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Orbital Angular Momentum and Generalized Transverse Momentum Distribution

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We show that, when boosted to the infinite momentum frame, the quark and gluon orbital angular momentum operators defined in the nucleon spin sum rule of X. S. Chen *et al.* are the same as those whose matrix elements correspond to the moments of generalized transverse momentum distributions. This completes the connection between the infinite momentum limit of each term in that sum rule and experimentally measurable observables. We also show that these orbital angular momentum operators can be defined locally, and discuss the strategies of calculating them in lattice QCD.

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Apportioning the longitudinal spin of a fast-moving nucleon among its quark and gluon partons is one of the most challenging issues in QCD. The quark spin measured in deep-inelastic scattering (DIS) experiments contributes about 25% to the proton spin [1], and a lot of experimental and theoretical effort has been devoted to determining the remaining pieces in the past 25 years. A recent global analysis [2] that includes the high-statistics 2009 STAR [3] and PHENIX [4] data shows evidence of non-zero gluon helicity in the proton. At $Q^2 = 10$ GeV^2, the polarized gluon distribution $\Delta g(x, Q^2)$ is found to be positive and away from zero in the momentum fraction range $0.05 \leq x \leq 0.2$. However, the result presented in [2] still has large uncertainty in the small-x region. Given that the integral value of $\Delta g(x, Q^2)$ from $x = 0.05$ to 0.2 is about 40% of the proton spin [2], there is still room for substantial contribution from the quark and gluon orbital angular momenta (OAM). To fully understand the proton spin structure, one needs to find the spin and OAM operators whose matrix elements give the partonic contributions that are both measurable in experiments and calculable in theory, especially, in lattice QCD which is a well-established non-perturbative approach to solving QCD. Compared to the quark spin and gluon helicity, parton OAM are still difficult to measure, while little progress has been made in their lattice calculation.

In this work, we address the question of orbital contributions, and show that the OAM operators in the gauge-invariant nucleon spin sum rule of Chen *et al.* [5, 6], when boosted to the infinite momentum frame (IMF), are equal to those whose matrix elements correspond to the moments of generalized transverse momentum distributions (GTMD). The latter contain information of parton OAM, and their measurement has been speculated in recent studies [7–9]. We also show that the OAM operators in [5, 6] can be defined locally on the lattice, and outline the ways to calculate their matrix elements. Therefore, our work demonstrates that the experimental observables for parton OAM are accessible through practical lattice calculations.

The quark spin and gluon helicity measured in DIS experiments can be incorporated into the naive nucleon spin sum rule by Jaffe and Manohar [10]:

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \mathcal{L}_q + \Delta G + \mathcal{L}_g \ , \tag{1}$$

where each individual term is defined to be the proton matrix element of canonical spin and OAM operators in the IMF (or on the light-cone) with the light-cone gauge $A^+ = 0$:

$$\begin{aligned} \vec{J}_{\rm can} \ =& \ \int d^3x \ \psi^\dagger \frac{\vec{\Sigma}}{2}\psi + \int d^3x \ \psi^\dagger \ \vec{x} \times (-i\vec{\nabla})\psi \\ & + \int d^3x \ \vec{E}_a \times \vec{A}^a + \int d^3x \ E^i_a \ \vec{x} \times \vec{\nabla} A^{i,a} \ . \end{aligned} \tag{2}$$

Here a and i are the color and spatial Lorentz indices.

As is well known, $\Delta\Sigma$ and ΔG are the first moments of polarized quark and gluon distributions which are given by light-cone correlation functions [11]. Meanwhile, there are attempts to define canonical OAM distributions and relate them to experiments [12]. One proposal based on quark models suggests that $\mathcal{L}_q$ is related to a transverse momentum distribution (TMD) function $h^\perp_{1T}$ [13–16] which is measurable in semi-inclusive DIS experiments [17, 18]. Furthermore, in a model-independent proposal, $\mathcal{L}_q$ is directly given by a Wigner distribution $W^q_{\rm LC}$, or GTMD $F^q_{1,4}$ [19–24]:

