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Cosmological Hints of Modified Gravity ?

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The recent measurements of Cosmic Microwave Background temperature and polarization anisotropies made by the Planck satellite have provided impressive confirmation of the ΛCDM cosmological model. However interesting hints of slight deviations from ΛCDM have been found, including a 95% c.l. preference for a "modified gravity" structure formation scenario. In this paper we confirm the preference for a modified gravity scenario from Planck 2015 data, find that modified gravity solves the so-called A_{lens} anomaly in the CMB angular spectrum, and constrains the amplitude of matter density fluctuations to $\sigma_8 = 0.815^{+0.032}_{-0.048}$, in better agreement with weak lensing constraints. Moreover, we find a lower value for the reionization optical depth of $\tau = 0.059 \pm 0.020$ (to be compared with the value of $\tau = 0.079 \pm 0.017$ obtained in the standard scenario), more consistent with recent optical and UV data. We check the stability of this result by considering possible degeneracies with other parameters, including the neutrino effective number, the running of the spectral index and the amount of primordial helium. The indication for modified gravity is still present at about 95% c.l., and could become more significant if lower values of τ were to be further confirmed by future cosmological and astrophysical data. When the CMB lensing likelihood is included in the analysis the statistical significance for MG simply vanishes, indicating also the possibility of a systematic effect for this MG signal.

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I. INTRODUCTION

The recent measurements of Cosmic Microwave Background (CMB) anisotropies by the Planck satellite experiment [1, 2] have fully confirmed, once again, the expectations of the standard cosmological model based on cold dark matter, inflation and a cosmological constant.

While the agreement is certainly impressive, some hints for deviations from the standard scenario have emerged that certainly deserve further investigation. In particular, an interesting hint for "modified gravity" (MG hereafter), i.e. a deviation of the growth of density perturbations from that expected under General Relativity (GR hereafter), has been reported in [3] using a phenomenological parametrization to characterize non-standard metric perturbations.

In past years, several authors (see e.g. [3–18]) have constrained possible deviations of the evolution of perturbations with respect to the ΛCDM model, by parametrizing the gravitational potentials Φ and Ψ and their linear combinations. Considering the parameter Σ, that modifies the lensing/Weyl potential given by the sum of the Newtonian and curvature potentials $\Psi + \Phi$, the analysis of [3] reported the current value of $\Sigma_0 - 1 = 0.28 \pm 0.15$ at 68% from Planck CMB temperature data, i.e. a deviation from the expected GR null value at about two standard deviations. The discrepancy with GR increases when weak lensing data is included, bringing the constrained value to $\Sigma_0 - 1 = 0.34^{+0.17}_{-0.14}$ (again, see [3]).

This result is clearly interesting and should be further investigated. Small systematics may still certainly be present in the data and a further analysis, expected by 2016, from the Planck collaboration could solve the issue. In the meantime, it is certainly timely to independently reproduce the result presented in [3] and to investigate its robustness, especially in view of other anomalies and tensions currently present in cosmological data.

Indeed, another anomaly seems to be suggested by the Planck data, i.e. the amplitude of gravitational lensing in the angular spectra. This quantity, parametrized by the lensing amplitude $A_{\rm lens}$ as firstly introduced in [19], is also larger than expected at the level of two standard deviations. The Planck+LowP analysis of [2] reports the value of $A_{\rm lens} = 1.22 \pm 0.10$ at 68% c.l.. This anomaly persists even when considering a significantly extended parameter space as shown in [20]. It is therefore mandatory to check if this deviation is in some way connected with the "Σ_0" anomaly performing an analysis by varying both parameters at the same time. This has been suggested but not actually done in [3].

Moreover, some mild tension seems also to be present between the large angular scale Planck LFI polarization data (that, alone, provides a constraint on the optical depth $\tau = 0.067 \pm 0.023$ [2]) and the Planck HFI small-scale temperature and polarization data that, when combined with large-scale LFI polarization, shifts the constraint to $\tau = 0.079 \pm 0.017$ [2]. Since the Planck constraints on τ are model-dependent, is meaningful to check if the assumption of MG could, at least partially, resolve the "τ" tension.

Another tension concerns the amplitude of the r.m.s. density fluctuations on scales of 8 Mpc h^{-1}, the so-called σ_8 parameter. The constraints on σ_8 derived by the

Planck data under the assumption of GR and Λ-CDM are in tension with the same quantity observed by low redshift surveys based on clusters counts, lensing and redshift-space distortions (see e.g. [21] and [2]). This tension appears most dramatic when considering the weak lensing measurements provided by the CFHTLenS survey (see discussion in [3]), which prefer lower values of σ_8 with respect to those obtained by Planck. Several solutions to this mild tension have been proposed, including dynamical dark energy [22], decaying dark matter [23, 24], ultralight axions [25], and voids [26]. It is therefore timely to further check if the "σ_8 tension" could be reconciled by assuming MG. This approach has already been suggested, for example, by [17].

Finally, there are also extra parameters such as the running of the spectral index $dn_S/dlnk$, the neutrino effective number N_{eff} (see e.g. [27]), and the helium abundance Y_p (see e.g. [28]) that could be varied and that could in principle be correlated with MG. Since the values of these parameters derived under Λ-CDM (see [2]) are consistent with standard expectations, it is crucial to investigate whether the inclusion of MG could change these conclusions.

This paper is organized as follows: in the next section we describe the MG parametrization that we consider, while in Section III we describe the data analysis method adopted. In Section IV, we present our results and in Section V we derive our conclusions.

II. PERTURBATION EQUATIONS

Let us briefly explain here how MG is implemented in our analysis, discussing the relevant equations. Assuming a flat universe, we can write the line element of the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric in the conformal Newtonian gauge as:

$$ds^2 = a(\tau)^2[-(1+2\Psi)d\tau^2 + (1-2\Phi)dx^i dx_i]\,, \qquad (1)$$

where a is the scale factor, τ is the conformal time, Ψ is the Newton's gravitational potential, and Φ the space curvature [1].

