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Stability of Einstein-Maxwell-Kalb-Ramond Wormholes

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This paper investigates a particular type of wormhole. The wormhole solutions studied are obtained by sewing together two static, spherically symmetric, charged black-hole metrics at their horizons. The charged wormholes are in a background Kalb-Ramond field, which is the source of the necessary tension in the gravitational field. The metric-tensor elements are studied by numerically solving Einstein's equations with stress-energy-tensor elements given by the combination of static electric and Kalb-Ramond fields. For a certain range of electric charge the tension is positive away from the wormhole throat, but the tension is negative near the throat, making it non-traversable. The wormholes are found to be quasi-stable against decay via gravitational instanton tunnelling.

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I. INTRODUCTION

Wormhole solutions of the Einstein equations were first studied by Einstein and Rosen [1], who called such solutions ‘bridges’. The solutions in [1] were offered as models for the elementary particles known at the time (the proton and the electron). Wormhole models of neutral and charged particles were also studied in [2, 3], where the relation between the mass and the charge which is characteristic of such models was first obtained [4]. These and subsequent attempts to identify elementary particles as wormholes have failed. Since wormholes can be constructed which join two separate space-times or places in a space-time, in a quantum theory of gravity they allow the formation of ‘baby universes’ [5]. Wormholes have also been proposed as a means of interstellar travel [6], although such objects would require some form of ‘exotic’ matter in order to allow interstellar travellers to safely pass through. The wormhole solutions studied in this paper are obtained by sewing together two ‘charged’ black hole metrics at their horizons. The resulting space-time consists of two congruent regions which are geometrically connected but are causally disconnected. The wormholes are assumed to be in a background field with tension, which is provided by a massless, antisymmetric-tensor field – the Kalb-Ramond (K-R) field [7]. Such fields occur naturally in string theory and should be a constituent of a gravitational multiplet if string theory is a viable theory of all physical phenomena [8–12].

II. CHARGED WORMHOLES

A. Metric Tensor Elements

Morris and Thorne [6] investigated traversable wormholes, finding that they require ‘exotic’ matter, with negative energy density (at least as seen by some observers) as well as significant levels of tension (negative pressure). Some types of fields allow tension, and Rahaman, Kalam, and Ghosh [13] showed that certain static, spherically symmetric solutions of the K-R field are wormholes (though they are not traversable). We study such (non-traversable) wormholes with a metric of the spherically symmetric form [6] (with $c = 1$; we also frequently use $G = 1$)

$$ds^2 = -e^{A(r)} dt^2 + e^{B(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (1)$$

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The functions $A(r)$ and $B(r)$ are determined by the Einstein equations (for brevity we use $'$ for d/dr)

$$\begin{aligned} \frac{e^{A(r)} B'(r)}{r e^{B(r)}} + \frac{e^{A(r)}}{r^2} - \frac{e^{A(r)}}{r^2 e^{B(r)}} &= 8\pi T_{00} \\ \frac{A'(r)}{r} - \frac{e^{B(r)}}{r^2} + \frac{1}{r^2} &= 8\pi T_{11} \\ -\frac{1}{2} \frac{r B'(r)}{e^{B(r)}} + \frac{1}{2} \frac{r A'(r)}{e^{B(r)}} + \frac{1}{2} \frac{r^2 A''(r)}{e^{B(r)}} + \frac{1}{4} \frac{r^2 (A'(r))^2}{e^{B(r)}} - \frac{1}{4} \frac{r^2 (A'(r)) B'(r)}{e^{B(r)}} \\ &= 8\pi T_{22} = 8\pi T_{33} / \sin^2(\theta). \end{aligned} \quad (2)$$

The action for such a wormhole in background electromagnetic and K-R fields is given by

$$S = \int_{\mathcal{M}} \sqrt{-g} \left(\frac{R}{16\pi G} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} \right) d^4x, \quad (3)$$

where g is the metric tensor determinant, R is the Ricci scalar, $F_{\mu\nu}$ is the electromagnetic-field tensor, and $H_{\mu\nu\lambda}$ is the totally antisymmetric K-R-field tensor. The energy-momentum tensor elements are given by

$$T_{\mu\nu} = F_{\mu\lambda} F_{\nu}{}^{\lambda} - \frac{1}{4} g_{\mu\nu} F_{\lambda\sigma} F^{\lambda\sigma} + \frac{1}{12} \left(3 H_{\lambda\sigma\mu} H^{\lambda\sigma}{}_{\nu} - \frac{1}{2} g_{\mu\nu} H_{\lambda\sigma\tau} H^{\lambda\sigma\tau} \right). \quad (4)$$

