{b\beta}_{\acute{y}} \; \mathbf{L}_{\acute{z}b} \\
& + \epsilon^{\alpha\gamma\delta} \; \mathbf{D}^{\mathsf{c}}_{\acute{u}\alpha} \; \mathbf{D}^{\mathsf{c}}_{\acute{v}\beta} \; \mathbf{U}^{\mathsf{c}}_{\acute{w}\gamma} \; \mathbf{D}^{\mathsf{c}}_{\acute{x}\delta} \; \mathbf{Q}^{a\beta}_{\acute{y}} \; \mathbf{L}_{\acute{z}a} + \epsilon^{\beta\gamma\delta} \; \mathbf{D}^{\mathsf{c}}_{\acute{u}\alpha} \; \mathbf{D}^{\mathsf{c}}_{\acute{v}\beta} \; \mathbf{Q}^{a\alpha}_{\acute{w}} \; \mathbf{L}_{\acute{x}a} \; \mathbf{U}^{\mathsf{c}}_{\acute{y}\gamma} \; \mathbf{D}^{\mathsf{c}}_{\acute{z}\delta} \Big] \\
& \times \sum_{p,q=1}^{2} \left(\sum_{M=1}^{8} \frac{U_{t_{pM}} \left\{ \sum_{s=1}^{2} f^{(10_s+)}_{\acute{w}\acute{x}} V_{t_{sM}} \right\}}{\mathsf{m}_{t_M}} \right) \mathcal{E}^{(pq)}_{\acute{u}\acute{v}} \left(\sum_{N=1}^{8} \frac{U_{t_{qN}} \left\{ \sum_{r=1}^{2} f^{(10_r+)}_{\acute{y}\acute{z}} V_{t_{rN}} \right\}}{\mathsf{m}_{t_N}} \right) \\
& + 8 \Big[2\epsilon^{\alpha\beta\gamma} \; \mathbf{U}^{\mathsf{c}}_{\acute{w}\alpha} \; \mathbf{D}^{\mathsf{c}}_{\acute{x}\beta} \; \mathbf{E}^{\mathsf{c}}_{\acute{y}} \; \mathbf{U}^{\mathsf{c}}_{\acute{z}\gamma} + 2\epsilon_{ab} \; \mathbf{U}^{\mathsf{c}}_{\acute{w}\alpha} \; \mathbf{D}^{\mathsf{c}}_{\acute{x}\beta} \; \mathbf{Q}^{a\beta}_{\acute{y}} \; \mathbf{Q}^{b\alpha}_{\acute{z}} \\
& + 2 \; \mathbf{Q}^{a\alpha}_{\acute{w}} \; \mathbf{L}_{\acute{x}a} \; \mathbf{E}^{\mathsf{c}}_{\acute{y}} \; \mathbf{U}^{\mathsf{c}}_{\acute{z}\alpha} - \epsilon_{bc} \epsilon_{\alpha\beta\gamma} \; \mathbf{Q}^{a\alpha}_{\acute{w}} \; \mathbf{L}_{\acute{x}a} \; \mathbf{Q}^{b\beta}_{\acute{y}} \; \mathbf{Q}^{c\gamma}_{\acute{z}} \Big]
\end{aligned}
$$

$$\times \sum_{N=1}^{8} \frac{\left\{\sum_{r=1}^{2} f^{(10_r+)}_{\acute{w}\acute{x}} U_{t_rN}\right\}\left\{\sum_{s=1}^{2} f^{(10_s+)}_{\acute{y}\acute{z}} V_{t_sN}\right\}}{\mathsf{m}_{t_N}}. \tag{43}$$

Here the top two lines give us the $\mathsf{B}-\mathsf{L}=-2$ operators with six matter fields while the bottom two lines give us $\mathsf{B}-\mathsf{L}=0$ operators with four matter fields which include B violating and L violating operators.

4.1.3 Evaluating W_1^{DT}

Here we have one Higgs field which is a doublet while the other field is a triplet. Since the doublet fields have both light and heavy modes while the triplets are all heavy, we get a combination of $\mathsf{B}-\mathsf{L}=-2$ operators with five fields and with six fields. Additionally we get $\mathsf{B}-\mathsf{L}=0$ operators with four fields as follows

$$\begin{aligned}
W_1^{DT\prime} = \quad & -\imath 8\sqrt{2}\left[\epsilon^{\alpha\beta\gamma}\ \mathbf{D}^{\mathsf{c}}_{\acute{w}\alpha}\ \mathbf{L}_{\acute{x}a}\ \mathbf{U}^{\mathsf{c}}_{\acute{y}\beta}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\gamma}\ \mathbf{H_u}{}^{a} + \mathbf{D}^{\mathsf{c}}_{\acute{w}\alpha}\ \mathbf{L}_{\acute{x}a}\ \mathbf{Q}^{b\alpha}_{\acute{y}}\ \mathbf{L}_{\acute{z}b}\ \mathbf{H_u}{}^{a}\right] \\
& \times \sum_{N=1}^{8} \frac{\left\{\sum_{p,q=1}^{2} U_{d_{p1}} \mathcal{E}^{(pq)}_{\acute{w}\acute{x}} U_{t_qN}\right\}\left\{\sum_{s=1}^{2} f^{(10_s+)}_{\acute{y}\acute{z}} V_{t_sN}\right\}}{\mathsf{m}_{t_N}} \\
& -64\left[\epsilon^{ab}\mathbf{L}_{\acute{u}a}\ \mathbf{D}^{\mathsf{c}}_{\acute{v}\alpha}\ \mathbf{E}^{\mathsf{c}}_{\acute{w}}\ \mathbf{L}_{\acute{x}b}\ \mathbf{Q}^{c\alpha}_{\acute{y}}\ \mathbf{L}_{\acute{z}c} + \epsilon^{\alpha\beta\gamma}\ \mathbf{L}_{\acute{u}a}\ \mathbf{D}^{\mathsf{c}}_{\acute{v}\alpha}\ \mathbf{Q}^{a\rho}_{\acute{w}}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\rho}\ \mathbf{U}^{\mathsf{c}}_{\acute{y}\beta}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\gamma}\right. \\
& \left.+\epsilon^{\alpha\beta\gamma}\epsilon^{ab}\ \mathbf{L}_{\acute{u}a}\ \mathbf{D}^{\mathsf{c}}_{\acute{v}\alpha}\ \mathbf{E}^{\mathsf{c}}_{\acute{w}}\ \mathbf{L}_{\acute{x}b}\ \mathbf{U}^{\mathsf{c}}_{\acute{y}\beta}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\gamma} + \mathbf{L}_{\acute{u}a}\ \mathbf{D}^{\mathsf{c}}_{\acute{v}\alpha}\ \mathbf{Q}^{a\beta}_{\acute{w}}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\beta}\ \mathbf{Q}^{b\alpha}_{\acute{y}}\ \mathbf{L}_{\acute{z}b}\right] \\
& \times \sum_{p,q=1}^{2}\left(\sum_{M=2}^{7} \frac{U_{d_pM}\left\{\sum_{s=1}^{2} f^{(10_s+)}_{\acute{w}\acute{x}} V_{d_sM}\right\}}{\mathsf{m}_{d_M}}\right) \mathcal{E}^{(pq)}_{\acute{u}\acute{v}} \left(\sum_{N=1}^{8} \frac{U_{t_qN}\left\{\sum_{r=1}^{2} f^{(10_r+)}_{\acute{y}\acute{z}} V_{t_rN}\right\}}{\mathsf{m}_{t_N}}\right) \\
& +8\left[2\epsilon^{\alpha\beta\gamma}\ \mathbf{U}^{\mathsf{c}}_{\acute{w}\alpha}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\beta}\ \mathbf{E}^{\mathsf{c}}_{\acute{y}}\ \mathbf{U}^{\mathsf{c}}_{\acute{z}\gamma} + 2\epsilon_{ab}\ \mathbf{U}^{\mathsf{c}}_{\acute{w}\alpha}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\beta}\ \mathbf{Q}^{a\beta}_{\acute{y}}\ \mathbf{Q}^{b\alpha}_{\acute{z}}\right. \\
& \left.+2\ \mathbf{Q}^{a\alpha}_{\acute{w}}\ \mathbf{L}_{\acute{x}a}\ \mathbf{E}^{\mathsf{c}}_{\acute{y}}\ \mathbf{U}^{\mathsf{c}}_{\acute{z}\alpha} - \epsilon_{bc}\epsilon_{\alpha\beta\gamma}\ \mathbf{Q}^{a\alpha}_{\acute{w}}\ \mathbf{L}_{\acute{x}a}\ \mathbf{Q}^{b\beta}_{\acute{y}}\ \mathbf{Q}^{c\gamma}_{\acute{z}}\right] \\
& \times \sum_{N=1}^{8} \frac{\left\{\sum_{r=1}^{2} f^{(10_r+)}_{\acute{w}\acute{x}} U_{t_rN}\right\}\left\{\sum_{s=1}^{2} f^{(10_s+)}_{\acute{y}\acute{z}} V_{t_sN}\right\}}{\mathsf{m}_{t_N}} \\
& -16\left[\mathbf{E}^{\mathsf{c}}_{\acute{w}}\ \mathbf{L}_{\acute{x}a}\ \mathbf{U}^{\mathsf{c}}_{\acute{y}\alpha}\ \mathbf{Q}^{a\alpha}_{\acute{z}} + \epsilon_{ab}\ \mathbf{Q}^{a\alpha}_{\acute{w}}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\alpha}\ \mathbf{U}^{\mathsf{c}}_{\acute{y}\beta}\ \mathbf{Q}^{b\beta}_{\acute{z}}\right] \\
& \times \sum_{N=2}^{7} \frac{\left\{\sum_{r=1}^{2} f^{(10_r+)}_{\acute{w}\acute{x}} U_{d_rN}\right\}\left\{\sum_{s=1}^{2} f^{(10_s+)}_{\acute{y}\acute{z}} V_{d_sN}\right\}}{\mathsf{m}_{d_N}}.
\end{aligned} \tag{44}$$

4.2 Operators arising from $\mathsf{M}_{\acute{x}i}\ \mathsf{H}^{(120)i}\ \mathsf{M}_{\acute{y}j}\ \mathsf{H}^{(120)j}$

The analysis of this case is very similar to that of W_2 and the two Higgs fields can be either both doublets, both triplets or one doublet and one triplet. Results are as below

$$\begin{aligned}
W_2 &= -\frac{1}{\mathbf{S}_{1_{\overline{126}}}}\imath 2\sqrt{\frac{5}{3}}\ \mathsf{M}_{\acute{x}i}\ \mathsf{H}^{(120)i}\ \left[f^{(120-)} f^{(\overline{126}+)^{-1}} f^{(120-)}\right]_{\acute{x}\acute{y}} \mathsf{M}_{\acute{y}j}\ \mathsf{H}^{(120)j} \\
&= W_2^{DD} + W_2^{TT} + W_2^{DT},
\end{aligned} \tag{45}$$

where

$$W_2^{DD} \equiv \mathcal{F}_{\acute{x}\acute{y}}\ \mathsf{M}_{\acute{x}a}\ \mathsf{M}_{\acute{y}b}\ \ \mathsf{H}^{(120)a}\ \mathsf{H}^{(120)b}, \tag{46}$$

$$W_2^{TT} \equiv \mathcal{F}_{\acute{x}\acute{y}}\ \mathsf{M}_{\acute{x}\alpha}\ \mathsf{M}_{\acute{y}\beta}\ \ \mathsf{H}^{(120)\alpha}\ \mathsf{H}^{(120)\beta}, \tag{47}$$

$$W_2^{DT} \equiv 2\mathcal{F}_{\acute{x}\acute{y}}\ \mathsf{M}_{\acute{x}a}\ \mathsf{M}_{\acute{y}\alpha}\ \ \mathsf{H}^{(120)a}\ \mathsf{H}^{(120)\alpha}, \tag{48}$$

and

$$\mathcal{F}_{\acute{x}\acute{y}} \equiv -\frac{1}{\mathbf{S}_{1_{\overline{126}}}}\left(\imath 2\sqrt{\frac{5}{3}}\right)\left[f^{(120-)} f^{(\overline{126}+)^{-1}} f^{(120-)}\right]_{\acute{x}\acute{y}}. \tag{49}$$

4.2.1 Evaluating W_2^{DD}

$$
\begin{aligned}
W_2^{DD\prime} = \quad & U_{d_{31}}^2 \mathcal{F}_{\acute{x}\acute{y}}\ \mathbf{L}_{\acute{x}a}\ \mathbf{L}_{\acute{y}b}\ \mathbf{H_u}^a\ \mathbf{H_u}^b \\
& + \frac{8}{3}\mathcal{F}_{\acute{u}\acute{v}} f^{(120-)}_{\acute{w}\acute{x}} f^{(120-)}_{\acute{y}\acute{z}} \Big[\mathbf{L}_{\acute{u}a}\ \mathbf{L}_{\acute{v}b}\ \mathbf{Q}^{a\alpha}_{\acute{w}}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\alpha}\ \mathbf{Q}^{b\beta}_{\acute{y}}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\beta} + \epsilon^{ac}\epsilon^{bd}\ \mathbf{L}_{\acute{u}a}\ \mathbf{L}_{\acute{v}b}\ \mathbf{E}^{\mathsf{c}}_{\acute{w}}\ \mathbf{L}_{\acute{x}c}\ \mathbf{E}^{\mathsf{c}}_{\acute{y}}\ \mathbf{L}_{\acute{z}d} \\
& + 2\epsilon^{bc}\ \mathbf{L}_{\acute{u}a}\ \mathbf{L}_{\acute{v}b}\ \mathbf{Q}^{a\alpha}_{\acute{w}}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\alpha}\ \mathbf{E}^{\mathsf{c}}_{\acute{y}}\ \mathbf{L}_{\acute{z}c} \Big] \left(\sum_{N=2}^{7} \frac{V_{d_{3N}} U_{d_{3N}}}{\mathsf{m}_{d_N}} \right)^2 \\
& - \frac{\imath 8}{\sqrt{3}} U_{d_{31}} \mathcal{F}_{\acute{w}\acute{x}} f^{(120-)}_{\acute{y}\acute{z}} \Big[\mathbf{L}_{\acute{w}a}\ \mathbf{L}_{\acute{x}b}\ \mathbf{Q}^{b\alpha}_{\acute{y}}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\alpha}\ \mathbf{H_u}^a + \epsilon^{bc}\ \mathbf{L}_{\acute{w}a}\ \mathbf{L}_{\acute{x}b}\ \mathbf{E}^{\mathsf{c}}_{\acute{y}}\ \mathbf{L}_{\acute{z}c}\ \mathbf{H_u}^a \Big] \sum_{N=2}^{7} \frac{V_{d_{3N}} U_{d_{3N}}}{\mathsf{m}_{d_N}}. \qquad (50)
\end{aligned}
$$

4.2.2 Evaluating W_2^{TT}

$$
\begin{aligned}
W_2^{TT\prime} = \quad & \frac{8}{3}\mathcal{F}_{\acute{u}\acute{v}} f^{(120-)}_{\acute{w}\acute{x}} f^{(120-)}_{\acute{y}\acute{z}} \Big[\mathbf{D}^{\mathsf{c}}_{\acute{u}\alpha}\ \mathbf{D}^{\mathsf{c}}_{\acute{v}\beta}\ \mathbf{Q}^{a\alpha}_{\acute{w}}\ \mathbf{L}_{\acute{x}a}\ \mathbf{Q}^{b\beta}_{\acute{y}}\ \mathbf{L}_{\acute{z}b} + \epsilon^{\alpha\beta\gamma}\epsilon^{\rho\sigma\lambda}\ \mathbf{D}^{\mathsf{c}}_{\acute{u}\alpha}\ \mathbf{D}^{\mathsf{c}}_{\acute{v}\rho}\ \mathbf{U}^{\mathsf{c}}_{\acute{w}\beta}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\gamma}\ \mathbf{U}^{\mathsf{c}}_{\acute{y}\sigma}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\lambda} \\
& + 2\epsilon^{\alpha\beta\gamma}\ \mathbf{D}^{\mathsf{c}}_{\acute{u}\rho}\ \mathbf{D}^{\mathsf{c}}_{\acute{v}\alpha}\ \mathbf{Q}^{a\rho}_{\acute{w}}\ \mathbf{L}_{\acute{x}a}\ \mathbf{U}^{\mathsf{c}}_{\acute{y}\beta}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\gamma} \Big] \left(\sum_{N=1}^{8} \frac{V_{t_{3N}} U_{t_{3N}}}{\mathsf{m}_{t_N}} \right)^2. \qquad (51)
\end{aligned}
$$

