Entanglement entropy at holographic interfaces

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Abstract

In this note we calculate the holographic entanglement entropy in the presence of a conformal interface for a geometric configuration in which the entangling region $\mathcal{A}$ lies on one side of the interface. For the supersymmetric Janus solution we find exact agreement between the holographic and CFT calculation of the entanglement entropy.

1 Introduction

In two-dimensional conformal field theories the folding trick [1, 2] allows one to map the problem of the construction of conformal interfaces between CFT_1 and CFT_2 to the construction of conformal boundary state in the folded product $CFT_1 \otimes \overline{CFT}_2$. This construction has been used to construct conformal interfaces for free compactified bosons and more general CFTs, see e.g. [2, 3, 4, 5, 6, 7, 8, 9, 10]. In general the entanglement entropy of a entangling region $\mathcal{A}$ in the presence of an interface depends on the location of the interface with respect to $\mathcal{A}$. For a region of length L which is placed symmetrically about the interface the entanglement entropy was calculated in [11], where it was found that the logarithmically divergent term is independent of the interface and the constant term was related to the boundary entropy or g function [12] of the folded boundary CFT.

In this note we are interested in a different geometrical setup where the entangling region $\mathcal{A}$ is the half-space lying on one side of the interface. This entanglement entropy has been calculated using the replica trick in [13] for system corresponding to a compact boson whose compactification radius R jumps across the interface. In [14] the analogous calculation was performed for interfaces in the two-dimensional Ising model. We will review some of these results in section 2.

Janus solutions [15, 16, 17, 18] are holographic realizations of conformal interfaces[1]. It is natural to use the Ryu-Takayanagi prescription [22, 23] to calculate entanglement entropy for these solutions and compare the results to the CFT calculation. For the symmetric entangling surface this was done in [11] using the non-supersymmetric Janus solution in three dimensions and in [24] using the supersymmetric Janus solution in six dimensions which is locally asymptotic to $AdS_3 \times S^3$. In this note we calculate the holographic entanglement entropy where the entangling region $\mathcal{A}$ is the half space which ends at the interface.

The structure of this note is as follows: In section 2 we review the CFT calculation of the entanglement entropy. In 3 we calculate the entanglement entropy for the non-supersymmetric Janus solution. In section 4 we perform the same calculation for the supersymmetric Janus solution. We present a discussion of our results and some possible avenues for future research in section 5. Some details of the supersymmetric solutions are delegated to appendix A.

2 CFT Interfaces and entanglement entropy

In this section we review the results of [13] and [14] on the CFT calculation of the entanglement entropy in the presence of a conformal interface. The entanglement entropy of a region

[1] See [19, 20, 21] for other approaches to describe interfaces in AdS.

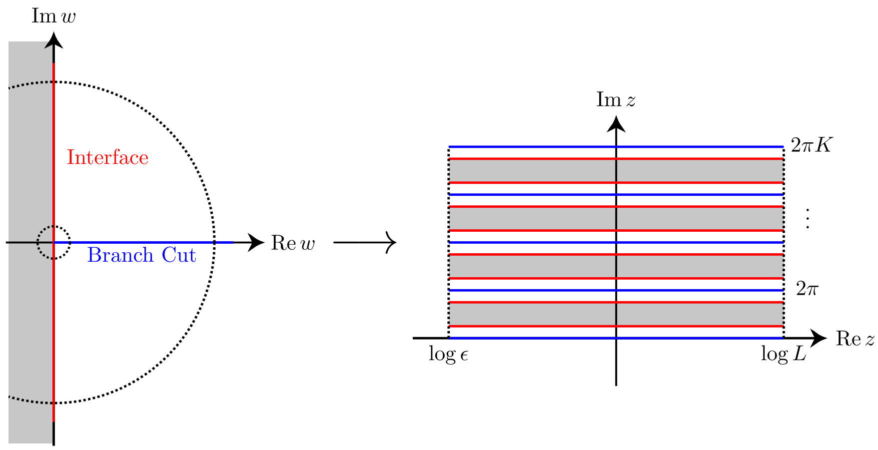

Figure 1: The map $z = \ln w$ maps the K-sheeted Riemann surface with interface at $\mathrm{Re}(w) = 0$ and the branch cut at $\mathrm{Im}(z) = 0, \mathrm{Re}(w) > 0$ to the geometry on the right. The dotted circles on the left correspond to the UV cutoff at $|w| = \epsilon$ and the IR cutoff at $|w| = L$. This figure was adapted from [14].

