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Pure de Sitter Supergravity

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Abstract

Using superconformal methods we derive an explicit de Sitter supergravity action invariant under spontaneously broken local $\mathcal{N} = 1$ supersymmetry. The supergravity multiplet interacts with a nilpotent goldstino multiplet. We present a complete locally supersymmetric action including the graviton and the fermionic fields, gravitino and goldstino, no scalars. In the global limit when the supergravity multiplet decouples, our action reproduces the Volkov-Akulov theory. In the unitary gauge where the goldstino vanishes we recover pure supergravity with the positive cosmological constant. The classical equations of motion, with all fermions vanishing, have a maximally symmetric solution: de Sitter space.

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1 Introduction

The cosmological constant is known to be negative or zero in pure supergravity, if there are no scalar fields [1]. Pure supergravity with a positive cosmological constant without scalars was not previously known. In this paper we present the locally $\mathcal{N} = 1$ supersymmetric action and transformation rules of such a theory. De Sitter space is a homogeneous solution of the bosonic equations of motion. Supersymmetry is spontaneously broken, so there is no conflict with no-go theorems that prohibit linearly realized supersymmetry [2]. [1]

The main motivation for this work is an increasing amount of observational evidence for an accelerating universe where a positive cosmological constant is a good fit to data. The next step toward a better understanding of dark energy is expected not before ESA space mission Euclid launches in 2020. It is therefore desirable to find a simple version of de Sitter supergravity as a natural source for the positive cosmological constant.

The KKLT uplifting procedure for constructing dS vacua in string theory was proposed in [3]. It was recently updated to the status of a manifestly supersymmetric uplifting using the $\overline{\text{D3}}$-brane on top of an O3-plane at the bottom of a warped throat [4, 5]. It corresponds to a globally supersymmetric Volkov-Akulov (VA) goldstino theory [6] coupled to a supergravity background. The global supersymmetry is realized non-linearly. This recent development indicates that a scalar independent de Sitter supergravity might exist. Another indication of the existence of such a supergravity was presented in [7], where the proposal to couple the VA goldstino theory [6] to supergravity was made. However, a complete action and transformation rules that describe this coupling have never been presented. The supersymmetric coupling of gravitino and goldstino in d=10 at the level quadratic in fermions was studied in [8, 9]. The curved superspace formulation of the VA goldstino theory was studied soon after the discovery of this theory, see for example a review paper [10] or an application of the constrained superfield formalism in superspace in [11]. The relation between the superspace approach and non-linearly realized supersymmetries was investigated in [12].

All earlier theories were not yet developed to the level of a component supergravity action with spontaneously broken local supersymmetry, generalizing the globally supersymmetric VA model. To construct such an action is the purpose of our paper. We will do this by decoding the superconformal action underlying dS supergravity, proposed in [13]. Such a decoding procedure, in addition to a standard gauge-fixing of local Weyl, R-symmetry and special supersymmetry requires an elimination of the auxiliary field F of the goldstino multiplet from the action which has a non-Gaussian dependence on F.

The important step for our ability to derive the complete action of a pure dS supergravity is the observation made in [14, 15] that VA theory can be described using a chiral superfield $S(x,\theta) = X + \sqrt{2}\,\theta\chi + \theta^2 F$ of global $\mathcal{N} = 1$ supersymmetry that satisfies the nilpotent constraint $S^2(x,\theta) = 0$. The constraint sets $X = \bar{\chi}P_L\chi/2F$ and thus eliminates the would-be fundamental scalar partner of the goldstino χ. The model constructed in this way [15] (KS) is equivalent to the original VA geometric model. That model with the action $\det E$, where E is a supersymmetric 1-form, is related to the model of [15] by the non-linear change

[1] Note that there exist $\mathcal{N}$-extended de Sitter superalgebras for even $\mathcal{N}$ but they have a non-compact R-symmetry group and therefore do not allow unitary representations.

of variables presented in [16]. The fact that X is Grassmann valued so that $XP_L\chi = 0$ greatly simplifies the construction of [15] and of our locally supersymmetric extension.

The superconformal approach to pure de Sitter supergravity suggested in [13] is the following: The model at the superconformal level contains the chiral compensating multiplet $\{X^0, \chi^0, F^0\}$, a chiral goldstino multiplet $\{X^1, \chi^1, F^1\}$, and a Lagrange multiplier multiplet $\{\Lambda, \chi^\Lambda, F^\Lambda\}$, interacting with the Weyl gravitational multiplet. The action is

$$\mathcal{L} = [N(X,\bar{X})]_D + [\mathcal{W}(X)]_F + \left[\Lambda(X^1)^2\right]_F . \tag{1.1}$$

where the notation of Chapter 16 of [17] is used[2] and all three chiral supermultiplets are unconstrained. All supersymmetries in (1.1) are linearly realized and manifest. Models of this type differ from the generic models in [17], and in other textbooks, in that the Kähler manifold of the embedding space $N(X,\bar{X})$ does not depend on the superfield Λ but does depend on X^I, $I = 0,1$. Therefore the equation of motion for Λ is algebraic and can be solved producing the superfield constraint $(X^1)^2 = 0$. This in turn leads to a non-generic supergravity: the elimination of the auxiliary field F^1 requires a more complicated procedure since its algebraic equation of motion contains both positive and negative powers of F^1, the latter due to the relation $X^1 = \bar{\chi}^1 P_L \chi^1/2F^1$ which arises as the solution of the constraint. Therefore, the knowledge of the Kähler potential K and the superpotential W at the supergravity level is not sufficient in presence of the nilpotent goldstino multiplet to produce the full fermionic action[3].

