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# Reviving bino dark matter with vectorlike fourth generation particles

Mohammad Abdullah and Jonathan L. Feng

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# MSSM4G: Reviving Bino Dark Matter with Vector-like 4th Generation Particles

Mohammad Abdullah\* and Jonathan L. Feng<sup>†</sup>

*Department of Physics and Astronomy,  
University of California, Irvine, California 92697, USA*

## Abstract

We supplement the minimal supersymmetric standard model (MSSM) with vector-like copies of standard model particles. Such 4th generation particles can raise the Higgs boson mass to the observed value without requiring very heavy superpartners, improving naturalness and the prospects for discovering supersymmetry at the LHC. Here we show that these new particles are also motivated cosmologically: in the MSSM, pure Bino dark matter typically overcloses the Universe, but 4th generation particles open up new annihilation channels, allowing BinOs to have the correct thermal relic density without resonances or co-annihilation. We show that this can be done in a sizable region of parameter space while preserving gauge coupling unification and satisfying constraints from collider, Higgs, precision electroweak, and flavor physics.

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\* maabdull@uci.edu

<sup>†</sup> jlf@uci.edu

## I. INTRODUCTION

Supersymmetric extensions of the standard model are well-motivated by three promising features that were first identified over three decades ago. First, supersymmetry (SUSY) softens the quadratically-divergent contributions to the Higgs boson mass, reducing the fine-tuning needed to explain the difference between the electroweak scale and the Plank scale [1–4]. Second, the minimal supersymmetric standard model (MSSM) provides the required new field content to improve the unification of gauge couplings [5–8]. And third, with the addition of  $R$ -parity, supersymmetric extensions contain stable neutralinos, which are natural candidates for weakly-interacting massive particle (WIMP) dark matter [9, 10].

The lack of direct evidence for supersymmetry, particularly after Run I of the LHC, has excluded some supersymmetric models, but not others [11, 12], and it remains important to develop supersymmetric models that continue to have the potential to realize the original motivating promises. In this work, we consider MSSM4G models in which the MSSM is extended to include vector-like copies of standard model particles. These models have been considered previously for their promise of raising the Higgs boson mass to the observed value without extremely heavy superpartners. We will show that these models also restore Bino-like neutralinos as excellent dark matter candidates in a broad range of parameter space that simultaneously preserves gauge coupling unification and satisfies all constraints from new physics searches and Higgs, electroweak, and flavor physics.

In the non-supersymmetric context, the possibility of a 4th generation of fermions has been considered at least since the 3rd generation was discovered. The multiple deaths and rebirths of this idea are nicely summarized in Ref. [13]. Briefly, in the 1990’s a 4th generation of *chiral*, or sequential, fermions was severely constrained by precision electroweak measurements at LEP, as parametrized, for example, by the  $S$ ,  $T$ , and  $U$  parameters of Peskin and Takeuchi [14, 15]. These constraints excluded degenerate chiral fermions, which have vanishing contributions to  $T$  [16], but non-degenerate chiral fermions that contribute to both  $S$  and  $T$  in a correlated way remained viable [17]. The status of chiral 4th generation fermions changed once again, however, with the advent of Higgs physics at the LHC. Since chiral fermions must get their mass from interactions with the Higgs boson, they contribute to Higgs production through gluon fusion if they are colored and to Higgs diphoton decay if they are electrically charged. These contributions are famously non-decoupling, and current

constraints exclude chiral 4th generation fermions up to perturbative values of the Yukawa couplings. Although loopholes still exist, for example, in models with extended Higgs sectors [18], even these possibilities are now severely constrained by the rapid improvements in precision Higgs measurements, and chiral 4th generation fermions are now essentially excluded.

The situation is completely different, however, for *vector-like* 4th generation fermions. They can be added in any combination, as vector-like fermions do not contribute to anomalies, and they may get masses without coupling to the Higgs boson, so their contributions to Higgs production and decay do decouple, and they may rather easily satisfy bounds from precision Higgs measurements. This also means that they do not contribute to electroweak symmetry breaking effects at leading order, which keeps them safe from precision electroweak constraints. Models with vector-like 4th generation fermions therefore remain viable, and such models have been studied for a variety of reasons [19].

In the context of supersymmetry, the possibility of vector-like 4th generation particles takes on added significance. As is well-known, the measured Higgs boson mass,  $m_h = 125.09 \pm 0.21 \pm 0.11$  GeV [20], implies there must be large radiative corrections [21–23]. In the MSSM, this typically requires heavy squarks, which, barring some explanation, strain naturalness. But 4th generation fermions and their scalar superpartners also contribute radiatively to the Higgs boson mass, reducing the need for very heavy superpartners. This was first noted long ago [24, 25] and has gained increasing attention through the years as the lower bound on the Higgs boson mass has grown [26–45]. At the same time, in supersymmetry, 4th generation extensions are highly constrained if one requires that they preserve gauge coupling unification and raise the Higgs mass significantly. These aspects have been discussed at length, for example, in Ref. [28], where the different possibilities for vector-like fermions were explored exhaustively with respect to their ability to increase the Higgs mass, while maintaining gauge coupling unification and avoiding bounds from electroweak precision data.

In this study, we show that, in supersymmetry, vector-like 4th generation particles are also motivated cosmologically. In many well-motivated supersymmetric models, renormalization group evolution or other effects imply that the Bino is the lightest gaugino, and so it is the lightest neutralino in “half” of parameter space (with the Higgsino being the lightest in the rest of parameter space). Pure Binors do not annihilate to  $W$  or  $Z$  bosons, and

they annihilate to standard model fermions only through  $t$ -channel sfermions. For these annihilation channels to be sufficiently efficient that Binos do not overclose the Universe, Binos must be lighter than about 300 GeV [46, 47]. Such light Binos are now excluded in many cases by results from the LHC. For example, searches for gluino pair production, followed by decays to neutralinos, exclude neutralino masses below 300 GeV, provided the gluinos are lighter than 1.4 TeV and not highly degenerate with the neutralinos [48, 49]. Light neutralinos produced in squark decays are similarly excluded [48, 49]. These bounds have loopholes. For example, if neutralinos are degenerate with staus to within 5%, they co-annihilate in the early Universe and may be as heavy as 600 GeV without overclosing the Universe [50, 51]. Such possibilities are currently viable, and will be probed completely in the upcoming LHC run [52–54]. However, barring such degeneracies and other accidental mass arrangements, Bino dark matter in the MSSM is now significantly constrained.