4.2.3 Evaluating W_2^{DT}

$$
\begin{aligned}
W_2^{DT\prime} = \quad & \frac{\imath 8}{\sqrt{3}} U_{d_{31}} \mathcal{F}_{\acute{w}\acute{x}} f^{(120-)}_{\acute{y}\acute{z}} \Big[\epsilon^{\alpha\beta\gamma} \mathbf{L}_{\acute{w}a}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\alpha}\ \mathbf{U}^{\mathsf{c}}_{\acute{y}\beta}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\gamma}\ \mathbf{H_u}^a + \mathbf{L}_{\acute{w}a}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\alpha}\ \mathbf{Q}^{b\alpha}_{\acute{w}}\ \mathbf{L}_{\acute{z}b}\ \mathbf{H_u}^a \Big] \sum_{N=1}^{8} \frac{V_{t_{3N}} U_{t_{3N}}}{\mathsf{m}_{t_N}} \\
& - \frac{32}{\sqrt{3}} \mathcal{F}_{\acute{u}\acute{v}} f^{(120-)}_{\acute{w}\acute{x}} f^{(120-)}_{\acute{y}\acute{z}} \Big[\epsilon^{ab} \mathbf{L}_{\acute{u}a}\ \mathbf{D}^{\mathsf{c}}_{\acute{v}\alpha}\ \mathbf{E}^{\mathsf{c}}_{\acute{w}}\ \mathbf{L}_{\acute{x}b}\ \mathbf{Q}^{c\alpha}_{\acute{y}}\ \mathbf{L}_{\acute{z}c} + \epsilon^{\alpha\beta\gamma}\ \mathbf{L}_{\acute{u}a}\ \mathbf{D}^{\mathsf{c}}_{\acute{v}\alpha}\ \mathbf{Q}^{a\rho}_{\acute{w}}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\rho}\ \mathbf{U}^{\mathsf{c}}_{\acute{y}\beta}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\gamma} \\
& + \epsilon^{\alpha\beta\gamma}\epsilon^{ab}\ \mathbf{L}_{\acute{u}a}\ \mathbf{D}^{\mathsf{c}}_{\acute{v}\alpha}\ \mathbf{E}^{\mathsf{c}}_{\acute{w}}\ \mathbf{L}_{\acute{x}b}\ \mathbf{U}^{\mathsf{c}}_{\acute{y}\beta}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\gamma} + \mathbf{L}_{\acute{u}a}\ \mathbf{D}^{\mathsf{c}}_{\acute{v}\alpha}\ \mathbf{Q}^{a\beta}_{\acute{w}}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\beta}\ \mathbf{Q}^{b\alpha}_{\acute{y}}\ \mathbf{L}_{\acute{z}b} \Big] \\
& \times \left(\sum_{M=2}^{7} \frac{V_{d_{3M}} U_{d_{3M}}}{\mathsf{m}_{d_M}} \right) \left(\sum_{N=1}^{8} \frac{V_{t_{3N}} U_{t_{3N}}}{\mathsf{m}_{t_N}} \right). \qquad (52)
\end{aligned}
$$

4.3 Operators arising from $\mathsf{M}_{\acute{x}i}\ \mathsf{H}^{(\overline{126})i}\ \mathsf{M}_{\acute{y}j}\ \mathsf{H}^{(\overline{126})j}$

$$
W_3 = \frac{1}{\mathbf{S}_{1_{\overline{126}}}} \frac{\imath}{4}\sqrt{\frac{3}{5}}\ \mathsf{M}_{\acute{x}i}\ \mathsf{H}^{(\overline{126})i}\ f^{(\overline{126}+)}_{\acute{x}\acute{y}}\ \mathsf{M}_{\acute{y}j}\ \mathsf{H}^{(\overline{126})j}. \qquad (53)
$$

No contribution. Firstly, because $U_{d_{41}} = 0$ and secondly because there is no $\overline{\mathsf{5}}$ of $\mathsf{SU(5)}$ in $\overline{\mathsf{126}}$ and hence a mass term involving $\mathsf{5}$ and $\overline{\mathsf{5}}$ cannot be written.

4.4 Operators arising from $\mathsf{M}_{\acute{x}i}\ \mathsf{H}^{(10_r)i}\ \mathsf{M}_{\acute{y}j}\ \mathsf{H}^{(120)j}$

The analysis of this case is similar to that of W_2 and W_3. Thus without further explanation we give the analysis below

$$
\begin{aligned}
W_4 &= -\frac{1}{\mathbf{S}_{1_{\overline{126}}}} \imath 2\sqrt{10}\ \mathsf{M}_{\acute{x}i}\ \mathsf{H}^{(10_r)i} \left[f^{(10_r+)} f^{(\overline{126}+)^{-1}} f^{(120-)} \right]_{\acute{x}\acute{y}} \mathsf{M}_{\acute{y}j}\ \mathsf{H}^{(120)j} \\
&= W_4^{DD} + W_4^{TT} + W_4^{DT} + W_4^{TD}, \qquad (54)
\end{aligned}
$$

where

$$
\begin{aligned}
W_4^{DD} &\equiv \mathcal{G}^{(r)}_{\acute{x}\acute{y}}\ \mathsf{M}_{\acute{x}a}\ \mathsf{M}_{\acute{y}b}\ \ \mathsf{H}^{(10_r)a}\ \mathsf{H}^{(120)b}, \qquad (55)\\
W_4^{TT} &\equiv \mathcal{G}^{(r)}_{\acute{x}\acute{y}}\ \mathsf{M}_{\acute{x}\alpha}\ \mathsf{M}_{\acute{y}\beta}\ \ \mathsf{H}^{(10_r)\alpha}\ \mathsf{H}^{(120)\beta}, \qquad (56)\\
W_4^{DT} &\equiv \mathcal{G}^{(r)}_{\acute{x}\acute{y}}\ \mathsf{M}_{\acute{x}a}\ \mathsf{M}_{\acute{y}\alpha}\ \ \mathsf{H}^{(10_r)a}\ \mathsf{H}^{(120)\alpha}, \qquad (57)\\
W_4^{TD} &\equiv \mathcal{G}^{(r)}_{\acute{x}\acute{y}}\ \mathsf{M}_{\acute{x}\alpha}\ \mathsf{M}_{\acute{y}a}\ \ \mathsf{H}^{(10_r)\alpha}\ \mathsf{H}^{(120)a}, \qquad (58)
\end{aligned}
$$

and

$$
\mathcal{G}^{(r)}_{\acute{x}\acute{y}} \equiv -\frac{1}{\mathbf{S}_{1_{\overline{126}}}} \left(\imath 2\sqrt{10}\right) \left[f^{(10_r+)} f^{(\overline{126}+)^{-1}} f^{(120-)} \right]_{\acute{x}\acute{y}}. \qquad (59)
$$

4.4.1 Evaluating W_4^{DD}

$$
\begin{aligned}
W_4^{DD\prime} = \quad & U_{d_{31}} \sum_{r=1}^{2} U_{d_{r1}} \mathcal{G}^{(r)}_{\acute{x}\acute{y}}\ \mathbf{L}_{\acute{x}a}\ \mathbf{L}_{\acute{y}b}\ \mathbf{H_u}^{a}\ \mathbf{H_u}^{b} \\
& +8\mathcal{I}_{\acute{w}\acute{x},\acute{y}\acute{z}} \Big[\epsilon^{ac}\epsilon^{bd}\ \mathbf{L}_{\acute{u}a}\ \mathbf{L}_{\acute{v}b}\ \mathbf{E}^{\mathsf{c}}_{\acute{w}}\ \mathbf{L}_{\acute{x}c}\ \mathbf{E}^{\mathsf{c}}_{\acute{y}}\ \mathbf{L}_{\acute{z}d} + \mathbf{L}_{\acute{u}a}\ \mathbf{L}_{\acute{v}b}\ \mathbf{Q}^{a\alpha}_{\acute{w}}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\alpha}\ \mathbf{Q}^{b\beta}_{\acute{w}}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\beta} \\
& +\epsilon^{ac}\ \mathbf{L}_{\acute{u}a}\ \mathbf{L}_{\acute{v}b}\ \mathbf{E}^{\mathsf{c}}_{\acute{w}}\ \mathbf{L}_{\acute{x}c}\ \mathbf{Q}^{b\alpha}_{\acute{y}}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\alpha} + \epsilon^{bc}\ \mathbf{L}_{\acute{u}a}\ \mathbf{L}_{\acute{v}b}\ \mathbf{Q}^{a\alpha}_{\acute{w}}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\alpha}\ \mathbf{E}^{\mathsf{c}}_{\acute{y}}\ \mathbf{L}_{\acute{z}c}\Big] \\
& +\imath 4U_{d_{31}} \Big[\epsilon^{ab}\ \mathbf{L}_{\acute{w}a}\ \mathbf{L}_{\acute{x}c}\ \mathbf{E}^{\mathsf{c}}_{\acute{y}}\ \mathbf{L}_{\acute{z}b}\ \mathbf{H_u}^{c} + \mathbf{L}_{\acute{w}a}\ \mathbf{L}_{\acute{x}b}\ \mathbf{Q}^{a\alpha}_{\acute{y}}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\alpha}\ \mathbf{H_u}^{b}\Big] \\
& \times \left[\sqrt{2}\sum_{N=2}^{7} \frac{\left(\sum_{r=1}^{2}\mathcal{G}^{(r)}_{\acute{w}\acute{x}}U_{d_{rN}}\right)\left(\sum_{s=1}^{2} f^{(10_s+)}_{\acute{y}\acute{z}}V_{d_{sN}}\right)}{\mathsf{m}_{d_N}} - \frac{1}{\sqrt{3}} f^{(120-)}_{\acute{y}\acute{z}} \sum_{N=2}^{7} \frac{\left(\sum_{r=1}^{2}\mathcal{G}^{(r)}_{\acute{w}\acute{x}}U_{d_{rN}}\right)V_{d_{3N}}}{\mathsf{m}_{d_N}}\right] \\
& +\imath 4 \sum_{r=1}^{2} U_{d_{r1}} \mathcal{G}^{(r)}_{\acute{x}\acute{y}} \Big[\epsilon^{bc}\ \mathbf{L}_{\acute{w}a}\ \mathbf{L}_{\acute{x}b}\ \mathbf{E}^{\mathsf{c}}_{\acute{y}}\ \mathbf{L}_{\acute{z}c}\ \mathbf{H_u}^{a} + \mathbf{L}_{\acute{w}a}\ \mathbf{L}_{\acute{x}b}\ \mathbf{Q}^{b\alpha}_{\acute{y}}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\alpha}\ \mathbf{H_u}^{a}\Big] \\
& \times \left[\sqrt{2}\sum_{N=2}^{7} \frac{U_{d_{3N}}\left(\sum_{s=1}^{2} f^{(10_s+)}_{\acute{y}\acute{z}}V_{d_{sN}}\right)}{\mathsf{m}_{d_N}} - \frac{1}{\sqrt{3}} f^{(120-)}_{\acute{y}\acute{z}} \sum_{N=2}^{7} \frac{U_{d_{3N}}V_{d_{3N}}}{\mathsf{m}_{d_N}}\right] \\
& +8\Big[\epsilon_{ab}\ \mathbf{Q}^{a\alpha}_{\acute{w}}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\alpha}\ \mathbf{U}^{\mathsf{c}}_{\acute{y}\beta}\ \mathbf{Q}^{b\beta}_{\acute{z}} + \mathbf{E}^{\mathsf{c}}_{\acute{w}}\ \mathbf{L}_{\acute{x}a}\ \mathbf{U}^{\mathsf{c}}_{\acute{y}\alpha}\ \mathbf{Q}^{a\alpha}_{\acute{z}}\Big] \\
& \times \left[2\sum_{N=2}^{7} \frac{\left\{\sum_{r=1}^{2} f^{(10_r+)}_{\acute{w}\acute{x}}U_{d_{rN}}\right\}\left\{\sum_{s=1}^{2} f^{(10_s+)}_{\acute{y}\acute{z}}V_{d_{sN}}\right\}}{\mathsf{m}_{d_N}} + \sqrt{\frac{2}{3}} f^{(120-)}_{\acute{w}\acute{x}} \sum_{N=2}^{7} \frac{\left\{\sum_{r=1}^{2} f^{(10_r+)}_{\acute{y}\acute{z}}U_{d_{rN}}\right\}V_{d_{3N}}}{\mathsf{m}_{d_N}}\right].
\end{aligned}
\tag{60}
$$

The coefficient $\mathcal{I}_{\acute{w}\acute{x},\acute{y}\acute{z}}$ is defined in Appendix E.

4.4.2 Evaluating W_4^{TT}

$$
\begin{aligned}
W_4^{TT\prime} = \quad & 8\mathcal{J}_{\acute{w}\acute{x},\acute{y}\acute{z}} \Big[\epsilon^{\alpha\gamma\delta}\epsilon^{\beta\mu\nu}\ \mathbf{D}^{\mathsf{c}}_{\acute{u}\alpha}\ \mathbf{D}^{\mathsf{c}}_{\acute{v}\beta}\ \mathbf{U}^{\mathsf{c}}_{\acute{w}\gamma}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\delta}\ \mathbf{U}^{\mathsf{c}}_{\acute{y}\mu}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\nu} + \mathbf{D}^{\mathsf{c}}_{\acute{u}\alpha}\ \mathbf{D}^{\mathsf{c}}_{\acute{v}\beta}\ \mathbf{Q}^{a\alpha}_{\acute{w}}\ \mathbf{L}_{\acute{x}a}\ \mathbf{Q}^{b\beta}_{\acute{y}}\ \mathbf{L}_{\acute{z}b} \\
& +\epsilon^{\alpha\gamma\delta}\ \mathbf{D}^{\mathsf{c}}_{\acute{u}\alpha}\ \mathbf{D}^{\mathsf{c}}_{\acute{v}\beta}\ \mathbf{U}^{\mathsf{c}}_{\acute{w}\gamma}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\delta}\ \mathbf{Q}^{a\beta}_{\acute{y}}\ \mathbf{L}_{\acute{z}a} + \epsilon^{\beta\gamma\delta}\ \mathbf{D}^{\mathsf{c}}_{\acute{u}\alpha}\ \mathbf{D}^{\mathsf{c}}_{\acute{v}\beta}\ \mathbf{Q}^{a\alpha}_{\acute{w}}\ \mathbf{L}_{\acute{x}a}\ \mathbf{U}^{\mathsf{c}}_{\acute{y}\gamma}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\delta}\Big] \\
& +4\Big[\epsilon_{ab}\ \mathbf{Q}^{a\alpha}_{\acute{u}}\ \mathbf{Q}^{b\beta}_{\acute{v}}\ \mathbf{U}^{\mathsf{c}}_{\acute{w}\alpha}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\beta} + \mathbf{E}^{\mathsf{c}}_{\acute{u}}\ \mathbf{U}^{\mathsf{c}}_{\acute{v}\alpha}\ \mathbf{Q}^{a\alpha}_{\acute{w}}\ \mathbf{L}_{\acute{x}a} + \epsilon^{\alpha\beta\gamma}\ \mathbf{E}^{\mathsf{c}}_{\acute{u}}\ \mathbf{U}^{\mathsf{c}}_{\acute{v}\alpha}\ \mathbf{U}^{\mathsf{c}}_{\acute{w}\beta}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\gamma}\Big] \\
& \times \Bigg[2\sum_{N=1}^{8} \frac{\left\{\sum_{r=1}^{2}\left(\mathcal{G}^{(r)}_{\acute{u}\acute{v}} + \mathcal{G}^{(r)}_{\acute{v}\acute{u}}\right)U_{t_{rN}}\right\}\left\{\sum_{s=1}^{2} f^{(10_s+)}_{\acute{w}\acute{x}}V_{t_{sN}}\right\}}{\mathsf{m}_{t_N}} \\
& -\sqrt{\frac{2}{3}} f^{(120-)}_{\acute{w}\acute{x}} \sum_{N=1}^{8} \frac{\left\{\sum_{r=1}^{2}\left(\mathcal{G}^{(r)}_{\acute{u}\acute{v}} + \mathcal{G}^{(r)}_{\acute{v}\acute{u}}\right)U_{t_{rN}}\right\}V_{t_{3N}}}{\mathsf{m}_{t_N}}\Bigg] \\
& +4\epsilon_{ab}\epsilon_{\alpha\beta\gamma}\ \mathbf{Q}^{a\alpha}_{\acute{u}}\ \mathbf{Q}^{b\beta}_{\acute{v}}\ \mathbf{Q}^{c\gamma}_{\acute{w}}\ \mathbf{L}_{\acute{x}c} \\
& \times \left[-2\sum_{N=1}^{8} \frac{\left\{\sum_{r=1}^{2}\mathcal{G}^{(r)}_{\acute{u}\acute{v}}U_{t_{rN}}\right\}\left\{\sum_{s=1}^{2} f^{(10_s+)}_{\acute{w}\acute{x}}V_{t_{sN}}\right\}}{\mathsf{m}_{t_N}} + \sqrt{\frac{2}{3}} f^{(120-)}_{\acute{w}\acute{x}} \sum_{N=1}^{8} \frac{\left\{\sum_{r=1}^{2}\mathcal{G}^{(r)}_{\acute{u}\acute{v}}U_{t_{rN}}\right\}V_{t_{3N}}}{\mathsf{m}_{t_N}}\right].
\end{aligned}
\tag{61}
$$

The coefficient $\mathcal{J}_{\acute{w}\acute{x},\acute{y}\acute{z}}$ is explicitly given in Appendix E.

