Heavy quarks in proton-nucleus collisions: The hybrid formalism

Tolga Altinoluk, Néstor Armesto, Guillaume Beuf, Alex Kovner, and Michael Lublinsky

Phys. Rev. D **93**, 054049 — Published 31 March 2016

DOI: 10.1103/PhysRevD.93.054049

Heavy quarks in proton-nucleus collisions - the hybrid formalism

Tolga Altinoluk[a], Néstor Armesto[a], Guillaume Beuf[b,c], Alex Kovner[d], Michael Lublinsky[b,d]
[a] *Departamento de Física de Partículas and IGFAE, Universidade de Santiago de Compostela, E-15706 Santiago de Compostela, Galicia-Spain*
[b] *Department of Physics, Ben-Gurion University of the Negev, Beer Sheva 84105, Israel*
[c] *European Centre for Theoretical Studies in Nuclear Physics and Related Areas (ECT*) and Fondazione Bruno Kessler, Strada delle Tabarelle 286, I-38123 Villazzano (TN), Italy*
[d] *Physics Department, University of Connecticut, 2152 Hillside Road, Storrs, CT 06269-3046, USA*

We explore the quark mass effects on inclusive hadron production in proton-nucleus collisions at high energies. We consider two processes. First, we compute the single inclusive cross-section for production of hadrons with open heavy flavour in the proton forward direction at leading order. Next, in the same kinematics, we calculate the heavy-quark contribution to single inclusive production of light or unidentified hadrons at next-to-leading-order. For both studies we exploit the hybrid formalism, that is the collinear factorisation on the proton side while high-density and high-energy effects are resummed on the side of the nucleus.

I. INTRODUCTION AND SUMMARY

Since the original suggestions [1], during the last three decades a lot of effort has been devoted to the study of hadronic structure at high energies. The main motivation for it is possible existence of a new regime of Quantum Chromodynamics (QCD) where partonic densities exhibit perturbative saturation. In this regime of partonic states become dense but the coupling constant is still small and the physics remains perturbative. The recent theoretical implementation of these ideas goes by the name of Color Glass Condensate (CGC) [2–7]. Besides the intense theoretical activity, experiments at the Relativistic Heavy Ion Collider (RHIC) at BNL and the Large Hadron Collider (LHC) at CERN offer new possibilities for searching and characterising such regime.

Admittedly, in spite of the fact that several saturation-based calculations describe data satisfactorily (e.g. [8–13]), there is no conclusive evidence for the existence of the saturated state in experimental data. One of the main reasons for this is that the accuracy of most calculations is still not sufficient to establish quantitative conclusions. Only a small (although important) part of next-to-leading order (NLO) corrections (the running coupling effects) is presently included in numerical implementations of high-energy evolution [14] even though the full set of NLO corrections is already available [15–18]. Calculation of various observables, like inclusive hadroproduction [19], photoproduction [20], etc. is confined at present to leading order (LO) in the strong coupling constant α_s.

Recently several papers have aimed to extend the accuracy of calculations in the CGC framework to NLO: deep inelastic scattering [21, 22] or single hadroproduction cross section at forward rapidities [23, 24] in the "hybrid" formalism [25]. Concerning the latter, numerical studies indicate very strong effects of the NLO corrections, with cross sections even becoming negative at moderate transverse momenta [26, 27], and even substantial next-to-next-to-leading (NNLO) effects are found [28]. More recent discussions focus on the eventual relevance of additional collinear resummations at small x [29, 30] and on the correct choice of the factorisation scale for the high-energy evolution [31, 32]. In the previous publication [33], we have introduced a restriction on the lifetime of the partonic fluctuations of the projectile and also revisited the choice of scales. These considerations led to modified NLO expressions which later were shown to improve considerably the stability of the results and their agreement with experimental data [34].

An important aspect of the calculations is the fact that in hadronic collisions one is forced to rely on factorisation schemes that separate hard scale perturbative processes from the non-perturbative structure of hadrons. Depending on the kinematics of the process under study, two main factorisation schemes are usually employed. The most common one is collinear factorisation [35] (see an introduction to heavy quark production in collinear factorisation in [36]). In the collinear scheme, one neglects the transverse momenta of incoming partons. Production cross sections are computed via a convolution in longitudinal momenta of partonic distribution functions with hard on-shell parton production matrix elements. The scheme is normally applicable when produced hadrons have large transverse momentum, and neglecting the transverse momenta of incoming partons is indeed a valid approximation. The scheme breaks down, however, when the produced system has relatively low transverse momentum such that either saturation or non-perturbative effects in at least one of the incoming hadrons become important.

An alternative scheme is based on k_T (or, more generally, high-energy) factorisation [37]. The k_T-factorisation scheme is built upon a separation between hard off-shell matrix elements and k_T-dependent unintegrated gluon densities. It is particularly tuned for central production of states with relatively small transverse energy. Heavy quark production within the high-energy factorisation formalism has been considered e.g. in Refs. [38–48]. The scheme breaks down in the forward kinematics, when Bjorken-x of one of the incoming partons is large.

Another factorisation scheme, the so called hybrid formalism, was introduced in Ref. [25]. In the present work we employ the hybrid framework. It is a combination of the previous two factorization approaches applied to asymmetric production, particularly when the inclusively produced state is measured in the forward direction of one of the colliding hadrons. In pA collisions, we focus on the hadron production in the proton forward direction. This implies that a relatively large fraction of the proton longitudinal momentum is taken by the incoming parton, and the collinear factorisation on the proton side can be applied. At the same time, the target nucleus is dense. Only its tail of small-x partons, with a typical transverse momenta of order Q_s, contributes to particle production. This is a kinematical regime for which the CGC formalism is most adequate. Technically, the hybrid formalism is realised in three steps:

1. A parton is collinearly factorised from the proton. Then, the contribution to its light-front wave function at first order in the QCD coupling constant g (including the gluon-to-quark-antiquark splitting in our case) is computed exactly in light-cone perturbation theory [49–51].

2. In the CGC, the propagation of gluons and light quarks is treated eikonally. That is, the $\hat{S}$-matrix element of all massless partons scattered off the fast nucleus is diagonal in coordinate space and simply given by a light-like Wilson line U in a relevant colour representation. In the high-energy limit, even massive quarks interact with the target via light-like Wilson lines [50], up to power-suppressed corrections, due to the Lorentz contraction of the target. However, when discussing the large mass limit, this approximation can break down. Then, our calculations are valid only as long as the energy of the collision is taken to be much larger than the large mass of the quark.

3. The partonic-level cross section has to be translated into the hadronic one. This requires a convolution with the proton parton distribution and parton fragmentation functions as in the usual collinear formalism. On the target nucleus side, we average the Wilson lines with respect to some given distribution $W^T[U]$, as in all CGC-type calculations.

As discussed above, single inclusive hadron production in the hybrid formalism has been the focus of a large number of recent publications [23, 24, 26–28, 31–34] . The result of this series of papers is a CGC-based computation performed at a full NLO accuracy in massless QCD. This opens a path for precise phenomenology based on saturation physics. In this paper, we further contribute to this effort by computing the heavy quark contribution to the NLO correction for this observable. We also calculate single inclusive heavy flavored hadron production at LO, mainly D or B mesons (analogous efforts for heavy quarkonia production can be found in [52–56]). This calculation is relevant for experimental data in the forward region. In this respect, LHCb has measured, in the region $2 < y < 4.5$ (5), B-meson production in pp collisions at $\sqrt{s} = 7$ [57, 58] and 8 [59] TeV, Λ_b production in pp collisions at $\sqrt{s} = 7$ TeV [60], and prompt charm production at $\sqrt{s} = 7$ [61] and 13 [62] TeV. ALICE has measured heavy flavour production through its decay into muons [63] in the forward rapidity range $2.5 < y < 4$.

It is still an open debate how to consistently treat heavy flavors in the parton model. The discussion has been mainly conducted in the framework of collinear factorisation and for the initial state. There are two basic alternatives (see [36, 64, 65]):

1. Fixed Flavor Number Scheme (nFFNS). n quarks q are considered as massless. Only for massless quarks and gluons there exist parton density functions (PDFs) that evolve according to massless splitting functions, radiation off massive quarks showing no collinear divergence [66]. In most implementations of nFFNS, heavy flavours Q are generated through gluon splitting $g \to q\bar{q}$ and appear at order $\mathcal{O}(\alpha_s)$. This scheme should be valid at moderate scales μ but collapse when $\mu \gg m_Q$. Indeed, logarithms of μ^2/m_Q^2 that may become large e.g. at large $\mu = p_\perp$, are not resummed. It is used in some PDF global fits [67].

2. Variable Flavor Number Scheme (VFNS): n light quarks q are considered as massless. They are evolved as massless up to $\mu^2 = m_Q^2$, where the heavy quark PDF appears through a matching, at this scale, to the results of the convolution of the matrix elements to produce heavy flavour with the light flavour PDFs. Above $\mu^2 = m_Q^2$, Q is treated as massless for the evolution and there is one additional PDF, so heavy flavour is $\mathcal{O}(1)$. This scheme resums properly the mentioned logarithms, thus it is correct for $\mu^2 \gg m_Q^2$, but close to threshold neglects powers of m_Q^2/μ^2. Therefore, a matching of FFNS and VFNS, generically known as Generalized Mass (GM-)VFNS, is nowadays commonly used. There are, at least, three versions of this matching used by different PDF fitting groups in their most recent analysis: TR in MMHT14 [68], ACOT in CT14 [69] and FONLL in NNPDF3.0 [70], see recent discussions in [65]. Note that while we have considered the initial state for the discussion, similar considerations hold for fragmentation functions (FFs) [71]. α_s has also to be matched at heavy quark thresholds.

As our aim is to provide results valid in the saturation regime for the target, we use the hybrid factorization formalism, and focus on the regime of moderate transverse momentum $\mathbf{p}_h$ of the produced hadron. Hence, logarithms of the type $\ln(\mathbf{p}_h{}^2/m_c^2)$ or $\ln(\mathbf{p}_h{}^2/m_b^2)$ will not be considered large, and accordingly we will use the 3FFNS. The introduction of intrinsic charm [72] is a open issue. We will follow the idea in [65] of including it through a non-evolving PDF.

The main results of our paper are the following:

- We provide results for the single inclusive cross section for extrinsic charm and beauty hadron production at LO, given in Eqs. (3.18), (3.19) and (3.20).[1]
- We discuss their heavy quark limit, whose final expression (3.32) is $\mathcal{O}(1/m_Q^4)$. Interestingly, in the heavy quark limit, the production cross section is linearly proportional to Q_s^2.
- We also provide the LO term for the intrinsic contribution to single inclusive heavy flavored hadron production (3.35).
- We compute the (extrinsic) heavy quark contribution to the NLO corrections to the single inclusive cross section for light or unidentified hadron production, including the heavy quark loop part (5.3).

Technical details are provided for all calculations. Note that part of our computations differ from the ones performed in k_T factorisation where heavy quarks appear at order α_s^2 and, thus, they also contain instantaneous contributions. We start with a moderate-x gluon in the proton and hence our computation is order α_s. Individual graphs with a quark loop do have a UV divergence, but these cancel in the sum over the graphs, at the amplitude level. Moreover, there is no collinear divergence in our calculation, because these are regulated by the heavy quark mass. Nevertheless, we will do the calculation fully in dimensional regularisation. Indeed, this facilitates the recovery of the massless limit in the $\overline{MS}$ factorization scheme, without complicating notably the calculations. Of course, it is safe to put $D = 4$ in any of our final expressions.

Furthermore, at variance with the previous calculation for massless partons [33], we do not introduce the Ioffe time restriction in the present paper. This restriction corresponds to the requirement that the coherence time of the produced fluctuation of the parent parton is larger than the size of the target, in order to guarantee that the fluctuation-target scattering is coherent. It acts as a regulator of soft divergencies due to gluon emission and its effects are twofold. On the one hand, it modifies soft non-divergent pieces; this modification amounts to power-suppressed contributions that are usually neglected, as contributions of such kind are not under control in the hybrid formalism. On the other hand, it makes it possible to absorb in a consistent way the soft divergencies into the small-x evolution of dipole amplitudes - the Balitsky-Kovchegov equation [3–5]. In the present work, there is no radiation of gluons in the leading-order heavy quark production as only $g \to q\bar{q}$ splitting processes are considered, thus no soft divergencies appear. Therefore, the introduction of the Ioffe time restriction is not required as it results only in the mentioned power-suppressed contributions that are usually neglected.

The structure of the paper is as follows. In the next Section (II), we introduce the light cone perturbation theory applied to a gluon splitting (merging) into a heavy quark pair. We compute the amplitude of the quark pair production in gluon scattering off the nucleus. Partonic and hadron level cross-sections are computed in Section III. Section IV is devoted to calculation of heavy quark loop contribution to gluon-to-gluon scattering amplitude. In Section V, results from previous sections are combined in order to provide the massive quark contribution to the NLO correction to single inclusive light or unidentified hadron production. Our conventions are summarised in Appendix A.

---

[1] While in principle these results could be extracted from the existing literature [40, 43, 45], we have chosen to provide an independent and self-consistent derivation resulting in a form suitable for direct numerical implementation.

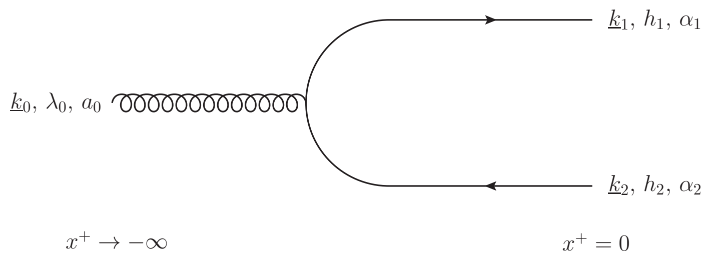

FIG. 1: Tree-level contribution to the $q\bar{q}$ Fock component of the incoming gluon state. λ_0, a_0 denote the gluon polarization and color index, and h_1, h_2 and α_1, α_2 the helicities and color indices of quark and antiquark respectively.

II. AMPLITUDE FOR HEAVY QUARK-ANTIQUARK PAIR PRODUCTION IN GLUON SCATTERING ON A BACKGROUND FIELD

A. Initial-state gluon wave function including heavy quarks

1. *Momentum space*

The Fock state decomposition of the physical (or dressed) state of the incoming gluon, at $x^+ = 0$, reads (see [49] and [50])

$$\begin{aligned} |g(\underline{k_0}, \lambda_0, a_0)_{\text{phys}}\rangle = \sqrt{Z_A}\Bigg[& a^\dagger(\underline{k_0}, \lambda_0, a_0)\,|0\rangle \\ & + \sum_{q\bar{q}\text{ states}} \Psi^{g_0}_{q_1\bar{q}_2}\;(t^{a_0})_{\alpha_1\,\alpha_2}\;\; b^\dagger(\underline{k_1}, h_1, \alpha_1)\, d^\dagger(\underline{k_2}, h_2, \alpha_2)\,|0\rangle \\ & + \sum_{gg\text{ states}} \Psi^{g_0}_{g_1 g_2}\;(T^{a_0})_{a_1\,a_2}\;\; a^\dagger(\underline{k_1}, \lambda_1, a_1)\, a^\dagger(\underline{k_2}, \lambda_2, a_2)\,|0\rangle + \;\cdots\Bigg]. \end{aligned} \tag{2.1}$$

We use the notation $\underline{k} \equiv (k^+, \mathbf{k})$, and $a^\dagger$, $b^\dagger$ and $d^\dagger$ are creation operators for gluons, quarks and antiquarks respectively. For later convenience, the fundamental (t^{a_0}) and adjoint (T^{a_0}) color generators have been extracted from the wave functions. Sum over repeated color indices is always implied. The sums over Fock states contain for each particle the sums over all the quantum numbers (apart from color) and the integration over momentum as

$$\int_0^{+\infty} \frac{dk^+}{(2\pi)2k^+} \int \frac{d^{D-2}\mathbf{k}}{(2\pi)^{D-2}}\;, \tag{2.2}$$

as well as the symmetry factor $1/n!$ every time that the Fock state contains n identical particles. Hence, there is an $1/2$ factor in the sum over gg states, but not in the one over the $q\bar{q}$ states.

The contribution of gluons and massless quarks to eq. (2.1) has already been calculated, for example in ref. [24, 33]. Here we are interested in the massive quark contributions. For simplicity, we perform the calculation in the case of QCD with a single massive quark flavor. Indeed, it is trivial to restore the flavor structure at the end of the calculation.