Here we will show that vector-like copies of 4th (and 5th) generation fermions open up new annihilation channels for the Bino, reducing its thermal relic density to the measured value or below. These new channels are extremely efficient, with even a single 4th generation lepton channel dominating over all MSSM channels combined. Binos are therefore restored as excellent dark matter candidates in regions of parameter space where naturalness is improved, gauge coupling unification is preserved, and all constraints are satisfied. Dark matter in 4th generation supersymmetry models has been discussed previously. In Refs. [55, 56], for instance, 4th generation neutrinos were considered as dark matter candidates. In Refs. [33, 34], neutralinos were shown to be viable dark matter candidates when highly degenerate with co-annihilating sleptons. To our knowledge (and surprise), there are no discussions in the literature of the effects of vector-like 4th generation particles on the thermal relic density of Binos in the generic, non-co-annihilating case, which is the focus of this study.

The paper is organized as follows. In Sec. II we present the particle content, simplifying assumptions, and existing bounds for the 4th generation models we will study. Simply requiring that the vector-like 4th generation particles preserve gauge coupling unification and contribute significantly to the Higgs boson mass reduces the number of models to consider to essentially two. We then examine these two models in detail in Secs. III and IV, where we present our results for the relic density and Higgs mass, respectively. In Sec. V we summarize our findings and comment on the experimental prospects for discovering supersymmetry in

these cosmologically-motivated models.

## II. THE MODEL

### A. Particle Content

The standard model, supplemented by right-handed neutrinos, includes quark isodoublets (doublets under the weak isospin  $SU(2)$  gauge group)  $Q$ , up-type quark isosinglets  $U$ , down-type quark isosinglets  $D$ , lepton isodoublets  $L$ , charged lepton isosinglets  $E$ , and neutrino isosinglets  $N$ . Beginning with the MSSM, we add vector-like copies of these fermions (and their superpartners). By this we mean adding both left- and right-handed versions of fermions whose  $SU(2) \times U(1)_Y$  charges are identical to one of the standard model fermions. As we are only considering vector-like extensions here, as a shorthand, we will list only one of the chiral fields, with the chiral partner implicitly included. Thus, for example, a model with an extra  $Q$  (or  $\mathbf{5}$ ) multiplet implicitly also includes its chiral partner  $\bar{Q}$  (or  $\bar{\mathbf{5}}$ ).

Gauge anomalies cancel within each vector-like pair, so there is no need to add a full generation at once. This would seem to lead to a Pandora's box of possibilities. However, the number of models to consider may be greatly reduced simply by requiring that the new particles preserve gauge coupling unification and contribute significantly to raising the Higgs boson mass.

To preserve gauge coupling unification, we begin by considering only full  $SU(5)$  multiplets, that is,  $\mathbf{1}$ ,  $\mathbf{5}$ , and  $\mathbf{10}$  multiplets. Using 1-loop renormalization group equations (RGEs), the gauge couplings remain perturbative up to the GUT scale with a full vector-like generation of  $\mathbf{5} + \mathbf{10}$ , but this is not true when 3-loop RGEs are used [28]. Thus, gauge coupling unification reduces the remaining possibilities to either one  $\mathbf{10}$  multiplet or one, two, or three  $\mathbf{5}$  multiplets (plus any number of singlets).

The  $\mathbf{5}$  multiplets contain  $D$  and  $E$  fields. To raise the Higgs boson mass, these fields must couple to the Higgs field. The  $D$  field would require a  $Q$  field, which would bring in an entire  $\mathbf{10}$ , ruining gauge coupling unification. The  $E$  field requires only an  $N$ , which is consistent with gauge coupling unification. However, as shown in Ref. [28], perturbativity up to the GUT scale requires that lepton Yukawa couplings be at most  $h = 0.75$ . The contribution of  $N_g = 3$  extra generations of leptons/sleptons to the Higgs boson mass scales as  $N_g h^4 \lesssim 1$ ;

this is to be compared with the contribution from  $N_c = 3$  colors of top quarks/squarks in the MSSM, which scales as  $N_c y_t^4 \approx 3$ . Extra lepton generations can therefore help raise the Higgs mass to its measured value only if the sleptons have extremely large masses, leading to extra fine-tuning, which defeats one of the primary purposes of adding a 4th generation [28]. This leaves us with only one possibility, adding a **10** and any number of **1s**. The singlets do not impact gauge coupling unification, cannot interact through Yukawa couplings with the Higgs boson in this model, and do not couple to Bino dark matter, and so have no effect; we will therefore omit them. The resulting model, known as the QUE model, is consistent with perturbative gauge coupling unification and can raise the Higgs boson mass through the  $H_u Q U$  interaction with a significant Yukawa coupling.

The additional particles in the QUE model are

$$\text{Dirac fermions: } T_4, B_4, t_4, \tau_4 \quad (1)$$

$$\text{Complex scalars: } \tilde{T}_{4L}, \tilde{T}_{4R}, \tilde{B}_{4L}, \tilde{B}_{4R}, \tilde{t}_{4L}, \tilde{t}_{4R}, \tilde{\tau}_{4L}, \tilde{\tau}_{4R} , \quad (2)$$

where the subscripts 4 denote 4th generation particles, upper- and lower-case letters denote isodoublets and isosinglets, respectively, and  $L$  and  $R$  denote scalar partners of left- and right-handed fermions, respectively. The SUSY-preserving interactions are specified by the superpotential

$$W_{\text{QUE}} = M_{Q_4} \hat{Q}_4 \hat{\bar{Q}}_4 + M_{t_4} \hat{t}_4 \hat{\bar{t}}_4 + M_{\tau_4} \hat{\tau}_4 \hat{\bar{\tau}}_4 + k \hat{H}_u \hat{Q}_4 \hat{\bar{t}}_4 - h \hat{H}_d \hat{Q}_4 \hat{\bar{t}}_4 , \quad (3)$$

where the carets denote superfields,  $\hat{Q}_4 = (\hat{T}_4, \hat{B}_4)$  is the quark isodoublet,  $\hat{t}_4$  and  $\hat{\tau}_4$  are the quark and lepton isosinglets, and the vector-like masses  $M_{Q_4}$ ,  $M_{t_4}$ , and  $M_{\tau_4}$  and the Yukawa couplings  $k$  and  $h$  are all free parameters. We also assume small but non-vanishing mixings of these fields with, say, 3rd generation fields, so that the 4th generation fermions decay and are not cosmologically troublesome. These have relevance for collider physics, but are not significant for the topics discussed here and so are not displayed. Finally, there are the soft SUSY-breaking terms