4.4.3 Evaluating W_4^{DT}

$$
\begin{aligned}
W_4^{DT\prime} = \quad & 16\sqrt{\frac{2}{3}} f^{(120-)}_{\acute{y}\acute{z}} \Big[\epsilon^{\alpha\beta\gamma}\ \mathbf{L}_{\acute{u}a}\ \mathbf{D}^{\mathsf{c}}_{\acute{v}\alpha}\ \mathbf{Q}^{a\delta}_{\acute{w}}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\delta}\ \mathbf{U}^{\mathsf{c}}_{\acute{y}\beta}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\gamma} + \epsilon^{ab}\ \mathbf{L}_{\acute{u}a}\ \mathbf{D}^{\mathsf{c}}_{\acute{v}\alpha}\ \mathbf{E}^{\mathsf{c}}_{\acute{w}}\ \mathbf{L}_{\acute{x}b}\ \mathbf{Q}^{c\alpha}_{\acute{y}}\ \mathbf{L}_{\acute{z}c} \\
& +\mathbf{L}_{\acute{u}a}\ \mathbf{D}^{\mathsf{c}}_{\acute{v}\alpha}\ \mathbf{Q}^{a\beta}_{\acute{w}}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\beta}\ \mathbf{Q}^{b\alpha}_{\acute{y}}\ \mathbf{L}_{\acute{z}b} + \epsilon^{ab}\epsilon^{\alpha\beta\gamma}\ \mathbf{L}_{\acute{u}a}\ \mathbf{D}^{\mathsf{c}}_{\acute{v}\alpha}\ \mathbf{E}^{\mathsf{c}}_{\acute{w}}\ \mathbf{L}_{\acute{x}b}\ \mathbf{U}^{\mathsf{c}}_{\acute{y}\beta}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\gamma}\Big]
\end{aligned}
$$

$$\times\left(\sum_{M=2}^{7}\frac{\left\{\sum_{r=1}^{2}\mathcal{G}^{(r)}_{\acute{u}\acute{v}}U_{d_{rM}}\right\}\left\{\sum_{s=1}^{2}f^{(10_s+)}_{\acute{w}\acute{x}}V_{d_{sM}}\right\}}{\mathsf{m}_{d_M}}\right)\left(\sum_{N=1}^{8}\frac{U_{t_{3N}}V_{t_{3N}}}{\mathsf{m}_{t_N}}\right)$$
$$+\imath\frac{4}{\sqrt{3}}\sum_{r=1}^{2}U_{d_{r1}}\mathcal{G}^{(r)}_{\acute{x}\acute{y}}f^{(120-)}_{\acute{y}\acute{z}}\left[\epsilon^{\alpha\beta\gamma}\ \mathbf{U}^{\mathsf{c}}_{\acute{w}\gamma}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\alpha}\ \mathbf{L}_{\acute{y}a}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\beta}\ \mathbf{H_u}^{a}+\mathbf{Q}^{a\alpha}_{\acute{w}}\ \mathbf{L}_{\acute{x}a}\ \mathbf{L}_{\acute{y}b}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\alpha}\ \mathbf{H_u}^{b}\right]\times\sum_{N=1}^{8}\frac{V_{t_{3N}}U_{t_{3N}}}{\mathsf{m}_{t_N}}$$
$$-16\left[\epsilon_{ab}\ \mathbf{Q}^{a\alpha}_{\acute{w}}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\alpha}\ \mathbf{U}^{\mathsf{c}}_{\acute{y}\beta}\ \mathbf{Q}^{b\beta}_{\acute{z}}+\mathbf{E}^{\mathsf{c}}_{\acute{w}}\ \mathbf{L}_{\acute{x}a}\ \mathbf{U}^{\mathsf{c}}_{\acute{y}\alpha}\ \mathbf{Q}^{a\alpha}_{\acute{z}}\right]\sum_{N=2}^{7}\frac{\left\{\sum_{r=1}^{2}f^{(10_r+)}_{\acute{w}\acute{x}}U_{d_{rN}}\right\}\left\{\sum_{s=1}^{2}f^{(10_s+)}_{\acute{y}\acute{z}}V_{d_{sN}}\right\}}{\mathsf{m}_{d_N}}. \tag{62}$$

4.4.4 Evaluating W_4^{TD}

$$\begin{aligned}W_4^{TD\prime}=\quad & -16\sqrt{\frac{2}{3}}f^{(120-)}_{\acute{y}\acute{z}}\left[\epsilon^{\alpha\beta\gamma}\ \mathbf{D}^{\mathsf{c}}_{\acute{u}\alpha}\ \mathbf{L}_{\acute{v}a}\ \mathbf{U}^{\mathsf{c}}_{\acute{w}\beta}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\gamma}\ \mathbf{Q}^{a\rho}_{\acute{y}}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\rho}+\epsilon^{ab}\epsilon^{\alpha\beta\gamma}\ \mathbf{D}^{\mathsf{c}}_{\acute{u}\alpha}\ \mathbf{L}_{\acute{v}a}\ \mathbf{U}^{\mathsf{c}}_{\acute{w}\beta}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\gamma}\ \mathbf{E}^{\mathsf{c}}_{\acute{y}}\ \mathbf{L}_{\acute{z}b}\right.\\&\left.+\epsilon^{ac}\ \mathbf{D}^{\mathsf{c}}_{\acute{u}\alpha}\ \mathbf{L}_{\acute{v}a}\ \mathbf{Q}^{b\alpha}_{\acute{w}}\ \mathbf{L}_{\acute{x}b}\ \mathbf{E}^{\mathsf{c}}_{\acute{y}}\ \mathbf{L}_{\acute{z}c}+\mathbf{D}^{\mathsf{c}}_{\acute{u}\alpha}\ \mathbf{L}_{\acute{v}a}\ \mathbf{Q}^{b\alpha}_{\acute{w}}\ \mathbf{L}_{\acute{x}b}\ \mathbf{Q}^{a\beta}_{\acute{y}}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\beta}\right]\\&\times\left(\sum_{M=1}^{8}\frac{\left\{\sum_{r=1}^{2}\mathcal{G}^{(r)}_{\acute{u}\acute{v}}U_{t_{rM}}\right\}\left\{\sum_{s=1}^{2}f^{(10_s+)}_{\acute{w}\acute{x}}V_{t_{sM}}\right\}}{\mathsf{m}_{t_M}}\right)\left(\sum_{N=2}^{7}\frac{U_{d_{3N}}V_{d_{3N}}}{\mathsf{m}_{d_N}}\right)\\&-\imath4\sqrt{2}U_{d_{31}}\left[\epsilon^{\alpha\beta\gamma}\ \mathbf{D}^{\mathsf{c}}_{\acute{w}\alpha}\ \mathbf{L}_{\acute{x}a}\ \mathbf{U}^{\mathsf{c}}_{\acute{y}\beta}\ \mathbf{D}^{\mathsf{c}}_{\acute{z}\gamma}\ \mathbf{H_u}^{a}+\mathbf{D}^{\mathsf{c}}_{\acute{w}\alpha}\ \mathbf{L}_{\acute{x}a}\ \mathbf{Q}^{b\alpha}_{\acute{y}}\ \mathbf{L}_{\acute{z}b}\ \mathbf{H_u}^{a}\right]\\&\times\sum_{N=1}^{8}\frac{\left\{\sum_{r=1}^{2}\mathcal{G}^{(r)}_{\acute{w}\acute{x}}U_{t_{rN}}\right\}\left\{\sum_{s=1}^{2}f^{(10_s+)}_{\acute{y}\acute{z}}V_{t_{sN}}\right\}}{\mathsf{m}_{t_N}}\\&+8\left[2\epsilon^{\alpha\beta\gamma}\ \mathbf{U}^{\mathsf{c}}_{\acute{w}\alpha}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\beta}\ \mathbf{E}^{\mathsf{c}}_{\acute{y}}\ \mathbf{U}^{\mathsf{c}}_{\acute{z}\gamma}+2\epsilon_{ab}\ \mathbf{U}^{\mathsf{c}}_{\acute{w}\alpha}\ \mathbf{D}^{\mathsf{c}}_{\acute{x}\beta}\ \mathbf{Q}^{a\beta}_{\acute{y}}\ \mathbf{Q}^{b\alpha}_{\acute{z}}+2\ \mathbf{Q}^{a\alpha}_{\acute{w}}\ \mathbf{L}_{\acute{x}a}\ \mathbf{E}^{\mathsf{c}}_{\acute{y}}\ \mathbf{U}^{\mathsf{c}}_{\acute{z}\alpha}\right.\\&\left.-\epsilon_{bc}\epsilon_{\alpha\beta\gamma}\ \mathbf{Q}^{a\alpha}_{\acute{w}}\ \mathbf{L}_{\acute{x}a}\ \mathbf{Q}^{b\beta}_{\acute{y}}\ \mathbf{Q}^{c\gamma}_{\acute{z}}\right]\sum_{N=1}^{8}\frac{\left\{\sum_{r=1}^{2}f^{(10_r+)}_{\acute{w}\acute{x}}U_{t_{rN}}\right\}\left\{\sum_{s=1}^{2}f^{(10_s+)}_{\acute{y}\acute{z}}V_{t_{sN}}\right\}}{\mathsf{m}_{t_N}}.\end{aligned} \tag{63}$$

4.5 Operators arising from $\mathsf{M}_{\acute{x}i}\ \mathsf{H}^{(120)i}\ \mathsf{M}_{\acute{y}j}\ \mathsf{H}^{(\overline{126})j}$

$$W_5=\frac{1}{\mathbf{S}_{1_{\overline{126}}}}\imath\sqrt{2}\ \mathsf{M}_{\acute{x}i}\ \mathsf{H}^{(120)i}\ f^{(120-)}_{\acute{x}\acute{y}}\ \mathsf{M}_{\acute{y}j}\ \mathsf{H}^{(\overline{126})j}. \tag{64}$$

This term does not generate any $\mathsf{B}-\mathsf{L}=-2$ operators involving only the SM fields because, firstly $U_{d_{41}}=0$ and secondly because there is no $\overline{5}$ of $\mathsf{SU}(5)$ in $\overline{126}$ and thus a mass term involving 5 and $\overline{5}$ cannot be written.

4.6 Operators arising from $\mathsf{M}_{\acute{x}i}\ \mathsf{H}^{(10_r)i}\ \mathsf{M}_{\acute{y}j}\ \mathsf{H}^{(\overline{126})j}$

$$W_6=\frac{1}{\mathbf{S}_{1_{\overline{126}}}}\imath\sqrt{3}\ \mathsf{M}_{\acute{x}i}\ \mathsf{H}^{(10_r)i}\ f^{(10_r+)}_{\acute{x}\acute{y}}\ \mathsf{M}_{\acute{y}j}\ \mathsf{H}^{(\overline{126})j}. \tag{65}$$

Just as in the preceding case this term also does not generate any $\mathsf{B}-\mathsf{L}=-2$ operators involving only the Standard Model fields.

5 Discussion of results

The analysis of Sec.(4)shows that there are a large number of sources of $\mathsf{B}-\mathsf{L}=-2$ operators arising from matter-Higgs interactions. As discussed in Sec.(4), the cubic matter-Higgs interactions consist of couplings of type $16\cdot16\cdot10$, $16\cdot16\cdot120$ and $16\cdot16\cdot\overline{126}$. After the singlet in $\overline{126}$ develops a VEV of size the GUT scale, the singlet in the 16-plet of matter gains a GUT size mass via the $16\cdot16\cdot\overline{126}$ coupling which violates $\mathsf{B}-\mathsf{L}$ by two units. The elimination of the singlet of the 16-plet results in a large number of $\mathsf{B}-\mathsf{L}=-2$ violating interactions, and thus it is useful to indicate all the sources of such terms. For convenience of the reader we summarize the sources of the $\mathsf{B}-\mathsf{L}$ violating dimension 5, dimension 7 and dimension 9 operators below:

	Effective Operator	B	L	dim$L_{\rm SM}$
4 Fields	$\mathbf{L}_a\ \mathbf{L}_b\ \mathbf{H_u}^a\ \mathbf{H_u}^b$	0	+2	5
5 Fields	$\epsilon^{\alpha\beta\gamma}\ \mathbf{D}^{\sf c}_\alpha\ \mathbf{D}^{\sf c}_\beta\ \mathbf{U}^{\sf c}_\gamma\ \mathbf{L}_a\ \mathbf{H_u}^a$	−1	+1	7
	$\epsilon^{ab}\ \mathbf{L}_a\ \mathbf{L}_b\ \mathbf{L}_c\ \mathbf{E}^{\sf c}\ \mathbf{H_u}^c$	0	+2	7
	$\mathbf{D}^{\sf c}_\alpha\ \mathbf{Q}^{a\alpha}\ \mathbf{L}_a\ \mathbf{L}_b\ \mathbf{H_u}^b$	0	+2	7
6 Fields	$\epsilon^{\alpha\beta\gamma}\epsilon^{\rho\sigma\lambda}\ \mathbf{D}^{\sf c}_\alpha\ \mathbf{D}^{\sf c}_\beta\ \mathbf{U}^{\sf c}_\gamma\ \mathbf{D}^{\sf c}_\rho\ \mathbf{D}^{\sf c}_\sigma\ \mathbf{U}^{\sf c}_\lambda$	−2	0	9
	$\epsilon^{ab}\epsilon^{cd}\ \mathbf{L}_a\ \mathbf{L}_b\ \mathbf{L}_c\ \mathbf{L}_d\ \mathbf{E}^{\sf c}\ \mathbf{E}^{\sf c}$	0	+2	9
	$\epsilon^{\alpha\beta\gamma}\epsilon^{ab}\ \mathbf{D}^{\sf c}_\alpha\ \mathbf{D}^{\sf c}_\beta\ \mathbf{U}^{\sf c}_\gamma\ \mathbf{L}_a\ \mathbf{L}_b\ \mathbf{E}^{\sf c}$	−1	+1	9
	$\epsilon^{\alpha\beta\gamma}\ \mathbf{D}^{\sf c}_\alpha\ \mathbf{D}^{\sf c}_\beta\ \mathbf{U}^{\sf c}_\gamma\ \mathbf{D}^{\sf c}_\rho\ \mathbf{Q}^{a\rho}\ \mathbf{L}_a$	−1	+1	9
	$\epsilon^{ab}\ \mathbf{D}^{\sf c}_\alpha\ \mathbf{Q}^{c\alpha}\ \mathbf{L}_a\ \mathbf{L}_b\ \mathbf{L}_c\ \mathbf{E}^{\sf c}$	0	+2	9
	$\mathbf{D}^{\sf c}_\alpha\ \mathbf{D}^{\sf c}_\beta\ \mathbf{Q}^{a\alpha}\ \mathbf{Q}^{b\beta}\ \mathbf{L}_a\ \mathbf{L}_b$	0	+2	9
4 Fields	$\epsilon_{\alpha\beta\gamma}\epsilon_{ab}\ \mathbf{Q}^{a\alpha}\ \mathbf{Q}^{b\beta}\ \mathbf{Q}^{c\gamma}\ \mathbf{L}_c$	+1	+1	6
	$\epsilon^{\alpha\beta\gamma}\ \mathbf{D}^{\sf c}_\alpha\ \mathbf{U}^{\sf c}_\beta\ \mathbf{U}^{\sf c}_\gamma\ \mathbf{E}^{\sf c}$	−1	−1	6
	$\epsilon_{ab}\ \mathbf{D}^{\sf c}_\alpha\ \mathbf{U}^{\sf c}_\beta\ \mathbf{Q}^{a\alpha}\ \mathbf{Q}^{b\beta}$	0	0	6
	$\mathbf{U}^{\sf c}_\alpha\ \mathbf{Q}^{a\alpha}\ \mathbf{L}_a\ \mathbf{E}^{\sf c}$	0	0	6

Table 5: Summary of the $\mathsf{B}-\mathsf{L}=0$ (bottom four entries) and $\mathsf{B}-\mathsf{L}=-2$ (all the remaining) operators that arise from matter-Higgs interactions involving $\mathsf{10}$, $\mathsf{120}$ and $\overline{\mathsf{126}}$ of Higgs fields. The object dim$L_{\rm SM}$ is the dimensionality of operators in the Lagrangian which have only the Standard Model particles with even R parity.

$\mathsf{B}-\mathsf{L}=-2$ dimension 5 operator: $\mathbf{L}_a\ \mathbf{L}_b\ \mathbf{H_u}^a\ \mathbf{H_u}^b$: This operator arises from a number of sources. The total contribution to it can be gotten from Eqs.(42, 50, 60). Next we consider the $\mathsf{B}-\mathsf{L}=-2$ dimension 7 operators. Here we have the following set of operators: (i) $\epsilon^{\alpha\beta\gamma}\ \mathbf{D}^{\sf c}_\alpha\ \mathbf{D}^{\sf c}_\beta\ \mathbf{U}^{\sf c}_\gamma\ \mathbf{L}_a\ \mathbf{H_u}^a$: Contribution to it arises from Eqs.(44, 52, 62, 63); (ii) $\epsilon^{ab}\ \mathbf{L}_a\ \mathbf{L}_b\ \mathbf{L}_c\ \mathbf{E}^{\sf c}\ \mathbf{H_u}^c$: Contribution to it arises from Eqs.(42, 50, 60); (iii) $\mathbf{D}^{\sf c}_\alpha\ \mathbf{Q}^{a\alpha}\ \mathbf{L}_a\ \mathbf{L}_b\ \mathbf{H_u}^b$: Contribution to it arises from Eqs.(42, 44, 50, 52, 60, 62, 63). From Table 4 we see that there are six $\mathsf{B}-\mathsf{L}=-2$ dimension nine operators: (a) $\epsilon^{\alpha\beta\gamma}\epsilon^{\rho\sigma\lambda}\ \mathbf{D}^{\sf c}_\alpha\ \mathbf{D}^{\sf c}_\beta\ \mathbf{U}^{\sf c}_\gamma\ \mathbf{D}^{\sf c}_\rho\ \mathbf{D}^{\sf c}_\sigma\ \mathbf{U}^{\sf c}_\lambda$: Contribution to it arises from Eqs.(43, 51, 61); (b) $\epsilon^{ab}\epsilon^{cd}\ \mathbf{L}_a\ \mathbf{L}_b\ \mathbf{L}_c\ \mathbf{L}_d\ \mathbf{E}^{\sf c}\ \mathbf{E}^{\sf c}$: Contribution to it arises from Eqs.(42, 50, 60); (c) $\epsilon^{\alpha\beta\gamma}\epsilon^{ab}\ \mathbf{D}^{\sf c}_\alpha\ \mathbf{D}^{\sf c}_\beta\ \mathbf{U}^{\sf c}_\gamma\ \mathbf{L}_a\ \mathbf{L}_b\ \mathbf{E}^{\sf c}$: Contribution to it arises from Eqs.(44, 52, 62, 63); (d) $\epsilon^{\alpha\beta\gamma}\ \mathbf{D}^{\sf c}_\alpha\ \mathbf{D}^{\sf c}_\beta\ \mathbf{U}^{\sf c}_\gamma\ \mathbf{D}^{\sf c}_\rho\ \mathbf{Q}^{a\rho}\ \mathbf{L}_a$: Contribution to it arises from Eqs.(43, 44, 51, 52, 61, 62, 63); (e) $\epsilon^{ab}\ \mathbf{D}^{\sf c}_\alpha\ \mathbf{Q}^{c\alpha}\ \mathbf{L}_a\ \mathbf{L}_b\ \mathbf{L}_c\ \mathbf{E}^{\sf c}$: Contribution to it arises from Eqs.(42, 44, 50, 52, 60, 62, 63); (f) $\mathbf{D}^{\sf c}_\alpha\ \mathbf{D}^{\sf c}_\beta\ \mathbf{Q}^{a\alpha}\ \mathbf{Q}^{b\beta}\ \mathbf{L}_a\ \mathbf{L}_b$: Contribution to it arises from Eqs.(42, 43, 44, 50, 51, 52, 60, 61, 62, 63).