At tree level in light-front perturbation theory, there is only one graph, see Fig. 1, contributing to the $q\bar{q}$ Fock state component of the wave function of the physical incoming gluon, which gives

$$\Psi^{g_0}_{q_1\bar{q}_2}\;\;(t^{a_0})_{\alpha_1\,\alpha_2} = \frac{\langle 0|\, d_2\, b_1\, V_I(0)\, a_0^\dagger\,|0\rangle}{\left[k_0^- - k_1^- - k_2^- + i\epsilon\right]}\;, \tag{2.3}$$

where $V_I(0)$ is the interaction part of the light-front QCD hamiltonian (see ref. [51]) evaluated at $x^+ = 0$ in the interaction picture. From the expressions (A1) and (A2) of the quantized free fields in the interaction picture, one finds the vertex

$$\begin{aligned} \langle 0|\, d_2\, b_1\, V_I(0)\, a_0^\dagger\,|0\rangle = & (2\pi)^{D-1}\delta^{(D-1)}(\underline{k_1}+\underline{k_2}-\underline{k_0})\,(\mu)^{2-\frac{D}{2}} \\ & \times g\,(t^{a_0})_{\alpha_1\,\alpha_2}\;\; \overline{u}(\underline{k_1}, h_1)\;\not{\epsilon}_{\lambda_0}(\underline{k_0})\; v(\underline{k_2}, h_2). \end{aligned} \tag{2.4}$$

Using the k^+ and $\mathbf{k}$ conservation, the energy denominator can be rewritten as

$$\begin{aligned}\left[k_0^- - k_1^- - k_2^- + i\epsilon\right] &= \left[\frac{\mathbf{k}_0^2}{2k_0^+} - \frac{\mathbf{k}_1^2+m^2}{2k_1^+} - \frac{\mathbf{k}_2^2+m^2}{2k_2^+} + i\epsilon\right]\\ &= -\frac{k_0^+}{2k_1^+\,k_2^+}\left[\left(\mathbf{k}_1 - \frac{k_1^+}{k_0^+}\,\mathbf{k}_0\right)^2 + m^2 - i\epsilon\right]\end{aligned} \tag{2.5}$$

and we can drop the $-i\epsilon$.

Moreover, using relations (A14), (A15) and (A17), one can make explicit all of the transverse momentum dependence of the $q\bar{q}g$ Dirac structure as

$$\begin{aligned}\overline{u}(\underline{k_1},h_1)\,\not\varepsilon_{\lambda_0}(\underline{k_0})\,v(\underline{k_2},h_2) &= \varepsilon_{\lambda_0}^j\;\overline{u_G}(k_1^+,h_1)\left[1+\left(\mathbf{k}_1^i\gamma^i + m\right)\,\frac{\gamma^+}{2k_1^+}\right]\left[-\gamma^j + \frac{\mathbf{k}_0^j}{k_0^+}\,\gamma^+\right]\\ &\quad\times\left[1+\frac{\gamma^+}{2k_2^+}\,\left(\mathbf{k}_2^l\gamma^l - m\right)\right]v_G(k_2^+,h_2)\\ &= -\frac{k_0^+}{2k_1^+\,k_2^+}\left(\mathbf{k}_1^i - \frac{k_1^+}{k_0^+}\,\mathbf{k}_0^i\right)\varepsilon_{\lambda_0}^j\;\overline{u_G}(k_1^+,h_1)\,\gamma^+\left[\frac{(k_0^+ - 2k_1^+)}{k_0^+}\,\delta^{ij} + i\,\sigma^{ij}\right]v_G(k_2^+,h_2)\\ &\quad -\frac{k_0^+}{2k_1^+\,k_2^+}\,m\,\varepsilon_{\lambda_0}^j\;\overline{u_G}(k_1^+,h_1)\,\gamma^+\gamma^j\,v_G(k_2^+,h_2),\end{aligned} \tag{2.6}$$

where

$$\sigma^{ij} = \frac{i}{2}\,[\gamma^i,\gamma^j]. \tag{2.7}$$

So, the tree-level amplitude for $q\bar{q}$ Fock state inside the incoming gluon wave function reads

$$\begin{aligned}\Psi_{q_1\bar{q}_2}^{g_0} &= \frac{(2\pi)^{D-1}\delta^{(D-1)}(\underline{k_1}+\underline{k_2}-\underline{k_0})}{\left[\left(\mathbf{k}_1 - \frac{k_1^+}{k_0^+}\,\mathbf{k}_0\right)^2 + m^2\right]}\,(\mu)^{2-\frac{D}{2}}\,g\\ &\quad\times\Bigg\{\left(\mathbf{k}_1^i - \frac{k_1^+}{k_0^+}\,\mathbf{k}_0^i\right)\varepsilon_{\lambda_0}^j\;\overline{u_G}(k_1^+,h_1)\,\gamma^+\left[\frac{(k_0^+ - 2k_1^+)}{k_0^+}\,\delta^{ij} + i\,\sigma^{ij}\right]v_G(k_2^+,h_2)\\ &\qquad + m\,\varepsilon_{\lambda_0}^j\;\overline{u_G}(k_1^+,h_1)\,\gamma^+\gamma^j\,v_G(k_2^+,h_2)\Bigg\}.\end{aligned} \tag{2.8}$$

Compared to the massless case, not only the mass now appears in the denominator, but we also have a new term in the wave function, associated with helicity flip for the quark.

2. *Mixed space*

Performing the Fourier transform of the Fock states to mixed space, defined by eq. (A6), we have

$$\begin{aligned}|g_{\text{phys}}(\underline{k_0},\lambda_0,a_0)\rangle &= \sqrt{Z_A}\Bigg[\int d^{D-2}\mathbf{x}_0\;\;e^{i\mathbf{k}_0\cdot\mathbf{x}_0}\;\;a^\dagger(k_0^+,\mathbf{x}_0,\lambda_0,a_0)\,|0\rangle\\ &\quad + \widetilde{\sum_{q\bar{q}\text{ states}}}\;\widetilde{\Psi}_{q_1\bar{q}_2}^{g_0}\;\;(t^{a_0})_{\alpha_1\,\alpha_2}\;\;b^\dagger(k_1^+,\mathbf{x}_1,h_1,\alpha_1)\,d^\dagger(k_2^+,\mathbf{x}_2,h_2,\alpha_2)\,|0\rangle\\ &\quad + \widetilde{\sum_{gg\text{ states}}}\;\widetilde{\Psi}_{g_1g_2}^{g_0}\;\;(T^{a_0})_{a_1\,a_2}\;\;a^\dagger(k_1^+,\mathbf{x}_1,\lambda_1,a_1)\,a^\dagger(k_2^+,\mathbf{x}_2,\lambda_2,a_2)\,|0\rangle + \cdots\Bigg].\end{aligned} \tag{2.9}$$

The tilde on the sum over Fock states indicates that we replace for each parton the phase space integration eq. (2.2) by

$$\int_0^{+\infty}\frac{dk^+}{(2\pi)2k^+}\int d^{D-2}\mathbf{x}\;. \tag{2.10}$$

For the massive quark-antiquark case, the Fourier-transformed amplitude reads

$$\begin{aligned}
\widetilde{\Psi}^{g_0}_{q_1\bar{q}_2} &\equiv \int \frac{d^{D-2}\mathbf{k}_1}{(2\pi)^{D-2}} \int \frac{d^{D-2}\mathbf{k}_2}{(2\pi)^{D-2}}\; e^{i\mathbf{k}_1\cdot\mathbf{x}_1 + i\mathbf{k}_2\cdot\mathbf{x}_2}\;\Psi^{g_0}_{q_1\bar{q}_2}\\
&= 2\pi\delta(k_1^+ + k_2^+ - k_0^+)\; e^{i\frac{\mathbf{k}_0}{k_0^+}\cdot(k_1^+\,\mathbf{x}_1 + k_2^+\,\mathbf{x}_2)}\;(\mu)^{2-\frac{D}{2}}\, g\\
&\quad\times\Bigg\{\varepsilon^j_{\lambda_0}\;\overline{u_G}(k_1^+,h_1)\,\gamma^+\left[\frac{(k_0^+ - 2k_1^+)}{k_0^+}\,\delta^{ij} + i\,\sigma^{ij}\right] v_G(k_2^+,h_2)\;\;\mathcal{B}^i_V(\mathbf{x}_{12},m)\\
&\qquad\qquad + m\,\varepsilon^j_{\lambda_0}\;\overline{u_G}(k_1^+,h_1)\,\gamma^+\gamma^j\,v_G(k_2^+,h_2)\;\;\mathcal{B}_S(\mathbf{x}_{12},m)\Bigg\}\,,
\end{aligned} \tag{2.11}$$

with the integrals

$$\begin{aligned}
\mathcal{B}^i_V(\mathbf{x}_{12},m) &\equiv \int \frac{d^{D-2}\mathbf{K}}{(2\pi)^{D-2}}\; e^{i\mathbf{K}\cdot\mathbf{x}_{12}}\;\frac{\mathbf{K}^i}{\mathbf{K}^2 + m^2}\\
&= \frac{i}{2\pi}\,\frac{\mathbf{x}^i_{12}}{\mathbf{x}^2_{12}}\left[2\pi\,\mathbf{x}^2_{12}\right]^{2-\frac{D}{2}}\;\left[m|\mathbf{x}_{12}|\right]^{\frac{D}{2}-1}\;\mathrm{K}_{\frac{D}{2}-1}\left(m|\mathbf{x}_{12}|\right)\,,
\end{aligned} \tag{2.12}$$

$$\begin{aligned}
\mathcal{B}_S(\mathbf{x}_{12},m) &\equiv \int \frac{d^{D-2}\mathbf{K}}{(2\pi)^{D-2}}\; e^{i\mathbf{K}\cdot\mathbf{x}_{12}}\;\frac{1}{\mathbf{K}^2 + m^2}\\
&= \frac{1}{2\pi}\left[2\pi\,\mathbf{x}^2_{12}\right]^{2-\frac{D}{2}}\;\left[m|\mathbf{x}_{12}|\right]^{\frac{D}{2}-2}\;\mathrm{K}_{\frac{D}{2}-2}\left(m|\mathbf{x}_{12}|\right)\,.
\end{aligned} \tag{2.13}$$

One recovers the same result as for the $q\bar{q}$ component of the transverse photon wave function (up to the color factor, obviously) as expected, with (in $D=4$) the K_1 and K_0 modified Bessel functions of the second kind.

B. Final state gluon to heavy quark pair splitting

1. *Momentum space*

The Fock state decomposition of the heavy quark-antiquark final state reads

$$\begin{aligned}
\langle\bar{q}(\underline{p_2},h_2,\beta_2)q(\underline{p_1},h_1,\beta_1)_{\text{phys}}| &= \left(\sqrt{Z_\Psi}\right)^2\Bigg[\langle 0|\,d(\underline{p_2},h_2,\beta_2)\,b(\underline{p_1},h_1,\beta_1)\\
&\qquad + \sum_{g\text{ states}}\Phi^{q_1\bar{q}_2}_{g_0}\;\;(t^{b_0})_{\beta_1\,\beta_2}\quad\langle 0|\,a(\underline{p_0},\lambda_0,b_0) + \;\cdots\Bigg].
\end{aligned} \tag{2.14}$$

The different terms in this expression have the following interpretation:

- First term: trivial contribution with the quark and antiquark directly emerging out of the target at $x^+=0$.
- Second term: contribution of gluon splitting to $q\bar{q}$ in the final state, see Fig. 2.
- Other terms: either they are of higher order in g, or they will not contribute to the $g+A\to q+\bar{q}+X$ amplitude in which we are interested.

At tree level, only the graph on Fig. 2 contributes to the final state wave function $\Phi^{q_1\bar{q}_2}_{g_0}$ for the one-gluon Fock component inside the $q\bar{q}$ final state. It gives

$$\Phi^{q_1\bar{q}_2}_{g_0}\;\;(t^{b_0})_{\beta_1\,\beta_2} = \frac{\langle 0|d(2)\,b(1)\,V_I(0)\,a^\dagger(0)\,|0\rangle}{\left[p_1^- + p_2^- - p_0^- + i\epsilon\right]}\,. \tag{2.15}$$

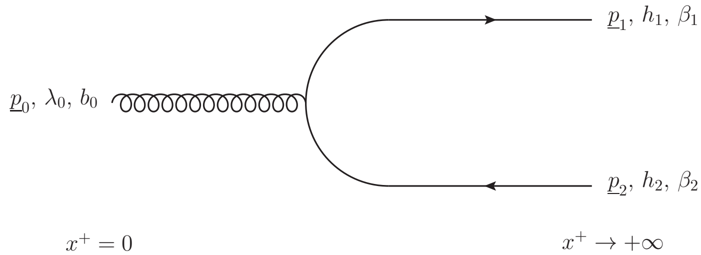

FIG. 2: Tree-level contribution to the Fock component of the outgoing $q\bar{q}$ state.

Up to the signs in the energy denominator and a trivial relabelling of the momentum variables and color indices, this is identical to its initial state analog $\Psi^{g_0}_{q_1\bar{q}_2}$, see eq. (2.3). Hence, from eq. (2.8) , we deduce

$$\Phi^{q_1\bar{q}_2}_{g_0} = -\frac{(2\pi)^{D-1}\delta^{(D-1)}(\underline{p_1}+\underline{p_2}-\underline{p_0})}{\left[\left(\mathbf{p}_1-\frac{p_1^+}{p_0^+}\,\mathbf{p}_0\right)^2+m^2\right]}\,(\mu)^{2-\frac{D}{2}}\,g$$
$$\times\Bigg\{\left(\mathbf{p}_1^i-\frac{p_1^+}{p_0^+}\,\mathbf{p}_0^i\right)\varepsilon^j_{\lambda_0}\;\overline{u_G}(p_1^+,h_1)\,\gamma^+\left[\frac{(p_0^+-2p_1^+)}{p_0^+}\,\delta^{ij}+i\,\sigma^{ij}\right]v_G(p_2^+,h_2)$$
$$+m\,\varepsilon^j_{\lambda_0}\;\overline{u_G}(p_1^+,h_1)\,\gamma^+\gamma^j\,v_G(p_2^+,h_2)\Bigg\} \tag{2.16}$$

and, thus, due to transverse and light-cone momentum conservation,

$$\Phi^{q_1\bar{q}_2}_{g_0} = -\frac{(2\pi)^{D-1}\delta^{(D-1)}(\underline{p_1}+\underline{p_2}-\underline{p_0})}{\left[\left(\mathbf{p}_2-\frac{p_2^+}{p_1^+}\,\mathbf{p}_1\right)^2+\left(\frac{p_0^+}{p_1^+}\right)^2m^2\right]}\,(\mu)^{2-\frac{D}{2}}\,g$$
$$\times\Bigg\{-\left(\frac{p_0^+}{p_1^+}\right)\left(\mathbf{p}_2^i-\frac{p_2^+}{p_1^+}\,\mathbf{p}_1^i\right)\varepsilon^j_{\lambda_0}\;\overline{u_G}(p_1^+,h_1)\,\gamma^+\left[\frac{(p_0^+-2p_1^+)}{p_0^+}\,\delta^{ij}+i\,\sigma^{ij}\right]v_G(p_2^+,h_2)$$
$$+\left(\frac{p_0^+}{p_1^+}\right)^2 m\,\varepsilon^j_{\lambda_0}\;\overline{u_G}(p_1^+,h_1)\,\gamma^+\gamma^j\,v_G(p_2^+,h_2)\Bigg\}. \tag{2.17}$$

2. Mixed space

Rewriting eq. (2.14) in mixed space, one gets

$$\langle\bar{q}(\underline{p_2},h_2,\beta_2)q(\underline{p_1},h_1,\beta_1)_{\text{phys}}| = \left(\sqrt{Z_\Psi}\right)^2$$
$$\times\Bigg[\int d^{D-2}\mathbf{x}_1\int d^{D-2}\mathbf{x}_2\;\;e^{-i\mathbf{p}_1\cdot\mathbf{x}_1-i\mathbf{p}_2\cdot\mathbf{x}_2}\;\;\langle 0|\,d(p_2^+,\mathbf{x}_2,h_2,\beta_2)\,b(p_1^+,\mathbf{x}_1,h_1,\beta_1)$$
$$+\widetilde{\sum_{g\text{ states}}}\,\widetilde{\Phi}^{q_1\bar{q}_2}_{g_0}\;\;(t^{b_0})_{\beta_1\,\beta_2}\;\;\langle 0|\,a(p_0^+,\mathbf{x}_0,\lambda_0,b_0)+\;\cdots\Bigg], \tag{2.18}$$

where

$$
\begin{aligned}
\widetilde{\Phi}_{g_0}^{q_1\bar{q}_2} &\equiv \int \frac{d^{D-2}\mathbf{p}_0}{(2\pi)^{D-2}}\; e^{-i\mathbf{p}_0\cdot\mathbf{x}_0}\;\Phi_{g_0}^{q_1\bar{q}_2}\\
&= \frac{-2\pi\delta(p_1^+{+}p_2^+{-}p_0^+)}{\left[\left(\mathbf{p}_2-\frac{p_2^+}{p_1^+}\,\mathbf{p}_1\right)^2+\left(\frac{p_0^+}{p_1^+}\right)^2 m^2\right]}\; e^{-i(\mathbf{p}_1+\mathbf{p}_2)\cdot\mathbf{x}_0}\;\;(\mu)^{2-\frac{D}{2}}\,g\\
&\quad\times\Bigg\{-\left(\frac{p_0^+}{p_1^+}\right)\left(\mathbf{p}_2^i-\frac{p_2^+}{p_1^+}\,\mathbf{p}_1^i\right)\varepsilon_{\lambda_0}^j\;\overline{u_G}(p_1^+,h_1)\,\gamma^+\left[\frac{(p_0^+-2p_1^+)}{p_0^+}\,\delta^{ij}+i\,\sigma^{ij}\right]v_G(p_2^+,h_2)\\
&\qquad\qquad+\left(\frac{p_0^+}{p_1^+}\right)^2 m\,\varepsilon_{\lambda_0}^j\;\overline{u_G}(p_1^+,h_1)\,\gamma^+\gamma^j\,v_G(p_2^+,h_2)\Bigg\}\,.
\end{aligned}
\tag{2.19}
$$