$\mathcal{A}$ is defined as the von Neumann entropy of the reduced density matrix $\rho_{\mathcal{A}} = tr_{\bar{\mathcal{A}}}|0\rangle\langle 0|$. A well known calculational method relates the entanglement entropy to a specific limit of Renyi entropies

$$S_{\mathcal{A}} = -\frac{\partial}{\partial K} \mathrm{tr}\ \rho_{\mathcal{A}}^K|_{K=1} \tag{2.1}$$

The Renyi entropies can be calculated by the replica trick in which the trace (2.1) is represented as a path integral over a K-sheeted Riemann surface where the branch cuts run along $\mathcal{A}$. The entanglement entropy can then be calculated from the partition function $Z(K)$ on the K-sheeted Riemann surface as follows

$$S_{\mathcal{A}} = (1 - \partial_K) \log Z(K)|_{K=1} \tag{2.2}$$

It has been shown in [13] that in the presence of an interface the K-sheeted partition function $Z(K)$ can be calculated by mapping the K-sheeted Riemann surface via $z = \ln w$ to a covering space (see figure 1). Introducing an UV cutoff ϵ and IR cutoff L and imposing periodic boundary conditions for simplicity, the K-th replica partition function becomes

$$Z(K) = Tr_{CFT_1}\left(I_{1,2}e^{-tH_2}I_{2,1}e^{-tH_1}\right)^K \tag{2.3}$$

Where $t = 2\pi^2/\log(L/\epsilon)$ and

$$H_i = L_0^{(i)} + \bar{L}_0^{(i)} - \frac{1}{12}c^{(i)}, \quad i = 1, 2 \tag{2.4}$$

are the Hamiltonians of CFT_1 and CFT_2 respectively. The interface operator $I_{1,2}$ maps states from CFT_1 to CFT_2 and the operator $I_{2,1} = (I_{1,2})^\dagger$ is the conjugate interface which maps CFT_2 to CFT_1.

In [13] the expression (2.3) has been determined and the entanglement entropy has been calculated for the $c = 1$ permeable interface of a compact boson whose radius jumps from R_1 to R_2, first introduced in [2]. After doubling the interface is mapped to a D1 brane inside a rectangular torus of radius R_1 and R_2 which is winding k_1 times around the R_1 cycle and k_2 times around the R_2 cycle. The result for the entanglement entropy calculated in [13] is

$$S_{\mathcal{A}} = \frac{1}{2}\sigma(|s|)\log\frac{L}{\epsilon} - \log|k_1 k_2| \tag{2.5}$$

Where s is given by

$$s = \sin 2\theta_+, \quad \theta_+ = \arctan\frac{k_2 R_2}{k_1 R_1} \tag{2.6}$$

The function $\sigma(x)$ can be expressed as an integral or in terms of dilogarithm functions

$$\begin{aligned}\sigma(x) &= \frac{x}{2} - \frac{2}{\pi^2}\int_0^\infty dz\, z\Big(\sqrt{1+(x/\sinh z)^2} - 1\Big)\\ &= \frac{1}{6} + \frac{x}{3} + \frac{1}{\pi^2}\Big((x+1)\log(x+1)\log x + (x-1)\mathrm{Li}_2(1-x) + (x+1)\mathrm{Li}_2(-x)\Big)\end{aligned} \tag{2.7}$$