We also summarize the sources of the $\mathsf{B}-\mathsf{L}=0$ operators. There are four of them as shown in Table 4. We list their sources as follows: (i) $\epsilon_{\alpha\beta\gamma}\epsilon_{ab}\ \mathbf{Q}^{a\alpha}\ \mathbf{Q}^{b\beta}\ \mathbf{Q}^{c\gamma}\ \mathbf{L}_c$: Contribution to it arises from Eqs.(44, 61, 63); (ii) $\epsilon^{\alpha\beta\gamma}\ \mathbf{D}^{\sf c}_\alpha\ \mathbf{U}^{\sf c}_\beta\ \mathbf{U}^{\sf c}_\gamma\ \mathbf{E}^{\sf c}$: Contribution to it arise from Eqs.(44, 61, 63); (iii) $\epsilon_{ab}\ \mathbf{D}^{\sf c}_\alpha\ \mathbf{U}^{\sf c}_\beta\ \mathbf{Q}^{a\alpha}\ \mathbf{Q}^{b\beta}$: Contribution to it arises from Eqs.(42, 44, 60, 61, 62, 63); (iv) $\mathbf{U}^{\sf c}_\alpha\ \mathbf{Q}^{a\alpha}\ \mathbf{L}_a\ \mathbf{E}^{\sf c}$: Contribution to it arises from Eqs.(42, 44, 60, 61, 62, 63).

It is instructive to trace back to the primitive $\mathsf{SU(5)}$ invariant effective structures in the superpotential from which the set of operators listed in Table 4 arise. Thus the $\mathsf{B}-\mathsf{L}=-2$ four field interaction arise from a primitive $\mathsf{SU(5)}$ structure $\mathsf{M}_i\mathsf{M}_j\mathsf{H_u}^i\mathsf{H_u}^j$ while the $\mathsf{B}-\mathsf{L}=-2$ five field interactions arise from the $\mathsf{SU(5)}$ structure $\mathsf{M}_i\mathsf{M}_j\mathsf{M}_k\mathsf{M}^{ij}\mathsf{H_u}^k$ and the $\mathsf{B}-\mathsf{L}=-2$ six field interactions arise from the primitive $\mathsf{SU(5)}$ structure $\mathsf{M}_i\mathsf{M}_j\mathsf{M}^{ij}\mathsf{M}_k\mathsf{M}_\ell\mathsf{M}^{k\ell}$. Here M_i are the $\bar{\mathsf{5}}$-plet of matter and M^{ij} are the 10-plet of matter fields and $\mathsf{H_u}^i$ is the 5-plet of light Higgs fields. The $\mathsf{B}-\mathsf{L}=0$ four field operators in Table 4 can be traced back to the primitive $\mathsf{SU(5)}$ structure $\epsilon_{ijklm}\mathsf{M}^{ij}\mathsf{M}^{kl}\mathsf{M}^{mn}\mathsf{M}_n$ in the superpotential. It is to be noted that the $\mathsf{B}-\mathsf{L}=-2$ four field and five field primitive $\mathsf{SU(5)}$ structures only contain up-Higgs, i.e., $\mathsf{H_u}$ and the down-Higgs $\mathsf{H_d}$ does not appear. This helps explain why only $\mathsf{H_u}$ appears in effective operators in Table 4. We note that $\mathsf{SU(5)}$ is, however, broken at the GUT scale, and thus these $\mathsf{SU(5)}$ structures are to be used only as mnemonics for book keeping to identify the origins of the effective operators at low energies.

There are different scales associated with the effective operators listed in Table 4. Thus suppose M stands for the GUT scale and denote all GUT scale masses by M. In this case the $\mathsf{B}-\mathsf{L}=-2$ four field effective operator is suppressed by the factor $1/M$ which leads to neutrino masses $O(<H>^2/M)$ which can lie in the desirable sub eV region. The $\mathsf{B}-\mathsf{L}=-2$ five field operator $\mathbf{D}^{\sf c}\mathbf{D}^{\sf c}\mathbf{U}^{\sf c}\mathbf{L}_a\mathbf{H_u}^a$ is suppressed by two powers of M in the superpotential. After dressing with loops involving a sparticle it will generate a dimension 7 operator in the effective Lagrangian suppressed by the factor $1/(M^2M_s)$ where M_s is the effective weak SUSY scale. This operator can produce $\mathsf{B}-\mathsf{L}=-2$ nucleon decay modes such $p\to\nu\pi^+$. In the usual GUT models, they would be suppressed relative to the proton decay arising from $\mathsf{B}-\mathsf{L}=0$ dimension six proton decay operators. The six field operator $\mathbf{D}^{\sf c}\mathbf{D}^{\sf c}\mathbf{U}^{\sf c}\mathbf{D}^{\sf c}\mathbf{D}^{\sf c}\mathbf{U}^{\sf c}$ is suppressed by $1/M^3$ in the superpotential and by a factor $1/(M^3 M_s^2)$ in the effective Lagrangian. It can induce $n-\bar{n}$ oscillations and such effects can be enhanced and become observable in models with low scales M.

6 Conclusion

The $\mathsf{SO(10)}$ models are among the prime candidates for a unified framework, which include the strong, the weak, and the electromagnetic interactions. However, like most grand unified models, the $\mathsf{SO(10)}$ models also suffer from the so called doublet-triplet problem, which means that after spontaneous breaking of the GUT symmetry, the Higgs doublets (as well as the Higgs triplets) will all be superheavy, requiring a huge fine-tuning to make the Higgs doublets light. The missing partners mechanism is one of the ways in which a grand unified model can resolve this severe doublet-triplet problem. Recently, a variety of $\mathsf{SO(10)}$ models were proposed in a supersymmetric framework, which include a missing partner mechanism. Here, we discussed the simplest of such models which contains a heavy sector consisting of $\mathsf{126}+\overline{\mathsf{126}}+\mathsf{210}$ Higgs multiplets, which breaks the GUT symmetry down to $\mathsf{SU(3)_C}\times\mathsf{SU(2)L}\times\mathsf{U(1)_Y}$. Combined with a light sector consisting of $2\times\mathsf{10}+\mathsf{120}$ and a mixing between the light and the heavy sectors, one finds that the model contains just a pair of light Higgs doublets while the remaining exotic fields in $2\times\mathsf{10}+\mathsf{120}$ become superheavy. We have carried out a detailed analysis of the Higgs sector and identified the exact linear combination of the fields that enter in the light Higgs doublet fields for this $\mathsf{SO(10)}$ missing partner model.

Further, in this work we have given a full classification of $\mathsf{B-L}=-2$ operators that arise from the matter-Higgs interactions. A list of these operators is given in Table 4 which include dimension five, seven and nine operators all of which are $\mathsf{B-L}$ violating. The dimension 5 operator is the well -known Weinberg operator which gives mass to the neutrinos, while the other operators can enter in GUT scale baryogenesis. Further, these operators can generate new kinds of proton decay modes [26]. Thus the conventional $\mathsf{B-L}=0$ baryon and lepton number violating operators give rise to the modes $p\to e^+\pi^0, p\to\bar{\nu}K^+$ while $\mathsf{B-L}=-2$ operators can generate proton decay modes such as $p\to\nu\pi^+$, $n\to e^-\pi^+, e^-K^+$. Further the $\mathsf{B-L}=-2$ dimension nine operator $\mathbf{D^cD^cU^cD^cD^cU^c}$ can induce $n-\bar{n}$ oscillations. Further, as discussed in Sec.(4), the decays of the heavy Higgs given by Eq.(29)- Eq.(35) which are $\mathsf{B-L}$ violating and which contain new sources of CP violation can be used to generate GUT scale baryogenesis which cannot be washed out by sphaleron interactions at the electroweak scale. In addition to the $\mathsf{B-L}=-2$ operators discussed here there are a large number of $\mathsf{B-L}=-2$ operators that arise from four field Higgs interactions such as from $(\mathsf{126}\times\mathsf{126})_r\cdot(X\times Y)_r$ and $(\mathsf{126}\times\overline{\mathsf{126}})_r\cdot(X\times Y)_r$ where $X,Y=\mathsf{10},\ \mathsf{45},\ \mathsf{54},\ \mathsf{120},\ \mathsf{126},\ \overline{\mathsf{126}},\ \mathsf{210}$ of Higgs fields(many of these operators are discussed in [25]). A complete analysis of $\mathsf{B-L}$ violating interactions from this set is outside the scope of this work and requires a separate analysis.

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Appendix A: Notation

In this Appendix we display the decomposition of $\mathsf{16}-$plet of matter and $\mathsf{10}-$, $\mathsf{120}-$, $\overline{\mathsf{126}}-$ and $\mathsf{210}-$plets of Higgs of $\mathsf{SO(10)}$ in terms of $\mathsf{SU(5)}$ representations. Thus we have

$$\begin{aligned}
\mathsf{16} &= \mathsf{1}(-5)\,[\mathsf{M}_{\acute{x}}]+\bar{\mathsf{5}}(3)\,[\mathsf{M}_{\acute{x}i}]+\mathsf{10}(-1)\left[\mathsf{M}_{\acute{x}}^{ij}\right],\\
\mathsf{10_r} &= \mathsf{5}(2)\left[\mathsf{H}^{(10_r)i}\right]+\bar{\mathsf{5}}(-2)\left[\mathsf{H}_i^{(10_r)}\right],\\
\mathsf{120} &= \mathsf{5}(2)\left[\mathsf{H}^{(120)i}\right]+\bar{\mathsf{5}}(-2)\left[\mathsf{H}_i^{(120)}\right]+\mathsf{10}(-6)\left[\mathsf{H}^{(120)ij}\right]+\overline{\mathsf{10}}(6)\left[\mathsf{H}_{ij}^{(120)}\right]+\mathsf{45}(2)\left[\mathsf{H}_k^{(120)ij}\right]\\
&\quad+\overline{\mathsf{45}}(-2)\left[\mathsf{H}_{ij}^{(120)k}\right],\\
\overline{\mathsf{126}} &= \mathsf{1}(10)\left[\mathsf{H}^{(\overline{126})}\right]+\mathsf{5}(2)\left[\mathsf{H}^{(\overline{126})i}\right]+\overline{\mathsf{10}}(6)\left[\mathsf{H}_{ij}^{(\overline{126})}\right]+\mathsf{15}(-6)\left[\mathsf{H}_{(S)}^{(\overline{126})ij}\right]+\overline{\mathsf{45}}(-2)\left[\mathsf{H}_{ij}^{(\overline{126})k}\right]\\
&\quad+\mathsf{50}(2)\left[\mathsf{H}_{lm}^{(\overline{126})ijk}\right],\\
\mathsf{210} &= \mathsf{1}(0)\left[\mathsf{H}^{(210)}\right]+\mathsf{5}(-8)\left[\mathsf{H}^{(210)i}\right]+\bar{\mathsf{5}}(8)\left[\mathsf{H}_i^{(210)}\right]+\mathsf{10}(4)\left[\mathsf{H}^{(210)ij}\right]+\overline{\mathsf{10}}(-4)\left[\mathsf{H}_{ij}^{(210)}\right]\\
&\quad+\mathsf{24}(0)\left[\mathsf{H}_j^{(120)i}\right]+\mathsf{40}(-4)\left[\mathsf{H}_l^{(210)ijk}\right]+\overline{\mathsf{40}}(4)\left[\mathsf{H}_{ijk}^{(210)l}\right]+\mathsf{75}(0)\left[\mathsf{H}_{kl}^{(210)ij}\right],
\end{aligned}\tag{66}$$

where $i,\ j=1,...,5$ are $\mathsf{SU}(5)$ indices, $\acute{u},\ \acute{v},\ \acute{w},\ \acute{x},\ \acute{y},\ \acute{z}=1,\ 2,\ 3$ represent generation indices and $r,\ s=1,\ 2$ count the number of 10 plet of $\mathsf{SO}(10)$ used in our model of the missing partner mechanism. The following are the Standard Model particle assignments

$$M_{\acute{x}}=\boldsymbol{\nu}^{\mathsf{c}}_{\acute{x}};\qquad \mathsf{M}_{\acute{x}\alpha}=\mathbf{D}^{\mathsf{c}}_{\acute{x}\alpha};\qquad \mathsf{M}_{\acute{x}a}=\begin{pmatrix}\mathbf{E}_{\acute{x}}\\ -\boldsymbol{\nu}_{\acute{x}}\end{pmatrix}=\mathbf{L}_{\acute{x}a};$$
$$\mathsf{M}^{a\alpha}_{\acute{x}}=\begin{pmatrix}\mathbf{U}^{\alpha}_{\acute{x}}\\ \mathsf{D}^{\alpha}_{\acute{x}}\end{pmatrix}=\mathbf{Q}^{a\alpha}_{\acute{x}};\qquad \mathsf{M}^{\alpha\beta}_{\acute{x}}=\epsilon^{\alpha\beta\gamma}\mathbf{U}^{\mathsf{c}}_{\acute{x}\gamma};\qquad \mathsf{M}^{ab}_{\acute{x}}=\epsilon^{ab}\mathbf{E}^{\mathsf{c}}_{\acute{x}}, \tag{67}$$

where $\alpha,\ \beta,\ \gamma=1,\ 2,\ 3$ are $\mathsf{SU}(3)$ color indices, while $a,\ b=1,\ 2$ are $\mathsf{SU}(2)$ weak indices and the superscript $^{\mathsf{c}}$ denotes charge conjugation.

Appendix B: Decomposition of SU(5) fields in terms of Standard Model states

In this appendix we give a decomposition of the 24, 45, 50 and 75-plets of $\mathsf{SU}(5)$ in terms of $\mathsf{SU}(3)_{\mathsf{C}}\times\mathsf{SU}(2)_{\mathsf{L}}\times\mathsf{U}(1)_{\mathsf{Y}}$ fields. These fields are needed in the spontaneous breaking of GUT and electroweak symmetry.