At the superconformal level, the dynamics of our pure dS supergravity model is specified by a quadratic Kähler potential and cubic superpotential:

$$N = \eta_{IJ}X^I\bar{X}^J = -X^0\bar{X}^0 + X^1\bar{X}^1 , \qquad \mathcal{W} = a\left(\frac{X^0}{\sqrt{3}}\right)^3 + b\left(\frac{X^0}{\sqrt{3}}\right)^2 X^1 . \tag{1.2}$$

After the superfield constraint $(X^1)^2 = 0$ is implemented, the last term in the action (1.1) vanishes. The parameters a, b are dimensionless as they must be in a conformal theory. One passes to the physical form of the theory by fixing the conformal gauge using $X^0 = \sqrt{3}/\kappa$, thus introducing Newton's constant $\kappa^2 = 8\pi G = M_{Pl}^{-2}$. It is then convenient[4] to redefine our parameters as follows: $a = \kappa\, m$ and $b = \kappa^2\, f$. The new parameters m and f have mass dimension one and two, respectively, and we take them real.[5] The cosmological constant $\mathbf{\Lambda}$ and the Lagrangian mass term of the gravitino are

$$\mathbf{\Lambda} = f^2 - 3\, m^2\, M_{Pl}^2, \qquad L_m = \frac{m}{2\kappa^2}\bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu , \tag{1.3}$$

[2]The superconformal action (1.1) without the Lagrange multiplier superfield Λ was studied in application to inflation and in de Sitter background in [18].

[3]See [19], eqs. (2.4) - (2.6) where the first supergravity model of this kind was presented. The fermion terms in this reference are incomplete, which gives an example of a supergravity where K and W is not sufficient for the determination of the complete action, only the bosonic one can be deduced from K and W.

[4]In cosmological applications one often works with Planck units, $M_{Pl} = \kappa^{-1} = 1$, however, here we would like to study also the flat space limit, therefore we keep κ consistently, in agreement with [17].

[5]In Sec. 3 and the appendix we give the formulas for complex a and b (or m and f). Then $b = \kappa^2\bar{f}$. One can take these two parameters real and positive after chiral rotations of the fields. If $a = |a|\mathrm{e}^{\mathrm{i}\theta_a}$ and $b = |b|\mathrm{e}^{\mathrm{i}\theta_b}$, the phases are removed by replacing $P_L\psi_\mu$ by $P_L\psi_\mu\mathrm{e}^{\mathrm{i}\theta_a/2}$, $P_L\chi$ by $P_L\chi\mathrm{e}^{\mathrm{i}(\theta_a/2-\theta_b)}$, and corresponding rotations on the composite expressions $X = X^1$ and F^I.

where ψ_μ has dimension 1/2.

The physics of the model depends on the relation between these quantities. When $m = 0,\ f \neq 0$, we have the pure de Sitter model with nonlinearly realized supersymmetry discussed above. When $m \neq 0,\ f = 0$, which requires that the fermion of the nilpotent multiplet vanishes, $\chi^1 = 0$, for consistency, we have the basic anti-de Sitter supergravity theory with linearly realized supersymmetry [1, 7]. In all other cases there is nonlinearly realized supersymmetry, and the sign of $\mathbf{\Lambda}$ determines whether the homogeneous bosonic geometry is de Sitter, Minkowski, or anti-de Sitter spacetime. Nonlinearly realized supersymmetry (essentially the same as spontaneous breaking) means that the vacuum expectation value of the SUSY transform of the goldstino field χ does not vanish, $\langle \delta\chi \rangle \neq 0$.

In Sec. 2 of the paper we present the main result, the novel pure dS supergravity action and its local supersymmetry. In Sec. 3 we explain the main logical steps in the derivation of the supergravity theory from the superconformal model in [13], with the details given in Appendix A. In Sec. 4 we study features of dS supergravity. We perform the limit of our new supergravity theory to flat spacetime, where fields of the gravity multiplet are decoupled and $m \to 0$. We show how the VA theory is recovered via its KS version. In the same section we look at the possible gauge-fixing of the local supersymmetry. In the unitary gauge with $\chi^1 = 0$ the gravitino wave operator in Euclidean signature has no zero-modes. In Discussion Sec. 5 we point out that the assumption of the mere existence of the nilpotent goldstino multiplet signifies a natural unavoidable spontaneous supersymmetry breaking, without the need for engineering, as e.g in the O'Raifeartaigh type models. Finally we note that the VA theory, when embedded in supergravity, leads to a positive cosmological constant term $\mathcal{L}_{SG} = -\sqrt{-\det g}\, f^2$. Without coupling to gravity and gravitino, without local supergravity, the vacuum energy term in the VA action $\mathcal{L}_{VA} = -f^2 + ...$ is a hint but not a reliable origin of the dark energy/cosmological constant, now in the context of dS supergravity it is!

2 Pure dS $\mathcal{N} = 1$ supergravity action and its local supersymmetry

The action invariant under spontaneously broken local supersymmetry is given by the following expression

$$
\begin{aligned}
e^{-1}\mathcal{L} \;=\;& \frac{1}{2\kappa^2}\left[R(\omega(e)) - \bar\psi_\mu\gamma^{\mu\nu\rho}D_\nu^{(0)}\psi_\rho + \mathcal{L}_{\rm SG,torsion}\right] + 3\frac{m^2}{\kappa^2} - f^2 \\
&+\frac{f}{\sqrt{2}}\bar\psi_\mu\gamma^\mu\chi + \frac{m}{2\kappa^2}\bar\psi_\mu\gamma^{\mu\nu}\psi_\nu + \frac{\kappa^2}{24}\chi^2\bar\chi^2 \\
&-\tfrac12\bar\chi\slashed{D}^{(0)}\chi - \tfrac{1}{32}{\rm i}\, e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar\psi_\mu\gamma_\nu\psi_\rho\bar\chi\gamma_*\gamma_\sigma\chi - \tfrac12\bar\psi_\mu P_R\chi\bar\psi^\mu P_L\chi \\
&+\frac{\bar\chi^2}{2f}A\frac{\chi^2}{2f} - \left(\frac{\chi^2}{2f}\bar B + \frac{\bar\chi^2}{2f}B\right) - \frac{\chi^2\bar\chi^2}{16f^4}\left(\frac{\Box\chi^2}{f} - 2B\right)\left(\frac{\Box\bar\chi^2}{f} - 2\bar B\right)\,,
\end{aligned}
\tag{2.1}
$$