Given the line element of the Eq. 1, we can use a phenomenological parametrization of the gravitational potentials Ψ and Φ and their combinations. We consider the parametrization used in the publicly available code `MGCAMB` [31, 32], introducing the scale-dependent function $\mu(k,a)$, that modifies the Poisson equation for Ψ:

$$k^2\Psi = -4\pi G a^2 \mu(k,a)\rho\Delta\,, \qquad (3)$$

where ρ is the dark matter energy density, Δ is the comoving density perturbation. Furthermore one can consider the function $\eta(k,a)$, that takes into account the presence of a non-zero anisotropic stress:

$$\eta(k,a) = \frac{\Phi}{\Psi}\,. \qquad (4)$$

We can then easily introduce the function $\Sigma(k,a)$, which modifies the lensing/Weyl potential $\Phi+\Psi$ in the following way:

$$-k^2(\Phi+\Psi) \equiv 8\pi G a^2 \Sigma(k,a)\rho\Delta\,, \qquad (5)$$

and that can be obtained directly from $\mu(k,a)$ and $\eta(k,a)$ as

$$\Sigma = \frac{\mu}{2}(1+\eta)\,. \qquad (6)$$

Of course, if we have GR then $\mu = \eta = \Sigma = 1$.

It is now useful to give an expression for μ and η. Following Ref.[3], we parametrize μ and η as:

$$\mu(k,a) = 1 + f_1(a)\frac{1+c_1(\lambda H/k)^2}{1+(\lambda H/k)^2}; \qquad (7)$$

$$\eta(k,a) = 1 + f_2(a)\frac{1+c_2(\lambda H/k)^2}{1+(\lambda H/k)^2}, \qquad (8)$$

where $H = \dot{a}/a$ is the Hubble parameter, c_1 and c_2 are constants and the $f_\mathrm{i}(a)$ are functions of time that characterize the amplitude of the deviation from GR.

Again, following [3] we choose a time dependence for these functions related to the dark energy density:

$$f_\mathrm{i}(a) = E_\mathrm{ii}\Omega_\mathrm{DE}(a)\,, \qquad (9)$$

where E_ii are, again, constants and $\Omega_\mathrm{DE}(a)$ is the dark energy density parameter. As discussed in Ref. [3], the inclusion of scale dependence does not change significantly the results, we can therefore consider the scale independent parametrization, in which $c_1 = c_2 = 1$. In other words, we modify the publicly available code `MGCAMB` [31, 32], by substituting to the original μ and η, the following parametrizations:

$$\mu(k,a) = 1 + E_{11}\Omega_\mathrm{DE}(a)\,; \qquad (10)$$

$$\eta(k,a) = 1 + E_{22}\Omega_\mathrm{DE}(a)\,. \qquad (11)$$

A detection of $E_\mathrm{ii} \neq 0$ could therefore indicate a departure of the evolution of density perturbations from GR. In order to further simplify the problem, we assume a cosmological constant for the background evolution.

III. METHOD

We consider flat priors listed in Table I on all the parameters that we are constraining. They are: the six

[1] In the synchronous gauge, that is the one adopted in boltzmann codes as `CAMB` [29], we have:

$$ds^2 = a(\tau)^2[-d\tau^2 + (\delta_{ij}+h_{ij})dx^i dx^j]\,, \qquad (2)$$

where h_{ij} are defined as in [30].

Parameter	Prior
$\Omega_b h^2$	$[0.005, 0.1]$
$\Omega_c h^2$	$[0.001, 0.99]$
Θ_s	$[0.5, 10]$
τ	$[0.01, 0.8]$
n_s	$[0.8, 1.2]$
$\log[10^{10} A_s]$	$[2, 4]$
E_{11}	$[-1, 3]$
E_{22}	$[-1.4, 5]$
$\frac{dn_s}{dlnk}$	[-1,1]
$N_{\rm eff}$	[0.05,10]
$A_{\rm lens}$	[0,10]
Y_P	[0.1,0.5]

TABLE I: External flat priors on the cosmological parameters assumed in this paper.

parameters of the ΛCDM model, i.e. the Hubble constant H_0, the baryon $\Omega_b h^2$ and cold dark matter $\Omega_c h^2$ energy densities, the primordial amplitude and spectral index of scalar perturbations, A_s and n_s respectively, (at pivot scale $k_0 = 0.05hMpc^{-1}$), and the reionization optical depth τ; the constant parameters of MG, E_{11} and E_{22}; the several extensions to ΛCDM model. In particular we vary the neutrino effective number $N_{\rm eff}$ (see e.g. [27]), the running of the scalar spectral index $dn_S/dlnk$, the primordial Helium abundance Y_P and the lensing amplitude in the angular power spectra $A_{\rm lens}$. We also vary foreground parameters following the same method of [33] and [2].

We constrain these cosmological parameters by using recent cosmological datasets. First of all, we consider the full Planck 2015 release on temperature and polarization CMB angular power spectra, including the large angular scale temperature and polarization measurement by the Planck LFI experiment and the small-scale temperature and polarization spectra by Planck HFI. We refer to the Planck HFI small angular scale temperature data plus large angular scale Planck LFI temperature and polarization data as *Planck TT*, while when we include small angular scale polarization from Planck HFI as *Planck pol* (see [33]). We also use information on CMB lensing from Planck trispectrum data (see [34]) and we refer to this dataset as *lensing*. Finally, we consider the weak lensing galaxy data from the CFHTlenS [35] survey with the priors and conservative cuts to the data as described in [2] and we refer to this dataset as WL.