B1: Decomposition of 24-plet of SU(5)

The 24-plet of $\mathsf{SU}(5)$, residing in 210-plet of $\mathsf{SO}(10)$, has the following $\mathsf{SU}(3)_{\mathsf{C}}\times\mathsf{SU}(2)_{\mathsf{L}}\times\mathsf{U}(1)_{\mathsf{Y}}$ decomposition

$$\mathsf{H}^{(210)i}_{j}(24)=(1,1,0)\mathbf{S}_{24_{210}}+(1,3,0)\mathbf{U}^{a}_{b}+(8,1,0)\mathbf{U}^{\alpha}_{\beta}+[(3,2,-5)\mathbf{U}^{\alpha}_{a}+c.c.], \tag{68}$$

where we have defined

$$\mathsf{H}^{(210)\alpha}_{\alpha}=-\mathsf{H}^{(210)a}_{a}\equiv\mathbf{S}_{24_{210}}. \tag{69}$$

The relationship above follows from the tracelessness condition on the tensor $\mathsf{H}^{(210)i}_{j}$. The reducible tensors of the 24-plet can be expressed in terms of the irreducible ones as follows:

$$\mathsf{H}^{(210)a}_{b}=\mathbf{U}^{a}_{b}-\frac{1}{2}\delta^{a}_{b}\mathbf{S}_{24_{210}};\qquad\qquad \mathsf{H}^{(210)\alpha}_{\beta}=\mathbf{U}^{\alpha}_{\beta}+\frac{1}{3}\delta^{\alpha}_{\beta}\mathbf{S}_{24_{210}}. \tag{70}$$

The kinetic energy of the 24-plet is given by

$$\begin{aligned}-\partial_A\mathsf{H}^{(210)i}_{j}\partial^A\mathsf{H}^{(210)i\dagger}_{j} \quad &= \quad -\Big[\partial_A\mathbf{S}_{24_{210}}\partial^A\mathbf{S}^{\dagger}_{24_{210}}+\partial_A\mathbf{U}^{\alpha}_{\beta}\partial^A\mathbf{U}^{\alpha\dagger}_{\beta}+\partial_A\mathbf{U}^{a}_{b}\partial^A\mathbf{U}^{a\dagger}_{b}+\partial_A\mathbf{U}^{\alpha}_{a}\partial^A\mathbf{U}^{\alpha\dagger}_{a}\\ &\qquad +\partial_A\mathbf{U}^{a}_{\alpha}\partial^A\mathbf{U}^{a\dagger}_{\alpha}\Big],\end{aligned} \tag{71}$$

so that the SM fields are normalized according to

$$\mathbf{S}_{24_{210}}\to\sqrt{\frac{6}{5}}\mathbf{S}_{24_{210}};\qquad \mathbf{U}^{\alpha}_{\beta}\to\mathbf{U}^{\alpha}_{\beta};\qquad \mathbf{U}^{a}_{b}\to\mathbf{U}^{a}_{b};\qquad \mathbf{U}^{\alpha}_{a}\to\mathbf{U}^{\alpha}_{a};\qquad \mathbf{U}^{a}_{\alpha}\to\mathbf{U}^{a}_{\alpha}. \tag{72}$$

B2: Decomposition of 45-plet of SU(5)

The 45-plet of $\mathsf{SU}(5)$, residing in 120 and 126-plets of $\mathsf{SO}(10)$, has the following $\mathsf{SU}(3)_{\mathsf{C}}\times\mathsf{SU}(2)_{\mathsf{L}}\times\mathsf{U}(1)_{\mathsf{Y}}$ decomposition

$$\begin{aligned}\begin{pmatrix}\mathsf{H}^{(120)ij}_{k}\\ \mathsf{H}^{(126)ij}_{k}\end{pmatrix}(45) \quad &= \quad (1,2,3)\begin{pmatrix}{}^{(45_{120})}\mathsf{D}^{a}\\ {}^{(45_{126})}\mathsf{D}^{a}\end{pmatrix}+(3,1,-2)\begin{pmatrix}{}^{(45_{120})}\mathsf{T}^{\alpha}\\ {}^{(45_{126})}\mathsf{T}^{\alpha}\end{pmatrix}+(3,3,-2)\mathbf{V}^{a\alpha}_{b}+(\overline{3},1,8)\mathbf{V}_{\alpha}+(\overline{3},2,-7)\mathbf{V}_{a\alpha}\\ &\quad +(\overline{6},1,-2)\mathbf{V}^{\alpha\beta}_{\gamma}+(8,2,3)\mathbf{V}^{\alpha a}_{\beta},\end{aligned} \tag{73}$$

where we have defined

$$\begin{pmatrix}\mathsf{H}^{(120)\beta a}_{\beta}\\ \mathsf{H}^{(126)\beta a}_{\beta}\end{pmatrix}=-\begin{pmatrix}\mathsf{H}^{(120)ba}_{b}\\ \mathsf{H}^{(120)ba}_{b}\end{pmatrix}\equiv\begin{pmatrix}{}^{(45_{120})}\mathsf{D}^{a}\\ {}^{(45_{126})}\mathsf{D}^{a}\end{pmatrix};\qquad\qquad \begin{pmatrix}\mathsf{H}^{(120)\beta\alpha}_{\beta}\\ \mathsf{H}^{(126)\beta\alpha}_{\beta}\end{pmatrix}=-\begin{pmatrix}\mathsf{H}^{(120)b\alpha}_{b}\\ \mathsf{H}^{(126)b\alpha}_{b}\end{pmatrix}\equiv\begin{pmatrix}{}^{(45_{120})}\mathsf{T}^{\alpha}\\ {}^{(45_{126})}\mathsf{T}^{\alpha}\end{pmatrix}. \tag{74}$$

The relationship above follows from the tracelessness condition on the tensor $\mathsf{H}_k^{(120)ij}$ and $\mathsf{H}_k^{(126)ij}$. The reducible tensors of the $\mathsf{45}$-plet can be expressed in terms of the irreducible ones as follows:

$$\begin{pmatrix}\mathsf{H}_b^{(120)a\alpha}\\ \mathsf{H}_b^{(126)a\alpha}\end{pmatrix} = \mathbf{V}_b^{a\alpha} - \frac{1}{2}\delta_b^a \begin{pmatrix}{}^{(45_{120})}\mathsf{T}^\alpha\\ {}^{(45_{126})}\mathsf{T}^\alpha\end{pmatrix}; \qquad \begin{pmatrix}\mathsf{H}_\beta^{(120)\alpha a}\\ \mathsf{H}_\beta^{(126)\alpha a}\end{pmatrix} = \mathbf{V}_\beta^{\alpha a} + \frac{1}{3}\delta_\beta^\alpha \begin{pmatrix}{}^{(45_{120})}\mathsf{D}^a\\ {}^{(45_{126})}\mathsf{D}^a\end{pmatrix};$$
$$\begin{pmatrix}\mathsf{H}_\alpha^{(120)ab}\\ \mathsf{H}_\alpha^{(126)ab}\end{pmatrix} = \epsilon^{ab}\mathbf{V}_\alpha; \qquad \begin{pmatrix}\mathsf{H}_a^{(120)\alpha\beta}\\ \mathsf{H}_a^{(126)\alpha\beta}\end{pmatrix} = \epsilon^{\alpha\beta\gamma}\mathbf{V}_{a\gamma}; \qquad \begin{pmatrix}\mathsf{H}_c^{(120)ab}\\ \mathsf{H}_c^{(126)ab}\end{pmatrix} = \delta_c^b \begin{pmatrix}{}^{(45_{120})}\mathsf{D}^a\\ {}^{(45_{126})}\mathsf{D}^a\end{pmatrix} - \delta_c^a \begin{pmatrix}{}^{(45_{120})}\mathsf{D}^b\\ {}^{(45_{126})}\mathsf{D}^b\end{pmatrix}$$
$$\begin{pmatrix}\mathsf{H}_\gamma^{(120)\alpha\beta}\\ \mathsf{H}_\gamma^{(126)\alpha\beta}\end{pmatrix} = \mathbf{V}_\gamma^{\alpha\beta} + \frac{1}{2}\left[\delta_\gamma^\alpha \begin{pmatrix}{}^{(45_{120})}\mathsf{T}^\beta\\ {}^{(45_{126})}\mathsf{T}^\beta\end{pmatrix} - \delta_\gamma^\beta \begin{pmatrix}{}^{(45_{120})}\mathsf{T}^\alpha\\ {}^{(45_{126})}\mathsf{T}^\alpha\end{pmatrix}\right]. \tag{75}$$

The kinetic energy of the $\mathsf{45}$-plet is given by

$$\begin{aligned} -\partial_A \begin{pmatrix}\mathsf{H}_k^{(120)ij}\\ \mathsf{H}_k^{(126)ij}\end{pmatrix} \partial^A \begin{pmatrix}\mathsf{H}_k^{(120)ij\dagger}\\ \mathsf{H}_k^{(126)ij\dagger}\end{pmatrix} \quad = \quad & -\Bigg[\partial_A \begin{pmatrix}{}^{(45_{120})}\mathsf{D}^a\\ {}^{(45_{126})}\mathsf{D}^a\end{pmatrix} \partial^A \begin{pmatrix}{}^{(45_{120})}\mathsf{D}^{a\dagger}\\ {}^{(45_{126})}\mathsf{D}^{a\dagger}\end{pmatrix} + \partial_A \begin{pmatrix}{}^{(45_{120})}\mathsf{T}^\alpha\\ {}^{(45_{126})}\mathsf{T}^\alpha\end{pmatrix} \partial^A \begin{pmatrix}{}^{(45_{120})}\mathsf{T}^{\alpha\dagger}\\ {}^{(45_{126})}\mathsf{T}^{\alpha\dagger}\end{pmatrix} \\ & + \partial_A \mathbf{V}_b^{a\alpha}\partial^A \mathbf{V}_b^{a\alpha\dagger} + \partial_A \mathbf{V}_\alpha \partial^A \mathbf{V}_\alpha^\dagger + \partial_A \mathbf{V}_{a\alpha}\partial^A \mathbf{V}_{a\alpha}^\dagger \\ & + \partial_A \mathbf{V}_\gamma^{\alpha\beta}\partial^A \mathbf{V}_\gamma^{\alpha\beta\dagger} + \partial_A \mathbf{V}_\beta^{\alpha a}\partial^A \mathbf{V}_\beta^{\alpha a\dagger}\Bigg], \end{aligned} \tag{76}$$

so that the SM fields are normalized according to

$$\begin{pmatrix}{}^{(45_{120})}\mathsf{D}^a\\ {}^{(45_{126})}\mathsf{D}^a\end{pmatrix} \to \frac{1}{2}\sqrt{\frac{3}{2}} \begin{pmatrix}{}^{(45_{120})}\mathsf{D}^a\\ {}^{(45_{126})}\mathsf{D}^a\end{pmatrix}; \qquad \begin{pmatrix}{}^{(45_{120})}\mathsf{T}^\alpha\\ {}^{(45_{126})}\mathsf{T}^\alpha\end{pmatrix} \to \frac{1}{\sqrt{2}} \begin{pmatrix}{}^{(45_{120})}\mathsf{T}^\alpha\\ {}^{(45_{126})}\mathsf{T}^\alpha\end{pmatrix};$$
$$\mathbf{V}_\alpha \to \frac{1}{\sqrt{2}}\mathbf{V}_\alpha; \quad \mathbf{V}_{a\alpha} \to \frac{1}{\sqrt{6}}\mathbf{V}_{a\alpha}; \quad \mathbf{V}_\beta^{\alpha a} \to \frac{1}{\sqrt{2}}\mathbf{V}_\beta^{\alpha a}; \quad \mathbf{V}_b^{a\alpha} \to \frac{1}{\sqrt{2}}\mathbf{V}_b^{a\alpha}; \quad \mathbf{V}_\gamma^{\alpha\beta} \to \frac{1}{\sqrt{2}}\mathbf{V}_\gamma^{\alpha\beta}. \tag{77}$$

One can now extend the above results to $\overline{\mathsf{45}}$ of $\mathsf{SU(5)}$ contained in $\mathsf{120}$ and $\overline{\mathsf{126}}$ plets.

B3: Decomposition of 50-plet of SU(5)

The $\mathsf{50}$-plet of $\mathsf{SU(5)}$, residing in $\overline{\mathsf{126}}$-plet of $\mathsf{SO(10)}$, has the following $\mathsf{SU(3)_C}\times\mathsf{SU(2)_L}\times\mathsf{U(1)_Y}$ decomposition

$$\begin{aligned} \mathsf{H}_{lm}^{(\overline{126})ijk}(\mathsf{50}) \quad = \quad & (1,1,-12)\mathbf{W} + (3,1,-2)^{(50_{\overline{126}})}\mathsf{T}^\alpha + (\bar{3},2,-7)\mathbf{W}_a^{\alpha\beta} + (\bar{6},3,-2)\mathbf{W}_{\gamma b}^{\alpha\beta a} + (6,1,8)\mathbf{W}_{\beta\gamma}^\alpha \\ & + (8,2,3)\mathbf{W}_\beta^{\alpha a}, \end{aligned} \tag{78}$$

where we have defined

$$\mathsf{H}_{ab}^{(\overline{126})ab\gamma} = \mathsf{H}_{\alpha\beta}^{(\overline{126})\alpha\beta\gamma} = -\mathsf{H}_{a\alpha}^{(\overline{126})a\alpha\gamma} \equiv {}^{(50_{\overline{126}})}\mathsf{T}^\alpha; \qquad \mathsf{H}_{\gamma a}^{(\overline{126})\gamma\alpha\beta} \equiv \mathbf{W}_a^{\alpha\beta}; \qquad \overline{\Delta}_{\gamma a}^{\gamma\alpha\beta} \equiv \mathbf{W}_\beta^{\alpha a}. \tag{79}$$

Again the first relationship follows from the tracelessness condition on the $\mathsf{SU(5)}$ irreducible tensor $\mathsf{H}_{lm}^{(\overline{126})ijk}$:

$$\mathsf{H}_{a\alpha}^{(\overline{126})a\alpha i} = -\frac{1}{2}\left[\mathsf{H}_{\alpha\beta}^{(\overline{126})\alpha\beta i} + \mathsf{H}_{ab}^{(\overline{126})abi}\right]. \tag{80}$$

The reducible tensors of the $\mathsf{50}$-plet can be expressed in terms of the irreducible ones as follows:

$$\mathsf{H}_{\rho\sigma}^{(\overline{126})\alpha\beta\gamma} = \frac{1}{2}\left[\delta_\rho^\alpha\delta_\sigma^{\beta\,(50_{\overline{126}})}\mathsf{T}^\gamma - \delta_\rho^\beta\delta_\sigma^{\alpha\,(50_{\overline{126}})}\mathsf{T}^\gamma - \delta_\rho^\alpha\delta_\sigma^{\gamma\,(50_{\overline{126}})}\mathsf{T}^\beta + \delta_\rho^\gamma\delta_\sigma^{\alpha\,(50_{\overline{126}})}\mathsf{T}^\beta + \delta_\rho^\beta\delta_\sigma^{\gamma\,(50_{\overline{126}})}\mathsf{T}^\alpha - \delta_\rho^\gamma\delta_\sigma^{\beta\,(50_{\overline{126}})}\mathsf{T}^\alpha\right];$$
$$\mathsf{H}_{\gamma\sigma}^{(\overline{126})\alpha\beta a} = \delta_\gamma^\alpha\mathbf{W}_\sigma^{\beta a} - \delta_\sigma^\alpha\mathbf{W}_\gamma^{\beta a} + \delta_\sigma^\beta\mathbf{W}_\gamma^{\alpha a} - \delta_\gamma^\beta\mathbf{W}_\sigma^{\alpha a}; \qquad \mathsf{H}_{ab}^{(\overline{126})\alpha\beta\gamma} = \epsilon^{\alpha\beta\gamma}\epsilon_{ab}\mathbf{W};$$
$$\mathsf{H}_{\sigma a}^{(\overline{126})\alpha\beta\gamma} = \delta_\sigma^\gamma\mathbf{W}_a^{\alpha\beta} - \delta_\sigma^\beta\mathbf{W}_a^{\alpha\gamma} + \delta_\sigma^\alpha\mathbf{W}_a^{\beta\gamma}; \qquad \mathsf{H}_{\beta\gamma}^{(\overline{126})\alpha ab} = \epsilon^{ab}\mathbf{W}_{\beta\gamma}^\alpha;$$
$$\mathsf{H}_{\gamma b}^{(\overline{126})\alpha\beta a} = \mathbf{W}_{\gamma b}^{\alpha\beta a} + \frac{1}{4}\delta_b^a\left[\delta_\gamma^{\alpha\,(50_{\overline{126}})}\mathsf{T}^\beta - \delta_\gamma^{\beta\,(50_{\overline{126}})}\mathsf{T}^\alpha\right]; \qquad \mathsf{H}_{c\beta}^{(\overline{126})ab\alpha} = \delta_c^a\mathbf{W}_\beta^{\alpha b} - \delta_c^b\mathbf{W}_\beta^{\alpha a};$$
$$\mathsf{H}_{cd}^{(\overline{126})ab\alpha} = \frac{1}{2}\left[\delta_c^a\delta_d^b - \delta_d^a\delta_c^b\right]{}^{(50_{\overline{126}})}\mathsf{T}^\alpha; \qquad \mathsf{H}_{bc}^{(\overline{126})\alpha\beta a} = \delta_c^a\mathbf{W}_b^{\alpha\beta} - \delta_b^a\mathbf{W}_c^{\alpha\beta}; \tag{81}$$

The kinetic energy of the $\mathsf{50}$-plet is given by

$$\begin{aligned} -\partial_A\mathsf{H}_{lm}^{(\overline{126})ijk}\partial^A\mathsf{H}_{lm}^{(\overline{126})ijk\dagger} \quad = \quad & -\Bigg[\partial_A\mathbf{W}\partial^A\mathbf{W}^\dagger + \partial_A{}^{(50_{\overline{126}})}\mathsf{T}^{\alpha\,(50_{\overline{126}})}\mathsf{T}^\alpha + \frac{1}{2!}\partial_A\mathbf{W}_a^{\alpha\beta}\partial^A\mathbf{W}_a^{\alpha\beta\dagger} + \frac{1}{2!}\partial_A\mathbf{W}_\beta^{\alpha a}\partial^A\mathbf{W}_\beta^{\alpha a\dagger} \\ & + \frac{1}{3!}\frac{1}{2!}\partial_A\mathbf{W}_{\gamma b}^{\alpha\beta a}\partial^A\mathbf{W}_{\gamma b}^{\alpha\beta a\dagger} + \frac{1}{2!}\partial_A\mathbf{W}_{\beta\gamma}^\alpha\partial^A\mathbf{W}_{\beta\gamma}^{\alpha\dagger}\Bigg], \end{aligned} \tag{82}$$

so that the SM fields are normalized according to

$$\mathbf{W} \to \frac{1}{2\sqrt{3}}\mathbf{W}; \qquad {}^{(50_{\overline{126}})}\mathsf{T}^{\alpha} \to \frac{1}{3}{}^{(50_{\overline{126}})}\mathsf{T}^{\alpha}; \qquad \mathbf{W}_a^{\alpha\beta} \to \frac{1}{2\sqrt{6}}\mathbf{W}_a^{\alpha\beta};$$
$$\mathbf{W}_\beta^{\alpha a} \to \frac{1}{4\sqrt{3}}\mathbf{W}_\beta^{\alpha a}; \qquad \mathbf{W}_{\gamma b}^{\alpha\beta a} \to \frac{1}{6\sqrt{2}}\mathbf{W}_{\gamma b}^{\alpha\beta a}; \qquad \mathbf{W}_{\beta\gamma}^{\alpha} \to \frac{1}{2\sqrt{3}}\mathbf{W}_{\beta\gamma}^{\alpha}. \tag{83}$$

One can now extend the above results to $\overline{\mathsf{50}}$ of $\mathsf{SU(5)}$ contained in $\mathsf{126}$ plet.

