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Earth's Stopping Effect in Directional Dark Matter Detectors

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We explore the stopping effect that results from interactions between dark matter and nuclei as the dark matter particles travel undergound towards the detector. Although this effect is negligible for heavy dark matter particles, there is parameter phase space where the underground interactions of the dark matter particles with the nuclei can create observable differences in the spectrum. Dark matter particles that arrive on the detector from below can have less energy from the ones arriving from above. These differences can be potentially detectable by upcoming directional detectors. This can unveil a large amount of information regarding the type and strength of interactions between nuclei and light dark matter candidates.

I. INTRODUCTION

There is strong evidence for the existence of Dark Matter (DM) nowdays. Searches for DM include efforts for laboratory production (e.g. LHC), possible indirect signals from the galaxy and beyond (e.g. due to annihilation or decay of DM to conventional photons or other Standard Model paticles), and direct detection where underground detectors could potentially register rare collisions between an incoming DM particle and a nucleus in the detector. Current direct search experiments can register events with a particular recoil energy, but they cannot identify the direction of the recoil. However, a new generation of experiments that can detect also the direction of the recoil is on the way [1–8]. The directional detection of these experiments is based on time projection diffuse gas chambers that have the capability of recostructing the nuclear recoil track, giving thus information about the direction of the incoming DM particle. Additionally there are directional detectors that are based on different techniques such as nuclear emulsion on solid detectors [9], and DNA detectors [10]. Although the above experiments are not yet competitive in setting DM limits with respect to the current non-directional underground detectors, they could start probing interesting DM parameter space in the near future.

Directional DM detectors once competitive to non-directional ones, can provide an immense amount of information that cannot be obtained otherwise. Conventional detectors register counts with particular recoil energies. Although the number of expected counts depends on the velocity distribution of the DM halo particles, it is hard to extract useful information regarding the form of the distribution function due to the fact that for a particular amount of nuclear recoil energy, all DM particles with velocities above a specific value could produce the observed recoil. The number of counts in the detector is proportional to an integral of the DM velocity distribution, thus making hard to reveal the exact shape of this distribution. On the contrary in directional detectors, the directional rate of counts per recoil energy is associated with the Radon transformation of the DM velocity distribution [11]. This can be in principle inverted and therefore one can obtain direct correspondence between the velocity distribution and the amount of registered counts on the detector. Furthermore, directional detection can help on two other fronts. On the one hand, it is much easier to eliminate background contamination with directional detectors. Known sources of contamination such as for example the sun can be easily eliminated. On the other hand, directional detectors can reveal information about possible substructure of the DM halo. Directional detectors could shed light on the possibility of DM streams and dark discs in the galaxy. This would be something almost impossible to probe with conventional detectors.

The recoil energy spectrum of DM scattering off nuclei in directional detectors has been studied extensively first in [11] and later in [12–22]. In all the above papers, the energy recoil spectrum has been studied for the two generic cases of spin-independent and spin-dependent DM-nucleon contact interactions. An extension to more generic non-relativistic scattering operators was studied in [23, 24].

In this paper we study the stopping effect of the earth in directional DM detectors. DM particles can arrive at the detector from different angles, having traveled different distances underground. Although DM particles are expected to interact feebly with nucleons, as it was pointed out in [25], there is DM parameter space especially for light DM candidates where DM-nuclei interactions as the particle travels underground might have an observable effect on the recoil energy spectrum of the detectors. There are two ways that underground DM-nuclei interactions can affect the spectrum. The first one is that particles traveling larger distances underground, might lose enough energy due to interactions, so by the time they reach the detector might not have enough energy to create a recoil above the threshold of the detector. This clearly creates an asymmetry between the amount of registered counts in the detector coming from above and from below. Additionally, DM-nuclei interactions

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can cause also the opposite effect for heavy enough DM particles. DM particles that interact undeground as they travel towards the detector, slow down. This reduction in the velocity might increase the DM-nucleus cross section and therefore the probability of detecting it. This is because in a variety of models the cross section is inversely proportional to some power of the velocity (e.g. for contact interactions $\sigma \sim 1/v^2$). The study of this stopping effect of the earth was studied in the context of conventional non-directional detectors in [25, 26]. In this case, since there is no way to know the direction of the recoiled nucleus, the effect can be seen indirectly via the observation of a diurnal modulated signal. As it was demonstrated in [25], since the earth moves with a nonzero velocity with respect to the rest frame of the galaxy, a daily varying DM signal is created because as the earth rotates around its own axis, the DM particles coming from the direction of the DM wind travel different distances underground at different times during a sidereal day. The observation of such a diurnal modulated signal can reveal information about the nature of DM-nucleon interactions. Additionally for detectors placed on shallow sites, this technique might be one of the few options available to probe light DM parameter space with long range forces that is currently inaccessible to detectors. Diurnal modulation has been investigated in the past in the context of Strongly Interacting Massive Particles [27, 28] and mirror DM [29, 30], as well as experimentally in the DAMA Collaboration [31, 32].

In this paper we study the same stopping effect of the earth in a more direct way, which is in the context of directional DM detectors. One should not have to rely on a diurnal modulated signal in order to probe the asymmetry in the spectrum between DM particles scattering from below and above. The paper is organised as follows: in section II we review the stopping power of DM particles due to DM-nuclei interactions. In section III we will derive the formalism for the directional recoil spectrum. In section IV we will present our results. Finally we conclude in section V.