C. Amplitude for quark-antiquark production in gluon scattering on the background field

We have the mixed-space Fock state decomposition (2.9) of the incoming physical gluon. It describes the partonic content of the gluon at $x^+=0$ right before scattering with the target. In the eikonal approximation, the instantaneous interaction with the target is described by the operator $\hat{S}_E$, which introduces a Wilson line for each parton present in the Fock state. More precisely, it acts as

$$
\begin{aligned}
\hat{S}_E\,|0\rangle &= |0\rangle\,,\\
\hat{S}_E\,a^\dagger(k^+,\mathbf{x},\lambda,a) &= U_A(\mathbf{x})_{ba}\;\;a^\dagger(k^+,\mathbf{x},\lambda,b)\,\hat{S}_E\,,\\
\hat{S}_E\,b^\dagger(k^+,\mathbf{x},h,\alpha) &= U_F(\mathbf{x})_{\beta\alpha}\;\;b^\dagger(k^+,\mathbf{x},h,\beta)\,\hat{S}_E\,,\\
\hat{S}_E\,d^\dagger(k^+,\mathbf{x},h,\alpha) &= \Big[U_F^\dagger(\mathbf{x})\Big]_{\alpha\beta}\;\;d^\dagger(k^+,\mathbf{x},h,\beta)\,\hat{S}_E\,.
\end{aligned}
\tag{2.20}
$$

After applying the operator $\hat{S}_E$ to the initial state (2.9), one only needs to project on the desired final state (2.18) in order to get the S-matrix element for the massive $q\bar{q}$ production by scattering of a gluon on the target background field. Extracting the delta function ensuring light-cone momentum conservation, we can define the amplitude $\mathcal{M}_{g\to q\bar{q}}$ for this process as

$$
\begin{aligned}
&\langle\bar{q}(\underline{p_2},h_2,\beta_2)q(\underline{p_1},h_1,\beta_1)_{\text{phys}}|\hat{S}_E|g_{\text{phys}}(\underline{k_0},\lambda_0,a_0)\rangle\\
&\qquad=(2k_0^+)(2\pi)\delta(p_1^+{+}p_2^+{-}k_0^+)\;i\,\mathcal{M}_{g\to q\bar{q}}\,.
\end{aligned}
\tag{2.21}
$$

Then, from the Fock state decompositions (2.9) and (2.18), the identities (2.20) and the commutation relations (A7), (A8) and (A9), it is straightforward to calculate the amplitude to leading order in the coupling g. One obtains

$$
i\,\mathcal{M}_{g\to q\bar{q}}=i\,\mathcal{M}_{g\to q\bar{q}}^{\text{bef}}+i\,\mathcal{M}_{g\to q\bar{q}}^{\text{aft}}\,,
\tag{2.22}
$$

where

$$
\begin{aligned}
i\,\mathcal{M}_{g\to q\bar{q}}^{\text{bef}} &= \frac{1}{2k_0^+}\,(\mu)^{2-\frac{D}{2}}\,g\int d^{D-2}\mathbf{x}_1\;e^{-i\mathbf{x}_1\cdot\left[\mathbf{p}_1-\frac{p_1^+}{k_0^+}\,\mathbf{k}_0\right]}\int d^{D-2}\mathbf{x}_2\;e^{-i\mathbf{x}_2\cdot\left[\mathbf{p}_2-\frac{p_2^+}{k_0^+}\,\mathbf{k}_0\right]}\\
&\times\Big[U_F(\mathbf{x}_1)\,t^{a_0}\,U_F^\dagger(\mathbf{x}_2)\Big]_{\beta_1\,\beta_2}\varepsilon_{\lambda_0}^j\Bigg\{\mathcal{B}_V^i(\mathbf{x}_{12},m)\;\;\overline{u_G}(p_1^+,h_1)\,\gamma^+\left[\frac{(k_0^+-2p_1^+)}{k_0^+}\,\delta^{ij}+i\,\sigma^{ij}\right]v_G(p_2^+,h_2)\\
&\qquad\qquad+\mathcal{B}_S(\mathbf{x}_{12},m)\;\;m\,\overline{u_G}(p_1^+,h_1)\,\gamma^+\gamma^j\,v_G(p_2^+,h_2)\Bigg\}
\end{aligned}
\tag{2.23}
$$

and

$$
\begin{aligned}
i\,\mathcal{M}^{\rm aft}_{g\to q\bar{q}} = -\frac{1}{2k_0^+}\,(\mu)^{2-\frac{D}{2}}\,g\,\int d^{D-2}\mathbf{x}_0\; e^{-i\mathbf{x}_0\cdot(\mathbf{p}_1+\mathbf{p}_2-\mathbf{k}_0)}\;& \frac{(t^{b_0})_{\beta_1\,\beta_2}\;U_A(\mathbf{x}_0)_{b_0\,a_0}}{\Big[\Big(\mathbf{p}_2-\frac{p_2^+}{p_1^+}\,\mathbf{p}_1\Big)^2+\Big(\frac{k_0^+}{p_1^+}\Big)^2\,m^2\Big]}\\
\times\varepsilon^j_{\lambda_0}\bigg\{-\left(\frac{k_0^+}{p_1^+}\right)\left(\mathbf{p}_2^i-\frac{p_2^+}{p_1^+}\,\mathbf{p}_1^i\right)\overline{u_G}(p_1^+,h_1)\,\gamma^+&\left[\frac{(k_0^+-2p_1^+)}{k_0^+}\,\delta^{ij}+i\,\sigma^{ij}\right]v_G(p_2^+,h_2)\\
+\left(\frac{k_0^+}{p_1^+}\right)^2 m\,\overline{u_G}(p_1^+,h_1)\,\gamma^+\gamma^j\,v_G(p_2^+,h_2)\bigg\}&
\end{aligned}
\tag{2.24}
$$

correspond to the contributions where the gluon splits into $q\bar{q}$ before or after crossing the target, respectively.

III. HEAVY FLAVORED HADRON PRODUCTION IN THE HYBRID FACTORIZATION

A. Partonic cross section for heavy quark production

From the amplitude $\mathcal{M}_{g\to q\bar{q}}$, one can obtain the partonic cross section for the process $g+A\to q+\bar{q}+X$ at LO, as (see ref. [50])

$$
\begin{aligned}
(2p_1^+)(2p_2^+)(2\pi)^{2D-2}\frac{d\sigma^{g+A\to q+\bar{q}+X}}{dp_1^+\,d^{D-2}\mathbf{p}_1\,dp_2^+\,d^{D-2}\mathbf{p}_2} &= (2k_0^+)(2\pi)\delta(p_1^++p_2^+-k_0^+)\,\frac{1}{d_A}\\
&\times\sum_{a_0,\,\beta_1,\,\beta_2}\frac{1}{D-2}\sum_{\lambda_0,\,h_1,\,h_2}\Big|\mathcal{M}_{g\to q\bar{q}}\Big|^2,
\end{aligned}
\tag{3.1}
$$

where, as usual, one has to sum over the colors and polarizations of final particles and average over the colors and polarizations of initial particles (note that $D-2$ is indeed the number of physical gluon polarizations in conventional dimensional regularization). d_A is the dimension of the adjoint representation of the gauge group i.e. $d_A=N_c^2-1$ for $SU(N_c)$.

Then, the single inclusive massive quark production cross section is obtained by integrating over the kinematics of the antiquark, as

$$
\begin{aligned}
(2p_1^+)(2\pi)^{D-1}\frac{d\sigma^{g+A\to q+X}}{dp_1^+\,d^{D-2}\mathbf{p}_1} = \int_0^{+\infty}\frac{dp_2^+}{(2\pi)2p_2^+}\int\frac{d^{D-2}\mathbf{p}_2}{(2\pi)^{D-2}}\;&(2p_1^+)(2p_2^+)(2\pi)^{2D-2}\\
\times\,&\frac{d\sigma^{g+A\to q+\bar{q}+X}}{dp_1^+\,d^{D-2}\mathbf{p}_1\,dp_2^+\,d^{D-2}\mathbf{p}_2}\,.
\end{aligned}
\tag{3.2}
$$

1. *Spin*

Note that both contributions to the amplitude, (2.23) and (2.24) are linear combinations of the same two spinor structures (spin-flip and spin non-flip), which contain all of the dependence on the helicities of the quarks, whereas the dependence on the gluon polarization always appears via a $\varepsilon^j_{\lambda_0}$ factor. The spin sum/average for the square of the spin-flip structure reads

$$
\begin{aligned}
&\frac{1}{D-2}\sum_{\lambda_0,\,h_1,\,h_2}\varepsilon^{j'\,*}_{\lambda_0}\varepsilon^j_{\lambda_0}\left[\overline{u_G}(p_1^+,h_1)\,\gamma^+\gamma^{j'}\,v_G(p_2^+,h_2)\right]^\dagger\overline{u_G}(p_1^+,h_1)\,\gamma^+\gamma^j\,v_G(p_2^+,h_2)\\
&=\frac{-g^{jj'}}{D-2}\sum_{h_1,\,h_2}-\,\overline{v_G}(p_2^+,h_2)\,\gamma^+\gamma^{j'}\,u_G(p_1^+,h_1)\overline{u_G}(p_1^+,h_1)\,\gamma^+\gamma^j\,v_G(p_2^+,h_2)\\
&=\frac{-g^{jj'}}{D-2}\,(-1)(2p_1^+)(2p_2^+)\;\,\mathrm{Tr}\big[\mathcal{P}_G\,\gamma^{j'}\,\gamma^j\big]\\
&=2(2p_1^+)(2p_2^+)\,.
\end{aligned}
\tag{3.3}
$$

For the square of the spin non-flip structure, one gets

$$\frac{1}{D-2}\sum_{\lambda_0,h_1,h_2}\varepsilon^{j'\,*}_{\lambda_0}\varepsilon^{j}_{\lambda_0}\left[\overline{u_G}(p_1^+,h_1)\,\gamma^+\left[\frac{(k_0^+-2p_1^+)}{k_0^+}\,\delta^{i'j'}+i\,\sigma^{i'j'}\right]v_G(p_2^+,h_2)\right]^\dagger$$

$$\times\,\overline{u_G}(p_1^+,h_1)\,\gamma^+\left[\frac{(k_0^+-2p_1^+)}{k_0^+}\,\delta^{ij}+i\,\sigma^{ij}\right]v_G(p_2^+,h_2)$$

$$=\frac{-g^{jj'}}{D-2}\,(2p_1^+)(2p_2^+)\mathrm{Tr}\Big[\mathcal{P}_G\left(\frac{(k_0^+-2p_1^+)}{k_0^+}\,\delta^{i'j'}-i\,\sigma^{i'j'}\right)\left(\frac{(k_0^+-2p_1^+)}{k_0^+}\,\delta^{ij}+i\,\sigma^{ij}\right)\Big]$$

$$=4(2p_1^+)(2p_2^+)\frac{(-g^{ii'})}{D-2}\left[\left(\frac{p_1^+}{k_0^+}\right)^2+\left(\frac{p_2^+}{k_0^+}\right)^2+\frac{D-4}{2}\right]. \tag{3.4}$$

For the interference between the spin flip and spin non-flip structures, the helicity sums lead to the trace of an odd number of gamma matrices, and thus vanish. Hence, as expected, there is no interference between the spin flip and spin non-flip contributions to the amplitude $\mathcal{M}_{g\to q\bar{q}}$.

2. *Squared amplitude*

It is now straightforward to calculate the spin and color sums/averages for the squared amplitude. First, for the *after* contribution, $\mathcal{M}^{\mathrm{aft}}_{g\to q\bar{q}}$, one gets

$$\frac{1}{d_A}\sum_{a_0,\beta_1,\beta_2}\frac{1}{D-2}\sum_{\lambda_0,h_1,h_2}\left|\mathcal{M}^{\mathrm{aft}}_{g\to q\bar{q}}\right|^2=\frac{g^2\,T_F\,(\mu^2)^{2-\frac{D}{2}}}{\left[\left(\mathbf{p}_2-\frac{p_2^+}{p_1^+}\,\mathbf{p}_1\right)^2+\left(\frac{k_0^+}{p_1^+}\right)^2m^2\right]^2}$$

$$\times\int d^{D-2}\mathbf{x}_0\int d^{D-2}\mathbf{x}_{0'}\;e^{-i(\mathbf{p}_1+\mathbf{p}_2-\mathbf{k}_0)\cdot\mathbf{x}_{00'}}\;S^A_{00'}\;\frac{p_1^+\,p_2^+}{(k_0^+)^2}$$

$$\times\left\{\frac{4}{D-2}\left(\frac{k_0^+}{p_1^+}\right)^2\left[\left(\frac{p_1^+}{k_0^+}\right)^2+\left(\frac{p_2^+}{k_0^+}\right)^2+\frac{D-4}{2}\right]\left(\mathbf{p}_2-\frac{p_2^+}{p_1^+}\,\mathbf{p}_1\right)^2+2\left(\frac{k_0^+}{p_1^+}\right)^4m^2\right\}, \tag{3.5}$$

with the adjoint dipole defined as

$$S^A_{01}\equiv\frac{1}{d_A}\,\mathrm{Tr}\big[U_A(\mathbf{x}_0)U_A^\dagger(\mathbf{x}_1)\big]\,. \tag{3.6}$$

Second, for the *before* contribution, $\mathcal{M}^{\mathrm{bef}}_{g\to q\bar{q}}$, one obtains

$$\frac{1}{d_A}\sum_{a_0,\beta_1,\beta_2}\frac{1}{D-2}\sum_{\lambda_0,h_1,h_2}\left|\mathcal{M}^{\mathrm{bef}}_{g\to q\bar{q}}\right|^2=(\mu^2)^{2-\frac{D}{2}}\,g^2\;\frac{p_1^+\,p_2^+}{(k_0^+)^2}$$

$$\times\int d^{D-2}\mathbf{x}_1\int d^{D-2}\mathbf{x}_{1'}\;e^{-i\left(\mathbf{p}_1-\frac{p_1^+}{k_0^+}\mathbf{k}_0\right)\cdot\mathbf{x}_{11'}}\int d^{D-2}\mathbf{x}_2\int d^{D-2}\mathbf{x}_{2'}\;e^{-i\left(\mathbf{p}_2-\frac{p_2^+}{k_0^+}\mathbf{k}_0\right)\cdot\mathbf{x}_{22'}}$$

$$\times\,\frac{1}{d_A}\,\mathrm{Tr}\big[U_F(\mathbf{x}_1)t^aU_F^\dagger(\mathbf{x}_2)U_F(\mathbf{x}_{2'})t^aU_F^\dagger(\mathbf{x}_{1'})\big]$$

$$\times\left\{\frac{4}{D-2}\left[\left(\frac{p_1^+}{k_0^+}\right)^2+\left(\frac{p_2^+}{k_0^+}\right)^2+\frac{D-4}{2}\right]\;\mathcal{B}^i_V(\mathbf{x}_{12},m)\;\;\mathcal{B}^{i\,*}_V(\mathbf{x}_{1'2'},m)\right.$$

$$\left.+2m^2\;\;\mathcal{B}_S(\mathbf{x}_{12},m)\;\;\mathcal{B}^*_S(\mathbf{x}_{1'2'},m)\right\}. \tag{3.7}$$

The multipole appearing in eq. (3.7) can be rewritten as a product of fundamental dipoles, and a N_c-suppressed fundamental quadrupole term. In single inclusive heavy quark production, this multipole will collapse to a dipole upon integration over the transverse momentum of the un-tagged produced particle.

Finally, for the interference between the before and after contributions to the amplitude, one gets

$$\frac{1}{d_A}\sum_{a_0,\,\beta_1,\,\beta_2}\frac{1}{D-2}\sum_{\lambda_0,\,h_1,\,h_2}\Big[\big(\mathcal{M}^{\rm aft}_{g\to q\bar{q}}\big)^\dagger\,\mathcal{M}^{\rm bef}_{g\to q\bar{q}}+c.c.\Big]=-g^2\,T_F\,(\mu^2)^{2-\frac{D}{2}}\,\frac{p_1^+\,p_2^+}{(k_0^+)^2}$$
$$\times\int d^{D-2}\mathbf{x}_0\int d^{D-2}\mathbf{x}_1\;e^{-i\left(\mathbf{p}_1-\frac{p_1^+}{k_0^+}\mathbf{k}_0\right)\cdot\mathbf{x}_{10}}\int d^{D-2}\mathbf{x}_2\;e^{-i\left(\mathbf{p}_2-\frac{p_2^+}{k_0^+}\mathbf{k}_0\right)\cdot\mathbf{x}_{20}}$$
$$\times\left\{\frac{-4}{D-2}\left[\left(\frac{p_1^+}{k_0^+}\right)^2+\left(\frac{p_2^+}{k_0^+}\right)^2+\frac{D-4}{2}\right]\left(\frac{k_0^+}{p_1^+}\right)\left[\mathbf{p}_2^i-\frac{p_2^+}{p_1^+}\,\mathbf{p}_1^i\right]\mathcal{B}_V^i(\mathbf{x}_{12},m)\right.$$
$$\left.+2m^2\left(\frac{k_0^+}{p_1^+}\right)^2\mathcal{B}_S(\mathbf{x}_{12},m)\right\}\frac{S_{120}}{\left[\left(\mathbf{p}_2-\frac{p_2^+}{p_1^+}\,\mathbf{p}_1\right)^2+\left(\frac{k_0^+}{p_1^+}\right)^2m^2\right]}\;+\;c.c.\,,\tag{3.8}$$

where we have defined the tripole operator as

$$S_{120}\equiv\frac{1}{d_F\,C_F}\,\mathrm{Tr}\big[U_F(\mathbf{x}_1)t^aU_F^\dagger(\mathbf{x}_2)t^b\big]\,U_A(\mathbf{x}_0)_{b\,a}\tag{3.9}$$

and we have used the identity $d_F\,C_F=d_A\,T_F$.