B4: Decomposition of $\mathbf{75}$-plet of $\mathbf{SU(5)}$

The $\mathsf{75}$-plet of $\mathsf{SU(5)}$, residing in $\mathsf{210}$-plet of $\mathsf{SO(10)}$, has the following $\mathsf{SU(3)_C \times SU(2)_L \times U(1)_Y}$ decomposition

$$\begin{aligned}\mathsf{H}_{kl}^{(210)ij}(\mathsf{75}) \quad = \quad & (1,1,0)\mathbf{S}_{75_{210}} + (8,1,0)\mathbf{X}_\beta^\alpha + (8,3,0)\mathbf{X}_{\beta b}^{\alpha a} + [(3,2,-5)\mathbf{X}_a^\alpha + (\bar{6},2,-5)\mathbf{X}_{\gamma a}^{\alpha\beta} \\ & + (\bar{3},1,-10)\mathbf{X}_\alpha + c.c.],\end{aligned} \tag{84}$$

where we have defined

$$\mathsf{H}_{ab}^{(210)ab} = \mathsf{H}_{\alpha\beta}^{(210)\alpha\beta} = -\mathsf{H}_{\alpha a}^{(210)\alpha a} \equiv \mathbf{S}_{75_{210}}; \qquad \mathsf{H}_{\alpha b}^{(210)ab} \equiv \mathbf{X}_a^\alpha; \qquad \mathbf{X}_\beta^\alpha \equiv \mathsf{H}_{\beta a}^{(210)\alpha a} + \frac{1}{3}\delta_\beta^\alpha \mathbf{S}_{75_{210}}. \tag{85}$$

The first relationship above follows from the double tracelessness condition on the tensor $\mathsf{H}_{kl}^{(210)ij}$:

$$\mathsf{H}_{\alpha a}^{(210)\alpha a} = -\frac{1}{2}\left(\mathsf{H}_{\alpha\beta}^{(210)\alpha\beta} + \mathsf{H}_{ab}^{(210)ab}\right). \tag{86}$$

The reducible tensors of the $\mathsf{75}$-plet can be expressed in terms of the irreducible ones as follows:

$$\begin{aligned}
\mathsf{H}_{cd}^{(210)ab} &= \frac{1}{2}\left(\delta_c^a\delta_d^b - \delta_d^a\delta_c^b\right)\mathbf{S}_{75_{210}}; & \mathsf{H}_{\gamma\sigma}^{(210)\alpha\beta} &= \frac{1}{2}\left(\delta_\sigma^\alpha\mathbf{X}_\gamma^\beta - \delta_\gamma^\alpha\mathbf{X}_\sigma^\beta\right) + \frac{1}{6}\left(\delta_\gamma^\alpha\delta_\sigma^\beta - \delta_\sigma^\alpha\delta_\gamma^\beta\right)\mathbf{S}_{75_{210}};\\
\mathsf{H}_{\beta b}^{(210)\alpha a} &= \mathbf{X}_{\beta b}^{\alpha a} + \frac{1}{2}\delta_b^a\mathbf{X}_\beta^\alpha - \frac{1}{6}\delta_b^a\delta_\beta^\alpha\mathbf{S}_{75_{210}}; & \mathsf{H}_{\gamma a}^{(210)\alpha\beta} &= \mathbf{X}_{\gamma a}^{\alpha\beta} - \frac{1}{2}\left(\delta_\gamma^\alpha\mathbf{X}_a^\beta - \delta_\gamma^\beta\mathbf{X}_a^\alpha\right);\\
\mathsf{H}_{ab}^{(210)\alpha\beta} &= \epsilon_{ab}\epsilon^{\alpha\beta\gamma}\mathbf{X}_\gamma; & \mathsf{H}_{bc}^{(210)a\alpha} &= \delta_b^a\mathbf{X}_c^\alpha - \delta_c^a\mathbf{X}_b^\alpha.
\end{aligned} \tag{87}$$

The kinetic energy of the $\mathsf{75}$-plet is given by

$$\begin{aligned}
-\partial_A\mathsf{H}_{kl}^{(210)ij}\partial^A\mathsf{H}_{kl}^{(210)ij\dagger} \quad = \quad & -\Big[\partial_A\mathbf{S}_{75_{210}}\partial^A\mathbf{S}_{75_{210}}^\dagger + \partial_A\mathbf{X}_\alpha\partial^A\mathbf{X}_\alpha^\dagger + \partial_A\mathbf{X}^\alpha\partial^A\mathbf{X}^{\alpha\dagger} + \partial_A\mathbf{X}_\beta^\alpha\partial^A\mathbf{X}_\beta^{\alpha\dagger}\\
& + \partial_A\mathbf{X}_a^\alpha\partial^A\mathbf{X}_a^{\alpha\dagger} + \partial_A\mathbf{X}_\alpha^a\partial^A\mathbf{X}_\alpha^{a\dagger} + + \frac{1}{2!}\frac{1}{2!}\partial_A\mathbf{X}_{\gamma a}^{\alpha\beta}\partial^A\mathbf{X}_{\gamma a}^{\alpha\beta\dagger}\\
& + \frac{1}{2!}\frac{1}{2!}\partial_A\mathbf{X}_{\alpha\beta}^{\gamma a}\partial^A\mathbf{X}_{\alpha\beta}^{\gamma a\dagger} + \frac{1}{2!}\frac{1}{2!}\partial_A\mathbf{X}_{\beta b}^{\alpha a}\partial^A\mathbf{X}_{\beta b}^{\alpha a\dagger}\Big],
\end{aligned} \tag{88}$$

so that the SM fields are normalized according to

$$\begin{aligned}
&\mathbf{S}_{75_{210}} \to \frac{1}{\sqrt{2}}\mathbf{S}_{75_{210}}; \qquad \mathbf{X}_\alpha \to \frac{1}{2}\mathbf{X}_\alpha; \qquad \mathbf{X}^\alpha \to \frac{1}{2}\mathbf{X}^\alpha; \qquad \mathbf{X}_\beta^\alpha \to \frac{1}{\sqrt{3}}\mathbf{X}_\beta^\alpha;\\
&\mathbf{X}_a^\alpha \to \frac{1}{\sqrt{6}}\mathbf{X}_a^\alpha; \qquad \mathbf{X}_\alpha^a \to \frac{1}{\sqrt{6}}\mathbf{X}_\alpha^a; \qquad \mathbf{X}_{\gamma a}^{\alpha\beta} \to \frac{1}{2\sqrt{2}}\mathbf{X}_{\gamma a}^{\alpha\beta}; \qquad \mathbf{X}_{\alpha\beta}^{\gamma a} \to \frac{1}{2\sqrt{2}}\mathbf{X}_{\alpha\beta}^{\gamma a};\\
&\mathbf{X}_{\beta b}^{\alpha a} \to \frac{1}{4}\mathbf{X}_{\beta b}^{\alpha a}.
\end{aligned} \tag{89}$$

Appendix C: Details of GUT symmetry breaking

Here, we give further details of the GUT symmetry breaking discussed in Sec.(2). As mentioned in Sec.(2) the fields that enter in the GUT symmetry breaking are $\mathsf{126} + \overline{\mathsf{126}} + \mathsf{210}$. A decomposition of the $\mathsf{SO(10)}$ invariant interaction for these fields: $\mathsf{210}\cdot\mathsf{210}$, $\mathsf{126}\cdot\overline{\mathsf{126}}$, $\mathsf{210}\cdot\mathsf{210}\cdot\mathsf{210}$ and $\mathsf{210}\cdot\mathsf{126}\cdot\overline{\mathsf{126}}$ in $\mathsf{SU(5)}$ fragments is exhibited below.

$$\begin{aligned}
W_{GUT} \quad &= \quad m_\Phi\Phi_{\mu\nu\sigma\xi}\Phi_{\mu\nu\sigma\xi} + m_\Delta\Delta_{\mu\nu\sigma\xi\zeta}\overline{\Delta}_{\mu\nu\sigma\xi\zeta} + \lambda\Phi_{\mu\nu\sigma\xi}\Phi_{\sigma\xi\rho\tau}\Phi_{\rho\tau\mu\nu} + \eta\Phi_{\mu\nu\sigma\xi}\Delta_{\mu\nu\rho\tau\zeta}\overline{\Delta}_{\sigma\xi\rho\tau\zeta}\\
&= \quad m_\Phi\frac{1}{2^4}\left[6\Phi_{c_ic_j\bar{c}_k\bar{c}_l}\Phi_{c_kc_l\bar{c}_i\bar{c}_j} + \cdots\right] + m_\Delta\frac{1}{2^5}\left[\Delta_{c_ic_jc_kc_lc_m}\overline{\Delta}_{\bar{c}_i\bar{c}_j\bar{c}_k\bar{c}_l\bar{c}_m} + \cdots\right]\\
&\quad + \lambda\frac{1}{2^6}\left[2\Phi_{c_ic_j\bar{c}_k\bar{c}_l}\Phi_{c_kc_l\bar{c}_m\bar{c}_n}\Phi_{c_mc_n\bar{c}_i\bar{c}_j} + 8\Phi_{c_ic_j\bar{c}_k\bar{c}_l}\Phi_{c_mc_k\bar{c}_n\bar{c}_i}\Phi_{c_lc_n\bar{c}_j\bar{c}_m} + \cdots\right]
\end{aligned}$$

$$
\begin{aligned}
&+\eta\frac{1}{27}\left[\Phi_{c_ic_j\bar{c}_k\bar{c}_l}\Delta_{c_kc_lc_mc_nc_p}\overline{\Delta}_{\bar{c}_i\bar{c}_j\bar{c}_m\bar{c}_n\bar{c}_p}+\cdots\right]\\
=\;& m_\Phi\left[\frac{3}{8}\mathsf{H}^{(210)ij}_{kl}\;\mathsf{H}^{(210)kl}_{ij}+\frac{1}{2}\mathsf{H}^{(210)i}_{j}\;\mathsf{H}^{(210)j}_{i}+\frac{3}{80}\mathbf{S}_{1_{210}}\mathbf{S}_{1_{210}}+\cdots\right]+m_\Delta\left[\frac{15}{2}\mathbf{S}_{1_{126}}\mathbf{S}_{1_{\overline{126}}}+\cdots\right]\\
&+\lambda\left[\frac{1}{32}\mathsf{H}^{(210)ij}_{kl}\;\mathsf{H}^{(210)kl}_{mn}\;\mathsf{H}^{(210)mn}_{ij}+\frac{1}{8}\mathsf{H}^{(210)ij}_{kl}\;\mathsf{H}^{(210)ln}_{jm}\;\mathsf{H}^{(210)mk}_{ni}-\frac{1}{8}\mathsf{H}^{(210)ij}_{kl}\;\mathsf{H}^{(210)kl}_{jm}\;\mathsf{H}^{(210)m}_{i}\right.\\
&\quad+\frac{1}{24}\mathsf{H}^{(210)ij}_{kl}\;\mathsf{H}^{(210)k}_{i}\;\mathsf{H}^{(210)l}_{j}+\frac{1}{80}\mathsf{H}^{(210)ij}_{kl}\;\mathsf{H}^{(210)kl}_{ij}\;\mathbf{S}_{1_{210}}+\frac{7}{108}\mathsf{H}^{(210)i}_{j}\;\mathsf{H}^{(210)j}_{k}\;\mathsf{H}^{(210)k}_{i}\\
&\quad\left.-\frac{1}{160}\mathsf{H}^{(210)i}_{j}\;\mathsf{H}^{(210)j}_{i}\;\mathbf{S}_{1_{210}}-\frac{3}{3200}\mathbf{S}^3_{1_{210}}+\cdots\right]\\
&+\eta\left[-\frac{3}{16}\mathbf{S}_{1_{126}}\mathbf{S}_{1_{\overline{126}}}\mathbf{S}_{1_{210}}+\cdots\right].
\end{aligned}\tag{90}
$$

We show only those fields whose VEVs enter in the spontaneous breaking of the GUT symmetry. These include the $\mathsf{75}$ ($\mathsf{H}^{(210)ij}_{kl}$) and $\mathsf{24}$ ($\mathsf{H}^{(210)i}_{j}$) in the $\mathsf{210}$-plet, and the singlets $\mathbf{S}_{1_{210}}$, $\mathbf{S}_{1_{126}}$, $\mathbf{S}_{1_{\overline{126}}}$ in the $\mathsf{210}$, $\mathsf{126}$-plet, $\overline{\mathsf{126}}$-plet, respectively. A further reduction of Eq.(90) gives Eq.(3).

Finally, we redisplay Eqs.(7-9) showing their explicit dependence on the parameters η and λ:

$$
\begin{aligned}
\mathbf{S}_{24_{210}}&\left[9\eta^3\lambda^3\mathbf{S}^3_{24_{210}}-24\eta^2\lambda^2\mathbf{S}^2_{24_{210}}\left(-28\lambda m_\Delta+45\eta m_\Phi\right)+64\eta\lambda\mathbf{S}_{24_{210}}\left(320\lambda^2m^2_\Delta-279\eta\lambda m_\Delta m_\Phi+972\eta^2m^2_\Phi\right)\right.\\
&\left.+3824\left(\lambda m_\Delta-2\eta m_\Phi\right)\left(4\lambda m_\Delta+3\eta m_\Phi\right)^2\right]=0,
\end{aligned}\tag{91}
$$

$$
\mathbf{S}_{75_{210}}=\frac{5\left[\eta\lambda\mathbf{S}^2_{24_{210}}+24\mathbf{S}_{24_{210}}\left(\lambda m_\Delta-2\eta m_\Phi\right)\right]}{6\left[\eta\lambda\mathbf{S}_{24_{210}}+8\left(4\lambda m_\Delta+3\eta m_\Phi\right)\right]},\tag{92}
$$

$$
\begin{aligned}
\mathbf{S}_{1_{126}}\cdot\mathbf{S}_{1_{\overline{126}}}=\;&\frac{1}{216\eta^3\left(\eta\lambda\mathbf{S}_{24_{210}}+32\lambda m_\Delta+24\eta m_\Phi\right)}\left[5\eta^3\lambda^2\mathbf{S}^3_{24_{210}}-32\eta^2\lambda\mathbf{S}^2_{24_{210}}\left(8\lambda m_\Delta+39\eta m_\Phi\right)\right.\\
&-1728\eta\mathbf{S}_{24_{210}}\left(7\lambda^2m^2_\Delta-7\eta\lambda m_\Delta m_\Phi-6\eta^2m^2_\Phi\right)\\
&\left.-13824m_\Delta\left(3\lambda m_\Delta-2\eta m_\Phi\right)\left(4\lambda m_\Delta+3\eta m_\Phi\right)\right].
\end{aligned}\tag{93}
$$

Appendix D: Couplings of light and heavy Higgs sectors

As discussed in Sec.(3), the doublet-triplet splitting involves couplings of light and heavy fields. In this appendix, we give further details of the couplings that enter in Eqs.(14) and (22). There are eight such couplings: $\mathsf{10}\cdot\mathsf{126}\cdot\mathsf{120}$, $\mathsf{10}\cdot\overline{\mathsf{126}}\cdot\mathsf{210}$, $\mathsf{120}\cdot\mathsf{126}\cdot\mathsf{210}$, $\mathsf{120}\cdot\overline{\mathsf{126}}\cdot\mathsf{210}$, $\mathsf{126}\cdot\overline{\mathsf{126}}$, $\mathsf{210}\cdot\mathsf{210}$, $\mathsf{210}\cdot\mathsf{210}\cdot\mathsf{210}$ and $\mathsf{210}\cdot\mathsf{126}\cdot\overline{\mathsf{126}}$. In the analysis below we first give the $\mathsf{SU(5)}$ decomposition of the relevant parts of these $\mathsf{SO(10)}$ invariant couplings and then further reduce them in the $\mathsf{SU(3)_C}\times\mathsf{SU(2)_L}\times\mathsf{U(1)_Y}$ invariant form to exhibit the Higgs doublets and Higgs triplets and their mixings. We here note that an analysis of these couplings in the Pati-Salam decomposition has been carried out in the first two papers of [37]. We give now the details of our analysis in $SU(5)\times U(1)$ decomposition.

