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A Proposed Search for the Detection of Gravitational Waves from Eccentric Binary Black Holes

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Most of compact binary systems are expected to circularize before the frequency of emitted gravitational waves (GWs) enters the sensitivity band of the ground based interferometric detectors. However, several mechanisms have been proposed for the formation of binary systems, which retain eccentricity throughout their lifetimes. Since no matched-filtering algorithm has been developed to extract continuous GW signals from compact binaries on orbits with low to moderate values of eccentricity, and available algorithms to detect binaries on quasi-circular orbits are sub-optimal to recover these events, in this paper we propose a search method for detection of gravitational waves produced from the coalescences of eccentric binary black holes (eBBH). We study the search sensitivity and the false alarm rates on a segment of data from the second joint science run of LIGO and Virgo detectors, and discuss the implications of the eccentric binary search for the advanced GW detectors.

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I. INTRODUCTION

The existence of gravitational waves (GWs) is the direct consequence of linearized gravity. Several endeavors have been undertaken for the detection of GWs. Laser Interferometer Gravitational-wave Observatory (LIGO) and Virgo Observatory are ground based interferometric detectors built for this purpose [1]. Of the many sources these detectors are designed to detect, compact binary coalescence (CBC) are the most sought after emitters of gravitational waves [1]. Binaries formed with large orbital separations and at low frequencies are expected to circularize before they enter the sensitivity band of ground-based GW detectors. [2], however, several dynamical-formation scenarios support the formation and merger of binary systems while retaining eccentricity throughout their lifetime. For example, significant number of eccentric binary black holes (eBBH) can form within galactic nuclei through 2-body scattering. The presence of a super massive black hole (SMBH) can create steep density cusps of stellar mass black holes providing suitable environment for runaway encounters. If two BHs loose sufficient energy in such an encounter they will form a bound system and merge within hours of its formation [3]. Another astrophysical scenario involves hierarchical triplets, modeled to consist of an inner and an outer binary. If the mutual inclination angle between the orbital planes of the inner and the outer binary is large enough, then the time averaged tidal force may induce oscillations in the eccentricity of the inner binary, known in the literature as the Kozai mechanism [4, 5]. Some other formation scenarios have been proposed for the formation of eBBH, suggesting eBBH as a potential GW source for the ground-based detectors [6, 7]. Overall, these scenarios suggest expected rate of coalescence detectable by advanced LIGO to be 1-2500 per year. The eccentricity of these sources, when they become visible to the detectors, depends on their formation mechanism. An eBBH formed in a galactic core is expected to have eccentricity larger than 0.9 at the time its orbital frequency is 5 Hz. On the contrary, eBBH formed in a globular cluster is expected to have low eccentricity and the eccentricity in the three body system is expected to oscillate. Detection of these sources offers rich information about their formation mechanics. Additionally, because of the high velocities involved during the periastron passage or presence of zoom-whirl behavior in the orbit can help probe into strong field regimes [8].

So far signals from merging stellar mass binary systems are searched mainly by matched-filtering the data with different families of templates. Separate searches have been conducted in the total mass range of 2 $M_\odot$ - 25 $M_\odot$ and 25 $M_\odot$ - 100 $M_\odot$ [9–13]. However, it has been shown that a large fraction of eBBH signals may be missed by the current template searches, which employ non-eccentric waveforms. Template searches are non-optimal for binaries with eccentricity more than 0.1 and currently only a dedicated search specifically targeting these systems can detect and study the rate of these sources [14–16].

Alternative methods have been proposed for the detection of eBBH (such as [17] for binaries with total mass less than 10 $M_\odot$). Moreover, there is an ongoing program that is developing a toolkit to detect and characterize eBBH events along with efforts in the GW modeling community [18, 19]. This paper is a significant step in that direction, in which we describe an, eBBH search using an excess energy (burst) method. These methods identify events by searching for a coincident appearance of the excess energy in two or more detector. Events surviving the consistency checks and criteria based on CBC model are admitted for further processing. Finally, we study the search sensitivity and the false alarm rates. We have performed a test search on the data obtained from the data obtained from the LIGO and Virgo detector from June of 2010 to October of 2010. Based on our results, we conclude that the advanced detectors can potentially detect eBBH signals, if the formation models predicting eBBH population hold true. In the event of null observation some of the models can be rejected at 90% confidence.