The electromagnetic and K-R fields satisfy the equations

$$\begin{aligned} \frac{1}{\sqrt{-g}} \partial_{\nu} (\sqrt{-g} F^{\mu\nu}) &= 0, \\ \frac{1}{\sqrt{-g}} \partial_{\nu} (\sqrt{-g} H^{\mu\lambda\nu}) &= 0. \end{aligned} \quad (5)$$

Many relationships among local properties look more familiar in a local orthonormal coordinate basis, the proper reference frame of observers at fixed r, θ, ϕ :

$$\begin{aligned} e_{\hat{t}} &= e^{-A(r)/2} e_t, \quad e_{\hat{r}} = e^{-B(r)/2} e_r \\ e_{\hat{\theta}} &= r^{-1} e_{\theta}, \quad e_{\hat{\phi}} = (r \sin(\theta))^{-1} e_{\phi}. \end{aligned} \quad (6)$$

In such a basis, pressure p is given by the equal $\hat{\theta}\hat{\theta}$ and $\hat{\phi}\hat{\phi}$ components of the stress-energy tensor, while energy density ρ and tension τ are given by the $\hat{t}\hat{t}$ and negative $\hat{r}\hat{r}$ components of the Einstein tensor respectively. We choose to consider the static field case in which the only independent non-zero components of $F^{\hat{\mu}\hat{\nu}}$ and $H^{\hat{\mu}\hat{\nu}\hat{\lambda}}$ are $F^{\hat{0}\hat{1}} = E(r) = Q/2\sqrt{\pi}r^2$, $H^{\hat{0}\hat{2}\hat{3}} = h(r)/(\sin(\theta))$. The energy-momentum tensor elements are then given by

$$\begin{aligned} T_{00} &= e^{A(r)} \left((E(r))^2 / 2 + (h(r))^2 / 2 \right) = e^{A(r)} T_{\hat{0}\hat{0}} \\ T_{11} &= e^{B(r)} \left(- (E(r))^2 / 2 + (h(r))^2 / 2 \right) = e^{B(r)} T_{\hat{1}\hat{1}} \\ T_{22} &= r^2 \left((E(r))^2 / 2 - (h(r))^2 / 2 \right) = r^2 T_{\hat{2}\hat{2}} = r^2 T_{\hat{3}\hat{3}}. \end{aligned} \quad (7)$$

The Einstein equations (2) cannot be solved analytically; however, the numerical solution of these equations shows that there is a singularity in the function $B(r)$ at a point r_0 determined in part by the value of the charge Q . This singularity at $r = r_0$ is handled by sewing together two congruent copies of the region $r \geq r_0$ (Fig.1). (Continuity at this junction can be seen by embedding the manifold in a five-dimensional spatially flat space-time with an additional coordinate z joining t, r, θ, ϕ ; the embedding $z = z(r)$ is defined by [6]

$$\frac{dz}{dr} = \pm \left(\sqrt{b(r) - 1} \right), \quad z(r_0) = 0, \quad (8)$$

where $b(r) = e^{B(r)}$; the $+/-$ sign distinguishes the two copies[13]. The numerical solution of the Einstein gravitational field equations allows the functions $A(r)$ and $b(r)$ defining the g_{00} and g_{11} metric tensor elements, respectively, to be plotted versus the radius r as shown in Figs. 2 and 3. The tension $\tau(r)$ is positive for $r > r_0$ if the electric charge Q is large enough (Fig. 4), but $\tau(r)$ is always less than the energy density $\rho(r)$ as can be seen from Eq. (7), so the wormhole described by this model is not traversable. The total electric and K-R charges for the space-time described by Eq. (1) are zero. However, a distant observer would be on either one side of the causal boundary (at r_0) or the other and would therefore detect a net electric charge of one sign or the other for each of the wormhole termini. The K-R ‘charge’ of the wormhole is zero due to the fact that the only non-zero component of $H^{\mu\nu\lambda}$ is transverse to the radial coordinate.