We have checked that eqs. (3.1), (3.5), (3.7) and (3.8) are equivalent to the collinear limit (provided in [45]) of the results for heavy $q\bar{q}$ pair production obtained in [40, 43, 45].

3. Partonic cross-section for single inclusive heavy quark production

The next step is to integrate over the momentum of the produced antiquark in order to obtain the single inclusive cross section at parton level, following the relation (3.2). Then, the contribution from the square of the before term reads

$$(2p_1^+)(2\pi)^{D-1}\frac{d\sigma^{g+A\to q+X}}{dp_1^+\,d^{D-2}\mathbf{p}_1}\bigg|_{\text{bef.-bef.}}=g^2\,T_F\,\theta(k_0^+-p_1^+)\,\frac{p_1^+}{k_0^+}\int d^{D-2}\mathbf{x}_1\int d^{D-2}\mathbf{x}_{1'}\,S^F_{11'}$$
$$\times\,e^{-i\left(\mathbf{p}_1-\frac{p_1^+}{k_0^+}\mathbf{k}_0\right)\cdot\mathbf{x}_{11'}}\left\{\frac{4}{D-2}\left[\left(\frac{p_1^+}{k_0^+}\right)^2+\left(1-\frac{p_1^+}{k_0^+}\right)^2+\frac{D-4}{2}\right]\mathcal{C}_1\left(|\mathbf{x}_{1'1}|,m\right)\right.$$
$$\left.+2m^2\,\mathcal{C}_0\left(|\mathbf{x}_{1'1}|,m\right)\right\},\tag{3.10}$$

where

$$\mathcal{C}_1\left(|\mathbf{r}|,m\right)\equiv(\mu^2)^{2-\frac{D}{2}}\int\frac{d^{D-2}\mathbf{K}}{(2\pi)^{D-2}}\,e^{-i\mathbf{K}\cdot\mathbf{r}}\,\frac{\mathbf{K}^2}{\big[\mathbf{K}^2+m^2\big]^2}$$
$$=\frac{1}{2\pi}\left(\frac{m^2}{4\pi\,\mu^2}\right)^{\frac{D}{2}-2}\left[\left(\frac{m|\mathbf{r}|}{2}\right)^{2-\frac{D}{2}}\mathrm{K}_{2-\frac{D}{2}}(m|\mathbf{r}|)-\left(\frac{m|\mathbf{r}|}{2}\right)^{3-\frac{D}{2}}\mathrm{K}_{3-\frac{D}{2}}(m|\mathbf{r}|)\right],\tag{3.11}$$
$$\mathcal{C}_0\left(|\mathbf{r}|,m\right)\equiv(\mu^2)^{2-\frac{D}{2}}\int\frac{d^{D-2}\mathbf{K}}{(2\pi)^{D-2}}\,e^{-i\mathbf{K}\cdot\mathbf{r}}\,\frac{1}{\big[\mathbf{K}^2+m^2\big]^2}$$
$$=\frac{1}{2\pi\,m^2}\left(\frac{m^2}{4\pi\,\mu^2}\right)^{\frac{D}{2}-2}\left(\frac{m|\mathbf{r}|}{2}\right)^{3-\frac{D}{2}}\mathrm{K}_{3-\frac{D}{2}}(m|\mathbf{r}|)\,,\tag{3.12}$$

with the fundamental dipole defined as

$$S^F_{01}\equiv\frac{1}{d_F}\,\mathrm{Tr}\big[U_F(\mathbf{x}_0)U_F^\dagger(\mathbf{x}_1)\big]\,.\tag{3.13}$$

The contribution from the square of the after term in the amplitude can be written as

$$(2p_1^+)(2\pi)^{D-1}\frac{d\sigma^{g+A\to q+X}}{dp_1^+\,d^{D-2}\mathbf{p}_1}\bigg|_{\text{aft.-aft.}} = g^2\,T_F\;\theta(k_0^+-p_1^+)\;\frac{k_0^+}{p_1^+}\;\int d^{D-2}\mathbf{x}_0\;\int d^{D-2}\mathbf{x}_{0'}\;S^A_{00'}$$
$$\times e^{-i\left(\frac{k_0^+}{p_1^+}\mathbf{p}_1-\mathbf{k}_0\right)\cdot\mathbf{x}_{00'}}\;\left\{\frac{4}{D-2}\left[\left(\frac{p_1^+}{k_0^+}\right)^2+\left(1-\frac{p_1^+}{k_0^+}\right)^2+\frac{D-4}{2}\right]\mathcal{C}_1\left(|\mathbf{x}_{00'}|,\frac{k_0^+}{p_1^+}\,m\right)\right.$$
$$\left.+2m^2\;\left(\frac{k_0^+}{p_1^+}\right)^2\;\mathcal{C}_0\left(|\mathbf{x}_{00'}|,\frac{k_0^+}{p_1^+}\,m\right)\right\}. \tag{3.14}$$

Finally, the interference contribution reads

$$(2p_1^+)(2\pi)^{D-1}\frac{d\sigma^{g+A\to q+X}}{dp_1^+\,d^{D-2}\mathbf{p}_1}\bigg|_{\text{interf.}} = -g^2\,T_F\;(\mu^2)^{2-\frac{D}{2}}\;\theta(k_0^+-p_1^+)$$
$$\times\int d^{D-2}\mathbf{x}_0\;\int d^{D-2}\mathbf{x}_1\;\int d^{D-2}\mathbf{x}_2\;S_{120}\;e^{-i\left(\mathbf{p}_1-\frac{k_0^+}{p_1^+}\mathbf{k}_0\right)\cdot\left(\mathbf{x}_{12}+\frac{k_0^+}{p_1^+}\mathbf{x}_{20}\right)}$$
$$\times\left\{\frac{-4}{D-2}\left[\left(\frac{p_1^+}{k_0^+}\right)^2+\left(1-\frac{p_1^+}{k_0^+}\right)^2+\frac{D-4}{2}\right]\mathcal{B}_V^i(\mathbf{x}_{12},m)\;\mathcal{B}_V^{i\,*}\left(\mathbf{x}_{20},\frac{k_0^+}{p_1^+}m\right)\right.$$
$$\left.+2m^2\;\left(\frac{k_0^+}{p_1^+}\right)\;\mathcal{B}_S(\mathbf{x}_{12},m)\;\mathcal{B}_S^*\left(\mathbf{x}_{20},\frac{k_0^+}{p_1^+}m\right)\right\}\;+\;c.c. \tag{3.15}$$

B. Hadron-level cross section for heavy quark production

Now, we want to use the results (3.10), (3.14) and (3.15) in order to write the cross section for single inclusive production of a heavy flavored hadron in the hybrid factorization, in a high-energy dense-dilute collision. The momentum of the projectile, target and produced hadron are denoted respectively P_P^μ, P_T^μ and p_h^μ. By choice of frame, we have $\mathbf{P}_P=\mathbf{P}_T=0$ and P_P^- and P_T^+ are negligible, whereas the Mandelstam s variable of the collision is given by $s\simeq 2P_P^+\,P_T^-$. The Feynman x_F variable is defined by $x_F\equiv p_h^+/P_P^+$.

Neglecting for the moment possible contributions from the eventual intrinsic heavy flavor content of the projectile, and from heavy quark production during jet fragmentation, the picture is the following: A large-x_B gluon with momentum k_0^μ is picked inside the projectile, then it collide on the target, producing a heavy quark of momentum p_1^μ, according to the partonic cross-section sum of (3.10), (3.14) and (3.15), and, finally, the heavy quark of momentum p_1^μ fragments into heavy flavored hadron h of momentum p_h^μ.

So, the hadronic cross section is obtained from the partonic cross section as

$$(2p_h^+)(2\pi)^{D-1}\frac{d\sigma^{p+A\to h+X}}{dp_h^+\,d^{D-2}\mathbf{p}_h} = \int_0^1 dx_B\;g^0(x_B)\int_0^1\frac{d\zeta}{\zeta^{D-2}}\;D^0_{h/q}(\zeta)$$
$$\times\;\;(2p_1^+)(2\pi)^{D-1}\frac{d\sigma^{g+A\to q+X}}{dp_1^+\,d^{D-2}\mathbf{p}_1}\,, \tag{3.16}$$

where, due to the collinear approximation, one has

$$k_0^+=x_B\,P_P^+\;,\quad \mathbf{k}_0=0\;,\quad p_1^+=\frac{p_h^+}{\zeta}\;\;\text{and}\quad \mathbf{p}_1=\frac{\mathbf{p}_h}{\zeta}\,. \tag{3.17}$$

Since we are interested only in the leading-order result for this channel, we can replace the bare PDFs and FFs by the renormalized ones.

All in all, one obtains from equations (3.10), (3.14) and (3.15) the following three contributions to the hadronic cross section:

$$(2\pi)^{D-2}\,\frac{d\sigma^{p+A\to h+X}}{dx_F\,d^{D-2}\mathbf{p}_h}\bigg|_{\text{bef.-bef.}} = \int_{x_F}^1 dx_B\;g(x_B,\mu^2)\int_{\frac{x_F}{x_B}}^1\frac{d\zeta}{\zeta^{D-2}}\;D_{h/q}(\zeta,\mu^2)\quad\alpha_s\,T_F\;\frac{x_F}{x_B\,\zeta}$$
$$\times\;\int d^{D-2}\mathbf{x}_1\;\int d^{D-2}\mathbf{x}_2\;S^F_{12}\;e^{-\frac{i}{\zeta}\,\mathbf{p}_h\cdot\mathbf{x}_{12}}$$
$$\times\left\{\frac{4}{D-2}\left[\left(\frac{x_F}{x_B\,\zeta}\right)^2+\left(1-\frac{x_F}{x_B\,\zeta}\right)^2+\frac{D-4}{2}\right]\mathcal{C}_1\left(|\mathbf{x}_{12}|,m\right)\;+2m^2\;\mathcal{C}_0\left(|\mathbf{x}_{12}|,m\right)\right\}, \tag{3.18}$$

$$(2\pi)^{D-2}\,\frac{d\sigma^{p+A\to h+X}}{dx_F\,d^{D-2}\mathbf{p}_h}\bigg|_{\text{aft.-aft.}} = \int_{x_F}^{1} dx_B\; g(x_B,\mu^2)\int_{\frac{x_F}{x_B}}^{1}\frac{d\zeta}{\zeta^{D-2}}\; D_{h/q}(\zeta,\mu^2)\quad \alpha_s\,T_F\;\frac{x_B\,\zeta}{x_F}$$
$$\times\int d^{D-2}\mathbf{x}_1\int d^{D-2}\mathbf{x}_2\; S^A_{12}\; e^{-i\frac{x_B}{x_F}\,\mathbf{p}_h\cdot\mathbf{x}_{12}}$$
$$\times\left\{\frac{4}{D-2}\left[\left(\frac{x_F}{x_B\,\zeta}\right)^2+\left(1-\frac{x_F}{x_B\,\zeta}\right)^2+\frac{D-4}{2}\right]\mathcal{C}_1\left(|\mathbf{x}_{12}|,\frac{x_B\,\zeta}{x_F}\,m\right)\right. \tag{3.19}$$
$$\left.+2m^2\,\left(\frac{x_B\,\zeta}{x_F}\right)^2\mathcal{C}_0\left(|\mathbf{x}_{12}|,\frac{x_B\,\zeta}{x_F}\,m\right)\right\}$$

and

$$(2\pi)^{D-2}\,\frac{d\sigma^{p+A\to h+X}}{dx_F\,d^{D-2}\mathbf{p}_h}\bigg|_{\text{interf.}} = \int_{x_F}^{1} dx_B\; g(x_B,\mu^2)\int_{\frac{x_F}{x_B}}^{1}\frac{d\zeta}{\zeta^{D-2}}\; D_{h/q}(\zeta,\mu^2)\;(-1)\,\alpha_s\,T_F\;\left(\mu^2\right)^{2-\frac{D}{2}}$$
$$\times\int d^{D-2}\mathbf{x}_0\int d^{D-2}\mathbf{x}_1\int d^{D-2}\mathbf{x}_2\; S_{120}\; e^{-\frac{i}{\zeta}\,\mathbf{p}_h\cdot\left[\mathbf{x}_{12}-\frac{x_B\,\zeta}{x_F}\,\mathbf{x}_{02}\right]}$$
$$\times\left\{\frac{-4}{D-2}\left[\left(\frac{x_F}{x_B\,\zeta}\right)^2+\left(1-\frac{x_F}{x_B\,\zeta}\right)^2+\frac{D-4}{2}\right]\mathcal{B}^i_V(\mathbf{x}_{12},m)\,\mathcal{B}^{i\,*}_V\left(\mathbf{x}_{20},\frac{x_B\,\zeta}{x_F}\,m\right)\right.$$
$$\left.+2m^2\,\left(\frac{x_B\,\zeta}{x_F}\right)\,\mathcal{B}_S(\mathbf{x}_{12},m)\,\mathcal{B}^*_S\left(\mathbf{x}_{20},\frac{x_B\,\zeta}{x_F}\,m\right)\right\}\;+\;c.c. \tag{3.20}$$

C. Large mass limit of the hadronic cross section

We also consider the large mass limit of the hadronic cross section. The before-before and after-after contributions to the total hadronic cross section are written in terms of the functions $\mathcal{C}_0(|\mathbf{r}|,m)$ and $\mathcal{C}_1(|\mathbf{r}|,m)$. These functions, when expanded in the large mass limit, can be approximated as

$$C_0(|\mathbf{r}|,m)\simeq\left(\mu^2\right)^{2-\frac{D}{2}}\,\frac{1}{m^4}\,\left(1+\frac{2}{m^2}\partial^2_{\mathbf{r}}\right)\,\delta^{(D-2)}(\mathbf{r})\;, \tag{3.21}$$
$$C_1(|\mathbf{r}|,m)\simeq\left(\mu^2\right)^{2-\frac{D}{2}}\,\frac{1}{m^4}\,\left(-\partial^2_{\mathbf{r}}\right)\,\delta^{(D-2)}(\mathbf{r})\;. \tag{3.22}$$

The interference contribution to the cross section is written in terms functions $\mathcal{B}^i_V(\mathbf{r},m)$ and $\mathcal{B}_S(\mathbf{r},m)$, whose leading term in the large mass limit reads

$$\mathcal{B}^i_V(\mathbf{r},m)\simeq\frac{1}{m^2}\,\left(-i\,\partial^i_r\right)\,\delta^{(D-2)}(\mathbf{r})\;, \tag{3.23}$$
$$\mathcal{B}_S(\mathbf{r},m)\simeq\frac{1}{m^2}\,\left(1+\frac{1}{m^2}\partial^2_{\mathbf{r}}\right)\,\delta^{(D-2)}(\mathbf{r})\;. \tag{3.24}$$