The paper is organized as follows: Section II contains information on the current ground based interferometric detectors. Section III presents an overview of the analysis. Section IV briefly describes the simulation studies. Section V reports the results of the test search and we conclude with discussions in Section VI.

II. DETECTORS

The LIGO and Virgo detectors are kilometer scale, ground-based GW detectors. LIGO detectors are located at Livingston, LA (L1) and Hanford, WA (H1) and Virgo detector is located at Casina, Italy (V1). Before the sixth scientific run, there was also a second detector operating in Hanford (H2). So far the detectors have conducted two joint runs and the last run ended in 2010. Since then the detectors underwent a period of upgrade to increase their sensitivity. As of October 2015 LIGO Livingston and Hanford detectors have started collecting data with around four fold increase in the sensitivity. The Virgo detector is expected to begin operation within the next year. The design sensitivity of the advanced detectors, expected to be achieved by the year 2019, is approximately an order of magnitude larger [20].

We performed a test eBBH search on the segment of data collected by the LIGO and Virgo detectors from June of 2010 to October of 2010 (S6D). We used three-fold L1-H1-V1 network to perform the search. The detector's output is affected by variety of noise sources of the environmental and instrumental origin, hence, only a subset of the original data surviving the data quality flags was used in the search [21, 22]. Data quality flags are classified into different categories. Data generated during detector malfunction or when the coupling between detector output and noise source is well understood is flagged. Data surviving these flags is searched for GWs. Events crossing a pre-selected threshold are saved for processing. Further, event-by-event flags are applied on the saved events. Flags where coupling between detector noise and noise source is not well understood are applied. Another set of data quality flags is used to remove events with weak environmental and instrumental correlations [23].

III. ANALYSIS OVERVIEW

A. Data Analysis Methodology

The eBBH search uses the coherent waveburst (cWB) method [24] to identify candidate events. cWB has been used for several burst searches on the S5-VSR1 and S6-VSR2/3 data [25–28]. Recently cWB has been upgraded extensively [29]. Use of multi-resolution Wilson-Daubechies transforms [30] was implemented to maximize collection of SNR from an event, resulting in recovery of more than 90% of the signal-to-noise ratio (SNR) for binaries with total mass 20 $M_\odot$ or more. cWB performs multi-resolution time-frequency (TF) analysis of the detector data and searches for coincidental appearance of excess energy in the network data. Excess energy TF pixels are extracted and are clustered to form an event. Thereafter, a likelihood analysis is performed on the event and the expected signal is reconstructed. Two major coherent statistics are obtained during the likelihood analysis. Overall consistency of the event is quantified by the network correlation coefficient (cc). The GW events are expected to have a cc value close to unity, its maximum possible value. The strength of an event is quantified by the coherent network amplitude (η). η is proportional to the SNR of the event.

We employ two model based constraint to suppress background induced events. The eBBH signals are expected to have elliptical polarization. Events that do not satisfy this constraint are discarded [29]. It is possible to identify significant events at this stage but as the eBBH signals are expected to have a chirping TF signature, only events with a chirping TF signature are admitted for further processing. The selection is made based on the reconstructed chirp mass and two goodness of fit parameters ("energy fraction" and "ellipticity") of the events. These parameters are estimated during the processing of the data [31].

B. Background Estimation

The false alarm rate of the background induced events are estimated by identifying events after performing relative time-slides between the data from different detectors. Data is shifted by a duration which is much longer than the maximum travel time of a GW between detectors, thereby removing existence of any coincident GWs. The cWB parameter space is not constrained, because of its eyes wide open approach and allows for the detection of multiple types of signals. Because of that, the cWB method is affected by background noises. Background induced events are suppressed by the application of elliptic polarization and reconstructed chirp mass constraint. Effect of these constraint is shown in Figure 1. The number of background events are reduced by three orders of magnitude.