$$\begin{aligned} \mathcal{L}_{\text{QUE}} = & -m_{\tilde{Q}_4}^2 |\tilde{Q}_4|^2 - m_{\tilde{\bar{Q}}_4}^2 |\tilde{\bar{Q}}_4|^2 - m_{\tilde{t}_4}^2 |\tilde{t}_4|^2 - m_{\tilde{\bar{t}}_4}^2 |\tilde{\bar{t}}_4|^2 - m_{\tilde{\tau}_4}^2 |\tilde{\tau}_4|^2 - m_{\tilde{\bar{\tau}}_4}^2 |\tilde{\bar{\tau}}_4|^2 \\ & - A_{t_4} H_u \tilde{Q}_4 \tilde{\bar{t}}_4 - A_{b_4} H_d \tilde{Q}_4 \tilde{\bar{t}}_4 - B_{Q_4} \tilde{Q}_4 \tilde{\bar{Q}}_4 - B_{t_4} \tilde{t}_4 \tilde{\bar{t}}_4 - B_{\tau_4} \tilde{\tau}_4 \tilde{\bar{\tau}}_4 , \end{aligned} \quad (4)$$

where all the coefficients are free, independent parameters.

If one drops the GUT multiplet requirement, there is another possibility consistent with perturbative gauge coupling unification [28]: the QDEE model, with the  $U$  of the **10** replaced by a  $D$ , and an additional (5th generation)  $E$ . This model also (accidentally) preserves gauge coupling unification and raises the Higgs mass through the  $H_d Q D$  interaction, and we will include it in our analysis.

With notation similar to that above, the QDEE model has the extra particles

$$\text{Dirac fermions: } T_4, B_4, b_4, \tau_4, \tau_5 \quad (5)$$

$$\text{Complex scalars: } \tilde{T}_{4L}, \tilde{T}_{4R}, \tilde{B}_{4L}, \tilde{B}_{4R}, \tilde{b}_{4L}, \tilde{b}_{4R}, \tilde{\tau}_{4L}, \tilde{\tau}_{4R}, \tilde{\tau}_{5L}, \tilde{\tau}_{5R} . \quad (6)$$

The superpotential is

$$W_{\text{QDEE}} = M_{Q_4} \hat{Q}_4 \hat{\bar{Q}}_4 + M_{b_4} \hat{b}_4 \hat{\bar{b}}_4 + M_{\tau_4} \hat{\tau}_4 \hat{\bar{\tau}}_4 + M_{\tau_5} \hat{\tau}_5 \hat{\bar{\tau}}_5 + k \hat{H}_u \hat{Q}_4 \hat{\bar{b}}_4 - h \hat{H}_d \hat{Q}_4 \hat{b}_4 , \quad (7)$$

and the soft SUSY-breaking terms are

$$\begin{aligned} \mathcal{L}_{\text{QDEE}} = & -m_{\tilde{Q}_4}^2 |\tilde{Q}_4|^2 - m_{\tilde{\bar{Q}}_4}^2 |\tilde{\bar{Q}}_4|^2 - m_{\tilde{b}_4}^2 |\tilde{b}_4|^2 - m_{\tilde{\bar{b}}_4}^2 |\tilde{\bar{b}}_4|^2 - m_{\tilde{\tau}_4}^2 |\tilde{\tau}_4|^2 - m_{\tilde{\bar{\tau}}_4}^2 |\tilde{\bar{\tau}}_4|^2 - m_{\tilde{\tau}_5}^2 |\tilde{\tau}_5|^2 - m_{\tilde{\bar{\tau}}_5}^2 |\tilde{\bar{\tau}}_5|^2 \\ & - A_{t_4} H_u \tilde{Q}_4 \tilde{\bar{b}}_4 - A_{b_4} H_d \tilde{Q}_4 \tilde{b}_4 - B_{Q_4} \tilde{Q}_4 \tilde{\bar{Q}}_4 - B_{b_4} \tilde{b}_4 \tilde{\bar{b}}_4 - B_{\tau_4} \tilde{\tau}_4 \tilde{\bar{\tau}}_4 - B_{\tau_5} \tilde{\tau}_5 \tilde{\bar{\tau}}_5 . \end{aligned} \quad (8)$$

## B. Simplifying Assumptions

Although we have reduced the number of models we consider to two fairly minimal ones, in each model there are still a large number of new parameters. To make progress and present our results, we make a number of simplifying assumptions about the weak-scale values of these parameters.

For both models, we choose the ratio of Higgs vacuum expectation values to be  $\tan \beta = 10$ , a moderate value that makes the tree-level Higgs mass near its maximal value. To maximize the radiative corrections from the 4th generation quark sector, we fix the up-type Yukawa couplings to be at their quasi-fixed point values:  $k = 1.05$  in the QUE model and 1.047 in the QDEE model [28]. The down-type Yukawa couplings  $h$  have lower quasi-fixed point values. They can boost the Higgs boson mass if  $h < 0$ , but their effects are suppressed by  $\tan \beta$  and so typically quite subdominant; for simplicity, we set  $h = 0$ . We also assume  $|\mu|$  is sufficiently large that the lightest neutralino is the Bino  $\tilde{B}$ . Finally, we choose  $A$ -parameters such that there is no left-right squark mixing, that is,  $A_{t_4} - \mu \tan \beta = 0$  and  $A_{b_4} - \mu \cot \beta = 0$ , and assume the 4th generation  $B$ -parameters are negligible.



For the QUE model, we assume spectra of the extra fermions and sfermions that can be specified by 4 parameters: the unified (weak-scale) squark, slepton, quark, and lepton masses

$$m_{\tilde{q}_4} \equiv m_{\tilde{T}_{4L}} = m_{\tilde{T}_{4R}} = m_{\tilde{B}_{4L}} = m_{\tilde{B}_{4R}} = m_{\tilde{t}_{4L}} = m_{\tilde{t}_{4R}} \quad (9)$$

$$m_{\tilde{\ell}_4} \equiv m_{\tilde{\tau}_{4L}} = m_{\tilde{\tau}_{4R}} \quad (10)$$

$$m_{q_4} \equiv m_{T_4} = m_{B_4} = m_{t_4} \quad (11)$$

$$m_{\ell_4} \equiv m_{\tau_4} . \quad (12)$$

Strictly speaking, some of these relations cannot be satisfied exactly, as quarks (squarks) that are in the same isodoublet have SU(2)-preserving masses specified by the same parameters, and their physical masses are then split by electroweak symmetry breaking. However, these splittings are small compared to the masses we will consider and so ignoring them will have little impact on our relic density results.