II. NUCLEAR STOPPING

DM particles can lose energy by interacting with nuclei or electrons as they travel underground towards the detector. DM particles from the halo do not have sufficient energy to ionize atoms as they travel underground. They can lose energy either by interactions with nuclei, or if allowed, by interactions with electrons. The latter can be either in the form of DM interactions with electrons in metallic layers of the earth, or in the form of DM-electron interactions that result in atomic excitations [25]. The determination of the most effective mode of decelerating DM particles depends strongly on the type of DM-nucleus interactions as well as the precise geological composition of the earth. For example contact or long range forces between DM and nuclei can result to different degrees of DM deceleration inside the earth. In this work here, we are going to consider only nuclear stopping. This is because nuclear stopping is quite insensitive to the geological composition of the earth. For example DM-electron interactions in metallic layers of the earth can give significant amounts of stopping because electrons there behave as a free Fermi gas that does not have an energy gap and therefore it can subtract energy from incoming DM particles by small bits at the time. However, they are model dependent, depending strongly on the geological morphology of the earth. For simplicity, we are going to consider contact spin-independent DM-nucleon interactions here. Our goal is to make a first generic estimate on the possibility of observing the stopping effect of the earth in directional detectors. Additional stopping modes for DM particles can only enhance the effect. Moreover we are going to assume a flat density for the earth of $\rho_e = 5.5\text{gr/cm}^3$. This will enable us to obtain more transparent results regarding the spectrum of the recoiled energy in directional detectors.

For a DM particle moving through a medium, the energy loss per distance traveled is given by

$$\frac{dE}{dx} = -\sum_i n_{N_i} \int_{E_{Ri}^{\min}}^{E_{Ri}^{\max}} \frac{d\sigma_i}{dE_R} E_R dE_R, \qquad (1)$$

where E_R is the recoil energy of the target nucleus, n_{N_i} is the number density of nuclei N_i and $d\sigma_i/dE_R$ is the differential cross section between N_i and DM which in the case of contact spin-independent interactions is given by

$$\frac{d\sigma}{dE_R} = \frac{m_N \sigma_N}{2\mu_N^2 v^2} F^2(E_R) = \frac{m_N \sigma_p A^2}{2\mu_p^2 v^2} F^2(E_R), \qquad (2)$$

where v is the DM velocity, m_N is the mass of the target nucleus, A the number of nucleons in the nucleus and μ_p (μ_N) the reduced mass between DM and proton (nucleus). σ_N and σ_p are correspondingly the DM-nucleus and DM-nucleon cross sections. $F^2(E_R)$ is the usual form factor that accounts for loss of coherence. We choose a simple form factor of the form

$$F^2(E_R) = e^{-E_R/Q_0}, \qquad (3)$$

where $Q_0 = 3/(2m_N r_0^2)$ and $r_0 = 0.3 + 0.91(m_N)^{1/3}$ is the radius of the nucleus measured in femtometers when m_N is in GeV [33]. The sum in Eq. (1) runs over all the elements found in the earth. We are going to include the three most abundant elements in the earth i.e. iron, oxygen and silicon. Once again, the error is in the right direction, i.e. extra contributions from other elements can only enhance the stopping effect of the earth we study here. The integral of Eq. (1) has lower and upper limits $E_{Ri}^{\min}$ and $E_{Ri}^{\max}$ respectively. $E_{Ri}^{\max} = 4m_X m_N E/(m_X + m_N)^2$ is the maximum recoil energy given a DM particle of energy E (m_X being the DM mass). For perfect contact interaction $E_{Ri}^{\min} = 0$. However in a realistic case, contact interactions might result by integrating out heavy mediators. For example in a Yukawa type of interaction between DM and nucleons where a mediator of mass m_ϕ is exchanged, DM and nucleon should come closer than a distance m_ϕ^{-1}. This requires the exchange of a mediator with energy determined by uncertainty principle of at least $E_R^{\rm min} = m_\phi^2/(2m_N)$. Upon writing $v^2 = 2E/m_X$, Eq. (1) can be integrated to

$$\frac{dE}{dx} = -\sum_N \frac{2n_N\sigma_p A^2\mu_N^4 E}{m_X m_N \mu_p^2}, \tag{4}$$

where we used $E_{Ri}^{\rm min} << E_{Ri}^{\rm max}$ and $E_{Ri}^{\rm max} << Q_0$ (the last is especially true for low DM masses that we are particularly interested). The sum runs over the three most abundant elements (iron, oxygen and silicon). The final trivial integration upon the assumption that the density and composition of the earth is constant, gives

$$\ln\frac{E_{in}}{E_f} = \sum_{N_s} \frac{2n_{N_s}\sigma_p A_s^2\mu_{N_s}^4 L}{m_X m_{N_s} \mu_p^2}, \tag{5}$$

where E_{in} and E_f are respectively the initial and final kinetic energies of the DM particle and L is the total length traveled underground. Note that we have added an index s in A , n_N , m_N and μ_N in order to distinguish the nucleus responsible for the deceleration of the DM particles (i.e. iron, oxygen and silicon) from the nucleus that serves as a target in the detector. Eq. (5) can be rewritten in terms of velocities as

$$v' = ve^{-\Delta L}, \tag{6}$$

where v' and v are the final velocity (after the particle has traveled L underground) and initial velocity (before the particle enters the earth) of the DM particle. Δ is

$$\Delta = \sum_{N_s} \frac{n_{N_s}\sigma_p A_s^2\mu_{N_s}^4}{m_X m_{N_s}\mu_p^2}. \tag{7}$$

III. RECOIL ENERGY SPECTRUM

We are going to consider now the energy recoil spectrum in directional detectors. Generally, the rate of counts (counts per time) per recoil energy per solid angle is [11]

$$\frac{d^2R}{dE_R d\Omega_q} = N_T n_\chi \int \frac{d^2\sigma}{dE_R d\Omega_q} f(v) v d^3v. \tag{8}$$