$$\begin{aligned} \mathcal{L}_q(x) \ =& \ \int d^2\vec{b}_\perp d^2\vec{k}_\perp (\vec{b}_\perp \times \vec{k}_\perp)^z W^q_{\rm LC}(x, \vec{b}_\perp, \vec{k}_\perp) \\ =& \ -\int d^2k_\perp \frac{\vec{k}^2_\perp}{M^2} F^q_{1,4}(x, 0, \vec{k}^2_\perp, 0, 0) \ , \end{aligned} \tag{3}$$

where M is the nucleon mass, $\vec{b}_\perp$ and $\vec{k}_\perp$ are the relative average transverse position and momentum of the quark, and "LC" stands for a specific choice of the gauge link that corresponds to semi-inclusive DIS or Drell-Yan processes. There is also similar relationship between $\mathcal{L}_g$ and the gluon Wigner distribution or GTMD. The definition of $\mathcal{L}_q$ in Eq. (3) has clear partonic interpretation, and it is suggested that $F^q_{1,4}$ can be measured in parity-violating deeply virtual Compton scattering processes that involve more than one hadronic reaction planes [7, 8]. Besides, it is also proposed that the gluon Wigner distribution can be measured at small x in hard diffractive dijet production [9].

Since the canonical quark and gluon OAM are defined in the light-cone plane and gauge which involve linear combinations of the spatial and and real time coordinates, they are not directly accessible in lattice QCD that uses imaginary time. Nevertheless, there has been recent progress on the lattice calculation of TMD functions [25–27], where a spatial gauge link is used in the calculation and the result is evolved to the IMF to obtain the light-cone correlator. This approach can be generalized to calculate $\mathcal{L}_q$ and $\mathcal{L}_g$ via Eq. (3).

A new perspective is provided in [28] which proves that the frame-dependent gluon spin operator in [5, 6] is equal to that defined from the first moment of $\Delta g(x, Q^2)$ in the IMF limit. Since the gluon spin operator does not depend on time, its matrix element can be calculated in lattice QCD, and is related to ΔG through a perturbative matching condition in the large momentum effective theory (LaMET) [29–32]. An attempt to calculate ΔG in lattice QCD with this gluon spin operator has been carried out recently [33, 34].

Inspired by this discovery, we explore the connection between GTMD and the OAM operators in the gauge-invariant nucleon spin sum rule by Chen *et al.* [5, 6],

$$\vec{J} = \int d^3x\, \psi^\dagger \frac{\vec{\Sigma}}{2}\psi + \int d^3x\, \psi^\dagger \vec{x}\times i\vec{D}_{\rm pure}\psi + \int d^3x\, \vec{E}\times\vec{A}_{\rm phys} + \int d^3x\, E^i(\vec{x}\times(-\vec{\mathcal{D}}_{\rm pure}))A^i_{\rm phys}\,, \tag{4}$$

where i is the spatial Lorentz index. Here $D^\mu_{\rm pure} = \partial^\mu - igA^\mu_{\rm pure}$ and $\mathcal{D}^\mu_{\rm pure} \equiv \partial^\mu - ig[A^\mu_{\rm pure},\ ]$ are the gauge-covariant derivatives acting on the fundamental and adjoint representations, respectively, with $A^\mu = A^\mu_{\rm phys} + A^\mu_{\rm pure}$, which is the QCD analogue of the well-known Helmholtz decomposition $\vec{A} = \vec{A}_\perp + \vec{A}_\parallel$ in electrodynamics. To make each term in Eq. (4) gauge invariant, one requires that under a gauge transformation $g(x)$,

$$\begin{aligned} A^\mu_{\rm phys}(x) &\to g(x)A^\mu_{\rm phys}(x)g^{-1}(x)\,, \\ A^\mu_{\rm pure}(x) &\to g(x)\left(A^\mu_{\rm pure}(x) + \frac{i}{g}\partial^\mu\right)g^{-1}(x)\,. \end{aligned} \tag{5}$$

In addition, to find a solution, it is suggested [6] that $A^\mu_{\rm phys}$ satisfies the non-Abelian transverse condition,

$$\mathcal{D}^i A^i_{\rm phys} \equiv \partial^i A^i_{\rm phys} - ig[A^i, A^i_{\rm phys}] = 0\,, \tag{6}$$

while $A^\mu_{\rm pure}$ is constrained by a null-field-strength condition,

$$F^{\mu\nu}_{\rm pure} \equiv \partial^\mu A^\nu_{\rm pure} - \partial^\nu A^\mu_{\rm pure} - ig[A^\mu_{\rm pure}, A^\nu_{\rm pure}] = 0. \tag{7}$$