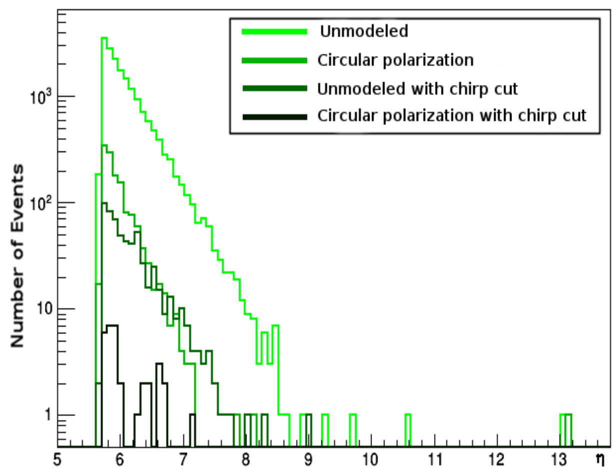

FIG. 1. Distribution of coherent network amplitude, η, for the time shift-analysis performed over S6D (LHV) network.

IV. SIMULATION

The search sensitivity is quoted as the visible volume for the eBBH mergers. Simulation studies are performed by processing the data injected with the eBBH waveforms. In the eBBH search we introduce an astrophysical model to populate the parameter space from which injection parameter are randomly derived. This gives an opportunity to quantify the sensitivity of the search based on the parameters defining the astrophysical model. The range of these parameters can be constrained by using the results obtained from the eBBH search.

A. Formation Model

It is now well established that SMBHs are ubiquitous in the nuclei of nearby galaxies [32]. Theoretical studies suggest that mass segregation from individual objects and satellite stellar systems may lead to a high density of compact objects in the galactic center cusp [33, 34]. Multiple works show that black holes with mass around 10 $M_\odot$ should segregate to the inner parsec [35–38].This dense stellar environment is ideally suited for the formation of eBBH, when runaway encounters can result is capture and quick merger of the binary. We model this astrophysical motivated scenario by implementing the model proposed by Kocsis and Loeb [3].

The parameter defining the astrophysical model is the mass of the SMBH and β, which defines the mass function as

$$\frac{\mathrm{d}N}{\mathrm{d}m} \propto m^{-\beta}\,, \tag{4.1}$$

where N is the number of binaries with mass m. The radial distribution of the stellar mass black holes in the galactic cusp is influenced the mass range of the stellar mass black holes. For the example eBBH search we fixed the mass range from 5 $M_\odot$ to 25 $M_\odot$. Events are generated for the provided radial and mass distribution of the stellar mass black hole inside the cusp and mass of the SMBH (events are generated based on Equation 31 of [3], we do not consider other corrections considered in the paper). Once a binary is formed, periastron distance (r_p) is evolved in time, until the burst frequency, defined as,

$$f_{\mathrm{avg}} = \frac{1}{\pi M}\frac{1}{\sqrt{2}r_p^{3/2}}, \tag{4.2}$$

is achieved, where M is the total mass of the binary.

B. Simulated Waveforms

We are using a waveform model that describes the inspiral, merger and ringdown of compact binaries on eccentric orbits [39]. The model is based on mapping the binary to an effective single black hole system described by a Kerr metric, thereby including certain relativistic effects such as zoom-whirl-type behavior. Mass and total angular momentum of the binary are identified with the mass and spin parameters of the effective Kerr space time and the orbital angular momentum and energy with that of the geodesics. The parameters are evolved with dissipation coming from the quadrupole radiation term. This approach has the advantage of reproducing the correct orbital dynamics in the Newtonian limit and general-relativistic test particle limit, while still incorporating strong-field phenomena such as pericenter precession, frame dragging and the existence of unstable orbits and related zoom-whirl dynamics. This search does not require waveforms to have accurate phase evolution. Moreover there was very weak dependence of the search sensitivity on the eccentricity of the binaries (discussed later). In terms of amplitude, the eBBH wavefrom model agreed with the numerical relativity waveforms to within 10% [40, 41].

