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Phys. Rev. D **92**, 103519 — Published 23 November 2015

DOI: [10.1103/PhysRevD.92.103519](https://doi.org/10.1103/PhysRevD.92.103519)

LUX likelihood and limits on spin-independent and spin-dependent WIMP couplings with LUXCalc

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We present **LUXCalc**, a new utility for calculating likelihoods and deriving WIMP-nucleon coupling limits from the recent results of the LUX direct search dark matter experiment. After a brief review of WIMP-nucleon scattering, we derive LUX limits on the spin-dependent WIMP-nucleon couplings over a broad range of WIMP masses, under standard assumptions on the relevant astrophysical parameters. We find that, under these and other common assumptions, LUX excludes the entire spin-dependent parameter space consistent with a dark matter interpretation of DAMA’s anomalous signal, the first time a single experiment has been able to do so. We also revisit the case of spin-independent couplings, and demonstrate good agreement between our results and the published LUX results. Finally, we derive constraints on the parameters of an effective dark matter theory in which a spin-1 mediator interacts with a fermionic WIMP and Standard Model fermions via axial-vector couplings. A detailed appendix describes the use of **LUXCalc** with standard codes to place constraints on generic dark matter theories.

I. INTRODUCTION

Evidence for a large amount of non-baryonic “dark matter” (DM) in the universe has been accumulating for decades [1, 2]. Recent observations of the cosmic microwave background have provided a precise measurement of the DM relic density, and also strongly support the idea of “cold” DM in the form of weakly interacting massive particles (WIMPs) [3–5]. The failure of the Standard Model (SM) of particle physics to adequately explain a variety of astrophysical observations has prompted the development of a large number of particle theories beyond the SM [6].

Concurrently with these theoretical developments, a large number of experiments have been conducted to search for DM annihilation in distant astrophysical objects, produce and observe DM particles in high energy particle collisions, or observe the direct interaction of particles of DM with Earth bound detectors. Although there are tantalising hints of DM signatures in one or more of these experiments, there is as yet no uncontroversial detection of (non-gravitational) WIMP interactions with ordinary matter [7]. Given a particular new theory of particle physics, it is sometimes challenging to assess the likelihood of the model (as a function of the model parameters) given the null results of these experiments.

In this paper, we present **LUXCalc**, a new utility for assessing the likelihood of new physics models given the recent results of the LUX experiment [8], the most constraining direct search experiment to date for a wide range of WIMP models. In addition to describing the

use of the tool for the general user, we apply it to derive LUX limits on spin-independent (SI) and spin-dependent (SD) WIMP-nucleon couplings. The former show good agreement with the official LUX results, thus validating our approach. Whilst SD limits have not been provided by the LUX collaboration, our results in the neutron-only coupling case are in close agreement with those calculated by Ref. [9].¹ We present for the first time the LUX SD proton-only limits and discuss the general SD mixed coupling case, finding that LUX alone fully excludes the SD-interacting DM interpretation of the anomalous signal seen in DAMA [11].

The paper is structured as follows. In Section II we provide a brief and self-contained review on the physics of WIMP-nuclear scattering, in both the SI and SD case. In Section III we review the LUX experiment, and present the methodology for our determination of the LUX likelihood and constraints. Section IV presents our SI and SD coupling limits as a function of the WIMP mass, including comparisons with the published LUX limits (where relevant), and those of other DM experiments. We also place constraints on an effective DM theory for which the SD constraints are particularly important: the case of fermionic DM interacting with a spin-1 mediator that has an axial-vector coupling to both the WIMP and to SM fermions. Finally, we present conclusions in Section V. The Appendix describes the use of the **LUXCalc** software.

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¹ Reference [10] first produced SD LUX limits, though only for a narrow range of WIMP masses around 10 GeV. Our full treatment of the LUX efficiencies provide more stringent limits than those from the more conservative analysis performed in that reference.

II. THEORY REVIEW

Direct detection experiments aim to observe the recoil of a nucleus in a collision with a DM particle [12]. After an elastic collision with a WIMP χ of mass m_χ , a nucleus of mass M recoils with energy $E = (\mu^2 v^2 / M)(1 - \cos \theta)$, where $\mu \equiv m_\chi M / (m_\chi + M)$ is the reduced mass of the WIMP-nucleus system, v is the speed of the WIMP relative to the nucleus, and θ is the scattering angle in the center of mass frame. The differential recoil rate per unit detector mass is

$$\frac{dR}{dE} = \frac{n_0}{M} \left\langle v \frac{d\sigma}{dE} \right\rangle = \frac{2\rho_0}{m_\chi} \int d^3v v f(\mathbf{v}, t) \frac{d\sigma}{dq^2}(q^2, v), \quad (1)$$

where $n_0 = \rho_0 / m_\chi$ is the number density of WIMPs, with ρ_0 the local DM mass density; $f(\mathbf{v}, t)$ is the time-dependent WIMP velocity distribution; and $\frac{d\sigma}{dq^2}(q^2, v)$ is the velocity-dependent differential cross-section, with $q^2 = 2ME$ the momentum exchange in the scatter. In the typical case that the target material contains more than one isotope, the differential rate is given by a mass-fraction weighted sum over contributions from the isotopes, each of the form given by Eqn. (1). Using the form of the differential cross-section for the most commonly assumed couplings,

$$\frac{dR}{dE} = \frac{1}{2m_\chi \mu^2} \sigma(q) \rho_0 \eta(v_{\min}(E), t), \quad (2)$$

where $\sigma(q)$ is an effective scattering cross-section and

$$\eta(v_{\min}, t) \equiv \int_{v > v_{\min}} d^3v \frac{f(\mathbf{v}, t)}{v} \quad (3)$$

is the mean inverse speed, with

$$v_{\min} = \sqrt{\frac{ME}{2\mu^2}} \quad (4)$$

the minimum WIMP velocity that can result in a recoil energy E . Equation (2) conveniently factorizes the differential rate into particle physics terms (σ) and astrophysics terms ($\rho_0 \eta$), which we describe separately in the following sections. More comprehensive reviews of direct detection can be found in Refs. [2, 13–15].

A. Particle physics: cross-section

For a SUSY neutralino and many other WIMP candidates, the dominant WIMP-quark couplings in direct detection experiments are the scalar and axial-vector couplings, which respectively give rise to SI and SD cross-sections [14]. In both cases,

$$\frac{d\sigma}{dq^2}(q^2, v) = \frac{\sigma_0}{4\mu^2 v^2} F^2(q) \Theta(q_{\max} - q) \quad (5)$$

to leading order. Here, Θ is the Heaviside step function, $q_{\max} = 2\mu v$ is the maximum momentum transfer in a collision at a relative velocity v , and the requirement $q < q_{\max}$ gives rise to the lower limit $v > v_{\min}$ in the integral for η in Eqn. (3). In the above equation, σ_0 is the scattering cross-section in the zero-momentum-transfer limit (we shall use σ_{SI} and σ_{SD} to represent this term in the SI and SD cases, respectively) and $F^2(q)$ is a form factor to account for the finite size of the nucleus. The WIMP coherently scatters off the entire nucleus when the momentum transfer is small, giving $F^2(q) \rightarrow 1$. However, as the de Broglie wavelength of the momentum transfer becomes comparable to the size of the nucleus, the WIMP becomes sensitive to the spatial structure of the nucleus and $F^2(q) < 1$, with $F^2(q) \ll 1$ at higher momentum transfers. It is traditional to define a form-factor corrected cross-section

$$\sigma(q) \equiv \sigma_0 F^2(q), \quad (6)$$

as was used in Eqn. (5) above. We note that this is an *effective* cross-section, whereas the *actual* scattering cross-section is given by $\int dq^2 \frac{d\sigma}{dq^2}(q^2, v)$ for a given relative velocity v . The total WIMP-nucleus scattering rate is then the sum over both the SI and SD contributions, each with its own form factor.