To perform the analysis, we use our modified version, according to the Eqs. 10, of the publicly available code `MGCAMB` [31, 32] that modifies the original publicly code `CAMB` [29] implementing the pair of functions $\mu(a, k)$ and $\eta(a, k)$, as defined in [32]. This code has been developed and tested in a completely independent way to the one used in [3].

We integrate `MGCAMB` in the latest July 2015 version of the publicly available Monte Carlo Markov Chain package `cosmomc` [36] with a convergence diagnostic based on the Gelman and Rubin statistic. This version includes

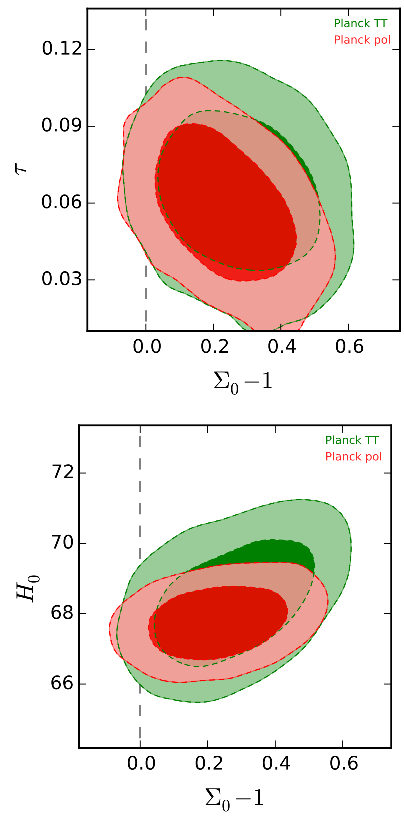

FIG. 1: Constraints at 68% and 95% confidence levels on the $\Sigma_0 - 1$ vs τ plane (top panel) and on the $\Sigma_0 - 1$ vs H_0 plane (bottom panel) from the Planck TT and Planck pol datasets. The 6 parameters of the ΛCDM model are varied. Notice that Σ_0 is different from one (dashed vertical line) at about 95 % confidence level. A small degeneracy is present between Σ_0 and τ: smaller optical depths are more compatible with the data if Σ_0 is larger than one (see top panel). Another degeneracy is present with the Hubble constant: larger values of the Hubble constant are more compatible with the considered data in case of Σ_0 different from one (bottom panel).

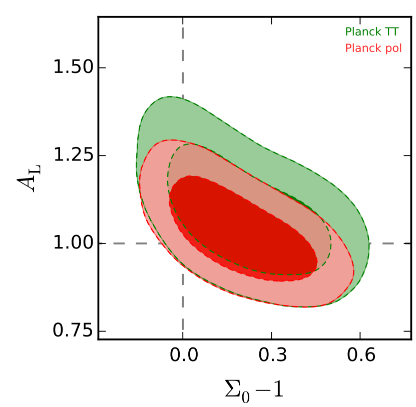

FIG. 2: Constraints at 68% and 95% confidence levels on the $\Sigma_0 - 1$ vs $A_{\rm lens}$ plane from the Planck TT and Planck pol datasets. A strong degeneracy is present between Σ_0 and $A_{\rm lens}$: larger values of $A_{\rm lens}$ are more compatible with the data if Σ_0 is smaller than one.

	Planck TT	Planck TT + WL	Planck TT + lensing	Planck pol	Planck pol + WL	Planck pol + lensing
E_{11}	$0.08^{+0.33}_{-0.72}$	$-0.18^{+0.19}_{-0.49}$	$0.08^{+0.34}_{-0.59}$	$0.06^{+0.33}_{-0.66}$	$-0.21^{+0.19}_{-0.45}$	$0.08^{+0.35}_{-0.54}$
E_{22}	$1.0^{+1.3}_{-1.6}$	$1.9^{+1.4}_{-1.0}$	$0.4^{+0.9}_{-1.4}$	$0.9^{+1.2}_{-1.5}$	$1.7^{+1.3}_{-1.0}$	$0.4^{+0.8}_{-1.3}$
$\mu_0 - 1$	$0.05^{+0.23}_{-0.50}$	$-0.13^{+0.13}_{-0.35}$	$0.05^{+0.24}_{-0.41}$	$0.04^{+0.23}_{-0.45}$	$-0.15^{+0.13}_{-0.32}$	$0.05^{+0.24}_{-0.38}$
$\eta_0 - 1$	$0.7^{+0.9}_{-1.2}$	$1.3^{+1.0}_{-0.7}$	$0.31^{+0.61}_{-0.94}$	$0.6^{+0.8}_{-1.0}$	$1.20^{+0.91}_{-0.68}$	$0.26^{+0.56}_{-0.86}$
$\Sigma_0 - 1$	0.28 ± 0.15	$0.34^{+0.16}_{-0.15}$	$0.11^{+0.09}_{-0.12}$	0.23 ± 0.13	0.27 ± 0.13	$0.10^{+0.09}_{-0.11}$
$\Omega_b h^2$	0.02251 ± 0.00027	0.02263 ± 0.00026	0.02238 ± 0.00024	0.02237 ± 0.00017	0.02243 ± 0.00017	0.02233 ± 0.00016
$\Omega_c h^2$	0.1175 ± 0.0024	0.1159 ± 0.0022	0.1171 ± 0.0021	0.1188 ± 0.0016	0.1180 ± 0.0015	0.1185 ± 0.0014
H_0	68.5 ± 1.1	69.2 ± 1.1	68.47 ± 0.99	67.78 ± 0.71	68.15 ± 0.69	67.83 ± 0.66
τ	0.065 ± 0.021	$0.061^{+0.020}_{-0.023}$	0.050 ± 0.019	0.059 ± 0.020	0.054 ± 0.019	0.045 ± 0.017
n_s	0.9712 ± 0.0071	0.9754 ± 0.0067	0.9706 ± 0.0062	0.9668 ± 0.0051	0.9689 ± 0.0050	0.9668 ± 0.0047
σ_8	$0.816^{+0.034}_{-0.052}$	$0.787^{+0.022}_{-0.039}$	$0.802^{+0.033}_{-0.039}$	$0.815^{+0.032}_{-0.048}$	$0.788^{+0.021}_{-0.035}$	0.803 ± 0.031

TABLE II: Constraints at 68% c.l. on the cosmological parameters assuming modified gravity (parametrized by E_{11} and E_{22}) and varying the 6 parameters of the standard ΛCDM model.