For the QDEE model, we also assume 4 unifying masses

$$m_{\tilde{q}_4} \equiv m_{\tilde{T}_{4L}} = m_{\tilde{T}_{4R}} = m_{\tilde{B}_{4L}} = m_{\tilde{B}_{4R}} = m_{\tilde{b}_{4L}} = m_{\tilde{b}_{4R}} \quad (13)$$

$$m_{\tilde{\ell}_4} \equiv m_{\tilde{\tau}_{4L}} = m_{\tilde{\tau}_{4R}} = m_{\tilde{\tau}_{5L}} = m_{\tilde{\tau}_{5R}} \quad (14)$$

$$m_{q_4} \equiv m_{T_4} = m_{B_4} = m_{b_4} \quad (15)$$

$$m_{\ell_4} \equiv m_{\tau_4} = m_{\tau_5} . \quad (16)$$

Finally, for both models, we assume that the Bino is lighter than all squarks and sleptons so that it is the lightest supersymmetric particle (LSP), but heavier than at least some fermions, so that it can annihilate to them and reduce its thermal relic density. For simplicity, we assume the mass ordering

$$m_{\tilde{q}_4}, m_{\tilde{\ell}_4}, m_{q_4} > m_{\tilde{B}} > m_{\ell_4} , \quad (17)$$

so that Binons annihilate to 4th generation leptons, but not 4th generation quarks. As we will see, the addition of the 4th generation lepton channels is enough to reduce the Bino relic density to allowed levels. This ordering also allows the colored new particles to be heavy enough to avoid LHC bounds.

### C. Existing Bounds

We have included a Higgs-Yukawa term for the vector-like up-type quarks, even though these already have vector-like masses. The motivation, of course, is to induce corrections to the Higgs boson mass. One has to worry, though, that such couplings could violate electroweak constraints. In Ref. [28], however, it is shown that already for 350 GeV vector-like up-type quarks, the contributions to the  $STU$  parameters are within the  $1\sigma$  exclusion contours, and the contributions are even smaller for the heavier masses that yield the correct relic density.

Another reason one might worry about the Higgs terms is constraints from Higgs physics, namely Higgs production and decay through triangle diagrams with fermions in the loop. As mentioned in the introduction, for chiral fermions, the linear relation between the fermion mass and the Higgs Yukawa slows down the decoupling of those triangle diagrams as the fermion mass is increased so that, by the time the experimental constraints are satisfied, the Yukawa coupling are non-perturbative [19]. Adding a vector-like mass makes these triangle diagrams decouple more quickly. However, there are still some limits from the LHC Higgs data, which we take from Ref. [19]. According to their analysis, vector-like quarks of about 1 TeV are (barely) safe from experimental limits. Note however that their fit is based on a model with both up- and down-type isosinglets, so their limits will be weaker when applied to our models, where either the down-type or up-type isosinglet is missing. The authors also perform a fit to the  $STU$  parameters that confirms our conclusions based on Ref. [28] that our model is safe.

Last, as noted above, to allow the 4th generation fermions to decay and so satisfy cosmological bounds, we assume that they mix with MSSM fields. In general, the 4th generation fields may then induce magnetic or electric dipole moments or mediate flavor-violating observables for fermions in the first 3 generations. We will assume that these mixings are minute, however, and dominantly with the 3rd generation, where bounds are weak and easily consistent with the lifetime requirement from cosmology.

### III. RELIC DENSITY

With the assumptions of Sec. II B, there are now new dark matter annihilation processes:  $\tilde{B}\tilde{B} \rightarrow \tau_i^+\tau_i^-$ , mediated by  $t$ - and  $u$ -channel sleptons  $\tilde{\tau}_{iL}$  and  $\tilde{\tau}_{iR}$ , where  $i = 4$  for the QUE model and  $i = 4, 5$  for the QDEE model. These new channels increase the thermally-averaged annihilation cross section  $\langle\sigma v\rangle$ , which may reduce the Bino thermal relic density  $\Omega_{\tilde{B}}h^2$  to acceptable levels even for large and viable Bino masses.

For the present purposes, it suffices to calculate the relic density using the approximation [57]

$$\Omega_{\tilde{B}}h^2 = 1.07 \times 10^9 \text{ GeV}^{-1} \frac{x_F}{\sqrt{g_*} M_{\text{Pl}} a [1 + b/(2ax_F)]} \quad (18)$$

$$x_F = \ln r - \frac{1}{2} \ln(\ln r) + \ln(1 + b/\ln r) \quad (19)$$

$$r = 0.038 \frac{g}{\sqrt{g_* M_{\text{Pl}} m_\chi a}}, \quad (20)$$

where  $x_F = m_{\tilde{B}}/T_F$ , the ratio of the dark matter mass to the freezeout temperature  $T_F$ ,  $g_*$  is the number of massless degrees of freedom at freezeout,  $g = 2$  is the number of degrees of freedom of the Bino,  $M_{\text{Pl}} \simeq 1.22 \times 10^{19} \text{ GeV}$  is the Planck mass, and  $a$  and  $b$  are the  $S$ - and  $P$ -wave cross section coefficients given below. For the parameters of interest here, we find  $x_F \approx 24$ , and so  $T_F$  is between the  $W$  and  $b$  masses and  $g_* \approx 87.25$ . The current bound on the dark matter relic density is  $\Omega_{\text{DM}}h^2 = 0.1199 \pm 0.0022$  [58]. Equation (18) is accurate to 5% [57] or better, and we will require  $\Omega_{\tilde{B}}h^2 = 0.12$  to within a fractional accuracy of 10%.