Note that the interface which corresponds to the Janus solution has $k_1 = k_2 = 1$ and hence the constant term in (2.5) vanishes. Furthermore the case of an identity defect (i.e. $R_1 = R_2$) then corresponds to $\theta_+ = \pi/4$ for which $\sigma(1) = 1/3$ and formula (2.5) agrees with the standard universal results for the entanglement entropy in a single vacuum CFT with $c = 1$. The complicated dependence of the entanglement entropy on s given by the function $\sigma(|s|)$ simplifies considerably if the free boson interface is combined with a free fermion in a supersymmetric fashion as pointed out in a recent paper [14]. This is due to an extensive cancellation between bosonic and fermionic oscillators in $Z(K)$. The entanglement entropy for a supersymmetric interface in a $c = 3/2$ CFT of a compact boson and a free fermion is given by [14]

$$S_{\mathcal{A}}^{susy} = \frac{1}{2}s\log\frac{L}{\epsilon} - \log|k_1 k_2| \tag{2.8}$$

3 Non-supersymmetric Janus solution

The three-dimensional Janus solution was constructed in [25]. The starting point is a three-dimensional gravity with negative cosmological constant coupled to a massless scalar (e.g. the dilaton field)

$$S = \frac{1}{16\pi G_N}\int d^3x\sqrt{g}\Big(R - \partial_\mu\phi\partial^\mu\phi + \frac{2}{L^2}\Big) \tag{3.1}$$

The Janus solution solves the equations of motion coming from this action and is given by

$$ds^2 = L^2\Big(d\mu^2 + f(\mu)\frac{dz^2 - dt^2}{z^2}\Big) \tag{3.2}$$

where

$$f(\mu) = \frac{1}{2}\Big(1 + \sqrt{1-2\gamma^2}\cosh(2\mu)\Big) \tag{3.3}$$

and

$$\phi(\mu) = \phi_0 - \sqrt{2}\,\tanh^{-1}\left(\frac{-1+\sqrt{1-2\gamma^2}}{\sqrt{2}\gamma}\tanh\mu\right) \tag{3.4}$$

The solution depends on one parameter γ. The holographic solution corresponds to an interface connecting two half spaces which are reached on the boundary of the spacetime by taking $\mu \to \pm\infty$. The massless scalar ϕ takes two asymptotic values in this limit and as shown in [24] the jump in ϕ can be identified with the jump in the radius of the free boson

$$\frac{R_2}{R_1} = \frac{\lim_{\mu\to+\infty} e^{-\phi/2}}{\lim_{\mu\to-\infty} e^{-\phi/2}} = \exp\left\{\sqrt{2}\,\tanh^{-1}\left(\frac{-1+\sqrt{1-2\gamma^2}}{\sqrt{2}\gamma}\right)\right\} \tag{3.5}$$

According to the Ryu-Takayanagi prescription the holographic entanglement entropy is determined by finding the area of a minimal surface (at constant time) which at the boundary of the bulk spacetime coincides with the boundary $\partial\mathcal{A}$ of the entangling region $\mathcal{A}$. In this note we calculate the entanglement entropy for the entangling region on one side of the interface. We give a sketch of this geometry (b) in figure 2 and contrast it with the symmetric case depicted in (a).

In three dimensions the minimal surface Γ at $t = 0$ is a curve and we have choose an embedding. The appropriate embedding for the case at hand turns out to be $\mu = \mu(z)$. For this choice the induced line element that leads to the following action

$$A[\Gamma] = \int dz\sqrt{\frac{f(\mu)}{z^2} + \left(\frac{\partial\mu}{\partial z}\right)^2} \tag{3.6}$$

The minimal area is found by solving the Euler-Lagrange equation which follows from (3.6)

$$f'(\mu)\left(\frac{1}{z^2} + \frac{(\partial_z\mu)^2}{f(\mu)+z^2(\partial_z\mu)^2}\right) - \frac{2}{z}\frac{f(\mu)\,(\partial_z\mu + z\partial_z^2\mu)}{f(\mu)+z^2(\partial_z\mu)^2} = 0 \tag{3.7}$$