Using these equations, we can obtain the large mass limit of the each contribution to the hadronic cross section that reads

$$(2\pi)^{D-2}\,\frac{d\sigma^{p+A\to h+X}}{dx_F\,d^{D-2}\mathbf{p}_h}\bigg|_{\text{bef.-bef.}} = \int_{x_F}^{1} dx_B\; g(x_B,\mu^2)\int_{\frac{x_F}{x_B}}^{1}\frac{d\zeta}{\zeta^{D-2}}\; D_{h/q}(\zeta,\mu^2)\quad \alpha_s\,T_F\;\frac{x_F}{x_B\,\zeta}$$
$$\times\,\left(\mu^2\right)^{2-\frac{D}{2}}\int d^{D-2}\mathbf{x}_1\int d^{D-2}\mathbf{x}_2\;\delta^{(D-2)}(\mathbf{x}_{12})$$
$$\times\left\{\frac{2}{m^2}-\frac{1}{m^4}\frac{\mathbf{p}_h^2}{\zeta^2}\left[4-\frac{4}{D-2}\left[\left(\frac{x_F}{x_B\,\zeta}\right)^2+\left(1-\frac{x_F}{x_B\,\zeta}\right)^2+\frac{D-4}{2}\right]\right]\right.$$
$$+\frac{1}{m^4}\left[4-\frac{4}{D-2}\left[\left(\frac{x_F}{x_B\,\zeta}\right)^2+\left(1-\frac{x_F}{x_B\,\zeta}\right)^2+\frac{D-4}{2}\right]\right]\left[\left(\partial^2_{\mathbf{x}_1}S^F_{12}\right)-2i\frac{1}{\zeta}\mathbf{p}^i_h\,\left(\partial^i_{\mathbf{x}_1}S^F_{12}\right)\right]$$
$$\left.+\mathcal{O}\left(\frac{1}{m^6}\right)\right\},$$

$$(2\pi)^{D-2}\,\frac{d\sigma^{p+A\to h+X}}{dx_F\,d^{D-2}\mathbf{p}_h}\bigg|_{\text{aft.-aft.}}=\int_{x_F}^{1}dx_B\;g(x_B,\mu^2)\int_{\frac{x_F}{x_B}}^{1}\frac{d\zeta}{\zeta^{D-2}}\;D_{h/q}(\zeta,\mu^2)\quad\alpha_s\,T_F\;\frac{x_F}{x_B\,\zeta}$$
$$\times\;\left(\mu^2\right)^{2-\frac{D}{2}}\int d^{D-2}\mathbf{x}_1\int d^{D-2}\mathbf{x}_2\;\delta^{(D-2)}(\mathbf{x}_{12})$$
$$\times\Bigg\{\frac{2}{m^2}-\frac{1}{m^4}\frac{\mathbf{p}_h^2}{\zeta^2}\left[4-\frac{4}{D-2}\left[\left(\frac{x_F}{x_B\,\zeta}\right)^2+\left(1-\frac{x_F}{x_B\,\zeta}\right)^2+\frac{D-4}{2}\right]\right]$$
$$+\frac{1}{m^4}\left(\frac{x_F}{x_B\,\zeta}\right)^2\left[4-\frac{4}{D-2}\left[\left(\frac{x_F}{x_B\,\zeta}\right)^2+\left(1-\frac{x_F}{x_B\,\zeta}\right)^2+\frac{D-4}{2}\right]\right]$$
$$\times\left[\left(\partial^2_{\mathbf{x}_1}S^A_{12}\right)-2i\frac{x_B}{x_F}\mathbf{p}_h^i\left(\partial^i_{\mathbf{x}_1}S^A_{12}\right)\right]+\mathcal{O}\left(\frac{1}{m^6}\right)\Bigg\},\tag{3.25}$$

$$(2\pi)^{D-2}\,\frac{d\sigma^{p+A\to h+X}}{dx_F\,d^{D-2}\mathbf{p}_h}\bigg|_{\text{interf.}}=\int_{x_F}^{1}dx_B\;g(x_B,\mu^2)\int_{\frac{x_F}{x_B}}^{1}\frac{d\zeta}{\zeta^{D-2}}\;D_{h/q}(\zeta,\mu^2)\;(-1)\,\alpha_s\,T_F$$
$$\times\left(\frac{x_F}{x_B\,\zeta}\right)\left(\mu^2\right)^{2-\frac{D}{2}}\int d^{D-2}\mathbf{x}_1\int d^{D-2}\mathbf{x}_2\int d^{D-2}\mathbf{x}_0\;\delta^{(D-2)}(\mathbf{x}_{12})\;\delta^{(D-2)}(\mathbf{x}_{02})$$
$$\times\Bigg\{\frac{2}{m^2}-\frac{1}{m^4}\frac{\mathbf{p}_h^2}{\zeta^2}\left[4-\frac{4}{D-2}\left[\left(\frac{x_F}{x_B\,\zeta}\right)^2+\left(1-\frac{x_F}{x_B\,\zeta}\right)^2+\frac{D-4}{2}\right]\right]$$
$$+\frac{1}{m^4}\left(\frac{x_F}{x_B\,\zeta}\right)\frac{4}{D-2}\left[\left(\frac{x_F}{x_B\,\zeta}\right)^2+\left(1-\frac{x_F}{x_B\,\zeta}\right)^2+\frac{D-4}{2}\right]$$
$$\times\left[\left(\partial^i_{\mathbf{x}_1}\partial^i_{\mathbf{x}_0}S_{120}\right)-\frac{i}{\zeta}\mathbf{p}_h^i\left(\partial^i_{\mathbf{x}_0}S_{120}\right)-\frac{i}{\zeta}\mathbf{p}_h^i\left(\frac{x_B\,\zeta}{x_F}\right)\left(\partial^i_{\mathbf{x}_1}S_{120}\right)\right]$$
$$+\frac{2}{m^4}\left[\left(\partial^2_{\mathbf{x}_1}S_{120}\right)-2\frac{i}{\zeta}\mathbf{p}_h^i\left(\partial^i_{\mathbf{x}_1}S_{120}\right)+\left(\frac{x_F}{x_B\,\zeta}\right)^2\left(\partial^2_{\mathbf{x}_0}S_{120}\right)-2\frac{i}{\zeta}\mathbf{p}_h^i\left(\frac{x_F}{x_B\,\zeta}\right)\left(\partial^i_{\mathbf{x}_0}S_{120}\right)\right]$$
$$+\mathcal{O}\left(\frac{1}{m^6}\right)\Bigg\}+c.c.\tag{3.26}$$

Each contribution to the total hadron-level cross section can be simplified. Any term with a single transverse derivative acting on an adjoint or fundamental dipole or the tripole operator can be dropped. This is due to the fact that each transverse derivative brings a generator of the $SU(N_c)$ group either in the fundamental or adjoint representation inside the trace. These terms become a trace of single generator by realising the delta functions, hence they vanish. Moreover, the delta functions also leads to simplifications on the tripole operator. It can be reduced to either the identity or a fundamental or adjoint dipole. Specifically, we have

$$S_{222}=\mathbb{1},\qquad S_{220}=S^A_{20},\qquad S_{122}=S^F_{12}\,.\tag{3.27}$$

After all these simplifications, the total hadron-level cross section in the large mass limit reads

$$(2\pi)^{D-2}\,\frac{d\sigma^{p+A\to h+X}}{dx_F\,d^{D-2}\mathbf{p}_h}=\int_{x_F}^{1}dx_B\;g(x_B,\mu^2)\int_{\frac{x_F}{x_B}}^{1}\frac{d\zeta}{\zeta^{D-2}}\;D_{h/q}(\zeta,\mu^2)\;\alpha_s\,T_F\left(\frac{x_F}{x_B\,\zeta}\right)\left(\mu^2\right)^{2-\frac{D}{2}}$$
$$\times\frac{1}{m^4}\int d^{D-2}\mathbf{x}_1\int d^{D-2}\mathbf{x}_2\int d^{D-2}\mathbf{x}_0\;\delta^{(D-2)}(\mathbf{x}_{12})\;\delta^{(D-2)}(\mathbf{x}_{02})$$
$$\times\frac{-4}{D-2}\left[\left(\frac{x_F}{x_B\,\zeta}\right)^2+\left(1-\frac{x_F}{x_B\,\zeta}\right)^2+\frac{D-4}{2}\right]$$
$$\times\left[\partial^2_{\mathbf{x}_1}S^F_{12}+\left(\frac{x_F}{x_B\,\zeta}\right)^2\partial^2_{\mathbf{x}_0}S^A_{02}+\left(\frac{x_F}{x_B\,\zeta}\right)\partial^i_{\mathbf{x}_1}\partial^i_{\mathbf{x}_0}\left[S_{120}+S_{210}\right]\right]+\mathcal{O}\left(\frac{1}{m^6}\right)\tag{3.28}$$

It is straightforward to calculate the action of the transverse derivatives on the dipole and tripole operators:

$$\partial^2_{\mathbf{x}_1} S^F_{12}\Big|_{\mathbf{x}_2\to\mathbf{x}_1} = -2\,\frac{C_F}{d_A}\,g^2\int_{-\infty}^{+\infty} dx^+\int_{-\infty}^{x^+} dz^+\left[\partial^i_{\mathbf{x}_1}\mathcal{A}^-_a(x^+,\mathbf{x}_1)\right] \times U_A(x^+,z^+;\mathbf{x}_1)^{ab}\left[\partial^i_{\mathbf{x}_1}\mathcal{A}^-_b(z^+,\mathbf{x}_1)\right]\,, \tag{3.29}$$

$$\partial^2_{\mathbf{x}_0} S^A_{02}\Big|_{\mathbf{x}_2\to\mathbf{x}_0} = -2\,\frac{C_A}{d_A}\,g^2\int_{-\infty}^{+\infty} dx^+\int_{-\infty}^{x^+} dz^+\left[\partial^i_{\mathbf{x}_0}\mathcal{A}^-_a(x^+,\mathbf{x}_0)\right] \times U_A(x^+,z^+;\mathbf{x}_0)^{ab}\left[\partial^i_{\mathbf{x}_0}\mathcal{A}^-_b(z^+,\mathbf{x}_0)\right]\,, \tag{3.30}$$

$$\partial^i_{\mathbf{x}_0}\partial^i_{\mathbf{x}_1} S^F_{120}\Big|_{\mathbf{x}_1\to\mathbf{x}_2;\,\mathbf{x}_0\to\mathbf{x}_2} = \frac{C_A}{d_A}\,g^2\int_{-\infty}^{+\infty} dx^+\int_{-\infty}^{x^+} dz^+\left[\partial^i_{\mathbf{x}_2}\mathcal{A}^-_a(x^+,\mathbf{x}_2)\right] \times U_A(x^+,z^+;\mathbf{x}_2)^{ab}\left[\partial^i_{\mathbf{x}_2}\mathcal{A}^-_b(z^+,\mathbf{x}_2)\right]\,. \tag{3.31}$$

Plugging (3.29), (3.30) and (3.31) in the total hadron-level cross section, (3.28), we get

$$\begin{aligned}(2\pi)^{D-2}\,\frac{d\sigma^{p+A\to h+X}}{dx_F\,d^{D-2}\mathbf{p}_h} &= \int_{x_F}^1 dx_B\;g(x_B,\mu^2)\int_{\frac{x_F}{x_B}}^1\frac{d\zeta}{\zeta^{D-2}}\;D_{h/q}(\zeta,\mu^2)\;\frac{\alpha_s\,T_F}{m^4}\left(\frac{x_F}{x_B\,\zeta}\right)(\mu^2)^{2-\frac{D}{2}}\\ &\times\frac{2}{D-2}\left[\left(\frac{x_F}{x_B\,\zeta}\right)^2+\left(1-\frac{x_F}{x_B\,\zeta}\right)^2+\frac{D-4}{2}\right]\left[\left(\frac{x_F}{x_B\,\zeta}\right)^2+\left(1-\frac{x_F}{x_B\,\zeta}\right)^2+\frac{2C_F-C_A}{C_A}\right]\\ &\times\frac{C_A}{C_F}\int d^{D-2}\mathbf{x}_1\int d^{D-2}\mathbf{x}_2\,\delta^{(D-2)}(\mathbf{x}_{12})(-1)\left(\partial^2_{\mathbf{x}_1}S^F_{12}\right)+\mathcal{O}\left(\frac{1}{m^6}\right).\end{aligned} \tag{3.32}$$

It is possible to further simplify the expression of the cross section by adopting a model for the fundamental dipole operator. Then, this model can be used to explicitly perform the transverse integrations. For instance, in the Golec-Biernat-Wüsthoff (GBW) model [73] (or in the McLerran-Venugopalan (MV) model [74] neglecting the logarithm in the exponent) the fundamental dipole is written as

$$S^F_{12} = e^{-\frac{\mathbf{x}_{12}^2 Q_s^2}{4}}. \tag{3.33}$$

Thus, the cross section in the GBW model reads

$$\begin{aligned}(2\pi)^{D-2}\,\frac{d\sigma^{p+A\to h+X}}{dx_F\,d^{D-2}\mathbf{p}_h} &= \int_{x_F}^1 dx_B\;g(x_B,\mu^2)\int_{\frac{x_F}{x_B}}^1\frac{d\zeta}{\zeta^{D-2}}\;D_{h/q}(\zeta,\mu^2)\;\frac{\alpha_s\,T_F}{m^4}\left(\frac{x_F}{x_B\,\zeta}\right)(\mu^2)^{2-\frac{D}{2}}\\ &\times\left[\left(\frac{x_F}{x_B\,\zeta}\right)^2+\left(1-\frac{x_F}{x_B\,\zeta}\right)^2+\frac{D-4}{2}\right]\\ &\times\left[\left(\frac{x_F}{x_B\,\zeta}\right)^2+\left(1-\frac{x_F}{x_B\,\zeta}\right)^2+\frac{2C_F-C_A}{C_A}\right]\frac{C_A}{C_F}\,Q_s^2\,S_\perp+\mathcal{O}\left(\frac{1}{m^6}\right).\end{aligned} \tag{3.34}$$

Here $S_\perp$ is the transverse area of the target, introduced to replace the $\mathbf{x}_1$ integration, as required due to the impact parameter independence of the GBW model.

D. About the intrinsic heavy flavor contribution

So far, we have considered only the extrinsic contribution to heavy flavored hadron production, where the heavy quarks are pair-produced perturbatively upon scattering on the target of an incoming gluon from the projectile. This is indeed expected to be usually the dominant contribution to heavy flavored hadron production, but maybe not the only one.

Another sizable contribution might come from the intrinsic heavy flavor content of the proton projectile at non-perturbative level, before any perturbative evolution. Such intrinsic heavy flavor contribution is known to be power-suppressed in the large mass limit. But, still, it might be relevant in the case of the charm quark and possibly also (but to a lesser extent) in the case of the bottom quark.

In a fixed flavor number scheme (FFNS) like the one we are using, where the heavy flavors are not considered active, the intrinsic charm and bottom contributions can be taken into account by providing charm and bottom PDFs inside the proton, and considering the relevant diagrams with an incoming charm or bottom quark. However, note that in this scheme, these heavy flavor PDF are independent of the factorization scale, and thus are not affected by perturbative evolution, as explained for example in Refs. [64, 65].

Then, the leading-order term for the intrinsic heavy flavor contribution to heavy flavored hadron production is identical to the quark channel leading-order term for unidentified hadron (or pion) production, up to the replacement of PDF and FF. One has

$$(2\pi)^{D\text{–}2}\,\frac{d\sigma^{p+A\to h+X}}{dx_F\,d^{D\text{–}2}\mathbf{p}_h}\bigg|_{\text{intr. heavy flavor}} = \int_0^1 dx_B\;Q(x_B)\int_0^1 \frac{d\zeta}{\zeta^{D-2}}\;D_{h/q}(\zeta,\mu^2)$$
$$\times x_B\,\delta\left(x_B-\frac{x_F}{\zeta}\right)\int d^{D\text{–}2}\mathbf{x}_1\int d^{D\text{–}2}\mathbf{x}_2\;S_{12}^F\;e^{-\frac{i}{\zeta}\,\mathbf{p}_h\cdot\mathbf{x}_{12}}\,, \tag{3.35}$$

where $Q(x_B)$ is the scale-invariant PDF for the intrinsic charm or bottom content of the proton projectile. It is a non-perturbative input which has to be modeled (see for example Refs. [72, 75]) and/or fitted on experimental data, like the initial condition at low μ for the PDFs of massless partons.

Formally, the intrinsic heavy flavor contribution (3.35) is of order $\mathcal{O}(1)$ in perturbation theory, whereas the extrinsic contribution starts at order $\mathcal{O}(\alpha_s)$, see eqs. (3.18), (3.19) and (3.20). Hence, from a formal point of view, if one wants to include both the extrinsic and intrinsic contributions, one should also calculate and include the $\mathcal{O}(\alpha_s)$ corrections to the intrinsic contribution (3.35). However, $Q(x_B)$ is expected to be much smaller than the gluon PDF, so that the the difference in perturbative order can be overcome.

In practice, one can estimate the leading-order term for each contribution in the kinematical range of interest thanks to the formulae (3.18), (3.19) and (3.20), and (3.35), and start to worry about the $\mathcal{O}(\alpha_s)$ corrections to the intrinsic contribution only when its $\mathcal{O}(1)$ term is non-negligible compared to the extrinsic contribution. The calculation of these $\mathcal{O}(\alpha_s)$ corrections to the intrinsic contribution can be done with the same method used in this paper. However, this is beyond the scope of this study, which focuses on the extrinsic contribution.

IV. HEAVY QUARK LOOP CORRECTION TO THE GLUON IN THE GLUON SCATTERING AMPLITUDE ON THE BACKGROUND FIELD

A. Final state heavy quark pair contribution to gluon merging

1. Momentum space

The Fock state decomposition of the one-gluon final state can be calculated directly following the rules of light-front perturbation theory. But, alternatively, it can also be obtained by taking the conjugate of the Fock state decomposition of the one-gluon initial state, Eq. (2.1). In both cases, one obtains

$$\begin{aligned}\langle g(\underline{p_f},\lambda_f,b_f)_{\text{phys}}| = \sqrt{Z_A}\Bigg[&\langle 0|\,a(\underline{p_f},\lambda_f,b_f)\\
&+\sum_{q\bar{q}\text{ states}}\left(\Psi^{g_0}_{q_1\bar{q}_2}\right)^\dagger\;(t^{b_f})_{\beta_2\,\beta_1}\;\langle 0|\,d(\underline{p_2},h_2,\beta_2)\,b(\underline{p_1},h_1,\beta_1)\\
&+\sum_{gg\text{ states}}\left(\Psi^{g_0}_{g_1 g_2}\right)^\dagger\;(T^{b_f})_{b_2\,b_1}\;\langle 0|\,a(\underline{p_2},\lambda_2,b_2)\,a(\underline{p_1},\lambda_1,b_1)+\;\cdots\Bigg].\end{aligned} \tag{4.1}$$

The different terms in this expression have the following interpretation:

- First term: trivial contribution with the gluon directly emerging out of the target at $x^+=0$.
- Second term: contribution of heavy quark-antiquark pair merging to a gluon in the final state, see Fig. 3.
- Third term: contribution of gluon pair merging to one single gluon in the final state.
- Other terms: either they are of higher order in g, or they will not contribute to the amplitudes in which we are interested.