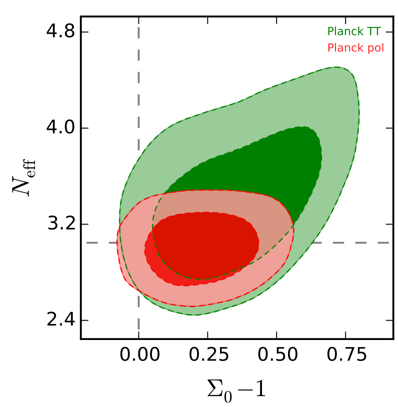

FIG. 3: Constraints at 68% and 95% confidence levels on the $\Sigma_0 - 1$ vs $N_{\rm eff}$ plane from the Planck TT and Planck polarization datasets. Notice that Σ_0 is different from unity (dashed vertical line) at about the 95 % confidence level. A small direction of degeneracy is present between Σ_0 and $N_{\rm eff}$: larger $N_{\rm eff}$ are more compatible with the data if Σ_0 is larger than one in case of the Planck TT dataset.

the support for the Planck data release 2015 Likelihood Code [33] (see `http://cosmologist.info/cosmomc/`) and implements an efficient sampling using the fast/slow parameter decorrelations [37].

IV. RESULTS

We first report the results assuming a modified gravity scenario parametrized by η and μ and varying only the 6 parameters of the standard ΛCDM model. The constraints on the several parameters are reported in Table II. When comparing the first and second column of our table, we see a complete agreement with the results presented in the first and third column of Table 6 of [3]. Namely we find evidence at $\sim 95\%$ c.l. for $\Sigma_0 - 1$ different from zero for the Planck TT dataset, and this indication is further confirmed when the WL dataset is included.

As fully discussed in [33], the Planck polarization HFI data at small angular scales fails to satisfy some of the internal checks in the data analysis pipeline. The results obtained by the inclusion of this dataset should therefore be considered as preliminary. We report the constraints from the Planck pol dataset in columns 4-6 in Table II. As we can see, the small angular scale HFI polarization data improves the constraints on Σ_0, also slightly shifting its value towards a better compatibility with standard ΛCDM. We can see however that the inclusion of small angular scale polarization does not alter substantially the conclusions obtained when using just the Planck TT dataset.

Considering just the Planck TT dataset, it is interesting to note that in this modified gravity scenario, the Hubble constant is constrained to be $H_0 = 68.5 \pm 1.1$

	Planck TT	Planck TT + WL	Planck TT + lensing	Planck pol	Planck pol + WL	Planck pol + lensing
E_{11}	$0.06^{+0.33}_{-0.65}$	$-0.15^{+0.22}_{-0.51}$	$0.08^{+0.33}_{-0.63}$	$0.07^{+0.33}_{-0.62}$	$-0.18^{+0.21}_{-0.47}$	$0.06^{+0.33}_{-0.63}$
E_{22}	$0.8^{+1.1}_{-1.7}$	$1.4^{+1.4}_{-1.3}$	$0.8^{+1.0}_{-1.5}$	$0.7^{+1.0}_{-1.6}$	1.4 ± 1.2	$0.8^{+1.1}_{-1.6}$
$\mu_0 - 1$	$0.04^{+0.23}_{-0.46}$	$-0.10^{+0.15}_{-0.36}$	$0.06^{+0.23}_{-0.44}$	$0.05^{+0.23}_{-0.43}$	$-0.12^{+0.15}_{-0.33}$	$0.04^{+0.22}_{-0.44}$
$\eta_0 - 1$	$0.6^{+0.7}_{-1.2}$	$1.0^{+1.0}_{-0.9}$	$0.5^{+0.7}_{-1.1}$	$0.5^{+0.7}_{-1.1}$	0.95 ± 0.81	$0.6^{+0.7}_{-1.1}$
$\Sigma_0 - 1$	$0.21^{+0.16}_{-0.21}$	$0.22^{+0.17}_{-0.22}$	$0.21^{+0.15}_{-0.17}$	$0.19^{+0.14}_{-0.18}$	$0.20^{+0.14}_{-0.18}$	$0.22^{+0.14}_{-0.16}$
$\Omega_b h^2$	0.02259 ± 0.00029	0.02273 ± 0.00028	0.02231 ± 0.00026	0.02239 ± 0.00017	0.02246 ± 0.00017	0.02229 ± 0.00016
$\Omega_c h^2$	0.1169 ± 0.0025	0.1152 ± 0.0023	0.1180 ± 0.0025	0.1187 ± 0.0016	0.1177 ± 0.0015	0.1191 ± 0.0015
H_0	68.8 ± 1.2	69.6 ± 1.1	68.1 ± 1.2	67.82 ± 0.73	68.26 ± 0.69	67.59 ± 0.70
τ	$0.059^{+0.021}_{-0.023}$	0.054 ± 0.021	0.059 ± 0.021	0.056 ± 0.020	$0.049^{+0.019}_{-0.022}$	0.057 ± 0.021
n_s	0.9730 ± 0.0073	0.9772 ± 0.0068	0.9687 ± 0.0070	0.9671 ± 0.0050	0.9694 ± 0.0049	0.9656 ± 0.0049
σ_8	$0.807^{+0.033}_{-0.049}$	$0.782^{+0.025}_{-0.038}$	$0.813^{+0.033}_{-0.046}$	$0.813^{+0.032}_{-0.044}$	$0.786^{+0.023}_{-0.035}$	$0.814^{+0.031}_{-0.046}$
$A_{\rm lens}$	$1.09^{+0.10}_{-0.13}$	$1.13^{+0.10}_{-0.14}$	$0.924^{+0.065}_{-0.089}$	$1.04^{+0.08}_{-0.10}$	$1.07^{+0.09}_{-0.11}$	$0.914^{+0.062}_{-0.078}$