C. Visible Volume

The visible volume, also called the sensitive volume, of the search is defined as,

$$V_{\rm vis}(m_1, m_2, e, r_p, \eta) = 4\pi \int_0^\infty \epsilon(m_1, m_2, e, r_p, \eta) r^2 {\rm d}r, \tag{4.3}$$

where ϵ is the detection efficiency of the search. Using following equations,

$$\epsilon = {\rm d}N_{\rm det}/{\rm d}N_{\rm inj}, \frac{1}{\rho_i} = \frac{4\pi r_i^2}{{\rm d}N_{\rm inj}/{\rm d}r}, \tag{4.4}$$

where $N_{\rm inj}/N_{\rm det}$ is the number of injected/detected injections, ${\rm d}N_{\rm inj}/{\rm d}r$ is the radial injection density and ρ_i is the volume density, Equation 4.3 becomes

$$V_{\rm vis} = \sum_i^{N_{\rm det}} \frac{1}{\rho_i}. \tag{4.5}$$

The index i runs over all the recovered injections. An elaborate discussion on the estimation of visible volume is also available in [12].

D. False Alarm Rate Density and Event Significance

The significance of a foreground event can be determined by estimating its false alarm rate, defined as

$${\rm FAR}(\eta) = \frac{N(\eta)}{T}, \tag{4.6}$$

where η is event's coherent network amplitude, T is the accumulated livetime and $N(\eta)$ are the number of background events with coherent network amplitude greater than η. However, FAR values can not be used to compare significance of events across different networks. Searches can be combined by using the False Alarm Rate Density (FAD) statistic, which is defined as,

$${\rm FAD}(\eta_j) = \min\left(\frac{{\rm FAR}(\eta_j)}{V_{\rm vis}(\eta_j)}, \frac{{\rm FAR}(\eta_{j-1})}{V_{\rm vis}(\eta_{j-1})}\right). \tag{4.7}$$

Events are ranked based on their FAD values with significant events having lower FAD rates.

To determine the event's significance, its FAD rate is compared to the time-volume product of the combined search given by

$$\nu = \sum_k T_{{\rm obs},k} V_{\rm vis}({\rm FAD}), \tag{4.8}$$

where the index k runs over all the detector networks. The mean of number of such events produced from background noise is

$$\mu({\rm FAD}) = {\rm FAD} \times \sum_k T_{{\rm obs},k} V_{\rm vis}({\rm FAD}). \tag{4.9}$$

Assuming FAD of the background events follows Poisson distribution, the false alarm probability (FAP) is given by

$${\rm FAP}(\eta) = 1 - \sum_{n=0}^{N-1} \frac{\mu^n}{n!} \exp(-\mu). \tag{4.10}$$

V. RESULTS

In this section we discuss the projected sensitivity and expected rates for eBBH mergers based on the results obtained in the test run. Figure 2 shows the sensitive distance for eBBH sources as a function of the component masses. Only recovered injections with FAR value of once in five years or less have been used to estimate the sensitive distance. The corresponding visible volume, estimated to be $\sim 10^7 {\rm Mpc}^3$, is expected to increase by more than three orders of magnitude for advanced detectors. The eBBH merger rate of $\sim 10^{-9}$ per galactic nuclei, when averaged over SMBH density, results in eBBH coalescence rate of $\sim 10^{-10} {\rm Mpc}^{-3}$ [42, 43]. With these numbers, advanced detectors are expected to observe an average of one detection per observation year. There are astrophysical models projecting per galactic merger rate to be as high as $\sim 10^{-5}$ (including a factor of ~ 30 due to variance in the central number density of BHs). If these models hold true we expect to detect multiple eBBH signals with the advanced detectors. On the other hand, in the event of a null observation some of the astrophysical models can be rejected with confidence [44, 45].