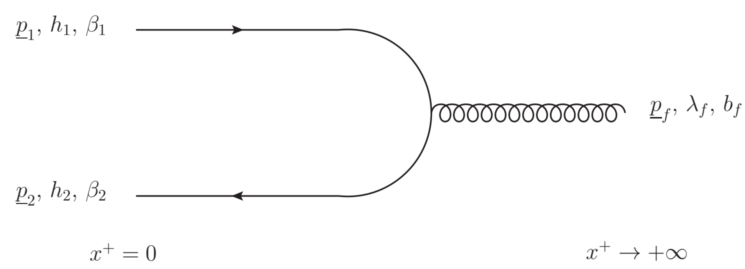

FIG. 3: Tree-level contribution to the $q\bar{q}$ Fock component of the outgoing g state.

Taking the conjugate of the wave function from Eq. (2.8), and using the notations from Fig. 3, one finds

$$\left(\Psi^{g_0}_{q_1\bar{q}_2}\right)^\dagger = \frac{(2\pi)^{D-1}\delta^{(D-1)}(\underline{p_1}+\underline{p_2}-\underline{p_f})}{\left[\left(\mathbf{p}_1-\frac{p_1^+}{p_f^+}\,\mathbf{p}_f\right)^2+m^2\right]}\,(\mu)^{2-\frac{D}{2}}\,g$$
$$\times\Bigg\{\left(\mathbf{p}_1^i-\frac{p_1^+}{p_f^+}\,\mathbf{p}_f^i\right)\varepsilon^{j\,*}_{\lambda_f}\,\overline{v_G}(p_2^+,h_2)\,\gamma^+\left[\frac{(p_f^+-2p_1^+)}{p_f^+}\,\delta^{ij}-i\,\sigma^{ij}\right]u_G(p_1^+,h_1)$$
$$-m\,\varepsilon^{j\,*}_{\lambda_f}\,\overline{v_G}(p_2^+,h_2)\,\gamma^+\gamma^j\,u_G(p_1^+,h_1)\Bigg\}\,. \tag{4.2}$$

2. Mixed space

In the mixed-space representation, the one-gluon final state (4.1) rewrites

$$\langle g_{\text{phys}}(\underline{p_f},\lambda_f,b_f)| = \sqrt{Z_A}\Bigg[\int d^{D-2}\mathbf{x}_0\;\; e^{-i\mathbf{p}_f\cdot\mathbf{x}_0}\;\;\langle 0|\,a(p_f^+,\mathbf{x}_0,\lambda_f,b_f)$$
$$+\widetilde{\sum_{q\bar{q}\text{ states}}}\left(\widetilde{\Psi}^{g_f}_{q_1\bar{q}_2}\right)^\dagger\;\;(t^{b_f})_{\beta_2\,\beta_1}\;\;\langle 0|\,d(p_2^+,\mathbf{x}_2,h_2,\beta_2)\,b(p_1^+,\mathbf{x}_1,h_1,\beta_1)$$
$$+\widetilde{\sum_{gg\text{ states}}}\left(\widetilde{\Psi}^{g_f}_{g_1g_2}\right)^\dagger\;\;(T^{b_f})_{b_2\,b_1}\;\;\langle 0|\,a(p_2^+,\mathbf{x}_2,\lambda_2,b_2)\,a(p_1^+,\mathbf{x}_1,\lambda_1,b_1)+\;\cdots\Bigg]\,, \tag{4.3}$$

where

$$\left(\widetilde{\Psi}^{g_f}_{q_1\bar{q}_2}\right)^\dagger = 2\pi\delta(p_1^++p_2^+-p_f^+)\;e^{-i\frac{\mathbf{p}_f}{p_f^+}\cdot(p_1^+\,\mathbf{x}_1+p_2^+\,\mathbf{x}_2)}\;\;(\mu)^{2-\frac{D}{2}}\,g$$
$$\times\Bigg\{\varepsilon^{j\,*}_{\lambda_f}\,\overline{v_G}(p_2^+,h_2)\,\gamma^+\left[\frac{(p_f^+-2p_1^+)}{p_f^+}\,\delta^{ij}-i\,\sigma^{ij}\right]u_G(p_1^+,h_1)\;\;\left[\mathcal{B}_V^i(\mathbf{x}_{12},m)\right]^*$$
$$-m\,\varepsilon^{j\,*}_{\lambda_f}\,\overline{v_G}(p_2^+,h_2)\,\gamma^+\gamma^j\,u_G(p_1^+,h_1)\;\;\left[\mathcal{B}_S(\mathbf{x}_{12},m)\right]^*\Bigg\}\,. \tag{4.4}$$

B. Heavy quark loop contribution to the gluon-to-gluon amplitude

1. Generic form

In the eikonal approximation, the scattering amplitude $\mathcal{M}_{g\to g}$ for a gluon on the target, and the corresponding S-matrix element, are related by

$$\langle g_{\rm phys}(\underline{p_f},\lambda_f,b_f)|\,\hat{S}_E\,|g_{\rm phys}(\underline{k_0},\lambda_0,a_0)\rangle = (2k_0^+)(2\pi)\delta(p_f^+-k_0^+)\; i\,\mathcal{M}_{g\to g}\;, \tag{4.5}$$

with the eikonal scattering operator $\hat{S}_E$ introduced in subsection II C. Calculating the left-hand side using the expansions (2.9) and (4.3), one finds

$$\begin{aligned}
&\langle g_{\rm phys}(\underline{p_f},\lambda_f,b_f)|\,\hat{S}_E\,|g_{\rm phys}(\underline{k_0},\lambda_0,a_0)\rangle = Z_A\\
&\times\Bigg\{(2k_0^+)(2\pi)\delta(p_f^+-k_0^+)\;\delta_{\lambda_f,\lambda_0}\int d^{D-2}\mathbf{x}_0\;\, e^{-i(\mathbf{p}_f-\mathbf{k}_0)\cdot\mathbf{x}_0}\;\,U_A(\mathbf{x}_0)_{b_f a_0}\\
&\quad+\widetilde{\sum_{q\bar{q}\text{ states}}}\left(\widetilde{\Psi}^{g_f}_{q_1\bar{q}_2}\right)^{\dagger}\,(t^{b_f})_{\beta_2\,\beta_1}\,U_F(\mathbf{x}_1)_{\beta_1\alpha_1}\left[U_F^\dagger(\mathbf{x}_2)\right]_{\alpha_2\beta_2}(t^{a_0})_{\alpha_1\,\alpha_2}\;\widetilde{\Psi}^{g_0}_{q_1\bar{q}_2}\\
&\quad+\widetilde{\sum_{gg\text{ states}}}\left(\widetilde{\Psi}^{g_f}_{g_1 g_2}\right)^{\dagger}\,(T^{b_f})_{b_2\,b_1}\,U_A(\mathbf{x}_1)_{b_1 a_1}\,U_A(\mathbf{x}_2)_{b_2 a_2}\,(T^{a_0})_{a_1\,a_2}\;\widetilde{\Psi}^{g_0}_{g_1 g_2}+\mathcal{O}\left(g^4\right)\Bigg\}.
\end{aligned}\tag{4.6}$$

Isolating the leading-order contribution

$$i\,\mathcal{M}^{LO}_{g\to g} = \delta_{\lambda_f,\lambda_0}\int d^{D-2}\mathbf{x}_0\;\, e^{-i(\mathbf{p}_f-\mathbf{k}_0)\cdot\mathbf{x}_0}\;\,U_A(\mathbf{x}_0)_{b_f a_0}\;, \tag{4.7}$$

one rewrites eq. (4.6) as

$$\begin{aligned}
(2k_0^+)(2\pi)\delta(p_f^+-k_0^+)\; i\,\left[\mathcal{M}_{g\to g}-\mathcal{M}^{LO}_{g\to g}\right] &= -(1-Z_A)\;(2k_0^+)(2\pi)\delta(p_f^+-k_0^+)\;\delta_{\lambda_f,\lambda_0}\\
&\qquad\times\int d^{D-2}\mathbf{x}_0\;\, e^{-i(\mathbf{p}_f-\mathbf{k}_0)\cdot\mathbf{x}_0}\;\,U_A(\mathbf{x}_0)_{b_f a_0}\\
+Z_A\Bigg\{\widetilde{\sum_{q\bar{q}\text{ states}}}&\left(\widetilde{\Psi}^{g_f}_{q_1\bar{q}_2}\right)^{\dagger}\,\mathrm{Tr}\big[t^{b_f}\,U_F(\mathbf{x}_1)\,t^{a_0}\,U_F^\dagger(\mathbf{x}_2)\big]\;\widetilde{\Psi}^{g_0}_{q_1\bar{q}_2}\\
&+\widetilde{\sum_{gg\text{ states}}}\left(\widetilde{\Psi}^{g_f}_{g_1 g_2}\right)^{\dagger}\,\mathrm{Tr}\big[T^{b_f}\,U_A(\mathbf{x}_1)\,T^{a_0}\,U_A^\dagger(\mathbf{x}_2)\big]\;\widetilde{\Psi}^{g_0}_{g_1 g_2}+\mathcal{O}\left(g^4\right)\Bigg\}.
\end{aligned}\tag{4.8}$$

2. Heavy quark contribution to the gluon wave function renormalization

The gluon renormalization constant Z_A, appearing in the Fock state decompositions (2.1), (2.9), (4.1) and (4.3), is determined by imposing the following normalization to the one-gluon asymptotic state:

$$\langle g(\underline{p_f},\lambda_f,b_f)_{\rm phys}|g(\underline{k_0},\lambda_0,a_0)_{\rm phys}\rangle = (2k_0^+)(2\pi)^{D-1}\delta^{(D-1)}(\underline{p_f}-\underline{k_0})\;\delta_{\lambda_f,\lambda_0}\;\delta_{b_f,a_0}\,. \tag{4.9}$$

Usually, the calculation of such renormalization constant is done in momentum space, inserting the expansions (2.1) and (4.1) into the normalization relation (4.9). But we will need a mixed space relation for Z_A, for coherence with the rest of the calculation, so that the Fock state expansions (2.9) and (4.3) are used instead, and one obtains

$$\begin{aligned}
(2k_0^+)(2\pi)^{D-1}\delta^{(D-1)}(\underline{p_f}-\underline{k_0})\,\delta_{\lambda_f,\lambda_0}\,\delta_{b_f,a_0}\,\frac{(1-Z_A)}{Z_A} &= \widetilde{\sum_{q\bar{q}\text{ states}}}\left(\widetilde{\Psi}^{g_f}_{q_1\bar{q}_2}\right)^{\dagger}(t^{b_f})_{\alpha_2\,\alpha_1}\,(t^{a_0})_{\alpha_1\,\alpha_2}\;\widetilde{\Psi}^{g_0}_{q_1\bar{q}_2}\\
&\quad+\widetilde{\sum_{gg\text{ states}}}\left(\widetilde{\Psi}^{g_f}_{g_1 g_2}\right)^{\dagger}\,(T^{b_f})_{a_2\,a_1}\,(T^{a_0})_{a_1\,a_2}\;\widetilde{\Psi}^{g_0}_{g_1 g_2}+\mathcal{O}\left(g^4\right)\;,
\end{aligned}\tag{4.10}$$

thanks to the commutation relations (A7), (A8) and (A9). Taking care of the color algebra, it results in

$$(2k_0^+)(2\pi)^{D-1}\delta^{(D-1)}(\underline{p_f}-\underline{k_0})\,\delta_{\lambda_f,\lambda_0}\,\frac{(1-Z_A)}{Z_A} = T_F \widetilde{\sum_{q\bar{q}\text{ states}}} \left(\widetilde{\Psi}^{g_f}_{q_1\bar{q}_2}\right)^\dagger\ \widetilde{\Psi}^{g_0}_{q_1\bar{q}_2}$$
$$+\,C_A \widetilde{\sum_{gg\text{ states}}} \left(\widetilde{\Psi}^{g_f}_{g_1g_2}\right)^\dagger\ \widetilde{\Psi}^{g_0}_{g_1g_2} + \mathcal{O}\left(g^4\right)\,. \tag{4.11}$$

In that expression, the first term correspond to the quark loop contribution, and the second one to the gluon loop contribution. Both are of order g^2, obviously.

3. Initial state/final state wave function overlap for a heavy quark loop

The next step is to evaluate the quantity

$$\widetilde{\sum_{q\bar{q}\text{ states}}} \left(\widetilde{\Psi}^{g_f}_{q_1\bar{q}_2}\right)^\dagger\ \widetilde{\Psi}^{g_0}_{q_1\bar{q}_2}\ F(\mathbf{x}_1,\mathbf{x}_2) \tag{4.12}$$

for a generic function $F(\mathbf{x}_1,\mathbf{x}_2)$. Indeed, for $F(\mathbf{x}_1,\mathbf{x}_2)\equiv 1$, it gives the quark loop contribution to the gluon wave function renormalization (4.11), and for

$$F(\mathbf{x}_1,\mathbf{x}_2) \equiv \mathrm{Tr}\Big[t^{b_f}\,U_F(\mathbf{x}_1)\,t^{a_0}\,U_F^\dagger(\mathbf{x}_2)\Big]\,, \tag{4.13}$$

it gives the resolved quark loop contribution to the scattering amplitude (4.8). Thanks to the expressions (2.11) and (4.4), one finds

$$\widetilde{\sum_{q\bar{q}\text{ states}}} \left(\widetilde{\Psi}^{g_f}_{q_1\bar{q}_2}\right)^\dagger \widetilde{\Psi}^{g_0}_{q_1\bar{q}_2}\ F(\mathbf{x}_1,\mathbf{x}_2) = \sum_{h_1,\,h_2}\int_0^{+\infty}\frac{dk_1^+}{(2\pi)2k_1^+}\int_0^{+\infty}\frac{dk_2^+}{(2\pi)2k_2^+}\int d^{D-2}\mathbf{x}_1\int d^{D-2}\mathbf{x}_2$$
$$\times F(\mathbf{x}_1,\mathbf{x}_2)(2\pi)\delta(k_1^+ + k_2^+ - k_0^+)\ (2\pi)\delta(k_1^+ + k_2^+ - p_f^+)\ e^{-i\left(\frac{\mathbf{p}_f}{p_f^+}-\frac{\mathbf{k}_0}{k_0^+}\right)\cdot(k_1^+\mathbf{x}_1+k_2^+\mathbf{x}_2)}\ (\mu^2)^{2-\frac{D}{2}}\ g^2$$
$$\times\varepsilon^{j'\,*}_{\lambda_f}\varepsilon^{j}_{\lambda_0}\ \overline{v_G}(k_2^+,h_2)\gamma^+\left\{\mathcal{B}_V^{i'\,*}(\mathbf{x}_{12},m)\ \left[\frac{(p_f^+-2k_1^+)}{p_f^+}\,\delta^{i'j'} - i\,\sigma^{i'j'}\right] - \mathcal{B}_S^*(\mathbf{x}_{12},m)\ m\ \gamma^{j'}\right\}$$
$$\times\, u_G(k_1^+,h_1)\overline{u_G}(k_1^+,h_1)\,\gamma^+\left\{\mathcal{B}_V^{i}(\mathbf{x}_{12},m)\ \left[\frac{(k_0^+-2k_1^+)}{k_0^+}\,\delta^{ij} + i\,\sigma^{ij}\right] + \mathcal{B}_S(\mathbf{x}_{12},m)\ m\ \gamma^{j}\right\}$$
$$\times\, v_G(k_2^+,h_2) \tag{4.14}$$
$$= (2\pi)\delta(k_0^+-p_f^+)\int d^{D-2}\mathbf{x}_1\ \int d^{D-2}\mathbf{x}_2\ \ F(\mathbf{x}_1,\mathbf{x}_2)\ (\mu^2)^{2-\frac{D}{2}}\,g^2\int_0^{k_0^+}\frac{dk_1^+}{(2\pi)}$$
$$\times\, e^{-i\left(\frac{\mathbf{p}_f-\mathbf{k}_0}{k_0^+}\right)\cdot(k_1^+\mathbf{x}_{12}+k_0^+\mathbf{x}_2)}\,\varepsilon^{j'\,*}_{\lambda_f}\varepsilon^{j}_{\lambda_0}\ \Bigg\{\mathcal{B}_V^{i'\,*}(\mathbf{x}_{12},m)\ \mathcal{B}_V^{i}(\mathbf{x}_{12},m)$$
$$\times\ \mathrm{Tr}\left[\mathcal{P}_G\left(\frac{(k_0^+-2k_1^+)}{k_0^+}\,\delta^{i'j'} - i\,\sigma^{i'j'}\right)\left(\frac{(k_0^+-2k_1^+)}{k_0^+}\,\delta^{ij} + i\,\sigma^{ij}\right)\right]$$
$$-\left|\mathcal{B}_S(\mathbf{x}_{12},m)\right|^2\ m^2\ \mathrm{Tr}\Big[\mathcal{P}_G\,\gamma^{j'}\,\gamma^{j}\Big]\Bigg\} \tag{4.15}$$
$$= (2k_0^+)(2\pi)\delta(k_0^+-p_f^+)\int d^{D-2}\mathbf{x}_1\ \int d^{D-2}\mathbf{x}_2\ \ F(\mathbf{x}_1,\mathbf{x}_2)\ (\mu^2)^{2-\frac{D}{2}}\,g^2\int_0^{k_0^+}\frac{dk_1^+}{(2\pi)(2k_0^+)}$$
$$\times\, e^{-i(\mathbf{p}_f-\mathbf{k}_0)\cdot\left(\mathbf{x}_2+\frac{k_1^+}{k_0^+}\mathbf{x}_{12}\right)}\,\varepsilon^{j'\,*}_{\lambda_f}\varepsilon^{j}_{\lambda_0}\ \Bigg\{2\,\mathcal{B}_V^{i'\,*}(\mathbf{x}_{12},m)\ \mathcal{B}_V^{i}(\mathbf{x}_{12},m)$$
$$\times\left[\left(\frac{k_0^+-2k_1^+}{k_0^+}\right)^2\,\delta^{i'j'}\,\delta^{ij} - \delta^{i'j'}\,\delta^{ij} + \delta^{i'i}\,\delta^{j'j}\right] + 2\,m^2\ \delta^{j'j}\ \left|\mathcal{B}_S(\mathbf{x}_{12},m)\right|^2\Bigg\}, \tag{4.16}$$

using, in order to simplify the calculation of the Dirac trace, the fact that $\mathcal{B}_V^{i'\,*}(\mathbf{x}_{12},m)\,\mathcal{B}_V^{i}(\mathbf{x}_{12},m)$ is invariant under the exchange of i and i'.