TABLE III: Constraints at 68% c.l. on the cosmological parameters assuming modified gravity (parametrized by E_{11} and E_{22}) and varying the 6 parameters of the standard ΛCDM model plus $A_{\rm lens}$.

at 68% c.l., i.e. a value significantly larger than the $H_0 = 67.3 \pm 0.96$ at 68% c.l. reported by the Planck collaboration assuming ΛCDM. Combining the Planck TT dataset with the HST prior of $H_0 = 73.0 \pm 2.4$ from the revised analysis of [42] as in [43] we found indeed an increased evidence for MG, with $\Sigma_0 - 1 = 0.33^{+0.18}_{-0.15}$ at 68% c.l..

Moreover, the amplitude of the r.m.s. mass density fluctuations σ_8 in our modified gravity scenario is constrained to be $\sigma_8 = 0.816^{+0.034}_{-0.052}$ at 68% c.l., i.e. a value significantly weaker (and shifted towards smaller values) than the value of $\sigma_8 = 0.829 \pm 0.014$ at 68% c.l. reported by the Planck collaboration again under ΛCDM assumption.

Considering the Planck pol dataset, the value of the optical depth is also significantly smaller in the MG scenario ($\tau = 0.059 \pm 0.020$ at 68% c.l.) respect to the value obtained under standard ΛCDM model of $\tau = 0.078 \pm 0.019$ at 68% c.l., i.e. reducing the tension with the Planck LFI large angular scale polarization constraint. Interestingly, a smaller value for the optical depth of $\tau \sim 0.05$ is in better agreement with recent optical and UV astrophysical data (see e.g. [44–46]) and the reionization scenarios presented in [48]. A value of $\tau > 0.07$ could imply unexpected properties for high-redshift galaxies. Assuming an external gaussian prior of $\tau = 0.05 \pm 0.01$ (at 68 % c.l..) as in [48] that would consider in a conservative way reionization scenarios where the star formation rate density rapidly declines after redshift $z \sim 8$ as suggested by [47], we find that the Planck TT dataset provides the constraint $\Sigma_0 - 1 = 0.30 \pm 0.14$ at 68% c.l., i.e. further improving current hints of MG. In this respect, future, improved, constraints on the value of τ from large-scale polarization measurements as expected from the Planck HFI experiment will obviously provide valuable information.

The degeneracies between Σ_0, H_0 and τ can be clearly seen in Figure 1 where we show the constraints at 68% and 95% confidence levels on the $\Sigma_0 - 1$ vs τ plane (top panel) and on the $\Sigma_0 - 1$ vs H_0 plane (bottom panel) from the Planck TT and Planck pol datasets. As we can see, a degeneracy is present between $\Sigma_0 - 1$ and τ: smaller optical depths are more compatible with the data if Σ_0 is larger than one (see top panel). As discussed, a second degeneracy is present with the Hubble constant: larger values of the Hubble constant are more compatible with the considered data in case of Σ_0 different from one (Bottom Panel).

As already noticed in [3] and as we will discuss in the next paragraph, the indication for MG from the Planck data is strictly connected with the $A_{\rm lens}$ anomaly, i.e. with the fact that Planck angular spectra show "more

	Planck TT	Planck TT + WL	Planck TT + lensing	Planck pol	Planck pol + WL	Planck pol + lensing
E_{11}	$0.07^{+0.31}_{-0.73}$	$-0.13^{+0.20}_{-0.58}$	$0.09^{+0.35}_{-0.64}$	$0.07^{+0.34}_{-0.66}$	$-0.21^{+0.19}_{-0.48}$	$0.08^{+0.34}_{-0.53}$
E_{22}	1.3 ± 1.4	$2.1^{+1.8}_{-1.0}$	$0.5^{+0.9}_{-1.5}$	$0.9^{+1.2}_{-1.5}$	$1.75^{+1.4}_{-1.0}$	$0.4^{+0.8}_{-1.2}$
$\mu_0 - 1$	$0.05^{+0.22}_{-0.53}$	$-0.09^{+0.15}_{-0.43}$	$0.06^{+0.25}_{-0.45}$	$0.05^{+0.23}_{-0.45}$	$-0.15^{+0.13}_{-0.33}$	$0.06^{+0.24}_{-0.37}$
$\eta_0 - 1$	0.96 ± 1.1	$1.5^{+1.3}_{-0.8}$	$0.3^{+0.6}_{-1.1}$	$0.59^{+0.8}_{-1.0}$	$1.22^{+0.96}_{-0.69}$	$0.24^{+0.56}_{-0.83}$
$\Sigma_0 - 1$	0.36 ± 0.18	$0.45^{+0.21}_{-0.17}$	$0.12^{+0.09}_{-0.14}$	$0.23^{+0.13}_{-0.15}$	$0.28^{+0.13}_{-0.15}$	$0.10^{+0.09}_{-0.10}$
$\Omega_b h^2$	$0.02294^{+0.00049}_{-0.00063}$	$0.02328^{+0.00048}_{-0.00063}$	$0.02252^{+0.00036}_{-0.00043}$	0.02234 ± 0.00025	0.02244 ± 0.00026	0.02224 ± 0.00024
$\Omega_c h^2$	0.1202 ± 0.0041	$0.1210^{+0.0041}_{-0.0046}$	0.1185 ± 0.0039	0.1184 ± 0.0030	0.1181 ± 0.0030	0.1173 ± 0.0030
H_0	$72.0^{+3.5}_{-4.8}$	$74.7^{+3.5}_{-4.9}$	$69.7^{+2.6}_{-3.2}$	67.6 ± 1.6	$68.2^{+1.6}_{-1.8}$	67.1 ± 1.5
τ	$0.072^{+0.023}_{-0.026}$	0.073 ± 0.024	$0.052^{+0.020}_{-0.025}$	$0.059^{+0.018}_{-0.021}$	$0.053^{+0.019}_{-0.021}$	$0.044^{+0.016}_{-0.019}$
n_s	$0.990^{+0.020}_{-0.025}$	$1.004^{+0.019}_{-0.025}$	$0.977^{+0.015}_{-0.017}$	0.9655 ± 0.0097	0.969 ± 0.010	0.9625 ± 0.0092
σ_8	$0.827^{+0.033}_{-0.062}$	$0.812^{+0.028}_{-0.054}$	$0.808^{+0.035}_{-0.048}$	$0.814^{+0.032}_{-0.049}$	$0.788^{+0.022}_{-0.038}$	$0.799^{+0.031}_{-0.037}$
$N_{\rm eff}$	$3.41^{+0.36}_{-0.46}$	$3.63^{+0.35}_{-0.48}$	$3.19^{+0.30}_{-0.34}$	3.02 ± 0.20	3.06 ± 0.21	2.95 ± 0.19