1. Spin-independent cross-section (SI)

The SI WIMP-nucleus interaction, which occurs through operators such as $(\bar{\chi}\chi)(\bar{q}q)$, has the cross-section

$$\sigma_{\text{SI}} = \frac{\mu^2}{\pi} \left[Z G_{\text{SI}}^{\text{p}} + (A - Z) G_{\text{SI}}^{\text{n}} \right]^2 = \frac{4\mu^2}{\pi} \left[Z f_{\text{p}} + (A - Z) f_{\text{n}} \right]^2, \quad (7)$$

where Z and $A - Z$ are the number of protons and neutrons in the nucleus, respectively, and f_{p} (f_{n}) is the effective coupling to the proton (neutron), with the alternate normalization $G_{\text{SI}}^{\text{N}} = 2f_{\text{N}}$ also found in the literature.² For neutralinos and most other WIMP candidates with a SI interaction arising through scalar couplings, $f_{\text{n}} \simeq f_{\text{p}}$ and the SI scattering cross-section of WIMPs with protons and neutrons are roughly comparable, $\sigma_{\text{n,SI}} \approx \sigma_{\text{p,SI}}$. For identical couplings ($f_{\text{n}} = f_{\text{p}}$), the SI cross-section can be written as

$$\sigma_{\text{SI}} = \frac{\mu^2}{\mu_{\text{p}}^2} A^2 \sigma_{\text{p,SI}}, \quad (8)$$

where μ_{p} is the WIMP-proton reduced mass. As neutralinos are the currently favored WIMP candidate, this assumption is widely made throughout the direct detection literature, though models can be constructed that

² Notably, G_{SI}^{N} are the G_F -like effective four-fermion coupling constants in the case of scalar interactions [16] and are the normalization used in `DarkSUSY` [17]. `micrOMEGAs` uses $\lambda_{\text{N}} = \frac{1}{2} G_{\text{SI}}^{\text{N}}$ [18].

violate this $f_n \simeq f_p$ condition (e.g. isospin-violating DM [19]). We will consider only identical SI couplings case when later examining the LUX results.

The SI cross-section grows rapidly with nuclear mass due to the A^2 factor in Eqn. (8), which arises from the fact that the total SI coupling of the WIMP to a nucleus is a coherent sum over the contributions from individual protons and neutrons within. Direct detection experiments therefore often use heavy nuclei to increase their sensitivity to WIMP scattering.

The SI form factor is essentially a Fourier transform of the mass distribution of the nucleus. A reasonably accurate approximation is the Helm form factor [13, 20]:

$$F(q) = 3e^{-q^2 s^2/2} \frac{\sin(qr_n) - qr_n \cos(qr_n)}{(qr_n)^3}, \quad (9)$$

where $s \simeq 0.9$ fm and $r_n^2 = c^2 + \frac{7}{3}\pi^2 a^2 - 5s^2$ is an effective nuclear radius with $a \simeq 0.52$ fm and $c \simeq 1.23A^{1/3} - 0.60$ fm. Further details on SI form factors can be found in Refs. [13, 21].

2. Spin-dependent cross-section (SD)

SD scattering is due to the interaction of a WIMP with the spin of the nucleus through operators such as $(\bar{\chi}\gamma_\mu\gamma_5\chi)(\bar{q}\gamma^\mu\gamma_5q)$, and takes place only in those detector isotopes with an unpaired proton and/or unpaired neutron. The SD WIMP-nucleus cross-section is

$$\begin{aligned} \sigma_{\text{SD}} &= \frac{4\mu^2}{\pi} \frac{(J+1)}{J} \left[G_{\text{SD}}^p \langle S_p \rangle + G_{\text{SD}}^n \langle S_n \rangle \right]^2 \\ &= \frac{32\mu^2 G_F^2}{\pi} \frac{(J+1)}{J} \left[a_p \langle S_p \rangle + a_n \langle S_n \rangle \right]^2, \end{aligned} \quad (10)$$

where G_F is the Fermi constant, J is the spin of the nucleus, $\langle S_p \rangle$ and $\langle S_n \rangle$ are the average spin contributions from the proton and neutron groups, respectively, and a_p (a_n) are the effective couplings to the proton (neutron) in units of $2\sqrt{2}G_F$; the alternative normalization $G_{\text{SD}}^N = 2\sqrt{2}G_F a_N$ is also found in the literature.³ Unlike the SI case, the two SD couplings a_p and a_n may differ substantially (though they are often of similar order of magnitude), so that a simplification comparable to Eqn. (8) is not made in the SD case. Because of the uncertain theoretical relation between the two couplings and following from the fact that one of $\langle S_p \rangle$ or $\langle S_n \rangle$ is often much smaller than the other, experiments typically only significantly constrain one of the two SD cross-sections, $\sigma_{p,\text{SD}}$ or $\sigma_{n,\text{SD}}$, but not both.

The SD form factor is given in terms of the structure function $S(q)$ as

$$F^2(q) = \frac{S(q)}{S(0)} \quad (11)$$

such that $F^2(0) = 1$ as previously prescribed. The $S(q)$ have the functional form

$$S(q) = a_p^2 S_{pp}(q) + a_n^2 S_{nn}(q) + a_p a_n S_{pn}(q). \quad (12)$$

The differential SD scattering cross-section can alternatively be written in terms of this structure function as⁴

$$\frac{d\sigma}{dq^2}(q^2, v) = \frac{8G_F^2}{v^2(2J+1)} S(q) \Theta(q_{\text{max}} - q). \quad (13)$$

In the limit of zero momentum transfer ($q \rightarrow 0$) the functions $S_{xy}(0)$ ($x, y = p, n$) can be related to expectation values of the proton/neutron spin [22, 23]:

$$\begin{aligned} \langle S_p \rangle^2 &= \frac{J}{(J+1)} \frac{\pi}{(2J+1)} S_{pp}(0) \quad \text{and} \\ \langle S_n \rangle^2 &= \frac{J}{(J+1)} \frac{\pi}{(2J+1)} S_{nn}(0). \end{aligned} \quad (14)$$

Under an alternative basis

$$a_0 = a_p + a_n \quad a_1 = a_p - a_n, \quad (15)$$

which is a more convenient basis for nuclear physics work,

$$S(q) = a_0^2 S_{00}(q) + a_1^2 S_{11}(q) + a_0 a_1 S_{01}(q), \quad (16)$$

where the structure functions in the two bases are related by

$$\begin{aligned} S_{pp} &= S_{00} + S_{11} + S_{01} \\ S_{nn} &= S_{00} + S_{11} - S_{01} \\ S_{pn} &= 2(S_{00} - S_{11}). \end{aligned} \quad (17)$$

The S_{ij} ($i, j = 0, 1$) are calculated in the literature. For the two non-zero-spin xenon isotopes (^{129}Xe and ^{131}Xe), we take these structure functions from Ref. [24], utilizing one plus two body (1b+2b) axial-vector currents where possible.

B. Astrophysics: dark matter distribution

The DM halo in the local neighborhood is most likely dominated by a smooth and well-mixed (virialized) component with an average density ρ_0 . The simplest model

³ Though we use a_N and G_{SD}^N to distinguish between the two normalizations here, a_N is frequently used within the literature for both cases. The G_{SD}^N normalization is used by **DarkSUSY** [16, 17], while **micrOMEGAs** uses $\xi_N = \frac{1}{2} G_{\text{SD}}^N$ [18].

⁴ The factor of $8G_F^2$ is omitted if Eqn. (12) is written in terms of G_{SD}^N rather than a_N . Again, sometimes the latter notation is used in the literature to represent the former normalization as defined here.

for this smooth component is often taken to be the Standard Halo Model (SHM) [25, 26], a non-rotating isothermal sphere with an isotropic, Maxwellian velocity distribution and most probable speed v_0 , where for the SHM, v_0 is equal to the disk rotation speed v_{rot} . The SHM velocity distribution in the Galactic (i.e. non-rotating) rest frame is

$$\tilde{f}(\mathbf{v}) = \begin{cases} \frac{1}{N_{\text{esc}}} \left(\frac{1}{\pi v_0^2} \right)^{3/2} e^{-\mathbf{v}^2/v_0^2}, & \text{for } |\mathbf{v}| < v_{\text{esc}} \\ 0, & \text{otherwise.} \end{cases} \quad (18)$$

Here,

$$N_{\text{esc}} = \text{erf}(z) - \frac{2}{\sqrt{\pi}} z e^{-z^2}, \quad (19)$$

with $z \equiv v_{\text{esc}}/v_0$, is a normalization factor. The Maxwellian distribution is truncated at the escape velocity v_{esc} to account for the fact that WIMPs with sufficiently high velocities escape the Galaxy's potential well.