According to [28], in the IMF limit, $A^+_{\rm pure} = A^+$. By setting $\nu = +$, one obtains a first-order linear equation for $A^\mu_{\rm pure}$ from Eq. (7),

$$\partial^+ A^{\mu,a}_{\rm pure} + gf^{abc}A^{+,b}A^{\mu,c}_{\rm pure} = \partial^\mu A^{+,a}\,. \tag{8}$$

The solution is given by

$$A^{\mu,a}_{\rm pure}(z^-) = -\frac{1}{2}\int dz'^-\mathcal{K}(z'^- - z^-) \times\left(\partial^\mu A^{+,b}(z'^-)\mathcal{L}^{ba}(z'^-, z^-)\right), \tag{9}$$

where the light-cone coordinates $z^\pm = (x^0 \pm x^3)/\sqrt{2}$. The kernel $\mathcal{K}(z'^- - z^-)$ is related to the boundary condition [21]. Here $\mathcal{L}$ is a light-cone gauge link defined in the adjoint representation. We can easily obtain $A^{\mu,a}_{\rm phys}$ by subtracting $A^{\mu,a}_{\rm pure}$ from $A^{\mu,a}$. After integration by parts, this solution for $A^{\mu,a}_{\rm phys}$ is identical to the one found by Hatta [21, 35],

$$A^\mu_{\rm phys}(z^-) = -\frac{1}{2}\int dy^-\mathcal{K}(y^- - z^-)\mathcal{W}^-_{zy}F^{+\mu}(y^-)\mathcal{W}^-_{yz}\,, \tag{10}$$

where the light-cone gauge link $\mathcal{W}^-_{zy}$ is defined in the fundamental representation. Note that it only requires $A^+_{\rm pure} = A^+$ or $A^+_{\rm phys} = 0$ in the IMF limit to get Eq. (8), thus Eq. (6) actually belongs to a universality class of conditions that can be used to fix $A^\mu_{\rm phys}$ and $A^\mu_{\rm pure}$ [29]. According to Eq. (9), if $A^+ = 0$, then $A^\mu_{\rm pure} = 0$, $A^\mu_{\rm phys} = A^\mu$. Therefore, the OAM operators of Chen *et al.* in Eq. (4), when boosted to IMF and fixed in the light-cone gauge, become those of Jaffe and Manohar in Eq. (2).

Under an infinite Lorentz boost along the z direction, the equal-time plane is tilted to the light-cone, and the bilinear operator $\bar{\psi}\gamma^0\cdots\psi$ transforms into $\bar{\psi}\gamma^+\cdots\psi$. The quark OAM L_q defined in Eq. (4) becomes

$$\lim_{P^z\to\infty} L^z_q = \int dz^- d^2z_T\, \bar{\psi}(z)\gamma^+(z^1 i\tilde{D}^2_{\rm pure} - z^2 i\tilde{D}^1_{\rm pure})\psi(z)\,, \tag{11}$$

where $\tilde{D}^\mu_{\rm pure}$ is the covariant derivative with $A^{\mu,a}_{\rm pure}(z^-)$ given in Eq. (9).

In the same limit, $E^i = F^{i0}$ with $i = 1, 2$ transforms into F^{i+}, while $E^3 = F^{+-}$ remains the same; whereas $A^3_{\rm phys}$ becomes $A^+_{\rm phys}$ that vanishes according to [28]. Therefore, the IMF limit of the gluon OAM L_g in Eq. (4) is

$$\lim_{P^z\to\infty} L^z_g = \int dz^- d^2z_T\, E^i(z)(-z^1\tilde{\mathcal{D}}^2_{\rm pure} + z^2\tilde{\mathcal{D}}^1_{\rm pure})A^i_{\rm phys}(z)\,. \tag{12}$$