TABLE IV: Constraints at 68% c.l. on the cosmological parameters assuming modified gravity (parametrized by E_{11} and E_{22}) and varying the 6 parameters of the standard ΛCDM model plus $N_{\rm eff}$.

lensing" than expected in the standard scenario. It is therefore not a surprise that when the Planck lensing data (obtained from a trispectrum analysis) that is on the contrary fully compatible with the standard expectations is included in the analysis the indication for modified gravity is significantly reduced to less than one standard deviation, as we can see from the third column of Table II. On the other hand, when weak lensing data from the WL dataset is included, the indication for MG increases, with $\Sigma_0 - 1$ larger than zero at more than 95% c.l..

In Tables III, IV, V, VI we report constraints assuming one single parameter extension to ΛCDM. In particular, we report constraints when adding as an extra parameter the lensing amplitude $A_{\rm lens}$ (Table III), the neutrino effective number $N_{\rm eff}$ (Table IV), the running of the scalar spectral index $dn_S/dlnk$ (Table V) and, finally, the Helium abundance Y_P (Table VI).

As expected, there is a main degeneracy between the $A_{\rm lens}$ parameter and Σ_0, as we can clearly see in Figure 2 where we report the 2D posteriors at 68% and 95% c.l. in the $\Sigma_0 - 1$ vs $A_{\rm lens}$ plane from the Planck TT and Planck pol datasets. In practice, the main effect of a modified gravity model is to enhance the lensing signal in the angular power spectrum. The same effect can be obtained by increasing $A_{\rm lens}$ and some form of degeneracy is clearly expected between the two parameters. As we see from the results in Table III, the value of the $A_{\rm lens}$ parameter, when MG is considered, is $A_{\rm lens} = 1.09^{+0.10}_{-0.13}$, fully consistent with 1, while for the standard ΛCDM the constraint is $A_{\rm lens} = 1.224^{+0.11}_{-0.096}$ at 68% c.l.. When also varying $A_{\rm lens}$ we found that the Planck pol datasets constraint the optical depth to $\tau = 0.056 \pm 0.020$ at 68% c.l.

On the other hand, by looking at the results in Tables IV, V, VI we do not see a significant degeneracy between the MG parameters and the new extra parameters. A small degeneracy is however present between Σ_0 and the effective neutrino number $N_{\rm eff}$. We see from Table IV that Planck TT data provides the constraint $N_{\rm eff} = 3.41^{+0.36}_{-0.46}$ at 68% c.l. that should be compared with $N_{\rm eff} = 3.13^{+0.30}_{-0.34}$ at 68% c.l. from the same dataset but assuming the standard ΛCDM model. While the possibility of an unknown "dark radiation" component (i.e. $N_{\rm eff} > 3.046$, see e.g. [38–40]) is therefore more viable in a MG scenario, it is however important to note that when adding polarization data the constraint on the neutrino number is perfectly compatible with the expectations of the standard three neutrino framework. The constraints at 68% and 95% c.l. in the $\Sigma_0 - 1$ vs $N_{\rm eff}$ planes are reported in Figure 3.

We also consider the possibility of a running of the scalar spectral index $dn_S/dlnk$. Results are reported in