The cross section for  $\tilde{B}\tilde{B} \rightarrow f^+f^-$  mediated by  $t$ - and  $u$ -channel sfermions  $\tilde{f}_{L,R}$  with masses  $m_{L,R}$  and hypercharges  $Y_{L,R}$  is

$$\left. \frac{d\sigma}{d\Omega} \right|_{\text{CM}} = \frac{1}{256\pi^2 s} \sqrt{\frac{s - 4m_f^2}{s - 4m_{\tilde{B}}^2}} \sum_{i,f} |\mathcal{M}|^2 \quad (21)$$

$$\begin{aligned} \sum_{i,f} |\mathcal{M}|^2 = & \frac{1}{4} g_Y^4 Y_L^4 \left[ \frac{(m_{\tilde{B}}^2 + m_f^2 - t)^2}{(m_L^2 - t)^2} + \frac{(m_{\tilde{B}}^2 + m_f^2 - u)^2}{(m_L^2 - u)^2} - \frac{2m_{\tilde{B}}^2(s - 2m_f^2)}{(m_L^2 - t)(m_L^2 - u)} \right] \\ & + \frac{1}{4} g_Y^4 Y_R^4 \left[ \frac{(m_{\tilde{B}}^2 + m_f^2 - t)^2}{(m_R^2 - t)^2} + \frac{(m_{\tilde{B}}^2 + m_f^2 - u)^2}{(m_R^2 - u)^2} - \frac{2m_{\tilde{B}}^2(s - 2m_f^2)}{(m_R^2 - t)(m_R^2 - u)} \right] \\ & + \frac{1}{2} g_Y^4 Y_L^2 Y_R^2 m_f^2 \left[ \frac{4m_{\tilde{B}}^2}{(m_L^2 - t)(m_R^2 - t)} + \frac{4m_{\tilde{B}}^2}{(m_L^2 - u)(m_R^2 - u)} \right. \\ & \quad \left. - \frac{s - 2m_{\tilde{B}}^2}{(m_L^2 - t)(m_R^2 - u)} - \frac{s - 2m_{\tilde{B}}^2}{(m_L^2 - u)(m_R^2 - t)} \right], \end{aligned} \quad (22)$$

where  $g_Y \simeq 0.35$  is the  $U(1)_Y$  gauge coupling.

Multiplying this differential cross section by the relative velocity  $v$ , expanding in powers of  $v$ , integrating over angles, and carrying out the thermal average, we find

$$\langle \sigma v \rangle = a + b x_F^{-1} \quad (23)$$

$$a = \frac{g_Y^4}{128\pi} \frac{m_f^2}{m_{\tilde{B}}} \sqrt{m_{\tilde{B}}^2 - m_f^2} \left[ \frac{Y_L^4}{\Delta_L^2} + \frac{Y_R^4}{\Delta_R^2} + \frac{2Y_L^2 Y_R^2}{\Delta_L \Delta_R} \right] \quad (24)$$

$$b = \frac{g_Y^4}{512\pi} \frac{1}{m_{\tilde{B}} \sqrt{m_{\tilde{B}}^2 - m_f^2}} \left[ \frac{Y_L^4}{\Delta_L^4} f_{LL} + \frac{Y_R^4}{\Delta_R^4} f_{RR} + \frac{Y_L^2 Y_R^2}{\Delta_L \Delta_R} m_f^2 f_{LR} \right], \quad (25)$$

where

$$\begin{aligned} f_{LL,RR} &= 13m_f^8 + m_f^6 (-26m_{L,R}^2 - 36m_{\tilde{B}}^2) + m_f^4 (70m_{L,R}^2 m_{\tilde{B}}^2 + 13m_{L,R}^4 + 49m_{\tilde{B}}^4) \\ &\quad + m_f^2 (-44m_{L,R}^2 m_{\tilde{B}}^4 - 26m_{L,R}^4 m_{\tilde{B}}^2 - 42m_{\tilde{B}}^6) + 16 (m_{L,R}^4 m_{\tilde{B}}^4 + m_{\tilde{B}}^8) \\ f_{LR} &= (18m_f^2 - 12m_{\tilde{B}}^2) \\ &\quad + \frac{8(m_{\tilde{B}}^2 - m_f^2)}{\Delta_L^2 \Delta_R^2} [-3m_f^8 + m_f^6 (8m_{\tilde{B}}^2 + 6m_L^2 + 6m_R^2) \\ &\quad + m_f^4 (-6m_{\tilde{B}}^4 - 17m_L^2 m_{\tilde{B}}^2 - 3m_L^4 - 17m_R^2 m_{\tilde{B}}^2 - 3m_R^4 - 12m_L^2 m_R^2) \\ &\quad + m_f^2 (6m_L^4 m_R^2 + 7m_L^4 m_{\tilde{B}}^2 + 16m_L^2 m_{\tilde{B}}^4 \\ &\quad + 6m_R^4 m_L^2 + 7m_R^4 m_{\tilde{B}}^2 + 16m_R^2 m_{\tilde{B}}^4 + 30m_L^2 m_R^2 m_{\tilde{B}}^2) \\ &\quad + m_{\tilde{B}}^8 - 5m_L^2 m_{\tilde{B}}^6 - 4m_L^4 m_{\tilde{B}}^4 - 9m_L^2 m_R^4 m_{\tilde{B}}^2 \\ &\quad - 5m_R^2 m_{\tilde{B}}^6 - 4m_R^4 m_{\tilde{B}}^4 - 9m_R^2 m_L^4 m_{\tilde{B}}^2 - 3m_L^4 m_R^4 - 18m_L^2 m_R^2 m_{\tilde{B}}^4], \end{aligned} \quad (27)$$

and  $\Delta_{L,R} = m_{\tilde{B}}^2 + m_{L,R}^2 - m_f^2$ .

Equations (23)–(27) are valid for sfermions with different masses and hypercharges. For degenerate vector-like sfermions with  $m_{\tilde{f}} \equiv m_L = m_R$  and  $Y_V \equiv Y_L = Y_R$ , the cross section coefficients simplify to

$$a = \frac{g_Y^4 Y_V^4}{32\pi} \frac{m_f^2}{m_{\tilde{B}}} \frac{\sqrt{m_{\tilde{B}}^2 - m_f^2}}{(m_{\tilde{B}}^2 + m_{\tilde{f}}^2 - m_f^2)^2} \quad (28)$$

$$\begin{aligned} b &= \frac{g_Y^4 Y_V^4}{128\pi} \frac{1}{m_{\tilde{B}}} \frac{1}{\sqrt{m_{\tilde{B}}^2 - m_f^2} (m_{\tilde{B}}^2 + m_{\tilde{f}}^2 - m_f^2)^4} \times \\ &\quad \left[ 17m_f^8 - 2m_f^6 (17m_{\tilde{f}}^2 + 20m_{\tilde{B}}^2) + m_f^4 (86m_{\tilde{B}}^2 m_{\tilde{f}}^2 + 17m_{\tilde{f}}^4 + 37m_{\tilde{B}}^4) \right. \\ &\quad \left. - 2m_f^2 (26m_{\tilde{B}}^4 m_{\tilde{f}}^2 + 11m_{\tilde{B}}^2 m_{\tilde{f}}^4 + 11m_{\tilde{B}}^6) + 8m_{\tilde{B}}^4 (m_{\tilde{f}}^4 + m_{\tilde{B}}^4) \right]. \end{aligned} \quad (29)$$