Ultimately, the velocity distribution in the Earth's rest frame is the one relevant for direct detection, which can be obtained after a Galilean boost:

$$f(\mathbf{v}) = \tilde{f}(\mathbf{v}_{\odot} + \mathbf{v}), \quad (20)$$

where $\mathbf{v}_{\odot} = \mathbf{v}_{\text{LSR}} + \mathbf{v}_{\odot, \text{pec}}$ is the motion of the Sun relative to the Galactic rest frame, $\mathbf{v}_{\text{LSR}} = (0, v_{\text{rot}}, 0)$ is the motion of the Local Standard of Rest in Galactic coordinates,⁵ and $\mathbf{v}_{\odot, \text{pec}}$ is the Sun's peculiar velocity. The additional time-dependent orbital motion of the Earth about the Sun is postulated to give rise to a measurable modulation in the signal [15, 26]; indeed, some experiments have claimed positive results for such signatures [27]. This small Earth orbital motion and resulting time dependence has been neglected here as it is not relevant to the LUX calculations.

The choice of values for the various DM distribution parameters is important for interpreting direct detection results. Due to the inability to directly observe the DM and various systematic issues in interpreting observations of the Galaxy, some of those parameters are known to limited precision. The range of estimates for these parameters are shown in Table I, as well as values commonly used for comparing direct detection results (“canonical”) and the default values used by LUXCalc. The LUXCalc default values for ρ_0 and v_{rot} are somewhat larger than the historical canonical values as more recent estimates tend to prefer the somewhat larger values. However, the canonical values are not inconsistent with the current observations and will thus continue to see wide usage. The parameter values can be easily changed in LUXCalc and we will use the canonical values ourselves when comparing LUX results with other experiments in later sections. In addition to these SHM parameters, we take $\mathbf{v}_{\odot, \text{pec}} = (11, 12, 7)$ km/s [39].

III. THE LUX EXPERIMENT AND ANALYSIS

In this section, we describe the basic operation of the LUX DM search detector and how their data is used to constrain WIMP parameters. The LUX experiment uses a liquid-xenon time projection chamber (TPC) to identify DM recoil events and distinguish them from other (background) events [40]. The first LUX results, released in 2013 [8], involved a fiducial exposure of 1.01×10^4 kg-days, comparable to that of the then-leading⁶ XENON100 experiment [41] (7.6×10^3 kg-days), which likewise used a TPC detector with a xenon target. The slightly larger exposure and somewhat better detector performance characteristics allowed LUX to overtake XENON100 and LUX is now the leading experiment in terms of sensitivity to a large variety of WIMPs (by about a factor of two over XENON100).

The principles of operation for a TPC are as follows. A recoiling xenon nucleus (or recoiling electron, in the case of some background processes) in the liquid xenon target will interact with other nearby atoms, inducing both excitations and ionization of those atoms. The relatively quick relaxation of the excitations releases photons that are collected by photomultiplier tubes (PMTs); this prompt scintillation is referred to as the S1 signal. While some of the ionized electrons can recombine (possibly producing excitations and contributing to the S1 prompt scintillation signal), many of the free electrons are drawn away from the interaction site to the surface of the liquid by the application of an electric field across the liquid. Above the liquid is a small region of gaseous xenon under a higher electric field. Electrons reaching this region rapidly accelerate and collide with xenon atoms in the gas, releasing photons and more electrons in a cascade process; this “proportional scintillation” is the S2 signal. The drift time of the ionized electrons across the liquid is substantially longer than the relaxation time of the xenon excitations, so the S1 and S2 scintillation signals are easily distinguishable. The benefit of observing two signals is that electron recoils, produced by background radiation, tend to produce a relatively larger amount of S2 for a given S1, so S2/S1 is used as a background discrimination parameter.

For a given WIMP spectrum, the average expected number of signal events in some analysis region is

$$\mu = MT \int_0^\infty dE \phi(E) \frac{dR}{dE}(E), \quad (21)$$

where MT is the detector mass \times time exposure and $\phi(E)$ is the fraction of recoil events of energy E that will be both observed and fall into the predefined analysis region. This $\phi(E)$ detection efficiency accounts for various trigger efficiencies, intrinsic statistical fluctuations and

⁵ Galactic coordinates are aligned such that $\hat{\mathbf{x}}$ is the direction to the Galactic center, $\hat{\mathbf{y}}$ is the direction of the local disk rotation, and $\hat{\mathbf{z}}$ is orthogonal to the plane of the disk.

⁶ In terms of sensitivity to WIMPs with SI interactions and masses above ~ 10 GeV.

	estimate	canonical	LUXCalc default	Refs.
ρ_0 [GeV/cm ³]	0.2 – 0.7	0.3	0.4	[28–33]
v_{rot} [km/s]	200 – 250	220	235	[34–37]
v_{esc} [km/s]	500 – 600	544	550	[38]

TABLE I. Values of parameters describing the Standard Halo Model (SHM). The columns show in order: ranges of values typically found in the literature, commonly used values for comparison of experimental results, and the default values used by LUXCalc. In the SHM, the final parameter v_0 is not independent, but fixed by the relation $v_0 = v_{\text{rot}}$.

the PMT response (i.e. detector resolution), and analysis cuts. The benefit of the above form is that all the complicated detector physics and responses are rolled into $\phi(E)$, which is independent of the type of WIMP interaction or spectrum. Thus, for a given experimental result and analysis region, $\phi(E)$ can be tabulated once and then used for analyzing arbitrary WIMPs. We use the TPCMC monte carlo [42] to model the detector response and generate the relevant $\phi(E)$ efficiency curves. TPCMC relies on NEST [43–45] for modeling the microphysics of a recoiling xenon atom.

We take as our analysis region in the S2/S1-S1 plane the region above the $S2 \geq 200$ PE threshold, below the nuclear recoil calibration data mean S2/S1 curve, and $2 \text{ PE} \leq S1 \leq 30 \text{ PE}$. This region matches that used by the LUX collaboration, except for the imposition of the hard S2/S1 cut that is not necessary for their profile likelihood analysis [46]. This region contains one observed event at an S1 of 3.1 PE with a mean expected background of $b = 0.64$ events. Ideally, a lower S2/S1 bound to the region should be imposed in the count-based analyses we will be using, but such a bound can often be set low enough that, in practice, the $\phi(E)$ are negligibly affected. The lower bound would serve more to exclude very low S2/S1 events that are almost certainly backgrounds; luckily, there are no such events in the LUX results and this issue is moot.⁷

Given an observed number of events N and expected background b — being 1 and 0.64 for this LUX analysis, respectively — a likelihood can be constructed from the Poisson distribution, with

$$\mathcal{L}(m_\chi, \sigma | N) = P(N | m_\chi, \sigma) = \frac{(b + \mu)^N e^{-(b+\mu)}}{N!}, \quad (22)$$

where $\mu = \mu(m_\chi, \sigma)$ is the expected number of signal events for a given WIMP mass m_χ and one or more scattering cross-sections σ (or, alternatively, couplings). This likelihood can be easily combined with those from a variety of other physics data, such as from colliders or indirect DM searches, in statistical scans of the Minimal Supersymmetric Standard Model (MSSM) or other new physics frameworks; see e.g. [47–51].

We derive constraints in a σ - m_χ parameter space via two methods: the Feldman-Cousins (Poisson-based) method [52] and Yellin’s maximum gap method [53]. These two techniques correspond to analyses with and without background subtraction, respectively, with the latter case commonly found in the literature due to the difficulty in reliably identifying and characterizing background contributions.

The Feldman-Cousins method derives a confidence interval in σ for each m_χ (resulting in a raster scan in the σ - m_χ plane) that is consistent with the observed number of events given the expected background. This confidence interval may be either one or two sided and, thus, is capable of excluding the zero-signal case when excess events are found. This method is relatively straightforward and easy to implement, but one of the drawbacks is the lack of spectral information in the analysis: only the total counts are used, not the distribution of S1 values (a coarse proxy for recoil energy). This spectral information can be useful in distinguishing between signal and background and can aid in constraining the mass of the WIMP in the event of a positive result, as heavier WIMPs induce more energetic xenon recoils and larger S1 values, on average. Analyses that make use of spectral information are substantially more complex to implement and are thus typically only performed by the experimental collaborations themselves; see e.g. Refs. [54, 55].