Mixing of light and heavy sector

D1: $\mathsf{10}\cdot\mathsf{126}\cdot\mathsf{210}$ Coupling

$$
\begin{aligned}
\mathrm{A}\;{}^1\Gamma_\mu\Delta_{\mu\nu\sigma\xi\zeta}\Phi_{\mu\sigma\xi\zeta}=\;&\frac{\imath}{5!}\mathrm{A}\left[\frac{1}{2\sqrt5}\mathsf{H}^{(10_1)i}\;\mathsf{H}^{(126)}\;\mathsf{H}^{(210)}_{i}+\frac{1}{4\sqrt{10}}\mathsf{H}^{(10_1)i}\;\mathsf{H}^{(126)lm}_{ijk}\;\mathsf{H}^{(210)jk}_{lm}\right.\\
&+\frac{1}{2\sqrt{10}}\mathsf{H}^{(10_1)i}\;\mathsf{H}^{(126)}_{j}\;\mathsf{H}^{(210)j}_{i}-\frac{\sqrt3}{10}\mathsf{H}^{(10_1)i}\;\mathsf{H}^{(126)}_{i}\;\mathsf{H}^{(210)}\\
&\left.+\frac{1}{4\sqrt5}\mathsf{H}^{(10_1)}_{i}\;\mathsf{H}^{(126)jk}_{l}\;\mathsf{H}^{(210)il}_{jk}+\frac{1}{\sqrt{30}}\mathsf{H}^{(10_1)}_{i}\;\mathsf{H}^{(126)ij}_{k}\;\mathsf{H}^{(210)k}_{j}+\cdots\right]\\
=\;&\frac{\imath}{5!}\mathrm{A}\left[\left(\frac{1}{2\sqrt5}\mathbf{S}_{1_{126}}\right){}^{(\overline{5}_{210})}\mathsf{D}_a{}^{(5_{10_1})}\mathsf{D}^a+\left(-\frac{\sqrt3}{10}\mathbf{S}_{1_{210}}-\frac{\sqrt3}{20}\mathbf{S}_{24_{210}}\right){}^{(\overline{5}_{126})}\mathsf{D}_a{}^{(5_{10_1})}\mathsf{D}^a\right.\\
&+\left(-\frac{1}{4\sqrt6}\mathbf{S}_{24_{210}}+\frac{1}{4\sqrt{15}}\mathbf{S}_{75_{210}}\right){}^{(\overline{5}_{10_1})}\mathsf{D}_a{}^{(45_{126})}\mathsf{D}^a
\end{aligned}
$$

$$
\begin{aligned}
&+ \left(\frac{1}{2\sqrt{5}}\mathbf{S}_{1_{126}}\right){}^{(\overline{5}_{210})}\mathsf{T}_\alpha{}^{(5_{10_1})}\mathsf{T}^\alpha + \left(-\frac{\sqrt{3}}{10}\mathbf{S}_{1_{210}} + \frac{1}{10\sqrt{3}}\mathbf{S}_{24_{210}}\right){}^{(\overline{5}_{126})}\mathsf{T}_\alpha{}^{(5_{10_1})}\mathsf{T}^\alpha \\
&+ \left(-\frac{1}{6\sqrt{2}}\mathbf{S}_{24_{210}} - \frac{1}{12\sqrt{5}}\mathbf{S}_{75_{210}}\right){}^{(\overline{5}_{10_1})}\mathsf{T}_\alpha{}^{(45_{126})}\mathsf{T}^\alpha \\
&+ \left(\frac{1}{12\sqrt{5}}\mathbf{S}_{75_{210}}\right){}^{(\overline{50}_{126})}\mathsf{T}_\alpha{}^{(5_{10_1})}\mathsf{T}^\alpha + \cdots\Bigg].
\end{aligned}
\tag{94}
$$

D2: $\mathbf{10}\cdot\overline{\mathbf{126}}\cdot\mathbf{210}$ Coupling

$$
\begin{aligned}
\mathrm{B_r}\ {}^r\Gamma_\mu\overline{\Delta}_{\mu\nu\sigma\xi\zeta}\Phi_{\mu\sigma\xi\zeta} &= -\frac{\imath}{5!}\mathrm{B_r}\Bigg[\frac{1}{2\sqrt{5}}\mathsf{H}_i^{(10_r)}\ \mathsf{H}^{(\overline{126})}\ \mathsf{H}^{(210)i} + \frac{1}{4\sqrt{10}}\mathsf{H}_i^{(10_r)}\ \mathsf{H}_{lm}^{(\overline{126})ijk}\ \mathsf{H}_{jk}^{(210)lm} \\
&\quad + \frac{1}{2\sqrt{10}}\mathsf{H}_i^{(10_r)}\ \mathsf{H}^{(\overline{126})j}\ \mathsf{H}_j^{(210)i} - \frac{\sqrt{3}}{10}\mathsf{H}_i^{(10_r)}\ \mathsf{H}^{(\overline{126})i}\ \mathsf{H}^{(210)} \\
&\quad + \frac{1}{4\sqrt{5}}\mathsf{H}^{(10_r)i}\ \mathsf{H}_{jk}^{(\overline{126})l}\ \mathsf{H}_{il}^{(210)jk} + \frac{1}{\sqrt{30}}\mathsf{H}^{(10_r)i}\ \mathsf{H}_{ij}^{(\overline{126})k}\ \mathsf{H}_k^{(210)j} + \cdots\Bigg] \\
&= -\frac{\imath}{5!}\mathrm{B_r}\Bigg[\left(\frac{1}{2\sqrt{5}}\mathbf{S}_{1_{\overline{126}}}\right){}^{(\overline{5}_{10_r})}\mathsf{D}_a{}^{(5_{210})}\mathsf{D}^a + \left(-\frac{\sqrt{3}}{10}\mathbf{S}_{1_{210}} - \frac{\sqrt{3}}{20}\mathbf{S}_{24_{210}}\right){}^{(\overline{5}_{10_r})}\mathsf{D}_a{}^{(5_{\overline{126}})}\mathsf{D}^a \\
&\quad + \left(-\frac{1}{4\sqrt{6}}\mathbf{S}_{24_{210}} + \frac{1}{4\sqrt{15}}\mathbf{S}_{75_{210}}\right){}^{(\overline{45}_{\overline{126}})}\mathsf{D}_a{}^{(5_{10_r})}\mathsf{D}^a \\
&\quad + \left(\frac{1}{2\sqrt{5}}\mathbf{S}_{1_{\overline{126}}}\right){}^{(\overline{5}_{10_r})}\mathsf{T}_\alpha{}^{(5_{210})}\mathsf{T}^\alpha + \left(-\frac{\sqrt{3}}{10}\mathbf{S}_{1_{210}} + \frac{1}{10\sqrt{3}}\mathbf{S}_{24_{210}}\right){}^{(\overline{5}_{10_r})}\mathsf{T}_\alpha{}^{(5_{\overline{126}})}\mathsf{T}^\alpha \\
&\quad + \left(-\frac{1}{6\sqrt{2}}\mathbf{S}_{24_{210}} - \frac{1}{12\sqrt{5}}\mathbf{S}_{75_{210}}\right){}^{(\overline{45}_{\overline{126}})}\mathsf{T}_\alpha{}^{(5_{10_r})}\mathsf{T}^\alpha \\
&\quad + \left(\frac{1}{12\sqrt{5}}\mathbf{S}_{75_{210}}\right){}^{(\overline{5}_{10_r})}\mathsf{T}_\alpha{}^{(50_{\overline{126}})}\mathsf{T}^\alpha + \cdots\Bigg].
\end{aligned}
\tag{95}
$$

D3: $\mathbf{120}\cdot\mathbf{126}\cdot\mathbf{210}$ Coupling

$$
\begin{aligned}
\mathrm{C}\ \Sigma_{\mu\nu\sigma}\Delta_{\nu\sigma\xi\zeta\rho}\Phi_{\mu\xi\zeta\rho} &= \frac{\imath}{5!}\mathrm{C}\Bigg[\frac{1}{4\sqrt{15}}\mathsf{H}_k^{(120)ij}\ \mathsf{H}_{jmn}^{(126)kl}\ \mathsf{H}_{il}^{(210)mn} + \frac{1}{8\sqrt{15}}\mathsf{H}_i^{(120)jk}\ \mathsf{H}_{jkl}^{(126)mn}\ \mathsf{H}_{mn}^{(210)il} \\
&\quad + \frac{1}{12\sqrt{10}}\mathsf{H}_k^{(120)ij}\ \mathsf{H}_{ijm}^{(126)kl}\ \mathsf{H}_l^{(210)m} - \frac{1}{12\sqrt{10}}\mathsf{H}_k^{(120)ij}\ \mathsf{H}_l^{(126)}\ \mathsf{H}_{ij}^{(210)kl} \\
&\quad - \frac{1}{12\sqrt{15}}\mathsf{H}_k^{(120)ij}\ \mathsf{H}_i^{(126)}\ \mathsf{H}_j^{(210)k} - \frac{1}{4\sqrt{30}}\mathsf{H}_{ij}^{(120)k}\ \mathsf{H}_k^{(126)lm}\ \mathsf{H}_{lm}^{(210)ij} \\
&\quad + \frac{1}{8\sqrt{5}}\mathsf{H}_{ij}^{(120)k}\ \mathsf{H}_l^{(126)ij}\ \mathsf{H}_k^{(210)l} + \frac{1}{12\sqrt{5}}\mathsf{H}_{ij}^{(120)k}\ \mathsf{H}_k^{(126)jl}\ \mathsf{H}_l^{(210)i} \\
&\quad - \frac{1}{20\sqrt{6}}\mathsf{H}_{ij}^{(120)k}\ \mathsf{H}_k^{(126)ij}\ \mathsf{H}^{(210)} - \frac{1}{8}\sqrt{\frac{3}{5}}\mathsf{H}^{(120)i}\ \mathsf{H}_j^{(126)}\ \mathsf{H}_i^{(210)j} \\
&\quad - \frac{1}{10\sqrt{2}}\mathsf{H}^{(120)i}\ \mathsf{H}_i^{(126)}\ \mathsf{H}^{(210)} - \frac{1}{4\sqrt{30}}\mathsf{H}_i^{(120)}\ \mathsf{H}_l^{(126)jk}\ \mathsf{H}_{jk}^{(210)il} \\
&\quad + \frac{1}{12\sqrt{5}}\mathsf{H}_i^{(120)}\ \mathsf{H}_k^{(126)ij}\ \mathsf{H}_j^{(210)k} - \frac{1}{\sqrt{30}}\mathsf{H}^{(120)i}\ \mathsf{H}^{(126)}\ \mathsf{H}_i^{(210)} + \cdots\Bigg] \\
&= \frac{\imath}{5!}\mathrm{C}\Bigg[\left(\frac{1}{48\sqrt{3}}\mathbf{S}_{24_{210}} + \frac{1}{12\sqrt{30}}\mathbf{S}_{75_{210}}\right){}^{(\overline{5}_{126})}\mathsf{D}_a{}^{(45_{120})}\mathsf{D}^a \\
&\quad + \left(-\frac{1}{20\sqrt{6}}\mathbf{S}_{1_{210}} - \frac{1}{240\sqrt{6}}\mathbf{S}_{24_{210}} - \frac{1}{12\sqrt{15}}\mathbf{S}_{75_{210}}\right){}^{(\overline{45}_{120})}\mathsf{D}_a{}^{(45_{126})}\mathsf{D}^a \\
&\quad + \left(-\frac{1}{10\sqrt{2}}\mathbf{S}_{1_{210}} + \frac{3}{40\sqrt{2}}\mathbf{S}_{24_{210}}\right){}^{(\overline{5}_{126})}\mathsf{D}_a{}^{(5_{120})}\mathsf{D}^a \\
&\quad + \left(-\frac{1}{48}\mathbf{S}_{24_{210}} - \frac{1}{12\sqrt{10}}\mathbf{S}_{75_{210}}\right){}^{(\overline{5}_{120})}\mathsf{D}_a{}^{(45_{126})}\mathsf{D}^a \\
&\quad - \left(\frac{1}{\sqrt{30}}\mathbf{S}_{1_{126}}\right){}^{(\overline{5}_{210})}\mathsf{D}_a{}^{(5_{120})}\mathsf{D}^a
\end{aligned}
$$

$$
\begin{aligned}
&+\left(\frac{1}{72}\mathbf{S}_{24_{210}}-\frac{1}{36\sqrt{10}}\mathbf{S}_{75_{210}}\right){}^{(\overline{5}_{126})}\mathsf{T}_{\alpha}{}^{(45_{120})}\mathsf{T}^{\alpha}\\
&+\left(-\frac{1}{20\sqrt{6}}\mathbf{S}_{1_{210}}-\frac{1}{40\sqrt{6}}\mathbf{S}_{24_{210}}\right){}^{(\overline{45}_{120})}\mathsf{T}_{\alpha}{}^{(45_{126})}\mathsf{T}^{\alpha}\\
&+\left(-\frac{1}{10\sqrt{2}}\mathbf{S}_{1_{210}}-\frac{1}{20\sqrt{2}}\mathbf{S}_{24_{210}}\right){}^{(\overline{5}_{126})}\mathsf{T}_{\alpha}{}^{(5_{120})}\mathsf{T}^{\alpha}\\
&+\left(-\frac{1}{24\sqrt{3}}\mathbf{S}_{24_{210}}-\frac{1}{12\sqrt{30}}\mathbf{S}_{75_{210}}\right){}^{(\overline{5}_{120})}\mathsf{T}_{\alpha}{}^{(45_{126})}\mathsf{T}^{\alpha}\\
&+\left(-\frac{1}{60\sqrt{6}}\mathbf{S}_{24_{210}}+\frac{1}{36\sqrt{15}}\mathbf{S}_{75_{210}}\right){}^{(\overline{50}_{126})}\mathsf{T}_{\alpha}{}^{(45_{120})}\mathsf{T}^{\alpha}\\
&-\left(\frac{1}{\sqrt{30}}\mathbf{S}_{1_{126}}\right){}^{(\overline{5}_{210})}\mathsf{T}_{\alpha}{}^{(5_{120})}\mathsf{T}^{\alpha}+\ \cdots\Bigg].
\end{aligned}
\tag{96}
$$

D4: $\mathbf{120}\cdot\overline{\mathbf{126}}\cdot\mathbf{210}$ Coupling

$$
\begin{aligned}
\overline{\mathrm{C}}\ \Sigma_{\mu\nu\sigma}\overline{\Delta}_{\nu\sigma\xi\zeta\rho}\Phi_{\mu\xi\zeta\rho} &= -\frac{\imath}{5!}\overline{\mathrm{C}}\Bigg[\frac{1}{4\sqrt{15}}\mathsf{H}^{(120)k}_{ij}\ \mathsf{H}^{(\overline{126})jmn}_{kl}\ \mathsf{H}^{(210)il}_{mn}+\frac{1}{8\sqrt{15}}\mathsf{H}^{(120)i}_{jk}\ \mathsf{H}^{(\overline{126})jkl}_{mn}\ \mathsf{H}^{(210)mn}_{il}\\
&\quad+\frac{1}{12\sqrt{10}}\mathsf{H}^{(120)k}_{ij}\ \mathsf{H}^{(\overline{126})ijm}_{kl}\ \mathsf{H}^{(210)l}_{m}-\frac{1}{12\sqrt{10}}\mathsf{H}^{(120)k}_{ij}\ \mathsf{H}^{(\overline{126})l}\ \mathsf{H}^{(210)ij}_{kl}\\
&\quad-\frac{1}{12\sqrt{15}}\mathsf{H}^{(120)k}_{ij}\ \mathsf{H}^{(\overline{126})i}\ \mathsf{H}^{(210)j}_{k}-\frac{1}{4\sqrt{30}}\mathsf{H}^{(120)ij}_{k}\ \mathsf{H}^{(\overline{126})k}_{lm}\ \mathsf{H}^{(210)lm}_{ij}\\
&\quad+\frac{1}{8\sqrt{5}}\mathsf{H}^{(120)ij}_{k}\ \mathsf{H}^{(\overline{126})l}_{ij}\ \mathsf{H}^{(210)k}_{l}+\frac{1}{12\sqrt{5}}\mathsf{H}^{(120)ij}_{k}\ \mathsf{H}^{(\overline{126})k}_{jl}\ \mathsf{H}^{(210)l}_{i}\\
&\quad-\frac{1}{20\sqrt{6}}\mathsf{H}^{(120)ij}_{k}\ \mathsf{H}^{(\overline{126})k}_{ij}\ \mathsf{H}^{(210)}-\frac{1}{8}\sqrt{\frac{3}{5}}\mathsf{H}^{(120)}_{i}\ \mathsf{H}^{(\overline{126})j}\ \mathsf{H}^{(210)i}_{j}\\
&\quad-\frac{1}{10\sqrt{2}}\mathsf{H}^{(120)}_{i}\ \mathsf{H}^{(\overline{126})i}\ \mathsf{H}^{(210)}-\frac{1}{4\sqrt{30}}\mathsf{H}^{(120)i}\ \mathsf{H}^{(\overline{126})l}_{jk}\ \mathsf{H}^{(210)jk}_{il}\\
&\quad+\frac{1}{12\sqrt{5}}\mathsf{H}^{(120)i}\ \mathsf{H}^{(\overline{126})k}_{ij}\ \mathsf{H}^{(210)j}_{k}-\frac{1}{\sqrt{30}}\mathsf{H}^{(120)}_{i}\ \mathsf{H}^{(\overline{126})}\ \mathsf{H}^{(210)i}+\cdots\Bigg]\\
&= -\frac{\imath}{5!}\overline{\mathrm{C}}\Bigg[\left(\frac{1}{48\sqrt{3}}\mathbf{S}_{24_{210}}+\frac{1}{12\sqrt{30}}\mathbf{S}_{75_{210}}\right){}^{(\overline{45}_{120})}\mathsf{D}_{a}{}^{(5_{\overline{126}})}\mathsf{D}^{a}\\
&\quad+\left(-\frac{1}{20\sqrt{6}}\mathbf{S}_{1_{210}}-\frac{1}{240\sqrt{6}}\mathbf{S}_{24_{210}}-\frac{1}{12\sqrt{15}}\mathbf{S}_{75_{210}}\right){}^{(\overline{45}_{\overline{126}})}\mathsf{D}_{a}{}^{(45_{120})}\mathsf{D}^{a}\\
&\quad+\left(-\frac{1}{10\sqrt{2}}\mathbf{S}_{1_{210}}+\frac{3}{40\sqrt{2}}\mathbf{S}_{24_{210}}\right){}^{(\overline{5}_{120})}\mathsf{D}_{a}{}^{(5_{\overline{126}})}\mathsf{D}^{a}\\
&\quad+\left(-\frac{1}{48}\mathbf{S}_{24_{210}}-\frac{1}{12\sqrt{10}}\mathbf{S}_{75_{210}}\right){}^{(\overline{45}_{\overline{126}})}\mathsf{D}_{a}{}^{(5_{120})}\mathsf{D}^{a}\\
&\quad-\left(\frac{1}{\sqrt{30}}\mathbf{S}_{1_{\overline{126}}}\right){}^{(\overline{5}_{120})}\mathsf{D}_{a}{}^{(5_{210})}\mathsf{D}^{a}\\
&\quad+\left(\frac{1}{72}\mathbf{S}_{24_{210}}-\frac{1}{36\sqrt{10}}\mathbf{S}_{75_{210}}\right){}^{(\overline{45}_{120})}\mathsf{T}_{\alpha}{}^{(5_{\overline{126}})}\mathsf{T}^{\alpha}\\
&\quad+\left(-\frac{1}{20\sqrt{6}}\mathbf{S}_{1_{210}}-\frac{1}{40\sqrt{6}}\mathbf{S}_{24_{210}}\right){}^{(\overline{45}_{\overline{126}})}\mathsf{T}_{\alpha}{}^{(45_{120})}\mathsf{T}^{\alpha}\\
&\quad+\left(-\frac{1}{10\sqrt{2}}\mathbf{S}_{1_{210}}-\frac{1}{20\sqrt{2}}\mathbf{S}_{24_{210}}\right){}^{(\overline{5}_{120})}\mathsf{T}_{\alpha}{}^{(5_{\overline{126}})}\mathsf{T}^{\alpha}\\
&\quad+\left(-\frac{1}{24\sqrt{3}}\mathbf{S}_{24_{210}}-\frac{1}{12\sqrt{30}}\mathbf{S}_{75_{210}}\right){}^{(\overline{45}_{\overline{126}})}\mathsf{T}_{\alpha}{}^{(5_{120})}\mathsf{T}^{\alpha}\\
&\quad+\left(-\frac{1}{60\sqrt{6}}\mathbf{S}_{24_{210}}+\frac{1}{36\sqrt{15}}\mathbf{S}_{75_{210}}\right){}^{(\overline{45}_{120})}\mathsf{T}_{\alpha}{}^{(50_{\overline{126}})}\mathsf{T}^{\alpha}\\
&\quad-\left(\frac{1}{\sqrt{30}}\mathbf{S}_{1_{126}}\right){}^{(\overline{5}_{210})}\mathsf{T}_{\alpha}{}^{(5_{120})}\mathsf{T}^{\alpha}+\ \cdots\Bigg].
\end{aligned}
\tag{97}
$$