The expansion in  $v$  assumes that  $v$  is the only small parameter. This is not true when  $f$  and  $\tilde{B}$  become degenerate and the annihilation is near threshold. In this limit, the expressions for  $b$  in Eqs. (27) and (29) become singular, signaling the breakdown of the expansion. The expansion is essentially an expansion in even powers of  $\alpha = v/\sqrt{1 - (m_f/m_{\tilde{B}})^2}$ . Requiring that the next omitted ( $D$ -wave) term be less than a 10% correction implies roughly  $\alpha^4 < 0.1$ . For characteristic velocities of  $v \sim 0.3$  at freezeout, this implies  $m_f < 0.85m_{\tilde{B}}$ . The case of near-threshold annihilation was considered in Ref. [47], where it was shown in a generic setting that corrections to  $\langle\sigma v\rangle$  above the few percent level may occur if  $m_f > 0.95m_{\tilde{B}}$ . There, alternative expressions valid in the degenerate limit were derived. Here, as we are primarily interested in the cosmologically preferred regions without accidental mass degeneracies, we will use Eqs. (28) and (29) and simply take care to avoid applying these cross section formulae to cases where the dark matter and final state fermion are in the degenerate region. We note also that an expression for  $\langle\sigma v\rangle$  was presented in Ref. [46] for degenerate sfermions. The expressions there differ from our result in Eq. (23) with  $m_L = m_R$ , but the disagreement is numerically small and at most at the 5% level.

The annihilation cross section has some interesting features. First, hypercharge enters to the fourth power. Isosinglet leptons have the largest hypercharge of any MSSM fields. As we show below, the squarks need to be above a TeV to achieve the correct Higgs mass. But leptons and sleptons can be relatively light. As a result, annihilation to leptons is particularly efficient, and it is fortunate that they exist in both the QUE and QDEE models. Note also that because the fermions are vector-like, there is no chiral suppression. This differs greatly from the MSSM, where annihilations to isosinglet leptons are hypercharge-enhanced, but extremely suppressed by the chiral suppression of the  $S$ -wave cross section, since all MSSM leptons are light. In both the QUE and QDEE models, there are heavy isosinglet leptons, and annihilation to them is neither hypercharge- nor chirality-suppressed. Annihilations to 4th generation particles therefore completely dominate over MSSM channels.

In Fig. 1, we show regions of the  $(m_{\tilde{\ell}_4}, m_{\tilde{B}})$  plane, with  $m_{\ell_4}$  fixed to the values indicated, where Bino dark matter freezes out with a relic density within 10% of the value required to be all of dark matter. These regions are bounded on all sides. We must require the mass ordering  $m_{\ell_4} < m_{\tilde{B}} < m_{\tilde{\ell}_4}$  so that the Binos are the LSPs, but may pair-annihilate to 4th generation leptons. The mass of  $\ell_4$  is bounded from below by heavy lepton searches. As this mass is increased, the Bino and slepton masses must also increase to maintain the mass

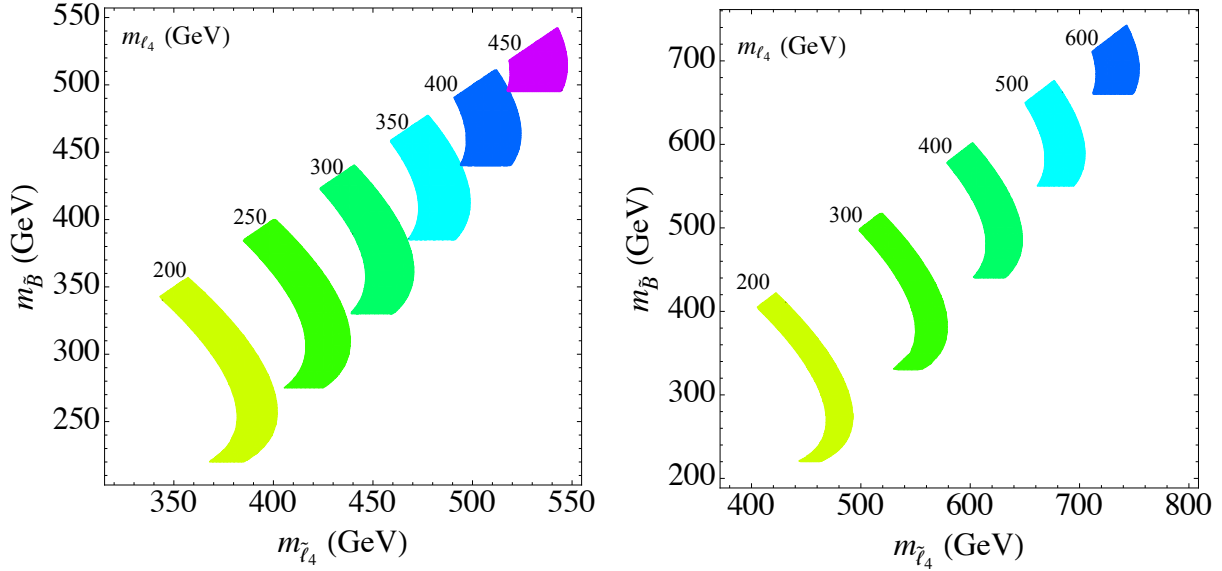


FIG. 1: Cosmologically preferred regions in the  $(m_{\tilde{\ell}_4}, m_{\tilde{B}})$  plane for the QUE (left) and QDEE (right) models. In each shaded region, the relic density is in the preferred range  $\Omega_{\tilde{B}} h^2 = 0.12 \pm 0.012$  for the value of  $m_{\ell_4}$  indicated.

ordering. As the masses increase, however, the annihilation cross section  $\langle \sigma v \rangle$  decreases, and at some point the thermal relic density of Binos is too large, providing an upper bound on all of these masses. To guarantee that the velocity expansion is reliable in the regions shown in Fig. 1, we have required  $m_{\ell_4} < 1.1 m_{\tilde{B}}$ . We have not included co-annihilation, which would be important for Binos and sleptons that are degenerate to more than 5%.