The maximum gap method makes no presumptions about the amount of background that might be contributing to the observed events, instead assuming any or all of the events might be signal to generate a conservative exclusion limit in σ at each m_χ : any cross-sections above this limit would yield too many events even if background contributions were ignored. This method does make use of the S1 distribution of the observed events, focusing on an S1 range where the expected number of events is largest relative to the number of observed events. Specifically, the maximum gap method breaks the full observable (S1) range into intervals separated by the observed events. If μ_k are the predicted number of events in each of these intervals, the “maximum gap” is the one where μ_k is largest. If $x \equiv \frac{\mu_{k,\text{max}}}{\mu}$ is the fraction of signal events expected in this interval, then

$$C_0(x, \mu) = \sum_{k=0}^{\lfloor \mu/x \rfloor} \frac{(kx - \mu)^k e^{-kx}}{k!} \left(1 + \frac{k}{\mu - kx} \right) \quad (23)$$

⁷ A conservative imposition of a lower S2/S1 cut corresponding to the 10% lower tail of the nuclear recoil calibration data (dashed red curve in Figure 4 of Ref. [8]) results in constraints that are weaker by ~ 20 -30%.

is the probability of the maximum gap being smaller than observed. In other words, a WIMP mass and cross-section(s) is excluded at greater than a 90% confidence level (CL) if $C_0 \geq 0.9$. To perform this calculation with the LUX result (with one event at an S1 of 3.1 PE), we divide the previously discussed LUX S2/S1-S1 analysis region into two parts, $S1 \in [2, 3.1]$ PE and $S1 \in [3.1, 30]$ PE, and use TPCMC to generate efficiency factors $\phi_1(E)$ and $\phi_2(E)$, respectively, for the two intervals.⁸ The expected number of events in an interval μ_k can then be calculated via Eqn. (21) under the replacement $\phi \rightarrow \phi_k$.

In tandem with this paper, we provide **LUXCalc**, a software package for performing the above-described LUX analyses. **LUXCalc** can be used as a standalone program or as a library to be called from other software packages such as **DarkSUSY** [17, 56] and **micrOMEGAs** [57–59]. A description of **LUXCalc** and its usage can be found in the Appendix. The software can be found at Ref. [60] or as ancillary files to the arXiv version of this paper.

Before turning to our results, we take a moment to stress that the $\phi(E)$ and $\phi_k(E)$ curves used here require a full statistical modeling of LUX’s TPC detector to generate and cannot be trivially generated from any efficiencies provided by LUX in Ref. [8]. One might be tempted to take the no-S2/S1 cut efficiency from Figure 1 of that reference and apply an additional factor of 0.5 to account for the fraction of nuclear recoils falling below the S2/S1 mean in the calibration data. There are two reasons why this is incorrect. First, the S2/S1 cut is not independent of the other cuts and, in fact, is very highly correlated with the $S1 \in [2, 30]$ PE cut near the boundaries. Second, the mean of the calibration recoil band in S2/S1-S1 space represents the distribution convolved over energy for that particular calibration spectrum; this does not imply that 50% of the events at any specific energy will fall below that mean. Finally, the efficiency curve provided by LUX (without the S2/S1 cut), applies only to the whole [2, 30] PE S1 interval and cannot be decomposed into the subintervals used for the maximum gap analysis. As we show in the next section, our efficiency curves nearly exactly reproduce LUX’s own low-mass constraints, an indication that those efficiency curves are correctly modeled (the naive approach of simply applying an additional 50% nuclear-recoil-median-cut acceptance would lead to constraints too weak by a factor of two at low masses). A tabulation of these efficiencies is also included as an ancillary file on the arXiv version of this paper.

One might note that the results of section III, i.e equations 21-23, are broadly applicable to any similar xenon-based TPC experiment. We refer the reader to work in progress [42] for exact methods on how to generate $\phi(E)$ for a general experiment.

⁸ By definition, $\phi(E) = \sum_k \phi_k(E)$.

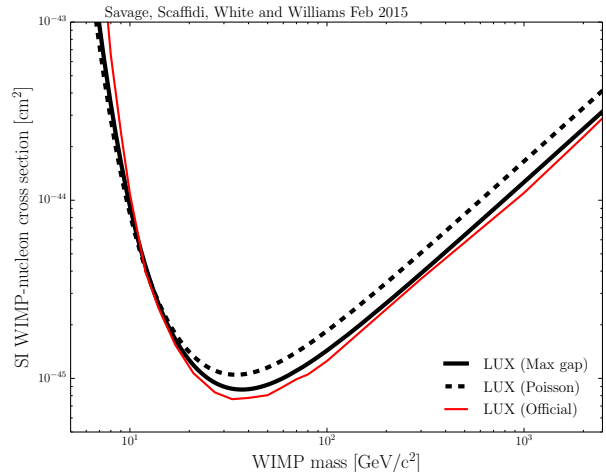


FIG. 1. Spin-independent cross-section constraints for the LUX experiment. Constraints are generated with and without background subtraction: a Poisson-based analysis with one observed event and 0.64 expected background events is used in the former case (dashed black), while the maximum gap method is used in the latter case (solid black); see the text for details. For comparison, the official LUX collaboration constraints are also shown (red), based upon an event-likelihood analysis. All constraints are at the 90% CL; cross-sections above the curves are excluded at greater than this level.

IV. PHYSICS RESULTS

In this section, we apply the previously described methods to analyze various physics models. We first examine in Section IV A the LUX constraints on generic WIMPs with SI and SD interactions, comparing our results with those from other DM searches and from the LUX collaboration itself. In Section IV B, we then apply LUX constraints to an effective theory model where SD interactions are particularly relevant.

A. Generic coupling limits

Our LUX SI scattering constraints are shown in Figure 1, with the maximum gap limit shown in solid black and the Poisson-based constraint shown in dashed black; constraints are at the 90% CL. The official LUX limits are also shown (thin red). Though they are not our preferred parameters, we use $v_0 = v_{\text{rot}} = 220$ km/s and $\rho_0 = 0.3$ GeV/cm³ to generate constraints in this and later figures to allow for a proper comparison with various other experimental constraints that use these values, such as the official LUX limits. The maximum gap limit is remarkably close to the LUX collaboration’s own limit, while the likelihood-based constraint is weaker by $\sim 30\%$ except at lower WIMP masses (which will be discussed

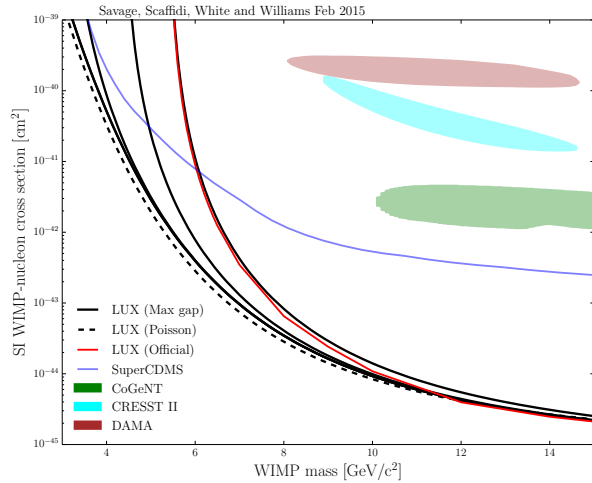


FIG. 2. Spin-independent cross-section constraints from various direct searches. Parameters that can reproduce the CoGeNT (90% CL), CRESST (2σ), and DAMA (3σ) anomalous signals are shown in the filled regions. SuperCDMS and LUX exclusion constraints are shown at the 90% CL. The LUX maximum gap constraints (solid black) are shown with nuclear recoil spectra limited to $E > 0, 1, 2, 3$ keV (strongest to weakest). The official LUX limit used a conservative 3 keV minimum.

momentarily).⁹ There are two main potential reasons why the likelihood-based constraint is weaker: (1) the analysis region for our analysis is the lower-half of the nuclear recoil band and thus contains only about half of the potential signal (while the LUX collaboration analysis uses all of it) and (2) our Poisson likelihood makes no use of spectral information (i.e. event S1). The first reason is not, in fact, a major issue in practice as the upper half of the nuclear recoil band that is being ignored is contaminated by background events and does not significantly improve the signal-to-noise in those analyses that use it.