where

$$
\begin{aligned}
\chi^2 \;&\equiv\; \bar\chi P_L\chi\,, \qquad D_\mu^{(0)} = \partial_\mu + \tfrac14\omega_\mu{}^{ab}(e)\gamma_{ab}\,, \\
\mathcal{L}_{\rm SG,torsion} \;&=\; -\tfrac{1}{16}\left[(\bar\psi^\rho\gamma^\mu\psi^\nu)(\bar\psi_\rho\gamma_\mu\psi_\nu + 2\bar\psi_\rho\gamma_\nu\psi_\mu) - 4(\bar\psi_\mu\gamma\cdot\psi)(\bar\psi^\mu\gamma\cdot\psi)\right]\,.
\end{aligned}
\tag{2.2}
$$

$$A = \Box + \mathrm{i}t^\mu\partial_\mu + \tfrac{1}{2}\mathrm{i}e^{-1}\partial_\mu(e\,t^\mu) + r\,, \qquad \Box = \frac{1}{\sqrt{g}}\partial_\mu\sqrt{g}g^{\mu\nu}\partial_\nu\,, \tag{2.3}$$

$$\begin{aligned} t^\mu &= \tfrac{1}{4}\mathrm{i}\bar\psi_\nu\gamma_*\gamma^{\nu\rho\mu}\psi_\rho\,, \quad r = -\tfrac{1}{6}\left[R(\omega(e)) - \bar\psi_\mu\gamma^{\mu\nu\rho}D^{(0)}_\nu\psi_\rho + \mathcal{L}_{\mathrm{SG,torsion}} - 8\kappa^2 f^2\right]\,, \\ B &= \frac{1}{\sqrt{2}}\left[-e^{-1}\partial_\mu\left(e\bar\psi_\nu\gamma^\mu\gamma^\nu P_L\chi\right) - \tfrac{2}{3}\bar\chi P_L\gamma^{\mu\nu}D_\mu\psi_\nu\right] + f\left(2\frac{m}{\kappa} + \frac{1}{2}\bar\psi_\mu\gamma^{\mu\nu}P_L\psi_\nu\right)\,, \end{aligned} \tag{2.4}$$

$$D_\mu\psi_\nu = \left(\partial_\mu + \tfrac{1}{4}\omega_\mu{}^{ab}(e,\psi)\gamma_{ab}\right)\psi_\nu\,. \tag{2.5}$$

The auxiliary fields F and A_μ of the supergravity multiplet were eliminated by their algebraic equations of motion. The cosmological constant in the 1st line is $\mathbf{\Lambda} = f^2 - 3\frac{m^2}{\kappa^2}$. In the 2nd line the gravitino couples to $f\gamma^\mu\chi$ which is the linear part of the supercurrent of the VA theory; nonlinear corrections are contained in B below. There is also a quadratic gravitino mass-like term and a quartic $\chi^2\bar\chi^2$ originating from elimination of A_μ. The 3d line of the action includes a goldstino kinetic term and quartic fermion interactions. The 4th line presents non-linear goldstino terms.

The supersymmetry transformations of the fields χ and e^a_μ, ψ_μ can be obtained from, respectively, (16.33), (16.45), and (16.47) of [17]. For the fields of the gravity multiplet we have

$$\begin{aligned} \delta e^a_\mu &= \frac{1}{2}\bar\epsilon\gamma^a\psi_\mu\,, & (2.6)\\ \delta P_L\psi_\mu &= P_L\left(\partial_\mu + \frac{1}{4}\omega(e,\psi)_{\mu ab}\gamma^{ab} - \frac{3}{2}iA_\mu + \frac{1}{2}i\gamma_\mu \not{A} + \frac{\kappa}{2\sqrt{3}}\gamma_\mu\bar F^0\right)\epsilon\,. & (2.7) \end{aligned}$$

with

$$F^0 = \overline{W}_0 = \sqrt{3}\,\frac{m}{\kappa} + \frac{2}{\sqrt{3}}f\,X = \sqrt{3}\,\frac{m}{\kappa} - \frac{1}{\sqrt{3}}\chi^2(1-\mathcal{A})\,. \tag{2.8}$$

and

$$A_\mu = \mathrm{i}\frac{\kappa^2}{6}\Big[(\bar X\partial_\mu X - X\partial_\mu\bar X) - \frac{1}{2}[\sqrt{2}\bar\psi_\mu(P_L\chi\bar X - P_R\chi X) + \bar\chi P_L\gamma_\mu\chi]\Big]. \tag{2.9}$$

Here

$$X = -\frac{\chi^2}{2f}(1-\mathcal{A})\,, \tag{2.10}$$

$$\mathcal{A} = \frac{\bar\chi^2}{2f^3}\left(A\,\frac{\chi^2}{2f} - B\right)\,. \tag{2.11}$$

The local supersymmetry transformation for the goldstino is

$$\delta P_L\chi = \frac{1}{\sqrt{2}}P_L\left[-f + (\not\partial - m)X - f\mathcal{A}\left(1 - 3\bar{\mathcal{A}} - \frac{\chi^2}{2f^3}\bar B\right)\right]\epsilon - \frac{1}{2}P_L\gamma^\mu\epsilon\bar\psi_\mu P_L\chi\,. \tag{2.12}$$

3 Derivation of pure dS supergravity

In this section we present the main steps in the derivation of dS supergravity from the underlying superconformal theory with linearly realized supersymmetry and Lagrange multiplier, as shown in eqs. (1.1), (1.2). Details are given in the Appendix.