4. Back to the heavy quark contribution to the gluon wave function renormalization

For the case $F(\mathbf{x}_1,\mathbf{x}_2)\equiv 1$ one can perform the integration over $\mathbf{x}_2$ in (4.16), while keeping $\mathbf{x}_{12}$ as independent integration variable. One gets

$$\widetilde{\sum_{q\bar{q}\text{ states}}}\left(\widetilde{\Psi}^{g_f}_{q_1\bar{q}_2}\right)^{\dagger}\;\widetilde{\Psi}^{g_0}_{q_1\bar{q}_2}\;=(2k_0^+)(2\pi)^{D-1}\delta^{(D-1)}(\underline{p_f}-\underline{k_0})\int d^{D-2}\mathbf{x}_{12}\;\;(\mu^2)^{2-\frac{D}{2}}$$
$$\times\, g^2\int_0^{k_0^+}\frac{dk_1^+}{(2\pi)(2k_0^+)}\;\varepsilon^{j'\,*}_{\lambda_f}\varepsilon^{j}_{\lambda_0}\;\Bigg\{2\,\mathcal{B}_V^{i'\,*}(\mathbf{x}_{12},m)\;\mathcal{B}_V^{i}(\mathbf{x}_{12},m)$$
$$\times\left[\left(\frac{k_0^+-2k_1^+}{k_0^+}\right)^2\delta^{i'j'}\,\delta^{ij}-\delta^{i'j'}\,\delta^{ij}+\delta^{i'i}\,\delta^{j'j}\right]+2\,m^2\,\delta^{j'j}\;\;|\mathcal{B}_S(\mathbf{x}_{12},m)|^2\Bigg\}\,. \tag{4.17}$$

Hence, the heavy quark loop contribution to the wave function renormalization obtained from eq. (4.11) is

$$\delta_{\lambda_f,\lambda_0}\,\frac{(1-Z_A)}{Z_A}\bigg|_{\text{quark loop}}=(\mu^2)^{2-\frac{D}{2}}\,g^2\,T_F\int d^{D-2}\mathbf{x}_{12}\;\int_0^{k_0^+}\frac{dk_1^+}{(2\pi)(2k_0^+)}\;\varepsilon^{j'\,*}_{\lambda_f}\varepsilon^{j}_{\lambda_0}$$
$$\times\Bigg\{2\,\mathcal{B}_V^{i'\,*}(\mathbf{x}_{12},m)\;\mathcal{B}_V^{i}(\mathbf{x}_{12},m)\;\left[\left(\frac{k_0^+-2k_1^+}{k_0^+}\right)^2\delta^{i'j'}\,\delta^{ij}-\delta^{i'j'}\,\delta^{ij}+\delta^{i'i}\,\delta^{j'j}\right]$$
$$+2\,m^2\,\delta^{j'j}\;\;|\mathcal{B}_S(\mathbf{x}_{12},m)|^2\Bigg\}\,. \tag{4.18}$$

5. Explicit expression for the heavy quark loop correction

Inserting the expressions (4.16) and (4.18) into the general expression (4.8) for the gluon to gluon scattering amplitude, and dropping the gluon loop contributions (and higher orders), one finds[2]

$$i\,\mathcal{M}_{g\to g}\bigg|_{\text{quark loop}}=(\mu^2)^{2-\frac{D}{2}}\,g^2\int_0^{k_0^+}\frac{dk_1^+}{(2\pi)(2k_0^+)}\;\int d^{D-2}\mathbf{x}_1\;\int d^{D-2}\mathbf{x}_2\;\;\varepsilon^{j'\,*}_{\lambda_f}\varepsilon^{j}_{\lambda_0}$$
$$\times\Bigg\{2\,\mathcal{B}_V^{i'\,*}(\mathbf{x}_{12},m)\;\mathcal{B}_V^{i}(\mathbf{x}_{12},m)\;\left[\left(\frac{k_0^+-2k_1^+}{k_0^+}\right)^2\delta^{i'j'}\,\delta^{ij}-\delta^{i'j'}\,\delta^{ij}+\delta^{i'i}\,\delta^{j'j}\right]$$
$$+2\,m^2\,\delta^{j'j}\;\;|\mathcal{B}_S(\mathbf{x}_{12},m)|^2\Bigg\} \tag{4.19}$$
$$\times\Bigg\{\mathrm{Tr}\Big[t^{b_f}\;U_F(\mathbf{x}_1)\;t^{a_0}\;U_F^{\dagger}(\mathbf{x}_2)\Big]\;e^{-i(\mathbf{p}_f-\mathbf{k}_0)\cdot\left(\mathbf{x}_2+\frac{k_1^+}{k_0^+}\mathbf{x}_{12}\right)}-T_F\;\;U_A(\mathbf{x}_1)_{b_f a_0}\;e^{-i(\mathbf{p}_f-\mathbf{k}_0)\cdot\mathbf{x}_1}\Bigg\}\,.$$

Note that the two terms in the last line cancel one each other when $\mathbf{x}_1$ and $\mathbf{x}_2$ coincide. Hence, there is a cancelation of UV divergences between the resolved quark loop graph and the quark loop contributions to the gluon wave function renormalization, leaving the UV finite result (4.19). This is to be expected since Z_A is determined by unitarity, without introducing a counterterm in the lagrangian (or hamiltonian).

---

[2] In order to get a more compact expression, the integration variable $\mathbf{x}_0$ has been relabelled $\mathbf{x}_1$.

V. HEAVY QUARK CONTRIBUTION TO THE NLO CORRECTION TO THE SINGLE INCLUSIVE HADRON PRODUCTION CROSS SECTION

A. Heavy quark loop contribution to the partonic cross section

Here, we are interested in the heavy quark loop contribution to the partonic cross section for the process $g+A\to g+X$. It is obtained from the overlap of the heavy quark loop contribution (4.19) to the $g+A\to g+X$ amplitude with the LO contribution (4.7), as

$$(2p_f^+)(2\pi)^{D-1}\frac{d\sigma^{g+A\to g+X}}{dp_f^+\,d^{D-2}\mathbf{p}_f}\bigg|_{\text{quark loop}} = (2k_0^+)(2\pi)\delta(p_f^+-k_0^+)$$
$$\times\frac{1}{d_A}\sum_{a_0,\,b_f}\frac{1}{D-2}\sum_{\lambda_0,\,\lambda_f}\bigg\{\big(i\,\mathcal{M}^{LO}_{g\to g}\big)^\dagger\; i\,\mathcal{M}_{g\to g}\Big|_{\text{quark loop}} + \;c.c.\bigg\}. \tag{5.1}$$

A straightforward calculation gives

$$(2p_f^+)(2\pi)^{D-1}\frac{d\sigma^{g+A\to g+X}}{dp_f^+\,d^{D-2}\mathbf{p}_f}\bigg|_{\text{quark loop}} = (2k_0^+)(2\pi)\delta(p_f^+-k_0^+)\;\alpha_s\,T_F\int_0^{k_0^+}\frac{dk_1^+}{k_0^+}\,(\mu^2)^{2-\frac{D}{2}}$$
$$\times\int d^{D-2}\mathbf{x}_0\int d^{D-2}\mathbf{x}_1\int d^{D-2}\mathbf{x}_2\bigg\{\bigg[\frac{4}{D-2}\bigg[\bigg(\frac{k_1^+}{k_0^+}\bigg)^2+\bigg(\frac{k_0^+-k_1^+}{k_0^+}\bigg)^2+\frac{D-4}{2}\bigg]$$
$$\times\,\mathcal{B}_V^{i\,*}(\mathbf{x}_{12},m)\,\mathcal{B}_V^{i}(\mathbf{x}_{12},m)\;+2\,m^2\;\big|\mathcal{B}_S(\mathbf{x}_{12},m)\big|^2\bigg]$$
$$\times\bigg[S_{120}\,e^{-i(\mathbf{p}_f-\mathbf{k}_0)\cdot\Big(\mathbf{x}_{20}+\frac{k_1^+}{k_0^+}\mathbf{x}_{12}\Big)}-S_{10}^A\,e^{-i(\mathbf{p}_f-\mathbf{k}_0)\cdot\mathbf{x}_{10}}\bigg] + \;c.c.\bigg\}. \tag{5.2}$$

B. Hadron-level cross section

The NLO corrections calculated in refs. [24, 33] for the single inclusive hadron production considered only the contributions from gluons and massless quarks. However, the inclusion of heavy quarks leads to additional NLO contributions.

First, there is the possibility of producing a heavy quark, which then fragments for example into a pion or an unidentified hadron (depending on the precise observable that one is considering). That contribution is given by the sum of the results (3.18), (3.19) and (3.20), up to the appropriate change of fragmentation function.

Second, there is the contribution from heavy quark loops. At partonic level, this corresponds to eq. (5.2). In order to transform it into a hadron level cross section, one can follow the same steps as in subsection III B, in particular eq. (3.16). One obtains

$$(2\pi)^{D-2}\frac{d\sigma^{p+A\to h+X}}{dx_F\,d^{D-2}\mathbf{p}_h}\bigg|_{\text{quark loop}} = \int_0^1 dx_B\;g(x_B,\mu^2)\int_0^1\frac{d\zeta}{\zeta^{D-2}}\;D_{h/g}(\zeta,\mu^2)\quad\alpha_s\,T_F$$
$$\times\,x_B\,\delta\bigg(x_B-\frac{x_F}{\zeta}\bigg)\int_0^1 dz\int d^{D-2}\mathbf{x}_0\int d^{D-2}\mathbf{x}_1\int d^{D-2}\mathbf{x}_2$$
$$\times\bigg\{\bigg[\frac{4}{D-2}\bigg[z^2+(1-z)^2+\frac{D-4}{2}\bigg]\,\mathcal{B}_V^{i\,*}(\mathbf{x}_{12},m)\,\mathcal{B}_V^{i}(\mathbf{x}_{12},m)\;+2\,m^2\;\big|\mathcal{B}_S(\mathbf{x}_{12},m)\big|^2\bigg]$$
$$\times\bigg[S_{120}\,e^{-\frac{i}{\zeta}\,\mathbf{p}_h\cdot(\mathbf{x}_{20}+z\,\mathbf{x}_{12})}-S_{10}^A\,e^{-\frac{i}{\zeta}\,\mathbf{p}_h\cdot\mathbf{x}_{10}}\bigg] + \;c.c.\bigg\}, \tag{5.3}$$

with $D_{h/g}(\zeta,\mu^2)$ the fragmentation function for gluon into pion or unidentified hadron.

On the other hand, one also expects a contribution from intrinsic charm or bottom in the proton. At LO, it writes the same as in the equation (3.35), but with the fragmentation function now into pion or unidentified hadron. This contribution is expected to be suppressed by the smallness of the intrinsic heavy flavor PDF $Q(x_B)$. But it is formally LO instead of NLO. Hence, if this contribution is not negligible, one may want to include also the corresponding NLO corrections, whose calculation is however beyond the scope of the present study.

Acknowledgments

We express our gratitude to the Department of Physics of Ben-Gurion University of the Negev, for warm hospitality during stays when parts of this work were done (TA, NA and AK) and for financial support as Distinguished Scientist Visitor (NA). ML thanks the Physics Department of the University of Connecticut for hospitality. This research was supported by the People Programme (Marie Curie Actions) of the European Union's Seventh Framework Programme FP7/2007-2013/ under REA grant agreement #318921; the DOE grant DE-FG02-13ER41989 (AK); the BSF grant #2012124 (ML and AK); the Kreitman Foundation (GB); the Israeli Science Foundation grant #87277111 (GB and ML); and the European Research Council grant HotLHC ERC-2011-StG-279579, Ministerio de Ciencia e Innovación of Spain under project FPA2014-58293-C2-1-P and Xunta de Galicia (Consellería de Educación) within the Strategic Unit AGRUP2015/11 (TA and NA).

Appendix A: Conventions

1. Fock space and the interaction picture

In the interaction picture the quark and gluon fields read

$$\Psi_\alpha(x) = \int_0^{+\infty} \frac{dk^+}{(2\pi)2k^+} \int \frac{d^{D-2}\mathbf{k}}{(2\pi)^{D-2}} \times \sum_{h=\pm\frac{1}{2}} \left[e^{-ik\cdot x}\, b(\underline{k},h,\alpha)\, u(\underline{k},h) + e^{+ik\cdot x}\, d^\dagger(\underline{k},h,\alpha)\, v(\underline{k},h) \right]\Bigg|_{k^-\equiv\frac{\mathbf{k}^2+m^2}{2k^+}} , \tag{A1}$$

$$A_a^\mu(x) = \int_0^{+\infty} \frac{dk^+}{(2\pi)2k^+} \int \frac{d^{D-2}\mathbf{k}}{(2\pi)^{D-2}} \times \sum_{\lambda} \left[e^{-ik\cdot x}\, a(\underline{k},\lambda,a)\, \epsilon_\lambda^\mu(\underline{k}) + e^{+ik\cdot x}\, a^\dagger(\underline{k},\lambda,a)\, \epsilon_\lambda^{\mu\,*}(\underline{k}) \right]\Bigg|_{k^-\equiv\frac{\mathbf{k}^2}{2k^+}} , \tag{A2}$$

respectively.

The commutation relations for creation and annihilation operators read

$$\left[a(\underline{k_1},\lambda_1,a_1), a^\dagger(\underline{k_2},\lambda_2,a_2)\right] = (2k_1^+)(2\pi)^{D-1}\delta^{(D-1)}(\underline{k_1}-\underline{k_2})\;\delta_{\lambda_1,\lambda_2}\;\delta_{a_1,a_2}\,, \tag{A3}$$

$$\left\{b(\underline{k_1},h_1,\alpha_1), b^\dagger(\underline{k_2},h_2,\alpha_2)\right\} = (2k_1^+)(2\pi)^{D-1}\delta^{(D-1)}(\underline{k_1}-\underline{k_2})\;\delta_{h_1,h_2}\;\delta_{\alpha_1,\alpha_2}\,, \tag{A4}$$

$$\left\{d(\underline{k_1},h_1,\alpha_1), d^\dagger(\underline{k_2},h_2,\alpha_2)\right\} = (2k_1^+)(2\pi)^{D-1}\delta^{(D-1)}(\underline{k_1}-\underline{k_2})\;\delta_{h_1,h_2}\;\delta_{\alpha_1,\alpha_2}\,. \tag{A5}$$

The Fourier transform from momentum space to mixed space is defined, for the gluon creation operator, as follows:

$$a^\dagger(\underline{k},\lambda,a) = \int d^{D-2}\mathbf{x}\;\; e^{i\mathbf{k}\cdot\mathbf{x}}\;\; a^\dagger(k^+,\mathbf{x},\lambda,a)\,, \tag{A6}$$

and analogously for other quantities.