The LUX SI scattering constraints for low WIMP masses are shown in Figure 2. For comparison, the parameters consistent with a DM interpretation of the anomalous signals seen in CoGeNT [61], CRESST [62], and DAMA [11, 63] are shown, as well as the exclusion constraints from SuperCDMS [64]. As the WIMP mass is lowered, the Poisson-based likelihood constraint (dashed black curve) becomes comparable to and then slightly better than the maximum gap limit (farthest left solid black curve, other curves discussed below). The low-mass improvement of the likelihood case relative to the maximum gap case is due to the fact that the single event in

the analysis becomes consistent with the expected WIMP spectrum, leading to a slight weakening in the sensitivity of the maximum gap method.

As measurements of the scintillation and ionization at very low nuclear recoil energies are limited and the theoretical models will eventually break down at sufficiently low energies (see e.g. Ref. [65]), the LUX collaboration conservatively ignores contributions from WIMP scattering events with $E < 3$ keV when analyzing their results. While this has little impact on constraints for WIMPs heavier than ~ 20 GeV, it becomes important for light WIMPs as light WIMPs can only induce low-energy recoils. In the figure, we show our own maximum gap constraints when considering only $E \geq 0, 1, 2$, and 3 keV (solid black curves from left to right, or most to least constraining). With the same $E \geq 3$ keV that LUX uses, the maximum gap constraint closely matches their constraint. To be clear, placing a minimum on the contributing E is not quite the same as defining the threshold in the detector. The actual trigger and S1 analysis thresholds are already built into the $\phi(E)$ efficiency term in Eqn. (21) regardless of the choice of lower bound in the integral over E . That efficiency falls rapidly below 3 keV, from 23% at 3 keV, to 4% at 2 keV and 0.04% at 1 keV. Even if the lower bound of integration is set to $E = 0$ keV, no recoil events with energies below 0.5 keV will contribute to the signal as $\phi(E)$ is zero at these energies. Placing a minimum requirement on E serves to avoid the (already suppressed) contributions from events where the NEST model has little experimental data to ensure its accuracy. For our remaining analyses, we do not apply this artificial cut on the low-energy recoil spectrum, though this choice has little impact on our conclusions and constraints with a cut applied can be easily generated with the LUXCalc code as described in the Appendix.

We now turn to the SD case, with WIMP-nucleon cross-section constraints shown in Figures 3 & 4 for neutron-only ($a_p = 0$) and proton-only ($a_n = 0$) couplings, respectively. As xenon has neutron-odd isotopes, LUX should be particularly sensitive to a WIMP with neutron-only SD couplings. In Figure 3, we show by the black curve the LUX constraints in this case, as determined via the method described in Section III. We also show constraints from other experiments with neutron-odd target materials: CDMS II [66] (at lower masses, also for the low-threshold analysis [67]), ZEPLIN-III [68], and XENON100 [69]. Here, the somewhat improved exposure and threshold of LUX over that of XENON100 is evident by the $\sim \times 2$ stronger constraint at heavy WIMP masses and the extension of the constraints to lower WIMP masses before losing sensitivity.

A SD proton-only interaction has historically been a means of producing DAMA's anomalous signal while evading the null results of other searches [70]: the proton-even target isotopes used by many experiments have suppressed interactions (and thus little expected signal) in this case, while the proton-odd sodium-iodide target used

⁹ While the likelihood analysis can produce two-sided limits, in this case the lower limit is simply zero cross-section, i.e. no WIMP signal: the LUX result is consistent with backgrounds alone at the 90% CL in this analysis.

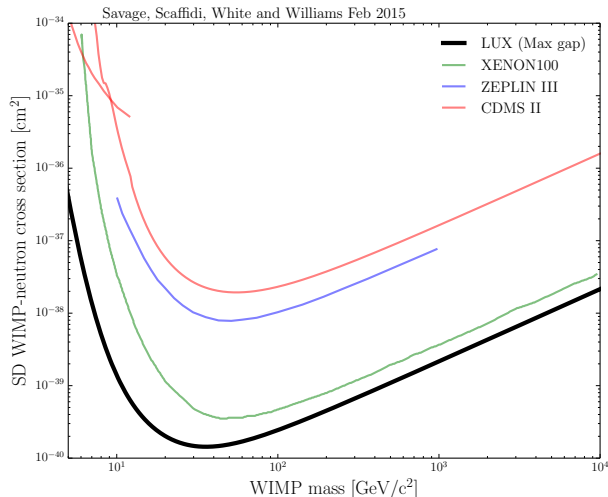


FIG. 3. Spin-dependent WIMP-neutron cross-section constraints for the neutron-only coupling case.

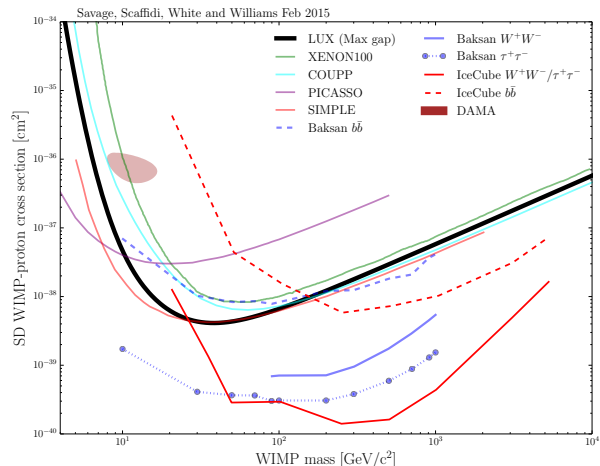


FIG. 4. Spin-dependent WIMP-proton cross-section constraints for the proton-only coupling case, including indirect search limits from the IceCube/DeepCore experiment. Note the IceCube/DeepCore W^+W^- constraint uses the tau channel for masses smaller than that of the W boson.

by DAMA ensures they remain sensitive to the WIMP. Recent results from the SIMPLE [71], PICASSO [72], and COUPP [73] experiments, which also have proton-odd target isotopes, are now in conflict with a SD proton-only coupling explanation for the DAMA signal [11, 63] as shown in Figure 4. Due to the ever-increasing detector sizes, even experiments with proton-even targets like XENON100 [69] are starting to probe DAMA's preferred parameter region. Our determination of the LUX constraints are shown by the thick black curve. We see here that LUX, even though it uses a proton-even xenon target material, fully excludes the DAMA region.

Indirect DM searches via neutrinos produced when WIMPs are caught and then annihilate in the Sun can place constraints on the SD WIMP-proton cross-section as collisions of WIMPs with hydrogen (protons) is part of the process for capturing WIMPs in the Sun [74]. Figure 4 shows constraints placed by the IceCube/DeepCore [75] and Baksan [76] neutrino detector searches for such neutrinos. The constraints depend on the annihilation channel and are shown here for the representative b -quark and W -boson channels. While neutrino searches can be very sensitive to WIMPs with SD proton couplings, the high thresholds in IceCube/DeepCore and some other neutrino experiments means they are often unable to probe for light WIMPs as LUX and other direct searches are capable of doing. Furthermore, the limits shown here assume the DM capture and annihilation processes in the Sun are in equilibrium, an assumption that may not be true for many DM candidates [77].

The exclusion of the DAMA region by LUX in the SD proton-only coupling case, the case where LUX limits are approximately at their weakest, suggests that the LUX result may exclude *any* SD explanation for the DAMA signal, the first time a single experiment would be able to do so. After a more careful examination over the mixed coupling case — not just the proton-only or neutron-only cases — this is indeed the case: the LUX likelihood-based limit at 90% CL excludes the entire SD parameter space consistent with the DAMA result within the 2σ CL, at least for the assumed halo model. As always, caveats apply. Various assumptions about detector behavior are made that, if incorrect, will affect the interpretation of the experimental results and alter the WIMP parameter space consistent with those results. See for example Ref. [78]. In addition, if the NEST model for low-energy events is inaccurate, the low-mass LUX limits may weaken. For the more conservative maximum gap analysis, a tiny part of the DAMA-compatible parameter space escapes the LUX bounds: $a_n/a_p = -0.16 \pm 0.04$ with $m_\chi \approx 10$ GeV. This remaining space will be excluded by the next LUX data release if excess events are not found.