Without co-annihilation, the largest possible masses are about  $m_{\ell_4} = 470$  GeV in the QUE model and 670 GeV in the QDEE model. To see this upper bound more clearly, in Fig. 2 we plot the relic density bands in the  $(m_{\tilde{\ell}_4}, m_{\ell_4})$  plane for fixed  $m_{\tilde{B}} = 1.2 m_{\ell_4}$ . Larger masses are allowed in the QDEE model, because there are two new annihilation channels, and since  $\langle \sigma v \rangle \sim m^{-2}$ , the upper bound on the masses is larger by roughly a factor of  $\sqrt{2}$ .

In Figs. 1 and 2 we have completely neglected the MSSM annihilation channels; including them would only move the preferred regions to slightly lower masses. For the reasons mentioned above, vector-like 4th generation particles are extremely efficient channels for annihilation and completely dominate the MSSM contributions in the case of Bino dark matter. As a result, the cosmologically-preferred Bino masses are significantly higher than in the MSSM and completely eliminate the tension between the relic density constraints and current LHC bounds on neutralino masses.

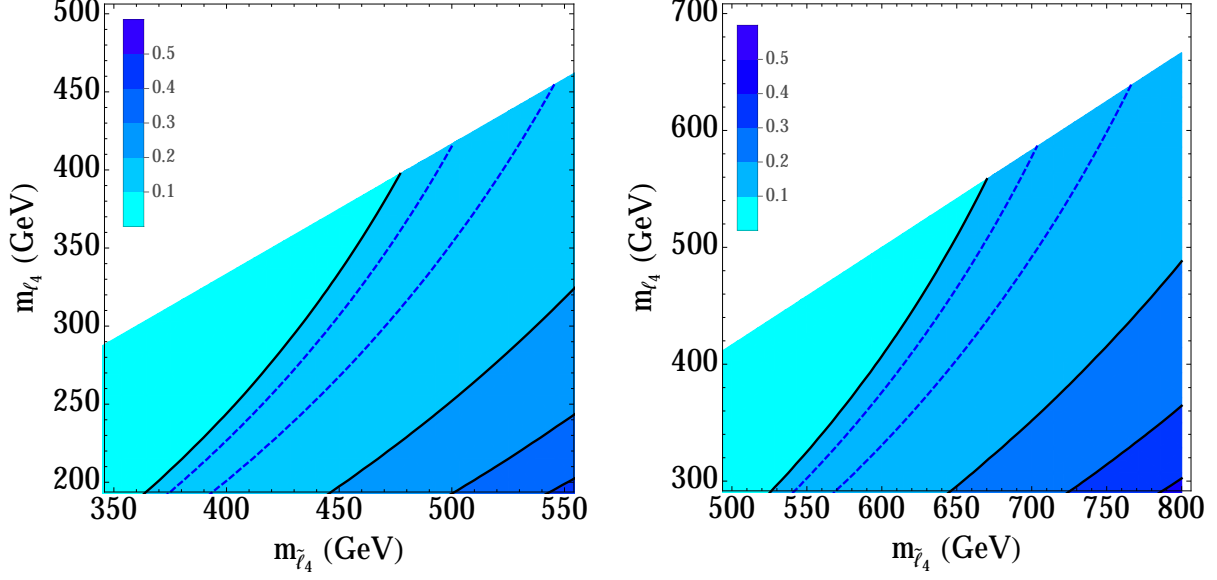


FIG. 2: Contours of constant relic density  $\Omega_{\tilde{B}} h^2$  for the QUE (left) and QDEE (right) models in the  $(m_{\tilde{l}_4}, m_{\ell_4})$  plane with fixed  $m_{\tilde{B}} = 1.2m_{\ell_4}$ . Between the dashed lines  $\Omega_{\tilde{B}} h^2 = 0.12 \pm 0.012$ .

#### IV. HIGGS BOSON MASS

In the MSSM, the Higgs boson mass is maximally the  $Z$  boson mass  $M_Z = 91$  GeV at tree level, but is raised by radiative corrections, dominantly from the diagrams with top quarks and squarks in loop. Up to 2-loop corrections, assuming no left-right stop mixing, the Higgs boson mass is [59]

$$m_h^2 = M_Z^2 \cos^2 2\beta \left( 1 - \frac{3}{8\pi^2} \frac{m_t^2}{v^2} t \right) + \frac{3}{4\pi^2} \frac{m_t^4}{v^2} \left[ t + \frac{1}{16\pi^2} \left( \frac{3}{2} \frac{m_t^2}{v^2} - 32\pi\alpha_3 \right) t^2 \right], \quad (30)$$

where

$$t = \ln \frac{M_{\tilde{t}}^2}{M_t^2} \quad (31)$$

$$m_t = \frac{M_t}{1 + \frac{4}{3\pi} \alpha_3(M_t)} \quad (32)$$

$$\alpha_3 = \frac{\alpha_3(M_Z)}{1 + \frac{b_3}{4\pi} \alpha_3(M_Z) \ln(M_t^2/M_Z^2)} \quad (33)$$

$$b_3 = 11 - 2N_f/3 = 7, \quad (34)$$

$M_t = 174$  GeV is the top quark mass,  $M_{\tilde{t}}$  characterizes the masses of the left- and right-handed top squarks,  $v = 174$  GeV is the Higgs vacuum expectation value,  $\alpha_3(M_Z) = 0.118$  is the strong gauge coupling at the  $Z$  pole, and  $b_3$  is the beta coefficient for the strong coupling

in the MSSM without the top quark and any extra matter. For  $\tan \beta = 10$ , the tree-level mass is near its maximal value, but even with top squark masses  $M_{\tilde{t}} = 2$  TeV, the Higgs mass is only 115 GeV, far short of the measured value of 125 GeV.

With the addition of vector-like quarks, however, this mass can be significantly increased. The contribution from a vector-like 4th generation of top quarks and squarks is [27, 28]

$$\Delta m_h^2 = \frac{N_c v^2}{4\pi^2} (k \sin \beta)^4 f(x) , \quad (35)$$

where  $N_c = 3$  is the number of colors,  $k$  is the up-type Yukawa coupling in Eqs. (3) and (7), and

$$f(x) = \ln x - \frac{1}{6} \left( 5 - \frac{1}{x} \right) \left( 1 - \frac{1}{x} \right) \quad (36)$$

$$x = \frac{m_{\tilde{q}_4}}{m_{q_4}} . \quad (37)$$

As a reminder,  $m_{q_4}$  and  $m_{\tilde{q}_4}$  are the physical masses of the 4th generation quarks and squarks, respectively, and we set  $k$  at its quasi-fixed point value  $k = 1.05$  and neglect the 4th generation down-type Yukawa  $h$ . Note that we are also neglecting 2-loop contributions from vector-like matter, since those contributions are small for  $m_{q_4}, m_{\tilde{q}_4} \gg m_h$  [28].