We will often use the notation for the physical multiplet $\{X^1, \chi^1, F^1\} \equiv \{X, \chi, F\}$. The role of the compensator multiplet $\{X^0, \chi^0, F^0\}$ is to fix the local Weyl and R-symmetry via the choice $X^0 = \bar{X}^0 = \frac{\sqrt{3}}{\kappa}$ and to fix the special local supersymmetry using $\chi^0 = 0$. But in a superconformal setting where equations depend covariantly on both multiplets $\{X^I, \chi^I, F^I\}$, $I = 0, 1$ we will use the original notation.

We first consider the component form of the Lagrange multiplier term in the action in (1.1) and solve the algebraic equations of motion for the superfield $\{\Lambda, \chi^\Lambda, F^\Lambda\}$. The $\Lambda(x)$ field equation is given in (A.5); its solution fixes

$$X = \frac{\chi^2}{2F}\,, \tag{3.1}$$

provided that $F \neq 0$. The equations of motion for $\chi^\Lambda,\ F^\Lambda$ are also satisfied without further constraints.

The detailed form of the first two terms in the superconformal action (1.1) is given in (A.7) which we then write as

$$\begin{aligned}
\mathcal{L} \;=\;& \eta_{IJ}\bar{X}^I\left[\partial_\mu\sqrt{g}g^{\mu\nu}\partial_\nu + \mathrm{i}\,e\,t_c^\mu\partial_\mu + \tfrac{1}{2}\mathrm{i}\,\partial_\mu(e\,t_c^\mu) + e\,r_0^c\right]X^J \\
&+e\,\eta_{IJ}\bar{X}^I\,B_c^J + e\,\eta_{IJ}X^I\bar{B}_c^J + e\,C_0^c + e\,\mathcal{L}_{1,F} + e\,\mathcal{L}_{W,\mathrm{ferm}}\,,
\end{aligned} \tag{3.2}$$

(up to total derivatives). The indices $I = 0, 1$ and the subscript c are a reminder that we are still in the superconformal setting with local conformal symmetry (and other symmetries) unbroken:

$$\begin{aligned}
t_c^\mu &= -2A^\mu + \tfrac{1}{4}\mathrm{i}\bar{\psi}_\nu\gamma_\star\gamma^{\nu\rho\mu}\psi_\rho\,,\\
r_0^c &= -\tfrac{1}{6}R(\omega(e)) + \tfrac{1}{6}\bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu^{(0)}\psi_\rho - A^\mu A_\mu - \tfrac{1}{6}\mathcal{L}_{\mathrm{SG,torsion}}\,,\\
B_c^I &= \frac{1}{\sqrt{2}}\left[-e^{-1}\partial_\mu\left(e\bar{\psi}_\nu\gamma^\mu\gamma^\nu P_L\chi^I\right) - \tfrac{2}{3}\bar{\chi}^I P_L\gamma^{\mu\nu}D_\mu\psi_\nu + \mathrm{i}A^\mu\bar{\psi}_\mu P_L\chi^I\right]\,,\\
C_0^c &= \eta_{IJ}\Big(-\tfrac{1}{2}\bar{\chi}^I\not{D}^{(0)}\chi^J + \tfrac{1}{4}\mathrm{i}\bar{\chi}^I\gamma_*\gamma^\mu\chi^J A_\mu\\
&\qquad -\tfrac{1}{32}\mathrm{i}\,e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_\nu\psi_\rho\bar{\chi}^I\gamma_*\gamma_\sigma\chi^J - \tfrac{1}{2}\bar{\psi}_\mu P_R\chi^I\bar{\psi}^\mu P_L\chi^J\Big)\,,\\
\mathcal{L}_{1,F} &= \eta_{IJ}F^I\bar{F}^J + \mathcal{W}_I F^I + \overline{\mathcal{W}}_{\bar{I}}\bar{F}^I\,,\\
\mathcal{L}_{W,\mathrm{ferm}} &= -\tfrac{1}{2}\mathcal{W}_{IJ}\bar{\chi}^I P_L\chi^J + \frac{1}{\sqrt{2}}\bar{\psi}_\mu\gamma^\mu\mathcal{W}_I P_L\chi^I + \tfrac{1}{2}\bar{\psi}_\mu P_R\gamma^{\mu\nu}\psi_\nu\,\mathcal{W} + \mathrm{h.c.}\,,
\end{aligned} \tag{3.3}$$

where $D_\mu^{(0)}$ and $\mathcal{L}_{\mathrm{SG,torsion}}$ are defined in (2.2) and $D_\mu\psi_\nu$ in (2.5).

The nilpotent fields X and $\bar{X}$ can appear in the Lagrangian (3.2) either linearly or as the bilinear $X\bar{X}$. Thus we look for a new form of $\mathcal{L}$ in which this behavior is manifest. This form is

$$e^{-1}\mathcal{L}(X,F) = (F + \bar{\mathcal{W}}_1)(\bar{F} + \mathcal{W}_1) - \bar{\mathcal{W}}_1\mathcal{W}_1 + \bar{X}\,A_c\,X + X\bar{B}_c + B_c\bar{X} + C_c\,. \tag{3.4}$$

Several simplifications based on the superconformal properties of the equations of motion were required to derive this form, as explained in Appendix A.2.