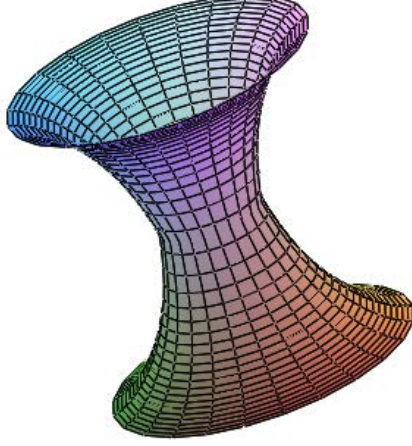


FIG. 1: The throat region of a wormhole obtained by sewing together two black holes at their horizons.

B. Einstein-Maxwell-Kalb-Ramond Action

The metric in Eq. (1) is part of the solutions of the field equations obtained from the Einstein-Maxwell-Kalb-Ramond action (Eq. (3)) on a manifold \mathcal{M} . We have assumed a static solution with spherical symmetry, which leads to fields with only one independent non-zero component each, F^{01} and H^{023} for the electromagnetic and K-R fields respectively. These fields are normalized as

$$\begin{aligned} F_{\mu\nu} F^{\mu\nu} &= -2 E(r)^2 \\ H_{\mu\nu\lambda} H^{\mu\nu\lambda} &= -6 h(r)^2. \end{aligned} \quad (9)$$

$F_{\mu\nu}$ and $H_{\mu\nu\lambda}$ satisfy the field equations given in Eq. (5).

The action is calculated using the procedure of [14]. For this, the form of the action is required to be first order in field derivatives [14]; this requires an integration by parts, leading to a non-trivial boundary term,

$$S_B = \frac{1}{8\pi G} \int_{\partial\mathcal{M}} \sqrt{-\gamma} K d^3x \quad (10)$$

where γ is the induced metric on the boundary $\partial\mathcal{M}$ and K is the trace of the second fundamental form, which is

$$K_{\mu\nu} = \nabla_\mu n_\nu - n_\lambda n^\lambda n_\mu n^\sigma \nabla_\sigma n_\nu. \quad (11)$$

Here, vector n_μ is a unit vector normal to the boundary $\partial\mathcal{M}$; in the present case $n_\mu = (0, e^{B(r)/2}, 0, 0)$. The action in Eq. (10) diverges when the radial coordinate becomes large, so it is renormalized by subtracting the flat-space value of the trace of the second fundamental form. Eq. (10) then becomes

$$S_B = \frac{1}{8\pi G} \int_{\partial\mathcal{M}} \sqrt{-\gamma} [K] d^3x \quad (12)$$

where $[K] = K - K^F|_{r \rightarrow \infty}$ and K^F is the second fundamental form for flat space.

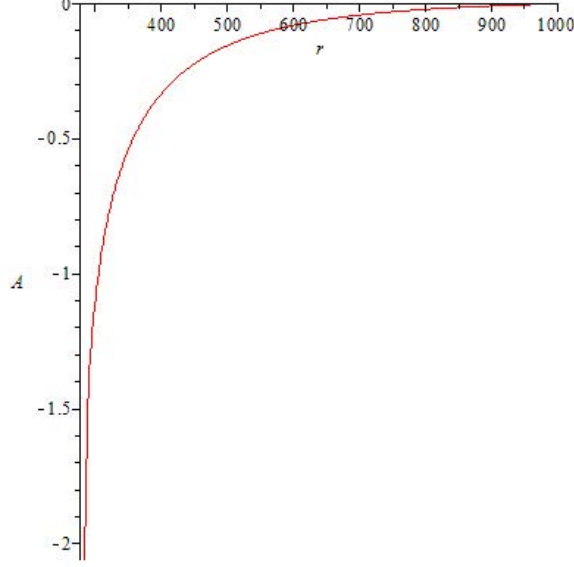


FIG. 2: The function $A(r) = \ln(-g_{00})$ plotted versus the radial coordinate r in dimensionless units.

The renormalized action is then

$$S = \int_{\mathcal{M}} d^4x \sqrt{-g} \left[\frac{R}{16\pi} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{12} H_{\rho\sigma\lambda} H^{\rho\sigma\lambda} \right] + \frac{1}{8\pi} \int_{\partial\mathcal{M}} [K] d\Sigma, \quad (13)$$

where Σ is the volume element of the compact Euclidean spatial hypersurface for a section with imaginary time τ and at fixed r [14]

$$d\Sigma = -ie^{A/2} r^2 \sin\theta d\tau d\theta d\phi. \quad (14)$$

The scalar curvature R equals

$$R = -\frac{2\pi}{3} H_{\rho\sigma\lambda} H^{\rho\sigma\lambda}, \quad (15)$$

giving a total action of

$$S = \int_{\mathcal{M}} d^4x \sqrt{-g} \left[-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} H_{\rho\sigma\lambda} H^{\rho\sigma\lambda} \right] + \frac{1}{8\pi} \int_{\partial\mathcal{M}} [K] d\Sigma. \quad (16)$$