A simple solution of the Euler-Lagrange equation is given by

$$\frac{\partial\mu}{\partial z} = 0, \quad f'(\mu) = 0 \tag{3.8}$$

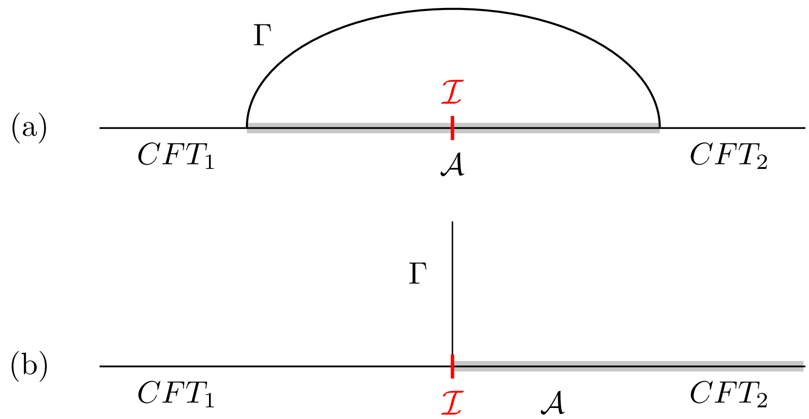

Figure 2: Two different geometries for the entangling region $\mathcal{A}$ and interface $\mathcal{I}$: (a) the entangling region is placed symmetrically about the interface, (b) the entangling surface is on one side of the interface. Γ is a sketch of the respective RT minimal surfaces in the bulk.

Hence μ is constant and the second equation is solved by $\mu = 0$. It is easy to see that this solution is indeed an absolute minimum for the length, as $\mu = 0$ minimizes the first term and $\partial_z \mu = 0$ minimizes the second term under the square root in the functional (3.6). The holographic entanglement entropy is then given by

$$\begin{aligned} S_{\text{hol}} &= \frac{L}{4G_N}\sqrt{f(0)}\int \frac{dz}{z} \\ &= \frac{c}{6\sqrt{2}}\sqrt{1+\sqrt{1-2\gamma^2}}\log\frac{L}{\epsilon} \end{aligned} \tag{3.9}$$

Where we have regulated the divergent integral over z and used $c = \frac{3L}{2G_N}$. In order to compare the functional dependence it is useful to expand the result as a power series in terms of small γ, for the holographic entanglement entropy one finds

$$S_{\text{hol}} = \left(1 - \frac{1}{4}\gamma^2 - \frac{5}{32}\gamma^4 + o(\gamma^6)\right)\frac{c}{6}\log\frac{L}{\epsilon} \tag{3.10}$$

We can compare this to the CFT result for the entanglement entropy (2.5). We set $k_1 = k_2 = 1$ which makes the constant term vanish, expanding (2.7) around $s = 1$ gives

$$\begin{aligned} \frac{1}{2}\sigma(s) &= \frac{1}{6} - \frac{1}{8}(1-s) - \frac{1}{4\pi^2}(1-s)^2 + o[(1-s)^3] \\ &= \frac{1}{6} - \frac{1}{16}\gamma^2 - \left(\frac{11}{192} + \frac{1}{16\pi^2}\right)\gamma^4 + o[\gamma^6] \end{aligned} \tag{3.11}$$

where we have used the expansion

$$s = 1 - \frac{\gamma^2}{2} - \frac{11}{24}\gamma^4 + o(\gamma^6) \tag{3.12}$$

which follows from (2.6) and (3.5). Using this expansion in the CFT entanglement entropy (2.5) and restoring a general value for the central charge (i.e. by considering c copies of the single boson) gives

$$S_{\rm CFT} = \left(1 - \frac{3}{8}\gamma^2 - \left(\frac{11}{32} + \frac{3}{8\pi^2}\right)\gamma^4 + o(\gamma^6)\right)\frac{c}{6}\log\frac{L}{\epsilon} \tag{3.13}$$

Comparing (3.11) and (3.13) shows that the two expressions only agree for the $\gamma = 0$ which corresponds to the case where no interface is present. This result is to be contrasted with result [11] for the symmetric entangling region where agreement of the CFT and the holographic entanglement entropy up to order γ^2 was found.