	Planck TT	Planck TT + WL	Planck TT + lensing	Planck pol	Planck pol + WL	Planck pol + lensing
E_{11}	$0.08^{+0.36}_{-0.79}$	$-0.21^{+0.20}_{-0.52}$	$0.05^{+0.34}_{-0.57}$	$0.06^{+0.34}_{-0.67}$	$-0.25^{+0.20}_{-0.43}$	$0.06^{+0.35}_{-0.53}$
E_{22}	$1.2^{+1.4}_{-1.9}$	$2.2^{+1.7}_{-1.1}$	$0.5^{+0.9}_{-1.3}$	$0.9^{+1.2}_{-1.6}$	$1.8^{+1.3}_{-1.0}$	$0.4^{+0.8}_{-1.2}$
$\mu_0 - 1$	$0.06^{+0.25}_{-0.55}$	$-0.15^{+0.14}_{-0.37}$	$0.03^{+0.24}_{-0.40}$	$0.04^{+0.24}_{-0.46}$	$-0.17^{+0.14}_{-0.30}$	$0.04^{+0.24}_{-0.36}$
$\eta_0 - 1$	$0.9^{+1.0}_{-1.3}$	$1.6^{+1.2}_{-0.8}$	$0.35^{+0.62}_{-0.94}$	$0.6^{+0.8}_{-1.1}$	$1.28^{+0.90}_{-0.69}$	$0.28^{+0.58}_{-0.85}$
$\Sigma_0 - 1$	0.31 ± 0.18	$0.38^{+0.20}_{-0.18}$	$0.11^{+0.10}_{-0.13}$	$0.22^{+0.13}_{-0.15}$	0.27 ± 0.13	$0.10^{+0.09}_{-0.11}$
$\Omega_b h^2$	$0.02267^{+0.00032}_{-0.00038}$	$0.02281^{+0.00033}_{-0.00039}$	0.02238 ± 0.00026	0.02238 ± 0.00018	0.02243 ± 0.00017	0.02232 ± 0.00017
$\Omega_c h^2$	0.1170 ± 0.0027	0.1154 ± 0.0024	0.1171 ± 0.0021	0.1188 ± 0.0016	0.1180 ± 0.0015	0.1186 ± 0.0015
H_0	$68.8^{+1.3}_{-1.4}$	$69.6^{+1.2}_{-1.3}$	68.5 ± 1.0	67.76 ± 0.72	68.12 ± 0.70	67.80 ± 0.66
τ	$0.068^{+0.022}_{-0.025}$	$0.064^{+0.021}_{-0.025}$	$0.051^{+0.019}_{-0.022}$	$0.060^{+0.019}_{-0.022}$	$0.054^{+0.020}_{-0.043}$	0.045 ± 0.017
n_s	0.9721 ± 0.0076	0.9765 ± 0.0073	0.9708 ± 0.0064	0.9665 ± 0.0051	0.9686 ± 0.0051	0.9669 ± 0.0050
σ_8	$0.816^{+0.036}_{-0.059}$	$0.784^{+0.022}_{-0.042}$	$0.800^{+0.033}_{-0.038}$	$0.815^{+0.033}_{-0.048}$	$0.785^{+0.021}_{-0.034}$	$0.803^{+0.030}_{-0.036}$
$\frac{dn_s}{dlnk}$	$-0.0073^{+0.0097}_{-0.0086}$	$-0.008^{+0.011}_{-0.009}$	0.0002 ± 0.0079	-0.0014 ± 0.0072	-0.0005 ± 0.0070	0.0016 ± 0.0070

TABLE V: Constraints at 68% c.l. on the cosmological parameters assuming modified gravity (parametrized by E_{11} and E_{22}) and varying the 6 parameters of the standard ΛCDM model plus $dn_S/dlnk$.

Table V and we find no degeneracy with MG parameters. The Planck TT constraint of $dn_S/dlnk = -0.0073^{+0.0097}_{-0.0086}$ at 68% c.l. is almost identical to the value $dn_S/dlnk = -0.0084 \pm 0.0082$ at 68% c.l. obtained using the same dataset but assuming standard ΛCDM.

We also considered variations in the primordial helium abundance Y_P since it affects small angular scale anisotropies. Our results are in Table VI. The Planck TT constraint is found to be $Y_P = 0.258 \pm 0.023$ at 68% c.l., slightly larger than the standard ΛCDM value of $Y_p = 0.252 \pm 0.021$ at 68% c.l. obtained using the same dataset. While a larger helium abundance is in better agreement with recent primordial helium measurements of [41], it is important to stress that the inclusion of polarization yields a constraint that is almost identical to the one obtained under ΛCDM. The constraints at 68% and 95% c.l. in the $\Sigma_0 - 1$ vs $dn_S/dlnk$ and $\Sigma_0 - 1$ vs Y_P planes are reported in Figure 4.

V. CONCLUSIONS

In this paper, we have further investigated the current hints for a "modified gravity" scenario from the recent Planck 2015 data release. We have confirmed that the statistical evidence for these hints, assuming the conservative dataset of Planck TT, is, at most, at ∼ 95% c.l., i.e. not extremely significant. The statistical significance increases when combining the Planck datasets with the WL cosmic shear dataset. Indeed, the Planck dataset seems to provide lower values for the σ_8 parameter with respect to those derived under the assumption of GR and Λ-CDM.

If future astrophysical or cosmological measurements will point towards a lower value of the optical depth of $\tau \sim 0.05$ or of the r.m.s. amplitude of mass fluctuations of $\sigma_8 \sim 0.78$ then the current hints for modified gravity could be further strenghtened.

However it also important to stress that when the CMB lensing likelihood is included in the analysis the statistical significance for MG simply vanishes.

We also investigated possible degeneracies with extra, non-standard parameters as the neutrino effective number, the running of the scalar spectral index and the primordial helium abundance showing that the results on these parameters assuming ΛCDM are slightly changed when considering the Planck TT dataset. Namely, under modified gravity we have larger values for the neutrino effective number, $N_{eff} = 3.41^{+0.36}_{-0.46}$ at 68% c.l., and for the helium abundance, $Y_p = 0.258 \pm 0.023$. at 68% c.l.. However, the constraints on these parameters are practically identical those obtained under GR when including