E_R is the recoil energy, Ω_q a solid angle around the recoil direction $\hat{q}$, $f(v)$ is the DM velocity distribution in the detector reference frame, N_T is the number of nuclei targets in the detector and $n_\chi = 0.3{\rm GeV\ cm}^{-3}/m_X$ is the DM number density in the earth's neighborhood. The directional differential cross section is related to the non-directional one as

$$\frac{d^2\sigma}{dE_R d\Omega_q} = \frac{1}{2\pi}\delta(\cos\theta - \frac{v_{\rm min}}{v'})\frac{d\sigma}{dE_R}, \tag{9}$$

where $v_{\rm min} = \sqrt{m_N E_R/(2\mu_N^2)}$ is the minimum velocity that can produce a recoil energy E_R. μ_N here is the reduced mass between DM and the target nucleus of the detector N, and θ is the angle between the velocity of the DM particle and the direction of the recoiled nucleus $\hat{q}$. Using Eqs. (2) and (9) in (8), we get

$$\frac{d^2R}{dE_R d\Omega_q} = \kappa \int \frac{1}{v'^2}\delta(\cos\theta - \frac{v_{\rm min}}{v'}) f'(x',v')v' d^3v', \tag{10}$$

where $\kappa = N_T n_\chi m_N \sigma_p A^2 F^2(E_R)/(4\pi\mu_p^2)$. One should keep in mind that N and A in the above equation refer to the target element of the detector. The reader should also notice that the $1/v'^2$ dependence inside the integral comes from the fact that the scattering between DM and nucleus takes place with a DM velocity v' which is smaller than the velocity of DM before enters the earth and is given by Eq. (6). Similarly the flux is given by the distribution of DM $f'(x',v')v'$ at the location of the detector, which is not the same as the DM flux at the surface of the earth $f(v)v$ (where no DM deceleration has taken place). We can find a relation between $f'(x',v')$ and $f(v)$ by using Liouville theorem. Let us approximately consider that DM moves on a straight line underground and DM-nuclei interactions inside the earth decelerate the particle but they don't deflect it from its path. This approximation is definitely valid in DM-nucleon long range interactions as well as in some types of contact interactions where the forward scattering is favored. In any case one can consider the straight line approximation as a conservative limit for the asymmetry we are going to study, i.e. the difference in the rate of events between DM particles that arrive at the detector from the top and the bottom. This is because deflection from the straight line of the upcoming DM particles will increase further the asymmetry. One can show on generic grounds that the number of deflected DM particles out of the path that leads to the detector is larger than that of particles that deflect inside the path of the detector as long as forward scattering is relatively favored. The distribution of DM as it enters the earth is governed by the Boltzmann equation

$$\frac{\partial f}{\partial t} + v_i\frac{\partial f}{\partial x_i} + a_i\frac{\partial f}{\partial v_i} = 0, \tag{11}$$

where we assumed that no collisions take place among DM particles. The acceleration a_i results from the force induced by the DM-nuclei undeground scatterings and this force is treated as an external one. Since we are interested in steady state solutions, one can set $\partial f/\partial t = 0$. This means that $f(x_i(t), v_i(t))$ remains constant along the trajectory of a DM particle (which is a straight line underground). This is a manifestation of the Liouville theorem and therefore $f(v) = f'(x',v')$, i.e. the distribution at the detector is equal to the one before DM enters the earth. Using this fact as well as $d^3v' = e^{-3\Delta L}d^3v$ (see Eq. (6)) we can rewrite Eq. (10) as

$$\frac{d^2R}{dE_R d\Omega_q} = \kappa \int \delta(\cos\theta - \frac{v_{\rm min}}{v}e^{\Delta L})\frac{f(v)}{v}e^{-2\Delta L}d^3v. \tag{12}$$

This is the main formula we are going to use in order to probe the stopping effect of the earth. In particular, we are going to consider the asymmetry in the directional rate between the two directions that give the largest possible difference, i.e. $\hat{q} = \hat{n}$ and $\hat{q} = -\hat{n}$, where $\hat{n}$ is the direction from the center of the earth to the position of the detector. This two directions correspond to particles that travel the shortest distance underground ($\hat{q} = -\hat{n}$) and the largest one ($\hat{q} = \hat{n}$).

We are going to use a truncated Maxwell distribution

$$f(v) = \frac{1}{\mathcal{N}} \exp\left[-\frac{(\vec{v} + \vec{v}_e)^2}{v_0^2}\right], \; v < v_{\rm esc} + v_e, \qquad (13)$$

where $\mathcal{N}$ is a normalization constant, v_e is the velocity of the earth with respect to the rest frame of the DM halo, and $v_{\rm esc} = 550{\rm km/\sec}$ is the escape velocity from Milky Way. It is understood that the velocity distribution is in the labaratory frame (boosted by $\vec{v}_e$). The length traveled underground by a DM particle is given by

$$\begin{aligned} L &= (R_\oplus - \ell_D)\cos\psi \\ &+ \sqrt{(R_\oplus - \ell_D)^2\cos^2\psi - (\ell_D^2 - 2R_\oplus \ell_D)}, \end{aligned} \qquad (14)$$

where $\cos\psi = \hat{v}\cdot\hat{n}$ represents the angle between the DM velocity and the upper direction of the detector $\hat{n}$. $R_\oplus$ and ℓ_D are the earth's radius and the depth of the detector respectively.