We can see from Eq. (35) that the 4th generation contribution to the Higgs boson mass is maximal when  $q_4$  is as light as possible. In Fig. 3 we show contours of the Higgs mass in the  $(m_{\tilde{t}_4}, m_{\tilde{t}})$  plane for fixed  $m_{t_4} = 1$  TeV. This choice of  $m_{t_4}$  is based partly on the  $\sim 700$  GeV limit on chiral 4th generation up-type quarks [16] and partly on the *STU* and Higgs constraints mentioned earlier. We see that, with the addition of 4th generation tops, the correct Higgs mass can be achieved for a range of  $m_{\tilde{t}_4}$  and  $m_{\tilde{t}}$  where both are below 3 TeV and discoverable at future runs of the LHC. One can see from Eq. (35) that the corrections from the vector-like matter are functions of  $x = m_{\tilde{t}_4}/m_{t_4}$ . One can use this to reinterpret Fig. 3 as determining the required ratio  $x$  to get the correct mass. For example, for  $m_{t_4}$  between 1 and 2 TeV, the correct Higgs boson mass can be obtained as long as  $x$  is between 2.5 and 2.

## V. CONCLUSIONS

In this work, we have considered the cosmology of MSSM4G models, in which the MSSM is extended by adding vector-like 4th (and 5th) generation particles. Remarkably, requiring



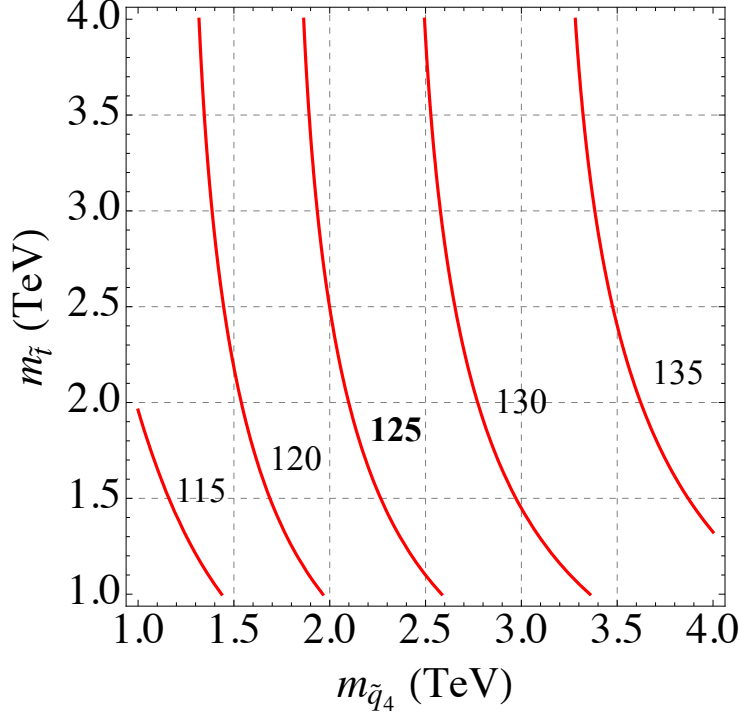


FIG. 3: Contours of constant Higgs boson mass (in GeV) in the  $(m_{\tilde{q}_4}, m_{\tilde{t}})$  plane, assuming no left-right squark mixings, for fixed  $m_{t_4} = 1$  TeV.

perturbative gauge coupling unification and that the extra particles raise the Higgs boson mass significantly reduces the number of MSSM4G models to two: the QUE and QDEE models.

Here we have shown that these models accommodate an excellent dark matter candidate, the Bino. In the MSSM, Bino dark matter must be lighter than 300 GeV to avoid overclosing the Universe. Such light Binos are in tension with constraints from the LHC in many scenarios. In contrast, in the MSSM4G models, Binos may annihilate to extra leptons through  $\tilde{B}\tilde{B} \rightarrow \ell_i^+ \ell_i^-$ , where  $i = 4$  in the QUE model, and  $i = 4, 5$  in the QDEE model. These annihilation channels are enhanced by the large hypercharges of lepton isosinglets, are not chirality-suppressed, and completely dominate over all of the MSSM annihilation channels combined. We have shown that these extra channels enhance the annihilation cross section to allow Bino masses as large as 470 GeV and 670 GeV in the QUE and QDEE models, respectively, without requiring co-annihilations or resonances. MSSM4G models are therefore motivated by dark matter also, as they accommodate Bino dark matter with the correct relic density in completely generic regions of parameter space.

An interesting question is how to discover supersymmetry if these MSSM4G models are realized in nature. As we have discussed, these models satisfy precision constraints from Higgs boson properties, electroweak physics, and low-energy observables; future improvements in these areas could see hints of anomalies from 4th generation particles, but this is not generic. These models also have improved naturalness relative to the MSSM, in the sense that the top squarks and 4th generation quarks and squarks, even without left-right mixing, may be lighter than 2 to 3 TeV and still give the correct Higgs boson mass. These are within reach of future runs of the LHC. As noted in Sec. [IV](#), however, it is also possible for the stop and 4th generation quarks and squarks to all be beyond the reach of the LHC.

However, the relic density does imply upper bounds on the masses of the 4th generation leptons and sleptons. Given this, it is very interesting to see how one could best search for these at both hadron and lepton colliders. Of course, Bino dark matter can also be searched for through direct and indirect dark matter searches. We plan to evaluate the efficacy of these searches in a future study [\[60\]](#).

Last, we note that there are many variations one could consider. We have assumed many mass unifications to simplify the presentation of our results; these could be relaxed. One could also contemplate left-right mixings for the squarks and their impact on the Higgs boson mass, or allow the lightest neutralino to include Higgsino or Wino components. We believe that the essential point is clear, though: the combination of supersymmetry and vector-like fourth generation particles accommodates an excellent Bino dark matter candidate even in its simplest realizations, and the QUE and QDEE models are among the more motivated and viable supersymmetric extensions of the standard model.

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