The main difference between dS supergravity and standard supergravities is now clear. In a generic theory the auxiliaries F^I appear as

$$\eta_{IJ}F^I\bar{F}^J + \mathcal{W}_I F^I + \overline{\mathcal{W}}_{\bar{I}}\bar{F}^I. \tag{3.5}$$

This behavior applies to F^0 in our theory, and this allows us to eliminate it via Gaussian integration; we give details and the forms of the coefficients in (3.4) in Appendix A.3.

The auxiliary field A_μ is also eliminated in this way (see Appendix A.4); its on-shell value, after superconformal gauge fixing

$$X^0 = \bar{X}^0 = \kappa^{-1}\sqrt{3}\,, \qquad \chi^0 = 0\,, \tag{3.6}$$

is given in (2.9). The Grassmann properties of X, $\bar{X}$ imply that on-shell effects of A_μ are far simpler than in a generic supergravity. Thus A_μ vanishes in B^I_c above, and the quadratic $A^\mu A_\mu$ term in r^c_0 with the term in C^c_0 produce the quartic $\chi^2\bar{\chi}^2$ in the second line of (2.1).

The action (3.4) reduces to the form

$$e^{-1}\mathcal{L} = (F+f)(\bar{F}+\bar{f}) - \bar{f}f + \bar{X}\,A\,X + X\bar{B} + B\bar{X} + C\,, \tag{3.7}$$

where, with (1.2) and (3.6),

$$f = \overline{\mathcal{W}}_1 = \kappa^{-2}\bar{b}\,, \tag{3.8}$$

and, for f and m real, A and B are the expressions in (2.3) and (2.4), and C is given in (A.22).

The elimination of $F^1 = F$ is a more complicated matter because the generic form no longer holds. To see this, one substitutes $X = \frac{\chi^2}{2F}$ in (3.7) to obtain

$$e^{-1}\mathcal{L} = (F+f)(\bar{F}+\bar{f}) - \bar{f}f + \frac{\bar{\chi}^2}{2\bar{F}}\,A\,\frac{\chi^2}{2F} + \frac{\chi^2}{2F}\bar{B} + B\frac{\bar{\chi}^2}{2\bar{F}} + C\,. \tag{3.9}$$

A closed form solution for the equations of motion for F, $\bar{F}$ is derived in Appendix A.5. We find that the equation of motion for F is solved by

$$F = -f\left[1 + \mathcal{A}\left(1 - 3\bar{\mathcal{A}} - \frac{\chi^2}{2f^2\bar{f}}\bar{B}\right)\right]\,. \tag{3.10}$$

where X and $\mathcal{A}$ are given in (2.10) and (2.11), respectively. On shell, the action (3.7) becomes

$$\begin{aligned}\mathcal{L} \quad = \quad & -f\bar{f} + \frac{\bar{\chi}^2}{2\bar{f}}A\frac{\chi^2}{2f} - \left(\frac{\chi^2}{2f}\bar{B} + \frac{\bar{\chi}^2}{2\bar{f}}B\right) + C \\ & -\frac{\chi^2\bar{\chi}^2}{16(f\bar{f})^2}\left(f^{-1}\Box\chi^2 - 2B\right)\left(\bar{f}^{-1}\Box\bar{\chi}^2 - 2\bar{B}\right)\,. \end{aligned} \tag{3.11}$$

This leads to our final result in (2.1).

4 Features of dS supergravity

In this section we discuss several features of the dS supergravity theory we constructed in the previous two sections. In the first subsection we discuss the flat spacetime limit and show that the theory reduces to the global Volkov-Akulov theory. In a next subsection we confirm that the de Sitter solution of the theory has no Killing spinors, i.e. there is no residual supersymmetry. Finally, in a third subsection we gauge-fix the local supersymmetry and show that the gravitino operator in a de Sitter background is well-defined.

4.1 The flat spacetime limit.

In the limit of the locally supersymmetric theory in which gravitational effects vanish, we expect to recover the Komargodski-Seiberg version [15] of the global VA theory. This is the limit in which the parameters κ, $m\kappa^{-1}$, the curvature R and the fields ψ_μ, A_μ all vanish. In this limit the action (2.1) reduces to

$$\mathcal{L} = -f^2 - \frac{1}{2}\bar{\chi}\partial\!\!\!/\chi + \frac{1}{4f^2}\bar{\chi}^2\Box\chi^2 - \frac{1}{16f^6}\chi^2\bar{\chi}^2(\Box\chi^2)(\Box\bar{\chi}^2)\,, \tag{4.1}$$

which is equivalent to eq. (3.6) of [15].

It is worth noting that the global limit of the fields of the constrained goldstino multiplet is given by

$$\{X = \frac{\chi^2}{2F},\ \chi\ , F = -f(1 + \frac{1}{4f^4}\bar{\chi}^2\Box\chi^2 - \frac{3}{16f^8}\chi^2\bar{\chi}^2\Box\chi^2\Box\bar{\chi}^2\}\,. \tag{4.2}$$

These constrained components of the Goldstino multiplet in (4.2) transform as though they are elementary, i.e.

$$\delta X = \frac{1}{\sqrt{2}}\bar{\epsilon}P_L\chi\,, \tag{4.3}$$

$$\delta\chi = \frac{1}{\sqrt{2}}P_L(\partial\!\!\!/X + F)\epsilon\,, \tag{4.4}$$

$$\delta F = \frac{1}{\sqrt{2}}\bar{\epsilon}\partial\!\!\!/P_L\chi\,. \tag{4.5}$$

This shows, above and beyond the call of duty, that the constraint $X^2 = 0$ is compatible with supersymmetry.[6]

[6]Note that the transformation rule (4.4) is exactly the flat limit of the transformation rule (2.12). This description of the global supersymmetry of the KS model appears to be new; an approximate form of $\delta\chi$ up to quadratic terms in χ was derived in eq. (15) of [16]. Our formula (4.4) is exact; it terminates at eighth order because of the Grassmann properties. Since F in (4.2) has been evaluated on shell, the SUSY transformation (4.5) must be checked using the equation of motion for $\partial\!\!\!/P_L\chi$.