Now recall that the quark GTMD function,

$$\begin{aligned} & f(x, \vec{k}_T, \vec{\Delta}_T) \\ \equiv & \int \frac{dz^- d^2z_T}{(2\pi)^3} e^{ix\bar{P}^+z^- - i\vec{k}_T\cdot\vec{z}_T}\langle P'S|\bar{\psi}(-\frac{z^-}{2}, -\frac{z_T}{2})\gamma^+ \\ & \times\mathcal{W}^-_{-\frac{z}{2},\pm\infty}\mathcal{W}^T_{-\frac{z_T}{2},\frac{z_T}{2}}\mathcal{W}^-_{\pm\infty,\frac{z}{2}}\psi(\frac{z^-}{2}, \frac{z_T}{2})|PS\rangle\,, \end{aligned} \tag{13}$$

where $P = \bar{P} - \Delta/2$, $P' = \bar{P} + \Delta/2$, $\Delta^+ = 0$, and $\mathcal{W}^T_{-\frac{z_T}{2},\frac{z_T}{2}}$ is a straight-line gauge link connecting the transverse fields at light-cone infinity. Here $\pm\infty$ correspond to the link choices for semi-inclusive DIS and Drell-Yan processes.

In [21], it is proved that

$$\begin{aligned} &\int dx d^2k_T\ k_T^i f(x,\vec{k}_T,\vec{\Delta}_T) \\ &= \frac{1}{2\bar{P}^+}\langle P'S|\bar{\psi}(0)\gamma^+(i\tilde{D}^i_{\rm pure} - i\overleftarrow{\tilde{D}}^i_{\rm pure})\psi(0)|PS\rangle \end{aligned} \quad (14)$$

where $\overleftarrow{\tilde{D}}^\mu_{\rm pure} = \overleftarrow{\partial}^\mu + igA^\mu_{\rm pure}$, and $A^\mu_{\rm pure}$ is exactly the solution in Eq. (9). Therefore, combining Eqs. (11) and (14), one obtains

$$\begin{aligned} \lim_{P^z\to\infty}\langle L_q^z\rangle &= \frac{1}{2P^+}\frac{1}{(2\pi)^3\delta^{(3)}(0)}\langle PS|\int dz^- d^2z_T \\ &\quad\times\bar{\psi}(z)\gamma^+\epsilon^{ij}z^i(i\tilde{D}^j_{\rm pure} - i\overleftarrow{\tilde{D}}^j_{\rm pure})\psi(z)|PS\rangle \\ &= \epsilon^{ij}\lim_{\Delta\to 0}\frac{\partial}{i\partial\Delta_T^i}\int dx d^2k_T\ k_T^j f(x,\vec{k}_T,\vec{\Delta}_T) \\ &= -\int d^2k_\perp\frac{\vec{k}_\perp^2}{M^2}F^q_{1,4}(x,0,\vec{k}^2_\perp,0,0)\ , \end{aligned} \quad (15)$$

with $\epsilon^{12} = -\epsilon^{21} = 1$, and $\epsilon^{11} = \epsilon^{22} = 0$.

Similarly, one can prove that

$$\begin{aligned} \lim_{P^z\to\infty}\langle L_g^z\rangle &= \frac{1}{2P^+}\frac{1}{(2\pi)^3\delta^{(3)}(0)}\langle PS|\int dz^- d^2z_T \\ &\quad\times F^+{}_\alpha(z)\epsilon^{ij}z^i(-\tilde{\mathcal{D}}^j_{\rm pure})A^\alpha_{\rm pure}(z)|PS\rangle \\ &= \epsilon^{ij}\lim_{\Delta\to 0}\frac{\partial}{i\partial\Delta_T^i}\int dx d^2k_T\ k_T^j g(x,\vec{k}_T,\vec{\Delta}_T) \\ &= -\int d^2k_\perp\frac{\vec{k}_\perp^2}{M^2}F^g_{1,4}(x,0,\vec{k}^2_\perp,0,0)\ , \end{aligned} \quad (16)$$

with the gluon GTMD [24, 36],

$$\begin{aligned} &g(x,\vec{k}_T,\vec{\Delta}_T) \\ &\equiv -\frac{i}{2x\bar{P}^+}\int\frac{dz^- d^2z_T}{(2\pi)^3}e^{ix\bar{P}^+z^- - i\vec{k}_T\cdot\vec{z}_T} \\ &\quad\times\langle P'S|F^{+\alpha}(-\frac{z^-}{2},-\frac{z_T}{2})\mathcal{W}^-_{-\frac{z}{2},\pm\infty}\mathcal{W}^T_{-\frac{z_T}{2},\frac{z_T}{2}} \\ &\quad\times\mathcal{W}^-_{\pm\infty,\frac{z}{2}}F^+{}_\alpha(\frac{z^-}{2},\frac{z_T}{2})|PS\rangle\ , \end{aligned} \quad (17)$$

where the gauge links are defined in the adjoint representation.