B. Application to effective theory

We now apply the analysis of this paper to a DM model for which the SD constraints are particularly important. Consider the case of Dirac DM annihilating through the s -channel exchange of a spin-1 mediator, V_μ , via an axial-vector interaction. Assuming that the mediator also has an axial-vector interaction with SM fermions, we obtain the Lagrangian:

$$\mathcal{L} \supset [g_{\chi a} \bar{\chi} \gamma^\mu \gamma^5 \chi + g_{fa} \bar{f} \gamma^\mu \gamma^5 f] V_\mu \quad (24)$$

for DM that is a Dirac Fermion, and

$$\mathcal{L} \supset \left[\frac{1}{2} g_{\chi a} \bar{\chi} \gamma^\mu \gamma^5 \chi + g_{fa} \bar{f} \gamma^\mu \gamma^5 f \right] V_\mu \quad (25)$$

for Majorana DM, where $g_{\chi a}$ and g_{fa} are unknown couplings [79, 80]. Assuming the limit of low-momentum exchange in the WIMP-nucleon scattering process, we can integrate out the mediator to obtain the following SD scattering cross-section for a Dirac WIMP:

$$\sigma_{(a,a)} = \frac{4\mu^2 g_{\chi a}^2}{\pi m_v^4} J(J+1) \left[\frac{\langle S_p \rangle}{J} \tilde{a}_p + \frac{\langle S_n \rangle}{J} \tilde{a}_n \right]^2 \quad (26)$$

where J is the spin of the nucleus, m_v is the mediator mass and μ is the reduced WIMP-nuclear mass. Here, \tilde{a}_p and \tilde{a}_n are the WIMP-proton and WIMP-neutron couplings, related to the couplings of Section II A 2 via $G_{SD}^N = 2\sqrt{2}G_{FN} = g_{\chi a}\tilde{a}_N/m_v^2$.¹⁰ For a Majorana WIMP, Eqn. (26) adopts an additional factor of 1/2. Though we focus here on direct searches for such a particle, colliders can also place constraints; see Ref. [80] for a discussion.

Expressions for \tilde{a}_p and \tilde{a}_n can be derived by starting from the WIMP-quark interactions and performing a weighted sum over the components of each nucleon. Since the heavy quarks and gluons contribute negligibly to the spin content of the nucleon, DM scattering off nucleons is dominated by the sum over light quarks. This allows us to write:

$$\tilde{a}_N = \sum_{q=u,d,s} g_{fa} \Delta_q^{(N)} \quad (27)$$

where $\Delta_q^{(N)}$ is the nuclear spin content of quark q , and we have assumed that the coupling between the WIMP and each quark is identical and given by the coupling in our above Lagrangian. The standard values for the various $\Delta_q^{(N)}$ are $\Delta_u^{(p)} = \Delta_d^{(n)} = 0.84$, $\Delta_u^{(n)} = \Delta_d^{(p)} = -0.43$ and $\Delta_s^{(p)} = \Delta_s^{(n)} = -0.09$ [7].

Due to the symmetry in the up and down quark contributions, $\tilde{a}_n = \tilde{a}_p$, so $a_n = a_p$. Neglecting the slight difference in the proton and neutron masses, $\sigma_{\chi p} \approx \sigma_{\chi n}$ with

$$\begin{aligned} \sigma_{\chi n} &= \frac{4\mu_{\chi n}^2 g_{\chi a}^2}{\pi m_v^4} \left(\frac{1}{2} \right) \left(\frac{3}{2} \right) \left[\frac{\frac{1}{2}}{\frac{1}{2}} \tilde{a}_n \right]^2 \\ &= \frac{3\mu_{\chi n}^2 g_{\chi a}^2 g_{fa}^2}{\pi m_v^4} \left[\sum_{q=u,d,s} \Delta_q^{(n)} \right]^2 \end{aligned} \quad (28)$$

the WIMP-neutron cross-section, obtained using $J = \frac{1}{2}$ and $\langle S_p \rangle = 0$ in Eqn. (26), and $\sigma_{\chi p}$ the WIMP-proton cross-section.

For a fixed WIMP mass, a limit on $\sigma_{\chi n}$ places a limit on $g_{\chi a}^2 g_{fa}^2 / m_v^4$, with that limit corresponding to a linear relationship between $\log(g_{\chi a} g_{fa})$ and $\log m_v$, as shown in Figure 5 (Figure 6) for a Dirac (Majorana) WIMP.

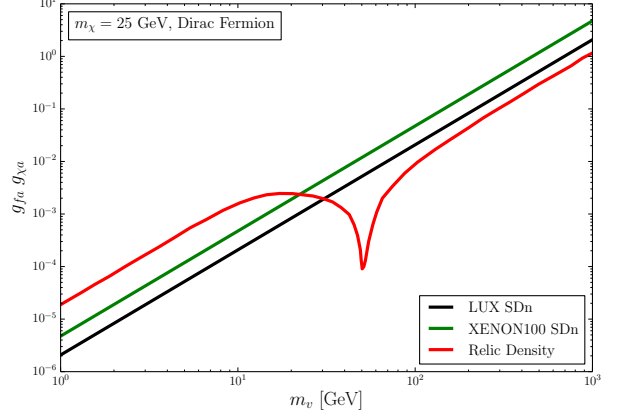


FIG. 5. Constraints on the parameters of the effective Lagrangian given in Eqn. (24) for a 25 GeV Dirac WIMP. The LUX SDn limit obtained by LUXCalc is shown in black, with coupling values above the black line excluded for a given mediator mass. Also shown are the results of a previous analysis utilising the XENON100 SDn limit (green line) [79]. The red line shows the parameter values required to obtain the correct DM relic density ($\Omega_{DM} = 0.268^{+0.013}_{-0.010}$) as measured by WMAP and Planck [79, 81].

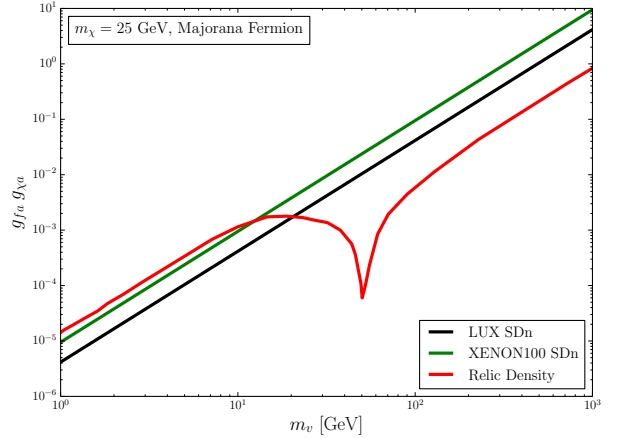


FIG. 6. Similar to Figure 5 but for a Majorana WIMP.

Here, we show LUX limits for a 25 GeV WIMP, using the 90% CL upper limit of $\sigma_{\chi n} < 1.26 \times 10^{-4}$ pb, determined by LUXCalc. We stress a subtle point here: the appropriate LUX limit on $\sigma_{\chi n}$ is determined using the $a_p = a_n$ relation expected for this model, not the $a_p = 0$ assumed in the SD neutron-only case shown in Figure 3, though in practice the two limits are similar. We also show in Figures 5 & 6 the results of a previous analysis [79] based on the XENON100 result. As expected, the LUX bounds are more stringent, raising the limit on the mass of a mediator consistent with relic density observations (red curves) from 20 GeV to 30 GeV for a Dirac WIMP, and from 10 GeV to 20 GeV for a Majorana WIMP.

¹⁰ Again, a_N is sometimes used in the literature to refer to the G_{SD}^N normalization used here, as is the case with Ref. [79].

V. CONCLUSIONS

The particle nature of DM is still unknown. There are several theoretical candidates for DM, however the WIMP hypothesis remains one of the most popular explanations of the phenomenon. The null results of direct detection experiments such as LUX provide limits on the physics of WIMP-nucleon interactions and therefore the parameter space of a given WIMP model.