Heavy sector

D5: $\mathbf{126}\cdot\overline{\mathbf{126}}$ Coupling

$$\begin{aligned}
m_\Delta \Delta_{\mu\nu\sigma\xi\zeta}\overline{\Delta}_{\mu\nu\sigma\xi\zeta} &= m_\Delta\left[2\mathsf{H}_i^{(126)}\ \mathsf{H}^{(\overline{126})i} + \mathsf{H}_k^{(126)ij}\ \mathsf{H}_{ij}^{(\overline{126})k} + \frac{1}{6}\mathsf{H}_{ijk}^{(126)lm}\ \mathsf{H}_{lm}^{(\overline{126})ijk} + \ \cdots\right] \\
&= m_\Delta\Big[2^{(\overline{5}_{126})}\mathsf{D}_a{}^{(5_{\overline{126}})}\mathsf{D}^a + {}^{(45_{126})}\mathsf{D}_a{}^{(\overline{45}_{\overline{126}})}\mathsf{D}^a + 2^{(\overline{5}_{126})}\mathsf{T}_\alpha{}^{(5_{\overline{126}})}\mathsf{T}^\alpha \\
&\quad + {}^{(45_{126})}\mathsf{T}_\alpha{}^{(\overline{45}_{\overline{126}})}\mathsf{T}^\alpha + \frac{1}{6}{}^{(\overline{50}_{126})}\mathsf{T}_\alpha{}^{(50_{\overline{126}})}\mathsf{T}^\alpha + \ \cdots\Big]. \qquad (98)
\end{aligned}$$

D6: $\mathbf{210}\cdot\mathbf{210}$ Coupling

$$\begin{aligned}
m_\Phi \Phi_{\mu\nu\sigma\xi}\Phi_{\mu\nu\sigma\xi} &= m_\Phi\left[2\mathsf{H}_i^{(210)}\ \mathsf{H}^{(210)i} + \ \cdots\right] \\
&= m_\Phi\left[2\ {}^{(\overline{5}_{210})}\mathsf{D}_a{}^{(5_{210})}\mathsf{D}^a + 2\ {}^{(\overline{5}_{210})}\mathsf{T}_\alpha{}^{(5_{210})}\mathsf{T}^\alpha + \ \cdots\right]. \qquad (99)
\end{aligned}$$

D7: $\mathbf{210}\cdot\mathbf{210}\cdot\mathbf{210}$ Coupling

$$\begin{aligned}
\lambda\Phi_{\mu\nu\sigma\xi}\Phi_{\sigma\xi\rho\tau}\Phi_{\rho\tau\mu\nu} &= \lambda\left[\frac{1}{\sqrt{2}}\mathsf{H}_i^{(210)}\ \mathsf{H}^{(210)j}\ \mathsf{H}_j^{(210)i} - \sqrt{\frac{3}{5}}\mathsf{H}_i^{(210)}\ \mathsf{H}^{(210)i}\ \mathsf{H}^{(210)} + \ \cdots\right] \\
&= -\lambda\Bigg[\left(\frac{3}{10\sqrt{2}}\mathbf{S}_{1_{210}} + \frac{1}{2}\sqrt{\frac{3}{5}}\mathbf{S}_{24_{210}}\right){}^{(\overline{5}_{210})}\mathsf{D}_a{}^{(5_{210})}\mathsf{D}^a \\
&\quad + \left(-\frac{3}{10\sqrt{2}}\mathbf{S}_{1_{210}} + \frac{1}{\sqrt{15}}\mathbf{S}_{24_{210}}\right){}^{(\overline{5}_{210})}\mathsf{T}_\alpha{}^{(5_{210})}\mathsf{T}^\alpha + \ \cdots\Bigg]. \qquad (100)
\end{aligned}$$

D8: $\mathbf{210}\cdot\mathbf{126}\cdot\overline{\mathbf{126}}$ Coupling

$$\begin{aligned}
\eta\Phi_{\mu\nu\sigma\xi}\Delta_{\mu\nu\rho\tau\zeta}\overline{\Delta}_{\sigma\xi\rho\tau\zeta} &= \eta\Bigg[\frac{1}{5}\mathsf{H}_i^{(210)}\ \mathsf{H}^{(\overline{126})i}\ \mathsf{H}^{(126)} + \frac{1}{5}\mathsf{H}_i^{(126)}\ \mathsf{H}^{(210)i}\ \mathsf{H}^{(\overline{126})} \\
&\quad + \frac{1}{10\sqrt{3}}\mathsf{H}_m^{(126)kl}\ \mathsf{H}_{ij}^{(\overline{126})m}\ \mathsf{H}_{kl}^{(210)ij} + \frac{1}{10\sqrt{2}}\mathsf{H}_i^{(126)kl}\ \mathsf{H}_{kl}^{(\overline{126})j}\ \mathsf{H}_j^{(210)i} \\
&\quad + \frac{1}{15\sqrt{2}}\mathsf{H}_l^{(126)jk}\ \mathsf{H}_{ik}^{(\overline{126})l}\ \mathsf{H}_j^{(210)i} + \frac{3}{10\sqrt{2}}\mathsf{H}^{(\overline{126})j}\ \mathsf{H}_i^{(126)}\ \mathsf{H}_j^{(210)i} \\
&\quad - \frac{2}{5\sqrt{15}}\mathsf{H}^{(\overline{126})i}\ \mathsf{H}_i^{(126)}\ \mathsf{H}^{(210)} - \frac{1}{30\sqrt{2}}\mathsf{H}_{ij}^{(\overline{126})klm}\ \mathsf{H}_m^{(126)}\ \mathsf{H}_{kl}^{(210)ij} \\
&\quad - \frac{1}{30\sqrt{2}}\mathsf{H}^{(\overline{126})m}\ \mathsf{H}_{ijm}^{(126)kl}\ \mathsf{H}_{kl}^{(210)ij} + \frac{1}{120\sqrt{3}}\mathsf{H}_{ij}^{(\overline{126})mnp}\ \mathsf{H}_{mnp}^{(126)kl}\ \mathsf{H}_{kl}^{(210)ij} \\
&\quad + \frac{1}{40\sqrt{3}}\mathsf{H}_{np}^{(\overline{126})klm}\ \mathsf{H}_{ijm}^{(126)np}\ \mathsf{H}_{kl}^{(210)ij} + \frac{1}{10\sqrt{3}}\mathsf{H}_{ip}^{(\overline{126})kmn}\ \mathsf{H}_{jmn}^{(126)lp}\ \mathsf{H}_{kl}^{(210)ij} \\
&\quad + \frac{1}{45\sqrt{2}}\mathsf{H}_{in}^{(\overline{126})klm}\ \mathsf{H}_{klm}^{(126)jn}\ \mathsf{H}_j^{(210)i} + \frac{1}{60\sqrt{15}}\mathsf{H}_{lm}^{(\overline{126})ijk}\ \mathsf{H}_{ijk}^{(126)lm}\ \mathsf{H}^{(210)} + \ \cdots\Bigg] \\
&= \eta\Bigg[\left(\frac{1}{5}\mathbf{S}_{1_{126}}\right){}^{(\overline{5}_{210})}\mathsf{D}_a{}^{(5_{\overline{126}})}\mathsf{D}^a + \left(\frac{1}{5}\mathbf{S}_{1_{\overline{126}}}\right){}^{(\overline{5}_{126})}\mathsf{D}_a{}^{(5_{210})}\mathsf{D}^a \\
&\quad + \left(-\frac{1}{6\sqrt{15}}\mathbf{S}_{24_{210}} + \frac{1}{15\sqrt{6}}\mathbf{S}_{75_{210}}\right){}^{(\overline{45}_{\overline{126}})}\mathsf{D}_a{}^{(45_{126})}\mathsf{D}^a \\
&\quad - \left(\frac{2}{5\sqrt{15}}\mathbf{S}_{1_{210}} + \frac{3}{20}\sqrt{\frac{3}{5}}\mathbf{S}_{24_{210}}\right){}^{(\overline{5}_{126})}\mathsf{D}_a{}^{(5_{\overline{126}})}\mathsf{D}^a \\
&\quad + \left(\frac{1}{5}\mathbf{S}_{1_{126}}\right){}^{(\overline{5}_{210})}\mathsf{T}_\alpha{}^{(5_{\overline{126}})}\mathsf{T}^\alpha + \left(\frac{1}{5}\mathbf{S}_{1_{\overline{126}}}\right){}^{(\overline{5}_{126})}\mathsf{T}_\alpha{}^{(5_{210})}\mathsf{T}^\alpha \\
&\quad + \left(-\frac{2}{5\sqrt{15}}\mathbf{S}_{1_{210}} + \frac{1}{10}\sqrt{\frac{3}{5}}\mathbf{S}_{24_{210}}\right){}^{(\overline{5}_{126})}\mathsf{T}_\alpha{}^{(5_{\overline{126}})}\mathsf{T}^\alpha
\end{aligned}$$

$$
- \left(\frac{2}{45}\mathbf{S}_{75_{210}}\right) {}^{(\overline{5}_{126})}\mathsf{T}_{\alpha}{}^{(50_{\overline{126}})}\mathsf{T}^{\alpha} - \left(\frac{2}{45}\mathbf{S}_{75_{210}}\right) {}^{(\overline{50}_{126})}\mathsf{T}_{\alpha}{}^{(5_{\overline{126}})}\mathsf{T}^{\alpha}
$$

$$
+ \left(\frac{1}{60\sqrt{15}}\mathbf{S}_{1_{210}} + \frac{1}{180\sqrt{15}}\mathbf{S}_{24_{210}} + \frac{1}{60\sqrt{6}}\mathbf{S}_{75_{210}}\right) {}^{(\overline{50}_{126})}\mathsf{T}_{\alpha}{}^{(50_{\overline{126}})}\mathsf{T}^{\alpha} + \cdots \Bigg]. \quad (101)
$$

Appendix E: Coefficients of $\mathsf{B}-\mathsf{L}=-2$ operators

In this appendix we exhibit some of the coefficients of the $\mathsf{B}-\mathsf{L}=-2$ operators appearing in Sec.(4). The coefficients $\mathcal{I}_{\acute{w}\acute{x},\acute{y}\acute{z}}$ and $\mathcal{J}_{\acute{w}\acute{x},\acute{y}\acute{z}}$ appearing in Eqs.(60) and (61) are given by

$$
\begin{aligned}
\mathcal{I}_{\acute{w}\acute{x},\acute{y}\acute{z}} = \; & 2\left(\sum_{M=2}^{7}\frac{\left\{\sum_{q=1}^{2}\mathcal{G}^{(q)}_{\acute{u}\acute{v}}U_{d_{qM}}\right\}\left\{\sum_{r=1}^{2}f^{(10_r+)}_{\acute{w}\acute{x}}V_{d_{rM}}\right\}}{\mathsf{m}_{d_M}}\right)\left(\sum_{N=2}^{7}\frac{U_{d_{3N}}\left\{\sum_{s=1}^{2}f^{(10_s+)}_{\acute{y}\acute{z}}V_{d_{sN}}\right\}}{\mathsf{m}_{d_N}}\right) \\
& + \frac{1}{3}f^{(120-)}_{\acute{w}\acute{x}}f^{(120-)}_{\acute{y}\acute{z}}\left(\sum_{M=2}^{7}\frac{\left\{\sum_{r=1}^{2}\mathcal{G}^{(r)}_{\acute{u}\acute{v}}U_{d_{rM}}\right\}V_{d_{3M}}}{\mathsf{m}_{d_M}}\right)\left(\sum_{N=2}^{7}\frac{U_{d_{3N}}V_{d_{3N}}}{\mathsf{m}_{d_N}}\right) \\
& - \sqrt{\frac{2}{3}}f^{(120-)}_{\acute{y}\acute{z}}\left(\sum_{M=2}^{7}\frac{\left\{\sum_{r=1}^{2}\mathcal{G}^{(r)}_{\acute{u}\acute{v}}U_{d_{rM}}\right\}\left\{\sum_{s=1}^{2}f^{(10_s+)}_{\acute{w}\acute{x}}V_{d_{sM}}\right\}}{\mathsf{m}_{d_M}}\right)\left(\sum_{N=2}^{7}\frac{U_{d_{3N}}V_{d_{3N}}}{\mathsf{m}_{d_N}}\right) \\
& - \sqrt{\frac{2}{3}}f^{(120-)}_{\acute{w}\acute{x}}\left(\sum_{M=2}^{7}\frac{\left\{\sum_{r=1}^{2}\mathcal{G}^{(r)}_{\acute{u}\acute{v}}U_{d_{rM}}\right\}V_{d_{3M}}}{\mathsf{m}_{d_M}}\right)\left(\sum_{N=2}^{7}\frac{U_{d_{3N}}\left\{\sum_{s=1}^{2}f^{(10_s+)}_{\acute{y}\acute{z}}V_{d_{sN}}\right\}}{\mathsf{m}_{d_N}}\right). \quad (102)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{J}_{\acute{w}\acute{x},\acute{y}\acute{z}} = \; & 2\left(\sum_{M=1}^{8}\frac{\left\{\sum_{q=1}^{2}\mathcal{G}^{(q)}_{\acute{u}\acute{v}}U_{t_{qM}}\right\}\left\{\sum_{r=1}^{2}f^{(10_r+)}_{\acute{w}\acute{x}}V_{t_{rM}}\right\}}{\mathsf{m}_{t_M}}\right)\left(\sum_{N=1}^{8}\frac{U_{t_{3N}}\left\{\sum_{s=1}^{2}f^{(10_s+)}_{\acute{y}\acute{z}}V_{t_{sN}}\right\}}{\mathsf{m}_{t_N}}\right) \\
& + \frac{1}{3}f^{(120-)}_{\acute{w}\acute{x}}f^{(120-)}_{\acute{y}\acute{z}}\left(\sum_{M=1}^{8}\frac{\left\{\sum_{r=1}^{2}\mathcal{G}^{(r)}_{\acute{u}\acute{v}}U_{t_{rM}}\right\}V_{t_{3M}}}{\mathsf{m}_{t_M}}\right)\left(\sum_{N=1}^{8}\frac{U_{t_{3N}}V_{t_{3N}}}{\mathsf{m}_{t_N}}\right) \\
& - \sqrt{\frac{2}{3}}f^{(120-)}_{\acute{y}\acute{z}}\left(\sum_{M=1}^{8}\frac{\left\{\sum_{r=1}^{2}\mathcal{G}^{(r)}_{\acute{u}\acute{v}}U_{t_{rM}}\right\}\left\{\sum_{s=1}^{2}f^{(10_s+)}_{\acute{w}\acute{x}}V_{t_{sM}}\right\}}{\mathsf{m}_{t_M}}\right)\left(\sum_{N=1}^{8}\frac{U_{t_{3N}}V_{t_{3N}}}{\mathsf{m}_{t_N}}\right) \\
& - \sqrt{\frac{2}{3}}f^{(120-)}_{\acute{w}\acute{x}}\left(\sum_{M=1}^{8}\frac{\left\{\sum_{r=1}^{2}\mathcal{G}^{(r)}_{\acute{u}\acute{v}}U_{t_{rM}}\right\}V_{t_{3M}}}{\mathsf{m}_{t_M}}\right)\left(\sum_{N=1}^{8}\frac{U_{t_{3N}}\left\{\sum_{r=1}^{2}f^{(10_r+)}_{\acute{y}\acute{z}}V_{t_{rN}}\right\}}{\mathsf{m}_{t_N}}\right). \quad (103)
\end{aligned}
$$

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