In terms of the functions defined in Eq. (9) the action is

$$S = \int_{\mathcal{M}} d^4x \sqrt{-g} \left(E(r)^2 + 6\pi h^2 \right) + \frac{1}{8\pi} \int_{\partial\mathcal{M}} [K] d\Sigma. \quad (17)$$

The function $h(r)$ is determined from the properties that the K-R field can be expressed as the dual of the derivative of a pseudoscalar $\Phi(r)$

$$H_{\mu\nu\lambda} = \epsilon_{\mu\nu\lambda}{}^\rho \partial_\rho \Phi(r) \quad (18)$$

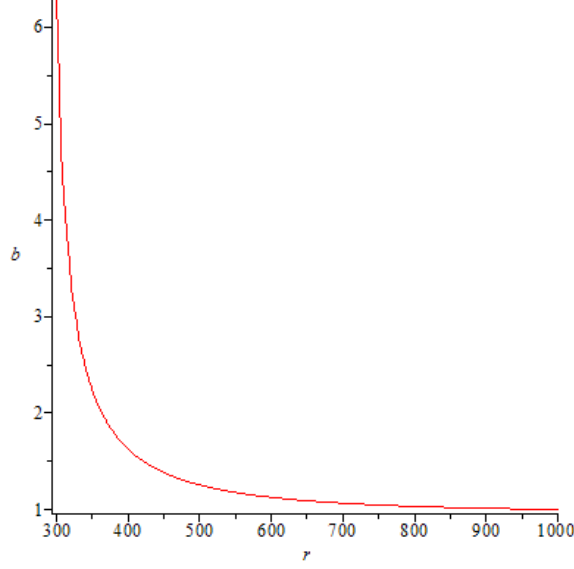


FIG. 3: The function $b(r) = g_{11} = e^{B(r)}$ plotted versus the radial coordinate r in dimensionless units.

and that $H_{\mu\nu\lambda}$ satisfies the Bianchi identity

$$\epsilon^{\mu\nu\lambda\alpha} \partial_\alpha H_{\mu\nu\lambda} = 0. \quad (19)$$

These relations require that

$$h(r) = \frac{h_0 e^{-A(r)/2}}{r^2}, \quad (20)$$

where h_0 is a constant. Substituting the above expressions for $E(r)$ and $h(r)$ into Eq. (17) and evaluating the surface term, the expression for the action becomes

$$S = i \frac{8\pi^2}{\kappa} \int_{r_0}^{\infty} e^{(A+B)/2} r^2 \left(\frac{Q^2}{2r^4} + 12\pi \frac{h_0^2 e^{-A(r)}}{r^4} \right) dr - i \frac{\pi}{\kappa} e^{A/2} r \left[2(e^{-B/2} - 1) - \frac{rA'}{2} e^{-B/2} \right], \quad (21)$$

where κ is the surface gravity evaluated at $r = r_0$

$$\kappa = \left[\left| \frac{A'}{2} \right| e^{(A-B)/2} \right] \Big|_{r_0}. \quad (22)$$

The functions $A(r)$ and $B(r)$ are determined by the Einstein equations and are given by

$$\begin{aligned} e^{-B(r)} &= 1 + c_1/r + \eta(r)/r - Q^2/8\pi r^2 \\ e^{A(r)} &= \frac{c_2 e^{B(r)}}{r^2} \exp \left[\int (2 + Q^2/4\pi r^2) \frac{e^{B(r)}}{r} dr \right], \end{aligned} \quad (23)$$

where c_1 and c_2 are constants of integration and the function $\eta(r)$ is defined by