4.2 No Killing spinors in dS

We assume that $\mathbf{\Lambda} = f^2 - 3m^2/\kappa^2 > 0$. Then the homogenous bosonic solution of the equations of motion of the theory defined by the action (2.1) is de Sitter space with curvature tensor

$$R_{\mu\nu}^{ab} = (e_\mu^a e_\nu^b - e_\nu^a e_\mu^b) H^2 \quad \text{and} \quad H^2 = \kappa^2 \mathbf{\Lambda}/3\,. \tag{4.6}$$

It is obvious that this solution has no residual supersymmetry. To see this one need only inspect the fermionic transformation rules (2.7) and (2.12). When ψ_μ and χ vanish, these rules simplify and give the conditions

$$\delta\psi_\mu = \hat{D}_\mu \epsilon \equiv (\partial_\mu + \frac{1}{4}\omega_{\mu ab}\gamma^{ab} + \frac{m}{2}\gamma_\mu)\epsilon = 0\ , \tag{4.7}$$

$$\delta\chi = -\frac{f}{\sqrt{2}}\epsilon = 0\,. \tag{4.8}$$

The second condition immediately tells us that there are no (non-vanishing) Killing spinors, indicating that the supersymmetry of the bosonic background is spontaneously broken.

The same conclusion follows from the integrability condition for (4.7). It may be useful to contrast this situation with the traditional Killing spinor analysis in anti-de Sitter space (see Sec 2.2.3 of [20]). The integrability condition for (4.7) is

$$[\hat{D}_\mu, \hat{D}_\nu]\epsilon = (\frac{1}{4}R_{\mu\nu ab}\gamma^{ab} + \frac{m^2}{2}\gamma_{\mu\nu})\epsilon = \frac{1}{2}\Big(H^2 + m^2\Big)\gamma_{\mu\nu}\epsilon = 0\,, \tag{4.9}$$

which shows again that there are no non-vanishing solutions.

4.3 Gauge-fixing local supersymmetry and gravitino in dS

The action (2.1) is locally supersymmetric. We now impose the unitary gauge condition $\chi = 0$ and the action becomes

$$e^{-1}\mathcal{L}_{\chi=0} = \frac{1}{2\kappa^2}\left[R(e,\omega(e)) - \bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu^{(0)}\psi_\rho + \mathcal{L}_{\text{SG,torsion}}\right] + \frac{3m^2}{\kappa^2} - f^2 + \frac{m}{2\kappa^2}\bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu\,. \tag{4.10}$$

In this Lagrangian, f is the measure of spontaneous supersymmetry breaking. When $f = 0$ the theory reduces to the well known AdS_4 supergravity [1]. The action (4.10) is locally supersymmetric uniquely in this case, so that the Lagrangian with $m \neq 0$, and $\mathbf{\Lambda} = -3m^2/\kappa^2$ has effectively zero physical gravitino mass [7]. The concept of the "mass spectrum" in AdS space is somewhat tricky, see for example a discussion of this issue with regard to the gravitino in [21]. It is suggested there that the spin 3/2 particle is massless in AdS space not when $m = 0$ but whenever gauge invariance appears. In the AdS case above, the gauge symmetry in the action (4.10) appears in case that $f = 0$ which means $\mathbf{\Lambda} = -3m^2/\kappa^2$.

For $\mathbf{\Lambda} = f^2 - 3m^2/\kappa^2 > 0$ we have dS supergravity with a positive cosmological constant. In this case, as long as $\mathbf{\Lambda} > 0$ there is no criterion to distinguish between "Lagrangian" mass m and a more "physical" mass. The reason is that at $f \neq 0$ the action in (4.10) never acquires a local supersymmetry unless the numerous goldstino dependent terms are added to the action and it becomes the expression in (2.1). In particular the restoration of gauge invariance requires a coupling between $\gamma^\mu\psi_\mu$ and a goldstino χ. Therefore the wisdom accumulated in studies of gravitino in AdS space, although non-trivial, cannot be applied for dS supergravity in (2.1). Of course, $\mathbf{\Lambda} = f^2 - 3m^2/\kappa^2 > 0$ describes a useful relation between the "Lagrangian" gravitino mass, the supersymmetry breaking scale and the cosmological constant.

We will confirm that the gravitino propagator[7] is well defined in dS space by showing that the wave operator in Euclidean signature has no zero-modes. Towards this end we consider the wave equation on S^4 which is the Wick rotation of dS_4. The radius of the sphere is given by $H^2 = \kappa^2\mathbf{\Lambda}/3$. Consider now the mode equation

$$\gamma^{\mu\nu\rho}\hat{D}_\nu\psi_\rho = \lambda\psi^\mu\,, \tag{4.11}$$

$$\hat{D}_\nu \equiv \partial_\nu + \frac{1}{4}\omega_{\nu ab}\gamma^{ab} + \frac{m}{2}\gamma_\nu\,. \tag{4.12}$$

We have moved the mass term into the definition of the traditional AdS covariant derivative [1] but note that $\omega_{\nu ab}$ is the spin connection on S^4. To clarify covariance issues below we include the Christoffel connection, and thus replace $\hat{D}_\nu \to \hat{\nabla}_\nu$.