This finishes our proof that the quark and gluon OAM in Eq. (4) in the IMF limit are the same as those whose matrix elements correspond to GTMD moments. Furthermore, when fixed to the light-cone gauge, they reduce to the canonical OAM in the Jaffe-Manohar sum rule. This is our main result.

Since the OAM operators in Eq. (4) do not depend on the light-cone coordinates, their matrix elements can be calculated in lattice QCD and related to the parton OAM in the IMF through a perturbative matching condition which has already been derived at one-loop order [32]. Therefore, we provide a new direction of theoretical determination of the parton OAM that can be measured in experiments.

As far as lattice calculation is concerned, solutions for $A^\mu_{\rm phys}$ and $A^\mu_{\rm pure}$ satisfying Eqs. (5–7) can be obtained through a gauge link fixed in the Coulomb gauge [37, 38]. To be specific, one starts from a link variable (or Wilson line) $U^\mu(x)\equiv\exp(-iagA^\mu(x))$ on the lattice that connects x to $x+a\hat{\mu}$, where a is the lattice spacing and $\hat{\mu}$ is the unit vector in the μ direction. Under a gauge transformation, $U^\mu(x)$ transforms as

$$U^\mu(x)\to U'^\mu(x) = g(x)U^\mu(x)g^{-1}(x+a\hat{\mu})\ , \quad (18)$$

under a gauge transformation $g(x)$. By finding a gauge transformation g_c that makes $U^\mu(x) = g_c(x)U^\mu_c(x)g_c^{-1}(x+a\hat{\mu})$ where $U^\mu_c(x)$ is fixed in the Coulomb gauge, one can define a new gauge link $U^\mu_{\rm pure}$ and obtain the solution for $A^\mu_{\rm phys}$ [38],

$$\begin{aligned} U^\mu_{\rm pure} &\equiv g_c(x)g_c^{-1}(x+a\hat{\mu}), \\ A^\mu_{\rm phys} &\equiv \frac{i}{ag}\left(U^\mu(x) - U^\mu_{\rm pure}(x)\right) \\ &= \frac{i}{ag}g_c(x)(U^\mu_c(x)-1)g_c^{-1}(x) + O(a) \\ &= g_c(x)A_c(x)g_c^{-1}(x) + O(a)\ . \end{aligned} \quad (19)$$

This is the finite version of the result in the continuum theory [37]. One can check that $A^\mu_{\rm phys}$ so defined satisfies the gauge transformation law in Eq. (5) with U^μ_c being unchanged and g_c transforming as $g'_c = gg_c$. A straight-forward calculation confirms that Eqs. (6–7) are also satisfied up to $O(a)$ corrections which vanish in the continuum limit. In this way, $A^\mu_{\rm phys}$ is a local operator after imposing the non-Abelian transverse condition and the OAM operator can be renormalized with standard perturbative or non-perturbative methods. This approach has been applied to the recent attempt to calculate ΔG [33, 34]. It can be used to calculate OAM from the operators defined in Eq. (4), too. Since the non-Abelian transverse condition is not the only one to define $A^\mu_{\rm phys}$, one can also try other possibilities like the axial gauge condition [29] on the lattice.