Let us consider in some detail the different directions and angles involved in the problem. Following [25] we define θ_l to be the latitude of the detector, and we choose the z-axis with direction south-north pole. α is the angle between $\vec{v}_e$ and the z-axis. We choose the orientation of the $x-y$ plane so $\vec{v}_e$ lies along the $x-z$ plane. In this reference system choice we have the following relations

$$\hat{n} = \hat{x}\cos\theta_l\cos\omega t + \hat{y}\cos\theta_l\sin\omega t \pm \hat{z}\sin\theta_l, \qquad (15)$$

$$\hat{v}_e = \hat{x}\sin\alpha + \hat{z}\cos\alpha, \qquad (16)$$

where ω is the angular velocity of the self-revolution of the earth. The $\pm$ corresponds to locations at the north and south hemisphere. We have chosen $t = 0$ the time where $\vec{v}_e$ and $\hat{n}$ align as much as possible, i.e. $\hat{n}$ is along the $x-z$ plane. Eq. (13) can now be rewritten as

$$f(v) = \frac{1}{\mathcal{N}} e^{-\frac{v^2+v_e^2}{v_0^2}} e^{-\frac{2vv_e}{v_0^2}\cos\delta}, \qquad (17)$$

where δ is the angle between $\vec{v}$ and $\vec{v}_e$. In order to find δ we express the WIMP velocity $\vec{v}$ as

$$\vec{v} = v(\hat{x}\sin\theta\cos\phi + \hat{y}\sin\theta\sin\phi + \hat{z}\cos\theta), \qquad (18)$$

where we use the usual polar angles θ and ϕ to characterize $\vec{v}$. The angle δ now reads

$$\cos\delta = \hat{v}\cdot\hat{v}_e = \sin\alpha\sin\theta\cos\phi + \cos\alpha\cos\theta. \qquad (19)$$

A comment is in order here. θ is the angle between the recoil direction $\hat{q}$ and $\hat{v}$. Since in this case $\hat{q} = \hat{n}, \theta$ is the angle between $\hat{n}$ and $\hat{v}$. However, as it can be seen from Eq. (15), the vector $\hat{n}$ has $\hat{x}$ and $\hat{y}$ time varying components, while $\hat{v}$ in Eq. (18) is expressed in spherical coordinates around the z-axis. Since we have chosen $\hat{q} = \hat{n}$, $\cos\theta$ should express the angle between $\hat{n}$ and $\hat{v}$ and not the angle between $\hat{z}$ and $\hat{v}$. In order to simplify our calculation and without introducing a big error in our estimate, we take the time average value of $\hat{n}$ which concides with $\hat{z}$. In other words within our approximation we have assumed that $\hat{n} = \hat{z}$ and therefore θ of Eq. (18) coincides with the definition of θ being the angle between $\hat{v}$ and the recoil direction $\hat{q}$.

We are interested in the difference on the directional detection rate between the direction $\hat{q} = \hat{n}$, i.e. the direction coming from the center of the earth to the detector and $\hat{q} = -\hat{n}$ (the opposite one). Practically speaking, we probe the asymmetry in the directional detection rate between events in the detector that come from below and from above. In the case where $\hat{q} = \hat{n}$, one can notice that $\psi = \theta$. Let us define $y = \cos\theta$, and y_+ that satisfies

$$y_+ = \frac{v_{\rm min}}{v} e^{\Delta L_+(y_+)}, \qquad (20)$$

where $L_+(y) = L$ defined in Eq. (14) (with $\cos\theta \to y$). y_+ in Eq. (20) is nothing else but the value of y (or $\cos\theta$) that makes the arguement inside the delta function of Eq. (12) zero. Eq. (12) can be written as

$$\frac{d^2R}{dE_R d\Omega_n} = \frac{\kappa}{\mathcal{N}} \int e^{-\frac{v^2+v_e^2+2vv_e\cos\delta}{v_0^2}} v e^{-2\Delta L_+} dv d\phi. \qquad (21)$$

Recall that $\exp[-2\Delta L_+] = v_{\rm min}^2/(v^2 y_+^2)$ (from Eq. (20)) and that y_+ is a function of v and E_R. Using this and Eq. (19), we have

$$\begin{aligned} \frac{d^2R}{dE_R d\Omega_n} &= \frac{\kappa}{\mathcal{N}} \int \exp\left[-\frac{v^2 + v_e^2 + 2vv_e(y_+\cos\alpha + \sin\alpha\sqrt{1-y_+^2}\cos\phi)}{v_0^2}\right] \frac{v_{\rm min}^2}{v y_+^2} dv d\phi \\ &= \frac{2\pi\kappa}{\mathcal{N}} \int_{v_1}^{v_{\rm esc}+v_e} \exp\left[-\frac{v^2 + v_e^2 + 2vv_e y_+\cos\alpha}{v_0^2}\right] \mathcal{I}_0\left(\frac{2vv_e}{v_0^2}\sin\alpha\sqrt{1-y_+^2}\right) \frac{v_{\rm min}^2}{v y_+^2} dv, \end{aligned} \qquad (22)$$

where we have used $\sin\theta = \sqrt{1-y_+^2}$, and we have integrated over ϕ in the second line. $\mathcal{I}_0$ is the modified Bessel function of the first kind. The minimum velocity v_1 is the solution of $v_1 = v_{\rm min}\exp\{\Delta L_+[y_+(v_1, E_R)]\}$. As we mentioned L_+ is a function of y_+ which is a function of v.

Let us look now on the directional rate from above ($\hat{q} = -\hat{n}$). In this case $\psi = \pi - \theta$ (and $\cos\psi = -\cos\theta$) and the distance traveled underground of Eq. (14) becomes

$$L_- = -(R_\oplus - \ell_D)\cos\theta + \sqrt{(R_\oplus - \ell_D)^2\cos^2\theta - (\ell_D^2 - 2R_\oplus \ell_D)}. \quad (23)$$

Since $\hat{q} = -\hat{n} \simeq -\hat{z}$ (the last equality holding as a time average of Eq. (15)), one should express $\hat{v}$ in spherical coordinates but with $\hat{z} \to -\hat{z}$. In this case the angle δ between $\hat{v}$ and $\hat{v}_e$ picks up a relative minus sign in the second term of Eq. (19), thus reading

$$\cos\delta = \hat{v}\cdot\hat{v}_e = \sin\alpha\sin\theta\cos\phi - \cos\alpha\cos\theta. \quad (24)$$