4 Supersymmetric Janus solution

The supersymmetric Janus solution of type IIB which is locally asymptotic to $AdS_3 \times S^3 \times M_4$ was constructed in [26] (see [27, 28] for some earlier work in this direction and [29, 30] for generalizations). Some aspects of the solutions are reviewed in appendix A for the convenience of the reader. The metric for the solution takes the following form

$$ds^2 = f_1^2 \frac{dz^2 - dt^2}{z^2} + f_2^2(d\phi_1^2 + \sin^2\phi_1 d\phi_2) + f_3^2 ds^2_{M_4} + \rho^2(dx^2 + dy^2) \tag{4.1}$$

We parametrize the minimal surface for the entanglement entropy by $t = 0$ and $x = x(z, y)$, i.e. the eight-dimensional surface is spanned by $\xi^a = \{z, y, \phi_1, \phi_2\}$ and the four coordinates of M_4. The induced metric is then given by

$$\gamma_{ab} = \frac{\partial x^\mu}{\partial \xi^a}\frac{\partial x^\nu}{\partial \xi^b} g_{\mu\nu} \tag{4.2}$$

and the action for the minimal surface is

$$\begin{aligned} S &= \int d^8\xi \sqrt{\det\gamma} && (4.3) \\ &= \int_{M_4} dV \int d\phi_1\, d\phi_2\, \sin\phi_1 \int dz\, dy\, \frac{1}{z} f_2^2 f_3^4 \rho \sqrt{f_1^2\,(1 + (\partial_y x)^2) + z^2\rho^2(\partial_z x)^2} && (4.4) \end{aligned}$$

The Euler-Lagrange equation following from (4.3) is given by

$$\begin{aligned} 0 = &\ \frac{1}{z}\partial_x \left(f_2^2 f_3^4 \rho \sqrt{f_1^2\,(1 + (\partial_y x)^2) + z^2\rho^2(\partial_z x)^2} \right) \\ & - \partial_z \left(\frac{f_2^2 f_3^4 \rho^3 z \partial_z x}{\sqrt{f_1^2\,(1 + (\partial_y x)^2) + z^2\rho^2(\partial_z x)^2}} \right) - \partial_y \left(\frac{f_2^2 f_3^4 \rho^2 f_1^2 \partial_y x}{z\sqrt{f_1^2\,(1 + (\partial_y x)^2) + z^2\rho^2(\partial_z x)^2}} \right) \end{aligned} \tag{4.5}$$

While it seems formidable to find a solution to (4.5) a simple solution can be found by setting

$$x(z,y) = x_0 \tag{4.6}$$

for which it is straightforward to verify that (4.5) reduces to

$$\partial_x \log\left(f_1^2 f_2^4 f_3^8 \rho^2\right)|_{x=x_0} = 0 \tag{4.7}$$

which has to be valid for all values of y. Plugging in the solution for the metric factors found in appendix A one finds

$$f_1^2 f_2^4 f_3^8 \rho^2 = 16L^4 \cosh^2\psi \cosh^2\theta \cosh^2(x+\psi)\sin^4 y \tag{4.8}$$

and hence (4.7) is satisfied if $x_0 = -\psi$. Since the expression under the square root in the action functional (4.3) is the sum of positive terms which are all minimized by the solution, we have indeed an absolute minimum as demanded by the Ryu-Takayanagi prescription. For the solution the area is given by

$$\begin{aligned} A &= \int_{M_4} dV\, Vol(S_2) \int dy \sin^2 y \int \frac{dz}{z}\, 4L^2 \cosh\psi \cosh\theta \\ &= Vol(S_3)Vol(M_4) 4L^2 \cosh\psi\cosh\theta \log\frac{L}{\epsilon} \\ &= 8\pi^2 L^2 Vol(M_4)\cosh\psi\cosh\theta\, \log\frac{L}{\epsilon} \end{aligned} \tag{4.9}$$