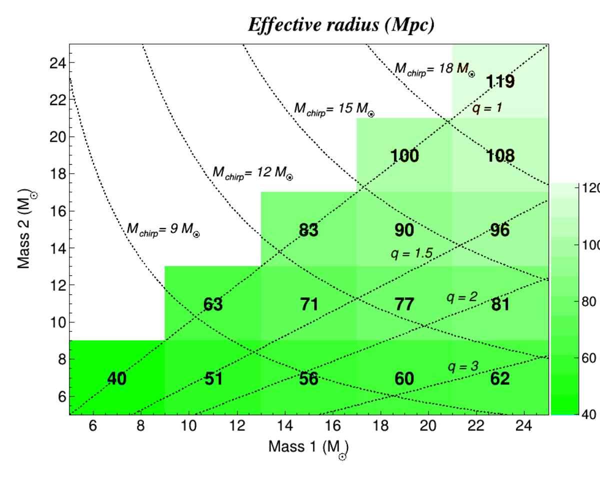

FIG. 2. The effective range $R_{\rm eff}$ in Mpc over component mass bin: S6D L1H1V1 network. The dotted contours represent constant mass ratio(q) and chirp mass($M_{\rm chirp}$). Overall, the sensitive distance increases with the increase in the chirp mass and decrease in the mass ratio (q).

A matched filtering search is an ideal choice for CBC sources, however, we can comment on the approximate fraction of eBBH signals the proposed search can detect.

Figure 3 plots the efficiency as a function of the eccentricity of binary at orbital frequency of 48 Hz. Efficiency is defined as the number of recovered injections divided by the number of injections. The injections have a fixed sky location. The efficiency does not show a visible trend for lower mass binaries. As expected, heavier binaries show minor increase in efficiency with increasing eccentricity (increased contribution from higher order modes). The search leaves the parameter space unconstrained in eccentricity, hence, the proposed eBBH search will also detect circular binaries with approximately equal efficiency. The effective radius for the example run is approximately 80% of the matched filtering search [12, 13] performed for circular binaries. Hence, we expect the proposed search to recover half of the events, which could have been otherwise recovered by a matched filtering search using accurate waveforms of binaries on eccentric orbits.

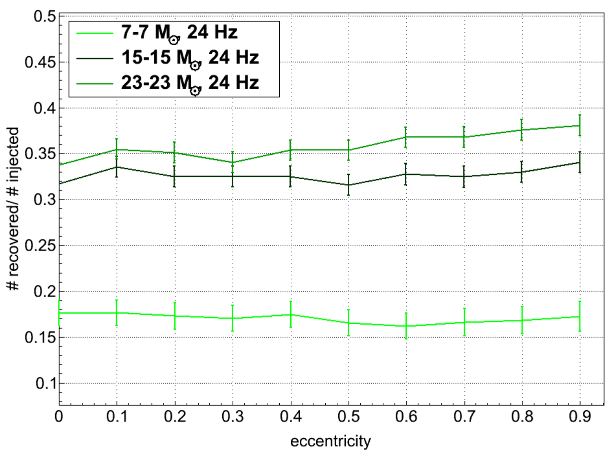

FIG. 3. Efficiency vs eccentricity at an orbital frequency of 24 Hz. Injection were made for three different masses. Heavier binaries show minor increase in efficiency with increasing eccentricity (efficiency values depend on the chosen injection distance)). There is no visible trend for lower mass binaries.

VI. DISCUSSION

We have introduced a novel search focused at the detection of GWs from eccentric binary black hole mergers. The search uses cWB algorithm to identify the events. A time-shift analysis is performed to estimate the background and simulation are performed to estimate sensitivity of the search. The search can use model based constraints, such as, polarization constraint and reconstructed chirp mass constraint to suppress the background. We show that these constraints suppress the background by three orders of magnitude. We describe FAD statistic which can be used to rank the events according to their significance.

We performed an example run and based on the obtained results we conclude that advanced detectors will detect multiple eBBH signals if the proposed astrophysical models hold true. The search will detect approximately half of the events a matched filter search would have detected. The search employs astrophysical model to populate the parameters space providing the opportunity to gauge the sensitivity of the search in terms of the parameters defining the astrophysical model. Hence, in the event of null observation it will become possible to reject some of the optimistic models.

VII. ACKNOWLEDGEMENT

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