We have developed and utilized the new tool **LUXCalc** to generate limits on the SI WIMP-nucleon cross-section at 90% CL both with Poisson and maximum gap based analyses for a WIMP mass in the range [5, 2500] GeV. We see that the maximum gap method generally agrees very well with the official LUX SI limits, whilst the Poisson likelihood-based constraint is weaker by $\sim 30\%$ above a WIMP mass of about 20 GeV. We then generate the LUX SI scattering limits for the low mass region [3, 15] GeV again using both the maximum gap and Poisson likelihood techniques. For the maximum gap method, we also show limits with the progressively more conservative exclusion of all contributions from events with energies below 1, 2, and 3 keV. These **LUXCalc**-generated limits are then compared with the official LUX result as well as the anomalous signal regions as seen by CoGeNT, CRESST, and DAMA and the exclusion curve from SuperCDMS. In this mass region, the Poisson-likelihood curve provided the strongest constraint.

We have used **LUXCalc** to generate for the first time the LUX limits on the SD WIMP-nucleon cross-section over the full range of WIMP masses. We show constraints for both neutron-only ($a_p = 0$) and proton-only ($a_n = 0$) couplings using the maximum gap method detailed in the text. We see that for the SD proton-only case, which is the SD case where the LUX limits are approximately at their weakest, the LUX limit fully excludes the DAMA

region. In fact, we find that the LUX likelihood-based limit at 90% CL excludes the entire SD parameter space consistent with the DAMA result within the 2σ CL (for the assumed parameters of the SHM). Furthermore, the more conservative maximum gap method excludes most of the DAMA parameter space, except for the small region $a_n/a_p = -0.16 \pm 0.04$ with $m_\chi \approx 10$ GeV.

We note that inelastic and/or exothermic scattering can have significant implications for the interpretation of Dark Matter direct detection experiments. The generalisation of our approach and of the **LUXCalc** package will be the subject of a future publication involving additional authors.

Finally, we have applied the main results of this work to an effective theory case where SD constraints on the WIMP-nucleon cross-section are particularly important. We see that the LUX bounds are more stringent than those of a previous study based on the XENON100 results [79] by a factor of ~ 2 in both the Dirac and Majorana WIMP cases.

We have made the **LUXCalc** tool publicly available for future studies involving the LUX results.

ACKNOWLEDGMENTS

CS thanks P. Sandick and the Department of Physics & Astronomy at the University of Utah for support. CS thanks C. Balázs and the School of Physics at Monash University, where part of this work was performed, for their hospitality. CS was partially supported by NSF award 10681111. The work of AS and AGW is supported by the Australian Research Council through the Centre of Excellence for Particle Physics at the Terascale. MW is supported by the Australian Research Council Future Fellowship FT140100244. We thank J. Cornell, P. Scott, and M. Szydagis for useful conversations.

Appendix A: LUXCalc

Here we describe the **LUXCalc** software package, which can be found at Ref. [60] or as ancillary files to the arXiv version of this paper. The package provides both a library and a standalone program for performing various likelihood and constraint calculations. The software is written in Fortran 95, but linking to the library can be easily performed from C++. We begin in Section A 1 by describing some basic usage of the standalone program, in Section A 2 we show how to link to the library from Fortran, and in Section A 3 we show how to do the same in C++. Finally, in Section A 4, we point out a few routines in other software packages (**DarkSUSY** and **micrOMEGAs**) that may be useful.

Both the library and program can be compiled by running ‘**make all**’; however, one of the **gfortran** or **ifort** compilers must be installed.

1. Program

The **LUXCalc** program is called in the following form:

```
./LUXCalc [mode] [options] [WIMP parameters]
```

<code>--verbosity=1</code>	Specifies how much detail to provide in the output. Increasing this above the default of 1 will cause a detailed header to be provided, as well as progressively more detailed data output. The additional data provided is mode specific.
<code>--rho=0.4</code>	The local density of the DM halo [GeV/cm ³].
<code>--vrot=235.0</code>	The local rotation speed of the galactic disk [km/s]. The velocity dispersion v_0 will be set to this value as expected for an isothermal spherical halo model (i.e. the Standard Halo Model), unless specified via the <code>--v0</code> option.
<code>--v0=235.0</code>	
<code>--vesc=550.0</code>	The velocity dispersion v_0 and local escape velocity v_{esc} (i.e. cutoff speed) [km/s] used to define the truncated Maxwellian velocity distribution (Eqn. (18)). Specifically, the parameter v_0 is the most probable speed of the distribution in the absence of any truncation.
<code>--Emin=0.0</code>	Only consider contributions to the expected signal from events with recoil energies above this value [keV]. Detector thresholds are already factored into the efficiencies, so contributions from low-energy events are suppressed regardless of this setting.
<code>--confidence-level=0.9</code>	
<code>--p-value=0.1</code>	Specifies the confidence level (CL) or the p-value ($p = 1 - \text{CL}$) to use for generating constraints.
<code>--m-tabulation=1.0,1000.0,-20</code>	For quantities that are tabulated by mass like cross-section limits, this option specifies the minimum and maximum masses in the tabulation [GeV], followed by the number of tabulation intervals. A negative number for the third value indicates number of intervals per decade.

TABLE II. Useful options for running the LUXCalc program. Any values shown are the default values. Some options are only useful in certain program modes.

where `[mode]` is a flag describing the type of calculation to be performed, `[options]` are optional flags that can be used to set various parameters or control the output, and `[WIMP parameters]` are the WIMP mass and scattering cross-section(s), necessary only in certain modes. Several of the program modes are described below and some of the most useful options are described in Table II. A full description of all modes and options can be found by running `./LUXCalc --help`.

Likelihood. The logarithm of the Poisson-based likelihood, as described in Section III, is given by

```
./LUXCalc --log-likelihood [options] [WIMP parameters]
```

The WIMP parameters are a list of values: the WIMP mass [GeV] followed by one, two, or four cross-sections [pb]. In the first case, the single cross-section is the SI WIMP-nucleon cross-section, assumed to be the same for protons and neutrons. In the second case, the two cross-sections are the SI and SD WIMP-nucleon cross-sections, again assuming identical couplings for protons and neutrons. In the last case, the four cross-sections are, in order, the SI WIMP-proton, SI WIMP-neutron, SD WIMP-proton, and SD WIMP-neutron cross-sections.

Maximum gap p-value. The maximum gap p-value, as described in Section III, is given by

```
./LUXCalc --log-pvalue [options] [WIMP parameters]
```

Specifically, this returns the quantity $1 - C_0$, where C_0 is defined in Eqn. (23) and is technically only an upper limit on the p-value, not the p-value itself. As opposed to the Poisson-based likelihood above, this analysis involves no background subtraction and is hence conservative. The WIMP parameters are as described for the likelihood mode above.

Likelihood constraints. Tabulated (by mass) upper and lower cross-section constraints (i.e. confidence intervals) as determined via the likelihood are generated by

```
./LUXCalc --constraints-SI [--theta-SI-pi=0.25] [options]
./LUXCalc --constraints-SD [--theta-SD-pi=0.25] [options]
```

where the two lines correspond to spin-independent (SI) and spin-dependent (SD) interactions. When determining constraints, the ratio between WIMP-neutron and WIMP-proton couplings will be kept fixed, with the ratio defined in terms of the polar angle θ in the G_n - G_p plane, i.e. $\tan \theta \equiv G_n/G_p = f_n/f_p = a_n/a_p$. The default behavior is to take the two WIMP-nucleon couplings to be identical; otherwise, θ can be specified (in units of π) via the options as shown. Increase the verbosity (e.g. `--verbosity=3`) to show the corresponding WIMP-neutron constraints in addition to the WIMP-proton constraints. The confidence level (CL) of the confidence intervals is specified via the `--confidence-level` option; the default is 90% CL.

Maximum gap limits. Tabulated (by mass) upper limits on the cross-section(s) as determined by the maximum gap method are generated by

```
./LUXCalc --limits-SI [--theta-SI-pi=0.25] [options]
./LUXCalc --limits-SD [--theta-SD-pi=0.25] [options]
```

where the options are as described for the likelihood constraints above.

2. Library: Fortran usage

LUXCalc is written as a single, self-contained Fortran 95 module. All floating point values are in the `REAL*8` format, while integers are of type `INTEGER`. The module must be loaded in any user routine that calls LUXCalc routines:

```
USE LUXCalc
```

Initialization. Before calling any routines, the module must first be initialized with

```
CALL LUXCalc_Init(intervals=.TRUE.)
```

The single argument here specifies if calculations should be performed for the intervals (gaps) between events. This is necessary for generating maximum gap limits, but is not required for any likelihood calculations. This initialization need only be performed once. To force only recoils of energy greater than E_{\min} to be considered in calculating rates, use

```
CALL LUXCalc_SetEmin(Emin=0d0)
```

where the argument is in keV. As noted elsewhere, detector thresholds are already factored into the efficiencies, so contributions from low-energy events are suppressed regardless of this setting.