As reviewed in appendix A the central charge of the dual CFT is given in terms of the parameters

$$c = \frac{3\times 32\pi^3 Vol(M_4) L^2}{\kappa_{10}^2}\cosh^2\psi \sinh^2\theta \tag{4.10}$$

Using the result of the area the holographic entanglement entropy can then be expressed as

$$\begin{aligned} S_A &= \frac{A}{4G_N^{(10)}} \\ &= \frac{1}{\cosh\theta\cosh\psi}\,\frac{c}{6}\log\frac{L}{\epsilon} \end{aligned} \tag{4.11}$$

where we used the identification $1/16G_N^{(10)} = 1/2\kappa_{10}^2$. In order to compare the holographic result (4.11) to the CFT (2.8) we have to set $\theta = 0$ which on the CFT side corresponds to an interface where only the radius of M_4 jumps and there is no jump of the RR modulus [24]. The jump of the radius can be identified with the parameter ψ of the supergravity solution as follows [24]

$$\frac{R_2}{R_1} = e^{\psi} \tag{4.12}$$

and hence

$$2\cosh\psi = \frac{r_+}{r_-} + \frac{r_-}{r_+} \tag{4.13}$$

The identification of s is given by

$$s = \sin 2\theta_+ = \frac{2r_+ r_-}{r_+^2 + r_-^2} = \frac{1}{\cosh\psi} \tag{4.14}$$

Hence in this special case the holographic entanglement entropy (4.11) becomes

$$S_A = \frac{c}{6} s \log\frac{L}{\epsilon} \tag{4.15}$$

which is in exact agreement with the CFT result (2.8) if we replace the value $c = 3/2$ for a real boson and a real fermion with the general value of the central charge. As far as this identification is concerned in our case the symmetric orbifold CFT which is dual to the supergravity on $AdS_3 \times S_3 \times M_4$ can simply be viewed as $4N = 4Q_5Q_1$ copies of the $c = 3/2$ system.

5 Discussion

In this note the holographic entanglement entropy was calculated for a surface $\mathcal{A}$ which lies on one side of a conformal interface. It is interesting to contrast the result (2.5) with the result for the entanglement entropy for a surface which is lying symmetrically across the interface:

$$S_{\mathcal{A}}^{\rm sym} = \frac{c}{6}\log\frac{L}{\epsilon} + g_B \tag{5.1}$$

Note that the for the geometric setup discussed in this note the logarithmically divergent term does not have an universal prefactor $c/6$ but depends on the parameters of the interface via the function $\sigma(|s|)$. This difference makes sense as the interface is located at the boundary between $\mathcal{A}$ and its complement, where the entanglement between the two regions is strongest.

It is also interesting to compare the holographic calculations of the entanglement entropy for the two cases. In [11] the non-supersymmetric Janus solution was used to calculate (5.1) and in particular the holographic boundary entropy g_B was calculated. A comparison with the CFT calculation led to an agreement of g_B to first nontrivial order in the deformation parameter γ. In section 3 we found that in our case the result disagrees even to the lowest nontrivial order in γ.

This state is to be contrasted with the supersymmetric Janus solution where both for the symmetric entangling region [24] and the one sided case calculated in section 4 the CFT and the holographic entanglement entropy agree. Note that the CFT and the gravity calculations are performed at very different points in the moduli space of the dual CFT. It is likely that the high degree of supersymmetry allows the extrapolation of the results from one point to the other.

The supersymmetric Janus solution depends on two parameters θ and ψ and we set $\theta = 0$ for the comparison. The paramater θ corresponds to an RR modulus and consequently to an twist field in the symmetric orbifold CFT. It would be interesting to see whether the CFT calculation can be performed for a general interface operator I_{12} which includes a jump in the twist field.