Our goal is to show that λ =0 is not an allowed eigenvalue, so that the wave operator is invertible. The first step is to multiply eq. (4.11) by γ_μ, obtaining

$$\gamma^{\nu\rho}\hat{\nabla}_\nu\psi_\rho = \frac{1}{2}\lambda\gamma\cdot\psi\,. \tag{4.13}$$

We next apply $\hat{\nabla}_\mu$ on (4.11), obtaining

$$\begin{aligned}\frac{1}{2}\gamma^{\mu\nu\rho}[\hat{\nabla}_\mu,\hat{\nabla}_\nu]\psi_\rho \;&=\; \frac{1}{2}\gamma^{\mu\nu\rho}[\frac{1}{4}R_{\mu\nu ab}\gamma^{ab} + \frac{m^2}{2}\gamma_{\mu\nu}]\psi_\rho = \lambda\hat{\nabla}\cdot\psi\,,\\ -\frac{3}{2}(H^2+m^2)\gamma\cdot\psi &= \lambda\hat{\nabla}\cdot\psi\,.\end{aligned} \tag{4.14}$$

We used $R_{\mu\nu}^{ab} = (e_\mu^a e_\nu^b - e_\nu^a e_\mu^b)H^2$ and some γ-algebra to obtain the last equality.

The original mode equation in (4.11) can be decomposed to read

$$[\gamma^\mu\gamma^{\nu\rho}\hat{\nabla}_\nu\psi_\rho + \gamma^\nu\hat{\nabla}_\nu\psi^\mu - \hat{\nabla}^\mu\gamma\cdot\psi] = \lambda\psi^\mu\,. \tag{4.15}$$

If we now suppose that ψ_μ is a putative zero-mode, this equation simplifies markedly. The right side vanishes and (4.13) and (4.14) imply that the first and third terms on the left side vanish as well. Thus a zero mode must satisfy the simple equation

$$\gamma^\nu\hat{\nabla}_\nu\psi^\mu = (\gamma^\nu\nabla_\nu + 2m)\psi_\mu = 0\,. \tag{4.16}$$

To finish the job we square the operator obtaining

$$\begin{aligned}&\gamma^\rho\gamma^\nu\hat{\nabla}_\rho\hat{\nabla}_\nu\psi^\mu = 0\;,\\ &[\hat{\nabla}^\nu\hat{\nabla}_\nu\psi^\mu - 3(H^2+m^2)\psi^\mu - H^2(\psi^\mu - \gamma^\mu\gamma\cdot\psi)] = 0\,.\end{aligned} \tag{4.17}$$

[7]See [21] for an application of gravitino propagators in dS and AdS spacetime to the problem of discontinuities in the massless limit.

The last (...) comes from the Christoffel term in $\hat{\nabla}$, in which $\gamma \cdot \psi$ vanishes since $\lambda = 0$. To see where we are going, let's make the temporary simplifying assumption that $m = 0$. We then multiply (4.17) by ψ^*_μ, integrate over the sphere, and integrate by parts to obtain

$$\int d^4x \sqrt{g}[\nabla_\nu \psi^*_\mu \nabla^\nu \psi^\mu + 4\, H^2 \psi^*_\mu \psi^\mu] = 0\,. \tag{4.18}$$

Since each term is non-negative we learn that any zero mode $\psi_\mu(x)$ vanishes identically.

When $m \neq 0$, there is a small complication. The first term in (4.17) becomes

$$\hat{\nabla}^\nu \hat{\nabla}_\nu \psi^\mu = (\nabla^\nu + \frac{m}{2}\gamma^\nu)(\nabla_\nu + \frac{m}{2}\gamma_\nu)\psi^\mu = (\nabla^\nu \nabla_\nu - m^2)\psi^\mu\,, \tag{4.19}$$

after (4.16) is used. We can then rewrite (4.17) as

$$[\nabla^\nu \nabla_\nu \psi^\mu - 4(H^2 + m^2)]\psi^\mu = 0\,. \tag{4.20}$$

The same "multiply and integrate" argument then implies that any zero mode vanishes identically!

In the unitary gauge the local supersymmetry of the supergravity action (2.1) is broken. The validity of this gauge-fixing in a dS background for the gravitino field equations in Euclidean signature of space-time was demonstrated above: there are no zero modes. In Lorentzian signature it means that the gravitino differential operator in dS space is invertible, by analytic continuation from the Euclidean signature.

Much more is known about the gravitino field equations in dS space, since the gravitino is one of the important factors in cosmology. During inflation the background is near dS and during the current acceleration, if caused by a cosmological constant, the background is a dS space. The classical gravitino equations which also follow from our gauge-fixed action (4.10) were studied in [18, 25] in a FRW metric as well as in a de Sitter background. The relatively simple form of the solution was obtained in the metric, conformal to flat, $ds^2 = a^2(d\eta^2 - d\vec{x}^2)$. The solution was found in the form of an expansion in momentum modes $\psi^\mu \sim \int d^3k e^{-i\vec{k}\cdot\vec{x}} \psi^\mu_{\vec{k}}(\eta)$ where an explicit dependence on the conformal time η enters via Hankel functions depending on $|k\eta|$, see for example eq. (10.5) in [18].

5 Discussion

In this paper we have derived the component Lagrangian and local SUSY transformation rules describing the coupling of the nonlinear Volkov-Akulov theory [6] to supergravity complete in all orders in fermions. The two keys to our construction were:

i) the reformulation [14, 15] of the global VA theory in terms of a chiral superfield $X = \{X, \chi, F\}$ subject to the constraint $X^2 = 0$, and

ii) the superconformal approach to $\mathcal{N} = 1,\ D = 4$ supergravity in the form largely developed in [18] and described in Chapter 16 of [17] where earlier references from the 80's on the superconformal approach to supergravity are also given.