The directional recoil rate can be written as

$$\begin{aligned}\frac{d^2R}{dE_R d\Omega_{-n}} &= \frac{\kappa}{\mathcal{N}}\int \exp\left[-\frac{v^2+v_e^2-2vv_e(y_-\cos\alpha - \sin\alpha\sqrt{1-y_-^2}\cos\phi)}{v_0^2}\right]\frac{v_{\rm min}^2}{vy_-^2}dvd\phi \\ &= \frac{2\pi\kappa}{\mathcal{N}}\int_{v_2}^{v_{\rm esc}+v_e}\exp\left[-\frac{v^2+v_e^2-2vv_e y_-\cos\alpha}{v_0^2}\right]\mathcal{I}_0\left(\frac{2vv_e}{v_0^2}\sin\alpha\sqrt{1-y_-^2}\right)\frac{v_{\rm min}^2}{vy_-^2}dv,\end{aligned} \quad (25)$$

where y_- is defined as the number that satisfies

$$y_- = \frac{v_{\rm min}}{v}e^{\Delta L_-(y_-)}. \quad (26)$$

v_2 is defined as the solution of $v_2 = v_{\rm min}\exp\{\Delta L_-[y_-(v_2, E_R)]\}$. In the second line of the equation we have performed the integration over ϕ. We are interested in the asymmetry on the directional recoil rate between $\hat{n}$ and $-\hat{n}$. However the two directional rates are not equal in the first place, even if we ignore the stopping effect completely. Since the earth is moving with respect to the rest frame of the DM halo, $\vec{v}_e$ defines a direction that breaks isotropy. The perspective of probing the forward-back asymmetry using directional detectors has been explored thorougly [12, 19, 33–35]. As a first step, we would like to estimate how big is the rate asymmetry between the direction $\hat{n}$ and $-\hat{n}$ due to the stopping effect we study compared to the pure forward-backward asymmetry due to the DM wind. This will gives us a sense of how easily this effect can be probed in directional detectors in the near future. Let us now calculate the forward-backward asymmetry due to the motion of the earth inside the galaxy. Following the steps from Eq. (12) to (22) and upon ignoring the stopping effect (i.e. $\Delta = 0$) we can derive the forward-backward asymmetry (i.e. the asymmetry between the directions $\hat{v}_e$ and $-\hat{v}_e$) as

$$\delta_0 = \frac{d^2R}{dE_R d\Omega_{-v_e}} - \frac{d^2R}{dE_R d\Omega_{v_e}} = \frac{4\pi\kappa}{\mathcal{N}}\int_{v_{\rm min}}^{v_{\rm esc}+v_e}\exp\left[-\frac{v^2+v_e^2}{v_0^2}\right]\sinh\left[\frac{2v_e v_{\rm min}}{v_0^2}\right]vdv. \quad (27)$$

We can now estimate the significance of the stopping effect with respect to the forward-backward asymmetry by considering the following ratio

$$R_1 = \frac{\frac{d^2R}{dE_R d\Omega_{-n}} - \frac{d^2R}{dE_R d\Omega_n}}{\delta_0}. \quad (28)$$

There is also another meaningful comparison we can make. We can compare the asymmetry due to the stopping effect compared to the pure asymmetry created in the flux by the DM wind evaluated in the up and down directions of the detector. In other words we get an estimate of the relevant importance of the stopping effect compared to that of the velocity by considering

$$R_2 = \frac{\delta R_s - \delta R_0}{\delta R_s}, \quad (29)$$

where

$$\delta R_s = \frac{d^2 R_s}{dE_R d\Omega_{-n}} - \frac{d^2 R_s}{dE_R d\Omega_n}, \quad (30)$$

$$\delta R_0 = \frac{d^2 R_0}{dE_R d\Omega_{-n}} - \frac{d^2 R_0}{dE_R d\Omega_n}. \quad (31)$$

The indices "s" and "0" refer to the directional recoil rates with stopping and after having ignored the stopping effect of the undeground atoms respectively. The latter is given by Eqs. (22) and (25) once we set $\Delta = 0$, $y_+ = y_- = v_{\min}/v$, and $v_1 = v_2 = v_{\min}$

$$\frac{d^2 R_0}{dE_R d\Omega_{-n}} - \frac{d^2 R_0}{dE_R d\Omega_n} = \frac{4\pi\kappa}{\mathcal{N}} \int_{v_{\min}}^{v_{\rm esc}+v_e} \exp\left[-\frac{v^2+v_e^2}{v_0^2}\right] \sinh\left[\frac{2v_e v_{\min}\cos\alpha}{v_0^2}\right] \mathcal{I}_0\left(\frac{2vv_e}{v_0^2}\sin\alpha\sqrt{1-\frac{v_{\min}^2}{v^2}}\right) vdv. \quad (32)$$

One can notice that by setting $\alpha = 0$, Eq. (32) reduces to Eq. (27). In addition to the previous ratios, it is important to estimate how big is the asymmetry compared to the total recoil rate, i.e. the number of counts per recoil energy after we integrate over the whole 4π solid angle. This can be probed by the ratio

$$\delta R = \frac{\delta R_s - \delta R_0}{dR_0/dE_R}\delta\Omega. \quad (33)$$

It is understood that dR_0/dE_R is the total rate that produces recoil energy E_R (upon ignoring the stopping effect), i.e. the total non-directional rate after one integrates over the whole solid angle of 4π. $\delta\Omega$ is the solid angle resolution for a typical directional detector. We take it here to be the solid angle of a cone with angle opening of $\pi/6$, i.e. $\delta\Omega = 2\pi(1-\cos[\pi/6])$. dR_0/dE_R can be easily estimated

$$\begin{aligned}\frac{dR_0}{dE_R} &= \frac{2\pi\kappa}{\mathcal{N}}\int \exp\left[-\frac{v^2+v_e^2+2vv_e\cos\theta}{v_0^2}\right] vdvd\cos\theta d\phi \\ &= \frac{\pi^{5/2}\kappa v_0^3}{\mathcal{N}v_e}\left(\text{erf}\left[\frac{v_{\rm esc}}{v_0}\right] - \text{erf}\left[\frac{2v_e+v_{\rm esc}}{v_0}\right] + \text{erf}\left[\frac{v_e-v_{\min}}{v_0}\right] + \text{erf}\left[\frac{v_e+v_{\min}}{v_0}\right]\right). \quad (34)\end{aligned}$$