$$\eta(r) = 4\pi \int r^2 h(r)^2 dr. \quad (24)$$

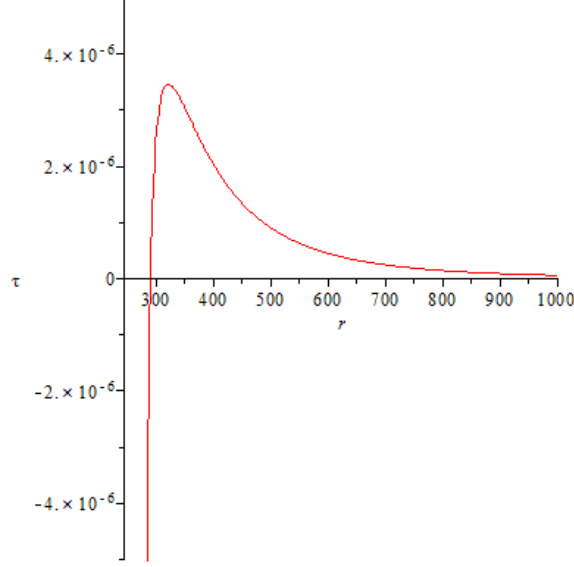


FIG. 4: The tension $\tau(r)$ plotted versus the radial coordinate r in dimensionless units.

This function satisfies the differential equation

$$\left[2r(r + c_1 + \eta) - \frac{Q^2}{4\pi}\right]\eta'' + 2\tau'(r + c_1 + \eta) - 2r\eta'(\eta' - 1) = 0, \quad (25)$$

and can be expressed as an infinite series

$$\eta(r) = \sum_{m=1}^{\infty} \frac{b_m}{r^m}, \quad (26)$$

whose expansion coefficients b_m satisfy the recursion relation

$$2m(m-1)b_m + 2(m-1)^2b_{m-1}c_1 - \frac{Q^2}{4\pi}(m-1)(m-2)b_{m-2} - 2\sum_{n=1}^{m-2}(m-2n-1)(n+1)b_{m-n-1}b_n = 0, \quad m \geq 2. \quad (27)$$

For $c_1 = 0$ an approximate solution can be obtained for $\eta(r)$ for small Q by making the change of variable

$$\eta(r) = -b/r + (Q^2/4\pi)\zeta(r), \quad (28)$$

where b is a wormhole parameter; for $Q = 0$ it is the throat radius. The equation for $\zeta(r)$ is

$$(r^2 - b)\zeta'' + (2r - 3b/r)\zeta' + (Q^2/4\pi)(r\zeta\zeta'' - r\zeta'^2 + \zeta\zeta') - (b/r^2)\zeta = 0. \quad (29)$$

When in this equation Q^2 is neglected as being much smaller than b , the solution to Eq. (29) is

$$\zeta(r) = \frac{a_1}{r} + \frac{a_2}{r} \left[\frac{1}{\chi(r)} + \arctan \chi(r) \right], \quad (30)$$

where a_1 and a_2 are constants of integration and $\chi(r) = \sqrt{b/(r^2 - b)}$. In the expression for $\zeta(r)$ the constant a_1 can be set to zero without loss of generality, or, equivalently, it can be absorbed into the constant b . With this approximate expression for $\zeta(r)$ the terms appearing in the action can be written as (to first order in Q^2)

$$h(r)^2 \approx \frac{4}{\kappa r^4} \left[b - (Q^2/4\pi) (a_2 \arctan \chi) \right], \quad (31)$$

$$e^B(r) \approx 1 + \chi^2 + \frac{Q^2}{4\pi b} \chi^2 (1 + \chi^2) \left[1 - 2a_2 \left(\frac{1}{\chi} + \arctan \chi \right) \right], \quad (32)$$

$$e^A(r) \approx 1 + \frac{Q^2}{8\pi b} \left[2a_2 \arctan \chi - \ln(1 + \chi^2) \right], \quad (33)$$

$$r_0 \approx \sqrt{b} + \frac{Q^2}{16\pi\sqrt{b}} (1 - \pi a_2), \quad (34)$$

$$\kappa \approx \left[\frac{Q^2}{8\pi b^{3/2}} \frac{\chi^2}{1 + \chi^2} |\chi - a_2| \right] \Big|_{r_0}, \quad (35)$$

Inserting these expressions into Eq. (21) and carrying out the integration over r gives

$$S = \left\{ \left(\frac{Q^2}{8\pi\sqrt{b}} + \frac{36\sqrt{b}}{\kappa} \right) \arctan \chi + \frac{9Q^2}{4\pi\kappa\sqrt{b}} \left[\chi - \arctan \chi - 2a_2 \chi \arctan \chi \right] \right\}_0^{\chi_{r_0}} - \frac{9Q^2}{4\pi\kappa\sqrt{b}} \int_0^{\chi_{r_0}} \frac{\ln(1 + \chi^2)}{1 + \chi^2} d\chi. \quad (36)$$