	Planck TT	Planck TT + WL	Planck TT + lensing	Planck pol	Planck pol + WL	Planck pol + lensing
E_{11}	$0.05^{+0.33}_{-0.71}$	$-0.18^{+0.20}_{-0.52}$	$0.05^{+0.35}_{-0.58}$	$0.08^{+0.34}_{-0.68}$	$-0.24^{+0.20}_{-0.44}$	$0.05^{+0.34}_{-0.52}$
E_{22}	1.2 ± 1.4	$2.1^{+1.6}_{-1.0}$	$0.5^{+0.9}_{-1.4}$	$0.6^{+0.8}_{-1.1}$	$1.9^{+1.3}_{-1.0}$	$0.4^{+0.8}_{-1.2}$
$\mu_0 - 1$	$0.04^{+0.24}_{-0.51}$	$-0.13^{+0.14}_{-0.37}$	$0.04^{+0.25}_{-0.41}$	$0.06^{+0.24}_{-0.47}$	$-0.17^{+0.14}_{-0.31}$	$0.04^{+0.23}_{-0.36}$
$\eta_0 - 1$	$0.9^{+1.0}_{-1.2}$	$1.5^{+1.2}_{-0.8}$	$0.36^{+0.62}_{-0.99}$	$0.6^{+0.8}_{-1.1}$	$1.30^{+0.91}_{-0.72}$	$0.29^{+0.57}_{-0.83}$
$\Sigma_0 - 1$	0.31 ± 0.16	$0.39^{+0.19}_{-0.15}$	$0.11^{+0.09}_{-0.12}$	$0.23^{+0.13}_{-0.16}$	0.29 ± 0.13	$0.10^{+0.09}_{-0.11}$
$\Omega_{\rm b}h^2$	$0.02269^{+0.00041}_{-0.00046}$	0.02293 ± 0.00042	0.02248 ± 0.00034	$0.02245^{+0.00024}_{-0.00026}$	0.02254 ± 0.00023	0.02236 ± 0.00023
$\Omega_{\rm c}h^2$	0.1167 ± 0.0028	0.1147 ± 0.0026	0.1169 ± 0.0023	0.1187 ± 0.0016	0.1178 ± 0.0015	0.1185 ± 0.0015
H_0	$69.1^{+1.5}_{-1.7}$	$70.2^{+1.5}_{-1.7}$	$68.8^{+1.2}_{-1.4}$	$67.98^{+0.84}_{-0.94}$	68.43 ± 0.81	67.92 ± 0.77
τ	$0.068^{+0.022}_{-0.024}$	$0.066^{+0.021}_{-0.025}$	$0.052^{+0.020}_{-0.023}$	$0.061^{+0.020}_{-0.022}$	$0.055^{+0.018}_{-0.022}$	0.046 ± 0.018
n_s	$0.979^{+0.014}_{-0.016}$	0.988 ± 0.015	0.975 ± 0.012	0.9700 ± 0.0086	0.9732 ± 0.0082	0.9681 ± 0.0080
σ_8	$0.816^{+0.033}_{-0.055}$	$0.791^{+0.022}_{-0.043}$	$0.803^{+0.035}_{-0.040}$	$0.819^{+0.032}_{-0.050}$	$0.788^{+0.021}_{-0.035}$	0.803 ± 0.031
Y_P	0.258 ± 0.023	0.268 ± 0.023	0.253 ± 0.021	0.252 ± 0.014	0.254 ± 0.013	0.248 ± 0.013

TABLE VI: Constraints at 68% c.l. on the cosmological parameters assuming modified gravity (parametrized by E_{11} and E_{22}) and varying the 6 parameters of the standard ΛCDM model plus Y_P.

the Planck HFI polarization data.

We have clearly shown that the slight Planck hints of MG are strongly degenerate with the anomalous lensing amplitude in the Planck CMB angular spectra parametrized by the $A_{\rm lens}$ parameter. Indeed, the $A_{\rm lens}$ anomaly disappears when MG is considered. Clearly, undetected small experimental systematics could be the origin of this anomaly. However our conclusions are that modified gravity could provide a physical explanation, albeit exotic, for this anomaly that has been pointed out already in pre-Planck CMB datasets [49], was present in the Planck 2013 data release [50] and seems still to be alive in the recent Planck 2015 release [2] [2].

An extra parameter we have not investigated here is the neutrino absolute mass scale Σm_ν. Since MG is degenerate with the $A_{\rm lens}$ we expect that in a MG scenario current constraints on the neutrino mass from CMB angular power spectra should be weaker. However a more detailed computation is needed and we plan to investigate it in a future paper ([52]).

During the submission process of our paper, another paper appeared [53], claiming an indication for MG from cosmological data. The dataset used in that paper is completely independent from the one used here and the MG parametrization is also different. Clearly a possible connection between the two results deserves future investigation.

[2] Another possible physical explanation for the $A_{\rm lens}$ anomaly has been also very recently proposed by [51] by considering the inclusion of compensated isocurvature perturbations.

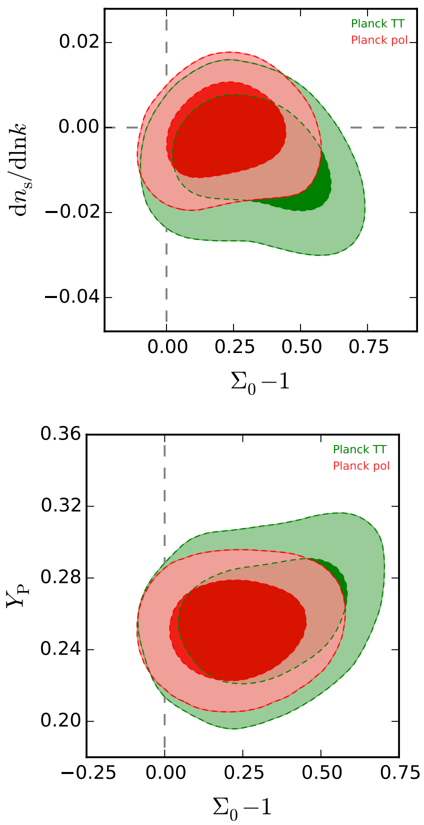

FIG. 4: Constraints at 68% and 95% confidence levels on the $\Sigma_0 - 1$ vs $dn_s/dlnk$ plane (top panel) and on the $\Sigma_0 - 1$ vs Y_p plane (bottom panel) from the Planck TT and Planck pol datasets. Notice that Σ_0 is different from unity (dashed vertical line) at about 95 % confidence level. There is virtually no degeneracy between Σ_0, the running of the scalar spectral index $dn_s/dlnk$ and the primordial helium abundance.

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