δR is an important parameter because it reflects the amount of data needed in order to probe the stopping effect of underground atoms on DM. It is the difference in the amount of events detected in a detector with a direction in the recoil within a cone (calibrated to a typical angle of $\pi/6$) pointing down and a cone pointing up, after subtracting the amount of the asymmetry due solely to the velocity of the earth with respect to the rest frame of the DM halo, over the total number of events (from all directions).

IV. RESULTS

We present the results of R_1, R_2 and δR in Figs. 1 to 5. In Fig. 1 we show the R_2 as a function of the DM-nucleon cross section. One can clearly see that the asymmetry increases with increasing cross section up to the point where the cross section becomes so strong that even DM particles coming from the top decelerate so much that cannot produce a recoil above the given values chosen in the figure (i.e. 0.1, 0.2 and 0.3 keV). In addition one

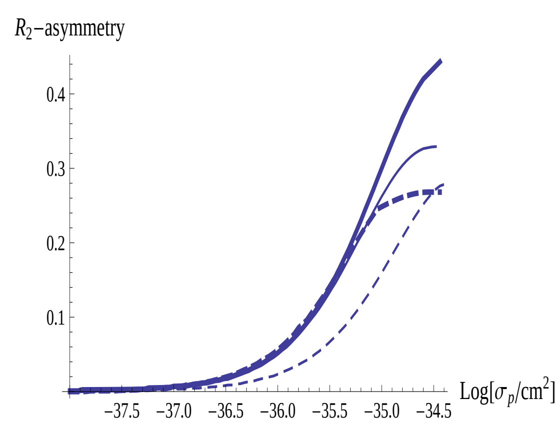

FIG. 1: R_2 asymmetry as a function of the DM-nucleon cross section for a DM particle of mass 1 GeV at recoil energies 0.1 keV (solid thick), 0.2 keV (solid thin), and 0.3 keV (thick dashed line). The thin dashed line corresponds to a 0.6 GeV DM particle with recoil 0.1 keV. We assume a CF_4 detector.

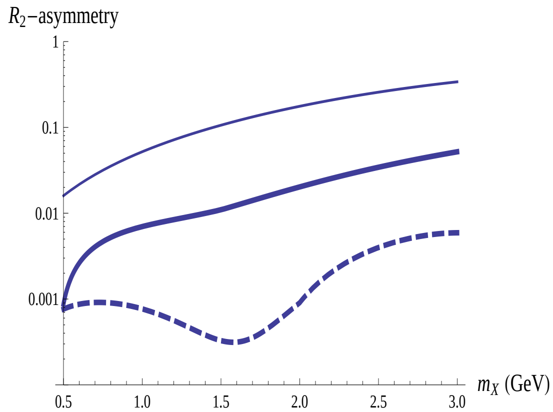

FIG. 2: R_2 asymmetry as a function of DM mass (in GeV) at a recoil energy of 0.1 keV for three values of DM-nucleon cross section 10^{-36}cm^2 (thin line), 10^{-37}cm^2 (thick line) and 10^{-38}cm^2 (dashed line).

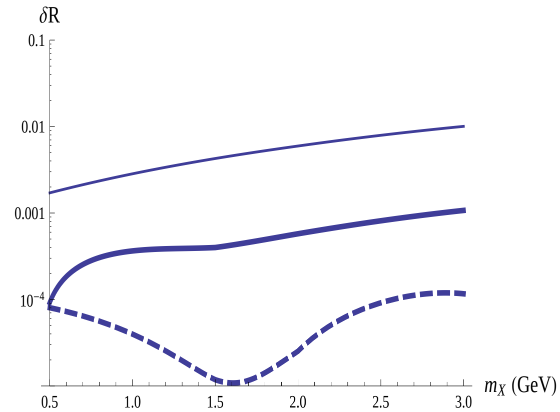

FIG. 5: δR for the parameters depicted in Fig. 2.

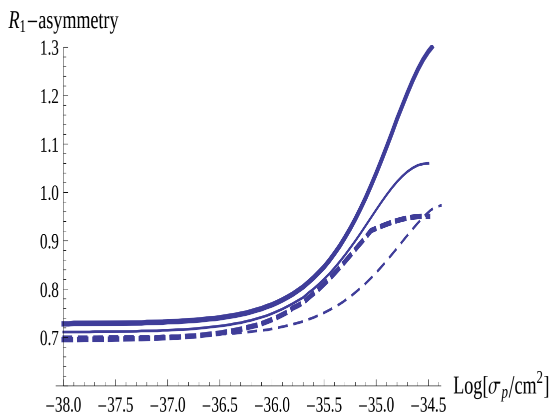

FIG. 3: R_1 asymmetry for the parameters depicted in Fig. 1.