At r_0 the expressions for $\chi(r)$ and κ become

$$\chi_0 \approx \sqrt{\frac{8\pi b}{Q^2(1 - \pi a_2)}} \gg 1, \quad (37)$$

and

$$\kappa \approx \frac{|Q|}{2b\sqrt{2\pi(1 - \pi a_2)}}. \quad (38)$$

With these expressions the action in Eq. (36) is approximately

$$S \approx i \frac{8\pi}{\kappa} \left\{ \frac{\pi}{8\pi} \left(\frac{Q^2}{2\sqrt{b}} + \frac{36\sqrt{b}}{\kappa} \right) - \frac{9Q^2}{4\pi\kappa b} \left[\pi \ln 2 + \frac{\pi}{2} - (1 - \pi a_2) \frac{2b^{3/2}}{Q^2} \kappa \right] \right\} - i \frac{\pi}{\kappa} \frac{a_2 Q^2}{4\pi\sqrt{b}}. \quad (39)$$

The constants a_2 and b can be related to the ADM mass of the wormhole and to the constant h_0 . The ADM mass is given by an integral over the $r \rightarrow \infty$ boundary B of the manifold,

$$M = \oint_B r^i (\partial_j h_i^j - \partial_i h_j^j) = -8\pi(c_1 + \eta)|_{r \rightarrow \infty} = -2Q^2 a_2 / \sqrt{b}. \quad (40)$$

where h_{ij} 's are the induced metric tensor elements on the boundary B , which, in an asymptotic Cartesian system, are given by $h^{ij} = -\frac{c_1 + \eta}{r^3} x^i x^j$. An approximate expression for h_0 can be obtained from Eqs. (20) and (24)

$$h_0^2 \approx \frac{b}{24\pi}. \quad (41)$$

The K-R 'charge' is zero for this wormhole, because the only non-zero component of the field is transverse to the radial direction. The K-R field in the wormhole must come from an external source.

C. Wormhole Stability

The stability of a wormhole can be analyzed within the context of quantum gravity by evaluating the partition function obtained from the summation over all histories

$$Z = \int [dg] \exp(i S[g]) \quad (42)$$

in units where $\hbar = c = 1$. The decay probability of a body is determined by the probability of a gravitational instanton [15] tunnelling from it to the vacuum. This probability can be written as [16, 17]

$$P \sim e^{i S_E / \hbar} \quad (43)$$

where S_E is the Euclidean action, which for our EMKR wormholes is given by Eq. (39). After substituting in for κ (Eq. (38)) and a_2 (Eq. (40)), the dominant term in the action (39) is

$$S_E \approx i \frac{144 \sqrt{2} \pi^{3/2} c^7 b^2 M}{G_N^{3/2} Q^3}. \quad (44)$$

where the fundamental constants c and G_N are now explicitly displayed. This expression shows that wormholes produced with small charge and any measurable mass and geometric size are quasi-stable, since the probability for decay is extremely small. The possibility of forming quasi-stable, microscopic wormholes is due to the presence of the K-R field with parameter $h_0 = \sqrt{b/24\pi}$. The horizon radius, r_0 , is determined, at least in the approximation used in Eq. (34), principally by the K-R field strength, h_0 , and the mass, M . Because of the presence of the K-R field, the charge to mass ratio no longer has to be < 1 , unlike the case for charged black holes with no K-R field.

III. CONCLUSIONS

Since wormholes may exist or perhaps can be created in nature or artificially as particles, as baby universes, or as mechanisms for time travel, an analysis of their stability is essential. We have shown in the above analysis that a particular type of wormhole can be quasi-stable through achieving a suitable adjustment of the parameters of the system. The parameter h_0 associated with the K-R field, in the approximation that $Q^2 \ll b$, essentially determines the radius of the wormhole. Moderate values for the K-R coupling strength, $h_0^2 \approx 10^{-18}$ erg cm, correspond to $r_0 \approx l_p$, where l_p is the Planck length, and to energy densities a few orders of magnitude less than that for a Planck mass. The mass of the wormhole is not determined by the K-R coupling strength and so could be anything between zero and the Planck mass. Wormholes with masses on the order of standard model masses (< 1 TeV) could be created by high energy cosmic rays or in an accelerator, providing a realization of the original attempt of Einstein and Rosen [1] to describe a particle as a wormhole, needing only the requirement that a K-R field of suitable strength is present.

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