Halo model. The parameters of the Standard Halo Model can be specified via

```
CALL LUXCalc_SetSHM(rho=0.4d0,vrot=235d0,v0=235d0, &
                    vesc=550d0)
```

where the arguments are the local DM density ρ_0 [GeV/cm^3], the disk rotation speed v_{rot} [km/s], the velocity dispersion v_0 [km/s], and the galactic escape speed v_{esc} [km/s]. The values shown here are the defaults, which are already set when `LUXCalc_Init()` is called, so the above function call is spurious. The halo parameters can be modified at any time.

WIMP mass and couplings. The WIMP parameters are specified by one of three routines:

```
CALL LUXCalc_SetWIMP_mfa(m,fp,fn,ap,an)
CALL LUXCalc_SetWIMP_mG(m,GpSI,GnSI,GpSD,GnSD)
CALL LUXCalc_SetWIMP_msigma(m,sigmapSI,sigmanSI, &
                             sigmapSD,sigmanSD)
```

Here, m is the WIMP mass m_χ [GeV]; fp and fn are SI WIMP-proton and WIMP-neutron couplings [GeV^{-2}], respectively; ap and an are SD WIMP-proton and WIMP-neutron couplings; $GpSI$, $GnSI$, $GpSD$, and $GnSD$ are WIMP-nucleon couplings [GeV^{-2}], differing from f and a only in normalization as discussed in Section II A; and the `sigma` arguments are WIMP-nucleon cross-sections [pb]. A negative value for a cross-section can be used to indicate the corresponding

coupling should be taken to be negative. The current mass, couplings, and cross-sections can be retrieved with the corresponding routines

```
CALL LUXCalc_GetWIMP_mfa(m,fp,fn,ap,an)
CALL LUXCalc_GetWIMP_mG(m,GpSI,GnSI,GpSD,GnSD)
CALL LUXCalc_GetWIMP_msigma(m,sigmapSI,sigmanSI, &
                             sigmapSD,sigmanSD)
```

The returned cross-section values will *not* be set negative for negative couplings.

Calculations. After any changes to the WIMP parameters and/or the halo distribution, the LUX rate calculations must be performed using

```
CALL LUXCalc_CalcRates()
```

This routine performs the various LUX rate calculations that are used for determining likelihoods and the various constraints. Thus, this routine *must* be called before obtaining expected events, likelihoods, p-values, etc. The relevant quantities are stored internally.

Events. The number of observed and expected events for the current WIMP are provided by the functions:

```
INTEGER FUNCTION LUXCalc_Events()
REAL*8 FUNCTION LUXCalc_Background()
REAL*8 FUNCTION LUXCalc_Signal()
REAL*8 FUNCTION LUXCalc_SignalSI()
REAL*8 FUNCTION LUXCalc_SignalSD()
```

In order, these return the observed number of events in LUX, the average expected background events, the average expected signal events, and the separate SI and SD contributions to the expected signal.

Likelihoods and p-values. The statistical functions for evaluating LUX results in the context of the current WIMP are:

```
REAL*8 FUNCTION LUXCalc_LogLikelihood()
REAL*8 FUNCTION LUXCalc_LogPValue()
REAL*8 FUNCTION LUXCalc_ScaleToPValue(logp)
```

The first function returns the log of the likelihood using a Poisson distribution in the number of observed events given the expected background and signal. The second function returns the logarithm of the p-value in a more conservative no-background-subtraction analysis. If `LUXCalc.Init()` was called with a `.TRUE.` argument, then the maximum gap method is used, with $p = 1 - C_0$, where C_0 is given by Eqn. (23). Otherwise, A Poisson distribution with zero background contribution is assumed. This function is only intended for determining conservative one-sided limits as the value returned is technically an upper limit on the p-value and not the p-value itself. The last function determines such a limit by identifying the factor x such that $\sigma = x\sigma_0$ gives the desired p-value (specified as $\log(p)$), with σ_0 the currently specified WIMP-nucleon cross-sections. The quantity $x\sigma_0$ is then the limit on the cross-sections at the given confidence level $(1 - p)$, assuming a fixed ratio of WIMP-nucleon couplings (e.g. $f_p = f_n$).

3. Library: C++ usage

To make usage of `LUXCalc` with C++ code easier, a C++ interface file `LUXCalc.hpp` is provided. The routines and functions are the same as those described for Fortran in the previous section, with identical names and signatures, though Fortran subroutines become C++ `void` functions. All arguments and return values are `bool`, `int` or `double`.

4. Useful software

Here we show how to extract the relevant DM parameters from two of the most popular software packages for examining DM in the context of SUSY: `DarkSUSY` [17, 56] and `micrOMEGAs` [18, 57–59]. `DarkSUSY` is written in Fortran 77, with the various G WIMP-nucleon couplings for a given SUSY model provided by the `dsddgpgn` routine. The WIMP (neutralino) mass must be retrieved from various common blocks. The necessary parameters can be retrieved from `DarkSUSY` and set in `LUXCalc` as shown in this minimal Fortran 95 example:

```
! Load LUXCalc module
USE LUXCALC
! Variables to store WIMP mass and couplings
REAL*8 :: M,GpSI,GnSI,GpSD,GnSD
! Calculated quantities (examples)
REAL*8 :: lnlike,signal
! DarkSUSY common blocks defined in 'dsmssm.h'
CHARACTER*8 :: pacodes_ctmp(0:50)
INTEGER :: pacodes_itmp(60),kn(4),lsp,kln
REAL*8 :: mass(0:50),mspctm_rtmp(6)
COMMON /PACODES/ pacodes_itmp(1:18),kn,pacodes_itmp(23:60), &
           pacodes_ctmp
COMMON /MSPCTM/ mass,mspctm_rtmp
COMMON /MSSMIUSEFUL/ lsp,kln
...
! Initialize LUXCalc
CALL LUXCalc_Init(.FALSE.)
...
! For each SUSY model, do following >>>>>>>
  ! Set WIMP parameters
  M = mass(kn(kln))
  CALL dsddgpgn(GpSI,GnSI,GpSD,GnSD)
  CALL LUXCalc_SetWIMP_mG(M,GpSI,GnSI,GpSD,GnSD)

  ! Calculate rates for current WIMP
  CALL LUXCalc_CalcRates()

  ! Get likelihood, expected signal events, etc.
  lnlike = LUXCalc_LogLikelihood()
  signal = LUXCalc_Signal()
  ...
```

The `micrOMEGAs` package provides the WIMP mass and couplings in the `Mcdm` global variable and `nucleonAmplitudes` routine, respectively; see Ref. [18] for a description of the relevant `micrOMEGAs` coupling routine and its arguments. A minimal C++ example using `micrOMEGAs` is:

```
// Initialize LUXCalc
LUXCalc_Init(false);
...
// For each SUSY model, do following >>>>>>>
  // Set WIMP parameters
  // separate particle/anti-particle couplings
  double lambdap[2],lambdan[2],xip[2],xin[2];
  // FeScLoop is micrOMEGAs-provided function
  nucleonAmplitudes(FeScLoop,lambdap,xip,lambdan,xin);
  double M = Mcdm; // Mcdm is global variable
  double GpSI = 2*lambdap[0];
  double GnSI = 2*lambdan[0];
  double GpSD = 2*xip[0];
  double GnSD = 2*xin[0];
```

```

LUXCalc_SetWIMP_mG(M,GpSI,GnSI,GpSD,GnSD);

// Calculate rates for current WIMP
LUXCalc_CalcRates();

// Get likelihood, expected signal events, etc.
double lnlike = LUXCalc_LogLikelihood();
double signal = LUXCalc_Signal();
...

```

We finally point out one subtlety: DarkSUSY and micrOMEGAs will yield somewhat different WIMP-nucleon scattering cross-sections for a given SUSY model due to different values chosen for the hadronic matrix elements that go into calculating WIMP-nucleon couplings from the WIMP-quark couplings [82]. This is more of an issue for the SI cross-sections, which typically differ by a factor of $\mathcal{O}(2)$. Both packages allow the matrix elements to be modified; see their respective manuals for further details.

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