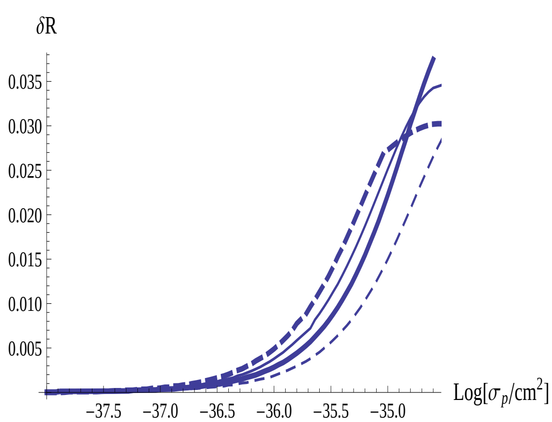

FIG. 4: δR for the parameters depicted in Fig. 1.

can notice that in the case of recoil energy 0.2 keV and in a more pronounced way in 0.3 keV, the asymmetry seems to flatten for a range of DM-nucleon cross section. The reason we have an almost flat asymmetry for that range of cross section is simple. The asymmetry increases as a function of the cross section up to the point where DM particles that come from below (traveling a distance of the earth's diameter) decelerate to low energies that cannot produce the given recoil. As the cross section increases further, the asymmetry is not affected simply because there are no more particles coming from below and therefore the asymmetry cannot increase further. At even larger cross section, the asymmetry drops only slightly because the underground interactions start to affect now the rate of events from above. However since the distance from above is not large (we have taken a typical 1.6 km), the significant drop in the number of events happens sharply at $\sim 10^{-34}\text{cm}^2$. Fig. 2 depicts the R_2 asymmetry as a function of the DM mass for three distinct values of the DM-nucleon cross section. We have chosen to plot the asymmetry up to $m_X = 3$ GeV, since CRESST already sets strong constraints on DM down to 2 GeV [36]. The constraints on smaller masses are not strict and therefore cross section of 10^{-36}cm^2 are allowed. Generally, one can conclude that especially for light enough DM particles where the allowed DM-nucleon cross section might not be so small since it is barely constrained by current direct detection experiments, the asymmetry in the up-down directional detection due to interactions of DM with underground atoms is a large fraction of the overall asymmetry that includes also the asymmetry due to the difference in the up-down DM flux caused by the motion of the earth in the galaxy. In Fig. 2 one can also observe that in the case of $\sigma_p = 10^{-38}\text{cm}^2$, the asymmetry drops as one reduces the DM mass, but then it starts increasing again for masses below ~ 1.6 GeV. In fact below this mass, the asymmetry switches sign thus becoming negative. This means that underground DM-nuclei interactions slow down the DM particle thus increasing the cross section (which scales as $\sim v^{-2}$) and the probability of scattering at the detector.

Fig. 3 represents the same parameter space as in Fig. 1 for the R_1 asymmetry instead of R_2. We have chosen to show also the R_1 asymmetry because it is the ratio between the asymmetry of up-down events over the forward-backward asymmetry (which is the one between the directions $\hat{v}_e$ and $-\hat{v}_e$). This comparison is important since as mentioned earlier, it is the most thorougly studied in the case of directional detectors. Fig. 3 also verifies the findings of the previous figures, i.e. for light DM particles with relatively strong DM-nucleon cross section, the stopping effect of underground atoms is significant. Finally Figs. 4 and 5 show that for DM-nucleon cross section of the order of 10^{-36}cm^2 or larger and for DM masses of 1 GeV or lower, the asymmetry can be of the order of a few percent with respect to the total non-directional detection rate. With the advent of new directional detectors with lower energy recoil thresholds, not only will be possible to probe lighter DM candidates but as we point out in this paper, we can gain significant information regarding the type and strength of DM-nucleon interactions. There is parameter phase space in the region of light DM where the stopping effect of underground atoms on DM particles might be statistically significant.

V. CONCLUSIONS

In this paper we make a first attempt to identify the importance of the stopping effect in the context of the directional DM detectors with respect to the well studied forward-backward directional asymmetry. We assume contact type DM-nucleon interactions, and a constant density for the earth. We derive formulas that give the energy loss of DM particles as they travel underground based on coherent scattering with the three most abundant nuclei in the earth. We also provide formulas that give the directional detection rate taking into account this effect assuming a typical CF_4 directional detector. We propose an up-down asymmetry in the directional detection rate as the best parameter one can use to study the significance of this stopping effect. We demonstrate that this up-down asymmetry in the directional detection rate can be a few percent of the total non-directional detection rate for a large range of DM-nucleon cross section and mass, and therefore it could be observed in upcoming direct detection experiments with directional detectors. In particular, as it was pointed out [33], 13 events will be sufficient to distinguish an isothermal distribution boosted with the velocity of the earth in the galaxy from a flat background. One would roughly expect that $13/R_2$ number of events will be sufficient to observe the DM stopping effect. We should emphasize that due to stringent constraints from direct DM search experiments, the above are valid for strictly light DM candidates below 1 GeV where the DM-nucleon cross section is less constrained. To get the asymmetry of the few percent, it would require a $\sim$ 0.5 GeV particle with a strong $\sigma_p \sim 10^{-36}$cm^2.

Although we have presented results for a CF_4 detector, our results are quite generic in the sense that one can easily use our formulas for different targets. The up-down asymmetry in directional detectors has two potential sources, i.e. the stopping effect and the asymmetry in the DM flux due to the velocity of the earth with respect to the DM halo. We demonstrate that there is phase space where the stopping effect represents a significant fraction of the overall asymmetry.

We leave several things for future work. One can include more elements than iron, oxygen and silicon for the DM stopping effect, a non-constant density profile for the earth, and different types of DM-atom interactions. For example long range DM-atom interactions or DM-electron interactions can have a significant amount of stopping if DM particles travel through metallic layers of the earth. In this paper we have assumed that DM moves underground in straight lines. A more precise study of the diffusion from the original path is required to get a more accurate estimate of the asymmetry.