Recently the CFT at the symmetric orbifold point has been conjectured to be dual to a higher spin theory [31, 32]. The region in moduli space where supergravity is valid is far removed from this point. Supersymmetry seems to make the result of the entanglement entropy independent of where on its moduli space the theory is. It would be interesting to investigate whether it is possible to construct the relevant interface theories in the Chern-Simons formulation following [33] and calculate the entanglement entropy following the proposals relating the entanglement entropy and the Wilson loop in higher spin theory [34, 35, 36].

Acknowledgements

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A Supersymmetric Janus solution

In this appendix we review the details of the supersymmetric Janus solution for the convenience of the reader. This solution was first constructed in [26] and generalized in [29, 30], where more details can be found. The ten-dimensional Janus metric is constructed as a fibration of $AdS_2 \times S^2 \times M_4$, where M_4 is either T_4 or K_3, over a two dimensional Riemann surface Σ

$$ds^2 = f_1^2 ds^2_{AdS_2} + f_2^2 ds^2_{S^2} + f_3^2 ds^2_{M_4} + \rho^2 dz d\bar{z} \tag{A.1}$$

All fields depend on the coordinates $z, \bar{z}$ of the surface Σ. For the supersymmetric Janus solution we choose Σ as an infinite strip as follows

$$w = x + iy, \qquad x \in [-\infty, +\infty], \quad y \in [0, \pi] \tag{A.2}$$

The boundaries of the strip are located at $y = 0, \pi$. The supersymmetric Janus solution depends on four parameters k, L, θ and ψ. The dilaton and axion are given by

$$e^{-2\phi} = k^4 \frac{\cosh^2(x+\psi)\mathrm{sech}^2\psi + \big(\cosh^2\theta - \mathrm{sech}^2\psi\big)\sin^2 y}{\big(\cosh x - \cos y \tanh\theta\big)^2} \tag{A.3}$$

$$\chi = -\frac{k^2}{2}\frac{\sinh 2\theta \sinh x - 2\tanh\psi \cos y}{\cosh x \cosh\theta - \cos y \sinh\theta} \tag{A.4}$$

The metric factors on Σ and M_4 are

$$\begin{aligned} \rho^4 &= e^{-\phi}\frac{L^2}{k^2}\frac{\cosh^2 x \cosh^2\theta - \cos^2 y \sinh^2\theta}{\cosh^2(x+\psi)}\cosh^4\psi \\ f_3^4 &= e^{-\phi}\frac{4}{k^2}\frac{\cosh x\cosh\theta - \cos y \sinh\theta}{\cosh x \cosh\theta + \cos y \sinh\theta} \end{aligned} \tag{A.5}$$

The following expressions for the AdS_2 and S^2 metric factors will be useful,

$$\begin{aligned} \frac{f_1^2}{\rho^2} &= \frac{\cosh^2(x+\psi)}{\cosh^2\theta\cosh^2\psi} \\ \frac{\rho^2}{f_2^2} &= \frac{1}{\sin^2 y} + \frac{\cosh^2\theta\cosh^2\psi - 1}{\cosh^2(x+\psi)} \end{aligned} \tag{A.6}$$

While the form of the antisymmetric tensor fields is not essential, we quote the expressions for the $D1$ and $D5$ brane charges from [26].

$$\begin{aligned} Q_{D5} &= 4\pi^2 k L Vol(M_4)\cosh\psi\ \cosh\theta \\ Q_{D1} &= \frac{16\pi^2 L}{k}\cosh\psi\ \cosh\theta \end{aligned} \tag{A.7}$$

The dual CFT is a $\mathcal{N}=(4,4)$ SCFT which, at a particular point of its moduli space, is a $(M^4)^{Q_{D1}Q_{D5}}/S_{Q_{D1}Q_{D5}}$ orbifold. The central charge c of this CFT takes the following form

$$c = \frac{6}{4\pi k_{10}^2} Q_{D1} Q_{D5} = \frac{3 \times 32\, \pi^3 Vol(M_4)\; L^2}{k_{10}^2} \cosh^2\psi\; \cosh^2\theta \tag{A.8}$$

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