Note that L_q resembles the mechanical OAM in the sum rule by Ji [39], except that the covariant derivative D^μ in the latter is replaced by $D^\mu_{\rm pure}$. Ji's sum rule is based on a gauge-invariant and frame-independent decomposition of the nucleon spin, so each individual contribution is the same in arbitrary frame including the IMF. This allows for a lattice calculation of the quark OAM and total gluon angular momentum near the rest frame of the proton, which has been carried out on a quenched lattice [40]. In practice, an operator with explicit dependence on the spatial coordinates poses a problem for the direct calculation of its forward matrix element due to periodic boundary conditions [41], so in [40] the quark OAM is not directly calculated. Instead, it is obtained by subtracting the quark spin from the total quark angular momentum. The latter is calculated from the Belinfante-improved symmetric energy-momentum tensor $\mathcal{T}_q^{\mu\nu}$,

$$J_q^i = \frac{1}{2}\epsilon^{ijk}\int d^3x \left(\mathcal{T}_q^{0k}x^j - \mathcal{T}_q^{0j}x^k\right) , \tag{20}$$

$$\mathcal{T}_q^{\mu\nu} = \frac{1}{2}\left[\bar{\psi}\gamma^{\{\mu}iD^{\nu\}}\psi - \bar{\psi}\gamma^{\{\mu}i\overleftarrow{D}^{\nu\}}\psi\right] , \tag{21}$$

where the angular momentum J_q is obtained from the forward limit of two form factors of $\mathcal{T}_q^{\mu\nu}$,

$$J_q = \frac{1}{2}\left[T_1(0) + T_2(0)\right] . \tag{22}$$

One might think of the same approach to obtain L_q^z by using the symmetric energy-momentum tensor

$$\mathcal{T}_q'^{\mu\nu} = \frac{1}{2}\left[\bar{\psi}\gamma^{\{\mu}iD_{\rm pure}^{\nu\}}\psi - \bar{\psi}\gamma^{\{\mu}i\overleftarrow{D}_{\rm pure}^{\nu\}}\psi\right] . \tag{23}$$

However, with this definition, Eq. (20) cannot be used; that is, the right-hand side of Eq. (20) will produce the quark angular momentum plus extra terms. Therefore, the sum rule in Eq. (22) cannot be used, either. If, however, one adopts the asymmetric energy-momentum tensor,

$$\mathcal{T}_{q-{\rm asy}}^{\mu\nu} = \bar{\psi}\gamma^\mu iD_{\rm pure}^\nu\psi , \tag{24}$$

one can obtain L_q via Eq. (20). Since the frame-dependence of $D_{\rm pure}^\mu$ in $\mathcal{T}_{q-{\rm asy}}^{\mu\nu}$ spoils the simple parametrization of its off-forward matrix element, one has to introduce a temporal vector $n^\mu = (1,0,0,0)$ in this case to include more Lorentz structures and form factors, which is analogous to that formulated on the light-cone [42]. If all the form factors can be calculated in lattice QCD, then the quark OAM can be obtained through a relevant sum rule.

Alternatively, one can directly calculate OAM from the off-forward matrix element by utilizing the relation

$$\begin{aligned} &\epsilon_{ij}\langle P'S|\int d^3x\ \bar{\psi}\gamma^0 x^i iD_{\rm pure}^j\psi|PS\rangle \\ &= \epsilon_{ij}\lim_{|\vec{q}|\to 0}\langle P'S|\int d^3x\ \bar{\psi}\gamma^0\frac{\partial}{\partial q^i}e^{i\vec{q}\cdot\vec{x}}D_{\rm pure}^j\psi|PS\rangle \end{aligned} \tag{25}$$

and the challenge here is to have a reliable extrapolation to the $|\vec{q}| \to 0$ limit.

The third lattice approach is the calculation of $\mathcal{L}_q$ and $\mathcal{L}_g$ from GTMD which can be generalized from the lattice study of TMD as mentioned in the introduction [25–27]. One of the challenges here is the renormalization of non-local gauge-link operators on the lattice.

In conclusion, we have proved that, in the IMF limit, the gauge-invariant quark and gluon OAM defined by Chen *et al.* in Eq. (4) are equal to those whose matrix elements correspond to GTMD moments. Furthermore, each term in the angular momentum sum rule in Eq. (4) reduces to their corresponding term in the Jaffe-Manohar sum rule in Eq. (2) in the light-cone gauge. The former can be calculated through local operators on the Euclidean lattice, and their matrix elements are matched to the physical quark and gluon OAM through LaMET. With the development of GTMD measurement in hard-exclusive processes, we should eventually be able to compare the theoretical and experimental results on the parton OAM.

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