In principle, if sufficient number of events is detected, this technique can be used as a "Dark matter tomography". One could study the density and composition profile of the earth based on the directional detection rate of DM that has traveled different distances and segments of the earth's interior, given that the DM-atom interactions have been identified and understood.

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VI. APPENDIX

Here we study the straight line approximation. For generic interactions one expects that DM particles will deflect from their original path as they start interacting with underground atoms, thus invalidating our straight line approximation upon we have derived our results. Firstly, we should emphasize that there are interactions that favor forward scattering. This is something that will validate by default the straight line approximation. Long range interactions are extremely forward since the scattering angle scales as $\sin^{-4}(\theta/2)$. Such an example is millicharged DM. In this case as it was pointed out in [25], the stopping power behaves similarly at low recoil energies upon identifying $16\pi\alpha^2\epsilon^2 a_0^4 m_x^2 Z^2 \to \sigma_p A^2$, where $\alpha = 1/137$ is the fine structure constant, ϵe the (milli)charge of DM, a_0 is the Bohr radius and Z the number of protons. It worth mentioning that in higher recoil energies, the stopping becomes larger than that of the corresponding contact interaction we studied here and therefore this enhances further the effect. The above obviously hold also for DM-nucleon interactions mediated by light mediators. Furthermore, even among the nonrelativistic contact interactions, there are types that favor forward scattering, thus validating the straight line assumption. Such an example is the operator O_8 in the list e.g. [24, 37].

However, one can show that the straight line approximation is a good approximation even in more generic not forward scatterings. In relative forward scattering, the straight line approximation gives a conservative estimate of the top-down asymmetry. Deflection of DM particles out of their straight line path will make the asymmetry between top and bottom incoming particles even larger. Within the straight line approximation there is no deflextion of the DM particles coming from the bottom. Any deflection (even partial one) will reduce further the number of events from the bottom, leading to an increase in the asymmetry. Therefore one should expect in general grounds that the straight line approximation is a conservative approximation. In principle one can argue that although DM particles moving up towards the detector deflect out of the path, other DM particles moving on different trajectories might deflect in the upward path, invalidating potentially the above argument regarding a conservative estimate of the asymmetry. However we argue below that this is not possible on generic grounds.

Although this can be easily generalized for any direction, let us consider for the simplicity of the argument here that we choose the top-down direction to be opposite of the DM wind. This means that DM particles that travel to the detector from the bottom have roughly parallel velocities. In this simplified picture, within the straight line approximation, the fraction of the DM flux crossing the earth that arrives at the detector is $\pi\ell^2/(\pi R_\oplus^2)$, where ℓ is the dimension of the detector and $R_\oplus$ the radius of the earth. Possible deflection of DM from the straight line will increase the asymmetry justifying our claim that this is a conservative limit, as long as the number of DM particles that deflect in path is smaller to the one of particles deflecting out. Let us assume that the DM trajectory is described by a random walk that after N scatterings has a probability

$$p(r')dr' = \frac{2}{\sqrt{\pi N}\delta\ell}\exp\left(-\frac{r'^2}{N\delta\ell^2}\right)dr', \qquad (35)$$

where r' defines the distance away from the straight path, N the number of scatterings and $\delta\ell$ the dispersion of the displacement between collisions. $\delta\ell$ is related to the mean free path of the DM particle underground multiplied by $\sqrt{2/3}$ simply because we consider the random walk in two out of the three dimensions. Now we can estimate the probability of a DM particle traveling initially in a straight path that is r away from the detector, to scatter after N collisions to a path passing through the detector. This is

$$P_d(r) = \int_0^{\arcsin(\ell/r)} \frac{d\phi}{2\pi}\int_r^{r+\ell} p(r')dr', \qquad (36)$$

where ϕ is the angle subtended by the detector from the point of consideration. The total flux of DM particles that are deflected in the path of the detector is simply

$$F_{in} = \int_\ell^{R_\oplus} P_d(r)\frac{2\pi r dr}{\pi R_\oplus^2} \qquad (37)$$

$$= \frac{1}{\pi R_\oplus^2}\int_\ell^{R_\oplus} r dr \int_0^{\arcsin(\ell/r)} d\phi \int_r^{r+\ell} p(r')dr' \qquad (38)$$

Similarly the flux of particles deflecting out of the path that lead to the detector is

$$F_{out} = \int_0^\ell \frac{r dr}{\pi R_\oplus^2}\int_0^{2\pi} d\phi \int_{\sqrt{\ell^2+r^2-2\ell r\cos\phi}}^{\infty} p(r')dr'. \qquad (39)$$

The straight line approximation is a conservative estimate of the asymmetry as long as $F_{in} < F_{out}$. We evaluated the ratio of the two quantities for a variety of values for the number of collisions N and $\delta\ell$ for a 10 meters detector. In all cases the inequality is satisfied making the straight line approximation a conservative estimate of the asymmetry.

The fact that we chose DM moving in parallel trajectories does not change the result. In scattering where forward scattering is favored, DM that approach the detector from high angles of attack can in principle scatter to the "right" angle and invalidate again the straight line approximation as a conservative estimate of the asymmetry. However, although in this case the DM particle does not have to walk randomly at large distances sideways, one of the scatterings must take place at a high angle. If $\delta\theta$ is the dispersion of the scattering angle, the probability to scatter at an angle $\delta\theta << \theta << 1$ will be

$$p(\theta) \sim \exp\left(\frac{-\theta^2}{N\delta\theta^2}\right). \qquad (40)$$

One can easily show that also in this case where forward scattering is favored, the flux of particles that enter the cone of detection due to deflection is much less than the one that is deflected out of the cone. Therefore the straight line approximation gives a conservative estimate in the asymmetry since it has a larger number of particles approaching from below.

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