Hence, we have the mixed-space commutation relations

$$\left[a(k_1^+,\mathbf{x}_1,\lambda_1,a_1), a^\dagger(k_2^+,\mathbf{x}_2,\lambda_2,a_2)\right] = \mathcal{D}(k_1^+,k_2^+)\;\delta^{(D-2)}(\mathbf{x}_1-\mathbf{x}_2)\;\delta_{\lambda_1,\lambda_2}\;\delta_{a_1,a_2}\,, \tag{A7}$$

$$\left\{b(k_1^+,\mathbf{x}_1,h_1,\alpha_1), b^\dagger(k_2^+,\mathbf{x}_2,h_2,\alpha_2)\right\} = \mathcal{D}(k_1^+,k_2^+)\;\delta^{(D-2)}(\mathbf{x}_1-\mathbf{x}_2)\;\delta_{h_1,h_2}\;\delta_{\alpha_1,\alpha_2}\,, \tag{A8}$$

$$\left\{d(k_1^+,\mathbf{x}_1,h_1,\alpha_1), d^\dagger(k_2^+,\mathbf{x}_2,h_2,\alpha_2)\right\} = \mathcal{D}(k_1^+,k_2^+)\;\delta^{(D-2)}(\mathbf{x}_1-\mathbf{x}_2)\;\delta_{h_1,h_2}\;\delta_{\alpha_1,\alpha_2}\,, \tag{A9}$$

with

$$\mathcal{D}(k_1^+,k_2^+) = (2k_1^+)(2\pi)\delta(k_1^+-k_2^+). \tag{A10}$$

2. Spinors

The projectors over good (G) and bad (B) components of a spinor Ψ are defined

$$\mathcal{P}_G \equiv \frac{\gamma^-\,\gamma^+}{2} = \frac{\gamma^0\,\gamma^+}{\sqrt{2}}\ ,$$
$$\mathcal{P}_B \equiv \frac{\gamma^+\,\gamma^-}{2} = \frac{\gamma^0\,\gamma^-}{\sqrt{2}}\ , \tag{A11}$$

so

$$\Psi_{G,B} \equiv \mathcal{P}_{G,B}\ \Psi\ . \tag{A12}$$

Note that

$$\overline{\Psi}\ \mathcal{P}_B = \overline{\Psi_G},\quad \overline{\Psi}\ \mathcal{P}_G = \overline{\Psi_B}. \tag{A13}$$

Concerning the solutions $u(\underline{k},h)$ and $v(\underline{k},h)$ of the free Dirac equation, the good and bad components are related through

$$u_B(\underline{k},h) = \frac{\gamma^+}{2k^+}\ \left(\mathbf{k}^j\gamma^j + m\right)\ u_G(k^+,h),$$
$$v_B(\underline{k},h) = \frac{\gamma^+}{2k^+}\ \left(\mathbf{k}^j\gamma^j - m\right)\ v_G(k^+,h). \tag{A14}$$

In this way, the dependence on $\mathbf{k}$ and m appears only in the bad components:

$$\overline{u_B}(\underline{k},h) = \overline{u_G}(k^+,h)\ \left(\mathbf{k}^j\gamma^j + m\right)\ \frac{\gamma^+}{2k^+}\ ,$$
$$\overline{v_B}(\underline{k},h) = \overline{v_G}(k^+,h)\ \left(\mathbf{k}^j\gamma^j - m\right)\ \frac{\gamma^+}{2k^+}\ . \tag{A15}$$

Finally, the completeness relations read

$$\sum_{h=\pm\frac{1}{2}} u_G(k^+,h)\ \overline{u_G}(k^+,h)\,\gamma^+ = \sum_{h=\pm\frac{1}{2}} v_G(k^+,h)\ \overline{v_G}(k^+,h)\,\gamma^+ = 2k^+\ \mathcal{P}_G\ . \tag{A16}$$

3. Polarization vectors

The polarization vectors in light-cone gauge $A^+ = 0$ are defined

$$\epsilon^+_\lambda(\underline{k}) = 0,$$
$$\epsilon^j_\lambda(\underline{k}) = \varepsilon^j_\lambda\ ,$$
$$\epsilon^-_\lambda(\underline{k}) = \frac{\mathbf{k}^j\,\varepsilon^j_\lambda}{k^+}\ , \tag{A17}$$

where the transverse vectors ε_λ obey the relations

$$\sum_\lambda \varepsilon^i_\lambda\,\varepsilon^{j\,*}_\lambda = -g^{ij},$$
$$-g_{ij}\,\varepsilon^i_{\lambda_1}\,\varepsilon^{j\,*}_{\lambda_2} = \delta_{\lambda_1,\lambda_2}\ . \tag{A18}$$

Note that, for arbitrary D, there are $D-2$ transverse polarizations λ. For $D = 4$, one can take

$$\varepsilon_\lambda = \frac{1}{\sqrt{2}}\left(\begin{array}{c}1\\ i\lambda\end{array}\right), \tag{A19}$$

with $\lambda = \pm 1$, so that λ coincides with the light-front helicity of the gluon.

[1] L.V. Gribov, E. Levin and M. Ryskin, Phys. Rep. **100**, 1 (1983); A. H. Mueller and J. Qiu, Nucl. Phys. B **268**, 427 (1986).

[2] A. Mueller, Nucl. Phys. B **335** 115 (1990); Nucl. Phys. B **415**, 373 (1994); Nucl. Phys. B **437**, 107 (1995).
[3] I. Balitsky, Nucl. Phys. B **463**, 99 (1996).
[4] I. Balitsky, Phys. Rev. Lett. **81** 2024 (1998); Phys. Rev. D **60** 014020 (1999).
[5] Y. V. Kovchegov, Phys. Rev. D **61**, 074018 (2000).
[6] J. Jalilian-Marian, A. Kovner, A. Leonidov and H. Weigert, Nucl. Phys. B **504**, 415 (1997); Phys. Rev. D **59**, 014014 (1998); J. Jalilian-Marian, A. Kovner and H. Weigert, Phys. Rev. D **59**, 014015 (1998); A. Kovner and J. G. Milhano, Phys. Rev. D **61**, 014012 (2000); A. Kovner, J. G. Milhano and H. Weigert, Phys. Rev. D **62**, 114005 (2000); H. Weigert, Nucl. Phys. A **703**, 823 (2002).
[7] E. Iancu, A. Leonidov and L. D. McLerran, Nucl. Phys. A **692**, 583 (2001); Phys. Lett. B **510**, 133 (2001); E. Ferreiro, E. Iancu, A. Leonidov and L. McLerran, Nucl. Phys. A **703**, 489 (2002).
[8] D. Kharzeev and E. Levin, Phys. Lett. B **523**, 79 (2001); D. Kharzeev, E. Levin and L. McLerran, Phys. Lett. B **561**, 93 (2003); D. Kharzeev, E. Levin and M. Nardi, Nucl. Phys. A **730**, 448 (2004); Phys. Rev. C **71**, 054903 (2005).
[9] E. Gotsman, E. Levin, M. Lublinsky and U. Maor, Eur. Phys. J. C **27**, 411 (2003).
[10] D. Kharzeev, Y. V. Kovchegov and K. Tuchin, Phys. Rev. D **68**, 094013 (2003).
[11] J. L. Albacete, N. Armesto, A. Kovner, C. A. Salgado and U. A. Wiedemann, Phys. Rev. Lett. **92**, 082001 (2004).
[12] E. Levin and A. H. Rezaeian, Phys. Rev. D **82**, 054003 (2010); Phys. Rev. D **82**, 014022 (2010).
[13] K. Dusling and R. Venugopalan, Phys. Rev. D **87**, 094034 (2013).
[14] J. L. Albacete and Y. V. Kovchegov, Phys. Rev. D **75**, 125021 (2007); J. L. Albacete, N. Armesto, J. G. Milhano and C. A. Salgado, Phys. Rev. D **80**, 034031 (2009); J. L. Albacete, N. Armesto, J. G. Milhano, P. Quiroga-Arias and C. A. Salgado, Eur. Phys. J. C **71**, 1705 (2011); J. Kuokkanen, K. Rummukainen and H. Weigert, Nucl. Phys. A **875**, 29 (2012); J. L. Albacete, A. Dumitru, H. Fujii and Y. Nara, Nucl. Phys. A **897**, 1 (2013); K. Dusling and R. Venugopalan, Phys. Rev. D **87**, 094034 (2013); T. Lappi and H. Mäntysaari, Phys. Rev. D **88**, 114020 (2013).
[15] I. Balitsky and G. A. Chirilli, Phys. Rev. D **77**, 014019 (2008).
[16] A. V. Grabovsky, JHEP **1309**, 141 (2013).
[17] I. Balitsky and G. A. Chirilli, Phys. Rev. D **88**, 111501 (2013).
[18] A. Kovner, M. Lublinsky and Y. Mulian, Phys. Rev. D **89**, 061704 (2014); JHEP **1408**, 114 (2014).
[19] J. L. Albacete and C. Marquet, Phys. Lett. B **687**, 174 (2010).
[20] F. Gelis and J. Jalilian-Marian, Phys. Rev. D **66**, 014021 (2002); J. Jalilian-Marian and A. H. Rezaeian, Phys. Rev. D **86**, 034016 (2012).
[21] I. Balitsky and G. A. Chirilli, Phys. Rev. D **83**, 031502 (2011).
[22] G. Beuf, Phys. Rev. D **85**, 034039 (2012).
[23] T. Altinoluk and A. Kovner, Phys. Rev. D **83**, 105004 (2011).
[24] G. A. Chirilli, Bo-Wen Xiao and Feng Yuan, Phys. Rev. D **86**, 054005 (2012).
[25] A. Dumitru, A. Hayashigaki and J. Jalilian-Marian, Nucl. Phys. A **765**, 464 (2006).
[26] J. Jalilian-Marian and A. H. Rezaeian, Phys. Rev. D **85**, 014017 (2012).
[27] A. M. Stasto, Bo-Wen Xiao and D. Zaslavsky, Phys. Rev. Lett. **112**, 012302 (2014).
[28] A. M. Stasto, Bo-Wen Xiao, Feng Yuan and D. Zaslavsky, Phys. Rev. D **90**, 014047 (2014).
[29] G. Beuf, Phys. Rev. D **89**, 074039 (2014).
[30] E. Iancu, J. D. Madrigal, A. H. Mueller, G. Soyez and D. N. Triantafyllopoulos, Phys. Lett. B **744**, 293 (2015).
[31] Z. B. Kang, I. Vitev and H. Xing, Phys. Rev. Lett. **113**, 062002 (2014).
[32] B. W. Xiao and F. Yuan, arXiv:1407.6314 [hep-ph].
[33] T. Altinoluk, N. Armesto, G. Beuf, A. Kovner and M. Lublinsky, Phys. Rev. D **91**, no. 9, 094016 (2015) [arXiv:1411.2869 [hep-ph]].
[34] K. Watanabe, B. W. Xiao, F. Yuan and D. Zaslavsky, Phys. Rev. D **92**, no. 3, 034026 (2015) [arXiv:1505.05183 [hep-ph]].
[35] J. C. Collins, D. E. Soper and G. F. Sterman, Nucl. Phys. B **261**, 104 (1985); J. C. Collins, D. E. Soper and G. F. Sterman, Nucl. Phys. B **263**, 37 (1986); J. C. Collins, D. E. Soper and G. F. Sterman, Adv. Ser. Direct. High Energy Phys. **5**, 1 (1989) [hep-ph/0409313].
[36] A. D. Martin, W. J. Stirling, R. S. Thorne and G. Watt, Eur. Phys. J. C **70** (2010) 51 [arXiv:1007.2624 [hep-ph]].
[37] J. C. Collins and R. K. Ellis, Nucl. Phys. B **360**, 3 (1991); S. Catani, M. Ciafaloni and F. Hautmann, Nucl. Phys. B **366**, 135 (1991). E. M. Levin, M. G. Ryskin, Y. M. Shabelski and A. G. Shuvaev, Sov. J. Nucl. Phys. **53**, 657 (1991) [Yad. Fiz. **53**, 1059 (1991)].
[38] S. P. Baranov, N. P. Zotov and A. V. Lipatov, Phys. Atom. Nucl. **67**,837 (2004). [Yad. Fiz. **67** (2004) 859].
[39] F. Gelis and R. Venugopalan, Phys. Rev. D **69**, 014019 (2004) [hep-ph/0310090].
[40] J. P. Blaizot, F. Gelis and R. Venugopalan, Nucl. Phys. A **743**, 57 (2004) [hep-ph/0402257].
[41] K. Tuchin, Phys. Lett. B **593**, 66 (2004) [hep-ph/0401022].
[42] Y. V. Kovchegov and K. Tuchin, Phys. Rev. D **74**, 054014 (2006) [hep-ph/0603055].
[43] H. Fujii, F. Gelis and R. Venugopalan, Nucl. Phys. A **780**, 146 (2006) [hep-ph/0603099].
[44] B. A. Kniehl, V. A. Saleev and A. V. Shipilova, Phys. Rev. D **81**, 094010 (2010) [arXiv:1003.0346 [hep-ph]].
[45] H. Fujii and K. Watanabe, Nucl. Phys. A **920**, 78 (2013) [arXiv:1308.1258 [hep-ph]].
[46] V. Saleev and A. Shipilova, Phys. Rev. D **86**, 034032 (2012) [arXiv:1201.4640 [hep-ph]].
[47] A. V. Karpishkov, M. A. Nefedov, V. A. Saleev and A. V. Shipilova, Phys. Rev. D **91**, no. 5, 054009 (2015) [arXiv:1410.7139 [hep-ph]].
[48] R. Maciula, A. Szczurek and M. Luszczak, Phys. Rev. D **92**, no. 5, 054006 (2015) [arXiv:1505.05038 [hep-ph]].
[49] J. B. Kogut and D. E. Soper, Phys. Rev. D **1**, 2901 (1970).

[50] J. D. Bjorken, J. B. Kogut and D. E. Soper, Phys. Rev. D **3**, 1382 (1971).
[51] S. J. Brodsky, H. C. Pauli and S. S. Pinsky, Phys. Rept. **301**, 299 (1998) [hep-ph/9705477].
[52] H. Fujii and K. Watanabe, Nucl. Phys. A **915**, 1 (2013) [arXiv:1304.2221 [hep-ph]].
[53] Z. B. Kang, Y. Q. Ma and R. Venugopalan, JHEP **1401**, 056 (2014) [arXiv:1309.7337 [hep-ph]].
[54] J. W. Qiu, P. Sun, B. W. Xiao and F. Yuan, Phys. Rev. D **89**, no. 3, 034007 (2014) [arXiv:1310.2230 [hep-ph]].
[55] K. Watanabe and B. W. Xiao, arXiv:1507.06564 [hep-ph].
[56] B. Ducloue, T. Lappi and H. Mantysaari, arXiv:1509.04856 [hep-ph].
[57] R. Aaij *et al.* [LHCb Collaboration], JHEP **1204** (2012) 093 [arXiv:1202.4812 [hep-ex]].
[58] R. Aaij *et al.* [LHCb Collaboration], JHEP **1308** (2013) 117 [arXiv:1306.3663, arXiv:1306.3663 [hep-ex]].
[59] R. Aaij *et al.* [LHCb Collaboration], Phys. Rev. Lett. **114** (2015) 13, 132001 [arXiv:1411.2943 [hep-ex]].
[60] R. Aaij *et al.* [LHCb Collaboration], JHEP **1408** (2014) 143 [arXiv:1405.6842 [hep-ex]].
[61] R. Aaij *et al.* [LHCb Collaboration], Nucl. Phys. B **871** (2013) 1 [arXiv:1302.2864 [hep-ex]].
[62] R. Aaij *et al.* [LHCb Collaboration], arXiv:1510.01707 [hep-ex].
[63] B. Abelev *et al.* [ALICE Collaboration], Phys. Lett. B **708** (2013) 265 [arXiv:1201.3791 [hep-ex]].
[64] R. D. Ball, V. Bertone, M. Bonvini, S. Forte, P. G. Merrild, J. Rojo and L. Rottoli, arXiv:1510.00009 [hep-ph].
[65] R. D. Ball, M. Bonvini and L. Rottoli, arXiv:1510.02491 [hep-ph].
[66] Y. L. Dokshitzer, V. A. Khoze and S. I. Troian, J. Phys. G **17** (1991) 1481; J. Phys. G **17** (1991) 1602.
[67] S. Alekhin, J. Bluemlein and S. Moch, Phys. Rev. D **89** (2014) 5, 054028 [arXiv:1310.3059 [hep-ph]].
[68] L. A. Harland-Lang, A. D. Martin, P. Motylinski and R. S. Thorne, Eur. Phys. J. C **75**, no. 5, 204 (2015) [arXiv:1412.3989 [hep-ph]].
[69] S. Dulat *et al.*, arXiv:1506.07443 [hep-ph].
[70] R. D. Ball *et al.* [NNPDF Collaboration], JHEP **1504**, 040 (2015) [arXiv:1410.8849 [hep-ph]].
[71] M. Cacciari, M. Greco and P. Nason, JHEP **9805** (1998) 007 [arXiv:hep-ph/9803400]; M. Cacciari, S. Frixione and P. Nason, JHEP **0103** (2001) 006 [arXiv:hep-ph/0102134].
[72] S. J. Brodsky, P. Hoyer, C. Peterson and N. Sakai, Phys. Lett. B **93**, 451 (1980).
[73] K. J. Golec-Biernat and M. Wusthoff, Phys. Rev. D **59**, 014017 (1998) [hep-ph/9807513].
[74] L. D. McLerran and R. Venugopalan, Phys. Rev. D **49**, 2233 (1994) [hep-ph/9309289]; Phys. Rev. D **49**, 3352 (1994) [hep-ph/9311205].
[75] J. Blümlein, arXiv:1511.00229 [hep-ph].