The combination of these two methods is successful because the Lagrange multiplier that enforces the constraint [13] maintains linearly realized local off-shell supersymmetry, so that superconformal methods govern the initial stages of the supergravity construction.

Nevertheless, one may distinguish between generic models of [17] in which a model is completely specified by its holomorphic superpotential $W(z^\alpha)$ and Kähler potential $K(z^\alpha, \bar{z}^{\bar{\alpha}})$ and models with one or more constrained superfields. In the first case the F^α appear in a universal quadratic fashion and they are easily eliminated. When there are constraints the dependence on the F^α is still algebraic, but more complicated. (See (3.5), (3.9) above.) Nevertheless, one can find F in closed form because the scalar component of the constrained multiplet is quadratic in the Grassmann valued goldstino, $X = -(1/2f)\chi^2 + \dots$, where f controls the cosmological constant.

The striking feature of our model is that it yields a pure de Sitter $\mathcal{N} = 1$ supergravity action in which the physical fields consist of the graviton, gravitino, and goldstino, but no scalars and no gauge multiplets. [8] Previous constructions of de Sitter supergravities require either a $U(1)$ gauge multiplet with Fayet-Illiopoulos coupling and a charged gravitino [24] with consequent anomaly problems, or O'Raifeartaigh-type models with multiple chiral multiplets, engineered to arrange a potential positive at a local minimum.
In our model de Sitter space is obtained as the homogeneous solution because spontaneous supersymmetry breaking is unavoidable in the presence of a fermionic goldstino. We hope that it will be helpful for describing dark energy.

Since supersymmetry is broken in our model there are no Killing spinors. There is a significant simplification of the action in the unitary gauge in which the goldstino vanishes and the nonlinearities associated with it disappear. We then find a very simple form of supergravity with the cosmological constant $\mathbf{\Lambda} = f^2 - 3\frac{m^2}{\kappa^2}$. The equation of motion for the gravitino in a de Sitter background has no zero modes and its solutions are known [18, 25].

Another feature of our new dS supergravity model is that it reduces in the flat space limit to the VA global theory [6] in the form given in [15]. We emphasize that the constrained components of the goldstino multiplet transform as a conventional chiral multiplet after elimination of F.

There is curious question for future work. The elegant geometric Lagrangian of the original form of the VA theory involves the determinant of a quadratic form in the goldstino, $\mathcal{L} = \mathrm{Det}(\delta^\mu_\nu + \bar{\chi}\gamma^\mu\partial_\nu\chi)$. It is known how to couple it to a supergravity background in the D-brane actions, however, the corresponding supersymmetry upon gauge-fixing local κ-symmetry, is still a rigid supersymmetry [4, 5]. It would be useful to know whether de Sitter supergravity with local supersymmetry presented in this paper may be brought to the geometric form of the global VA theory: this could generate further insights into the nature of fundamental symmetries and the origin of the positive cosmological constant.

So far we have explicitly constructed only the complete pure dS supergravity action with local supersymmetry. More general explicit supergravity models with constrained super-

[8] In [22] it has been shown that other superconformal constructions of such theories are dual to ours. In [23] the same method as ours was used, but using a different gauge fixing than the one given in eq. (3.6).

fields interacting with general matter multiplets, to all orders in fermions, still have to be constructed. The corresponding superconformal action was already proposed in [13], for any number of chiral multiplets X^I, with generic Kähler manifold and generic superpotential together with constraints on functions of chiral multiplets determined by Lagrange multipliers Λ^k:

$$\mathcal{L} = [N(X,\bar{X})]_D + [\mathcal{W}(X)]_F + \left[\Lambda^k A_k(X)\right]_F . \tag{5.1}$$

None of the Λ^k can appear in the Kähler potential and the $A_k(X)$ must be algebraic functions of X^I. The superconformal action in (5.1) must be decoded and the theory expressed in physical form. Extension of the procedures of this paper will be needed to investigate the physics of this more general framework.

In closing we note that pure and complete anti-de Sitter supergravity [1] was first formulated in 1977, but the pure and complete de Sitter supergravity is first constructed now, 38 years later. The action and its local supersymmetry transformation are presented in Sec. 2 of this paper.

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A From superconformal action to supergravity

In order to write the D terms in (1.1), we use the relation that for a chiral multiplet $(X, P_L\chi, F)$ of Weyl weight 1, the D-action can be written in the form of an F-action:

$$[X\bar{X}]_D = \tfrac{1}{2}[X\bar{F}]_F \,, \tag{A.1}$$

where $\bar{F}$ is the lowest component of a chiral multiplet of Weyl weight 2 since it transforms only under $P_L\epsilon$. The components of this multiplet are given in [17, (16.36)]:

$$(\bar{F}, \not{D} P_R\chi, \Box^C \bar{X}) \,. \tag{A.2}$$

The explicit expression of the superconformal covariant derivative is given in [17, (16.34)] and of the superconformal d'Alembertian on a scalar field of Weyl weight 1 in [17, (16.37)]. These steps are performed separately for the X^0 multiplet and for the X^1 multiplet. Therefore, we write the Lagrangian as

$$\mathcal{L} = [\tfrac{1}{2}\eta_{IJ}X^I\bar{F}^J]_F + [\mathcal{W}(X^I)]_F + \left[\Lambda(X^1)^2\right]_F . \tag{A.3}$$

The superconformal F-type action is given in [17, (16.35)]. The first term of (A.3) is identical to [17, (16.39)], where pure $\mathcal{N}=1$ supergravity was explained, and the $\mathcal{W}$ term was written in [17, (17.19)].

A.1 Solution of the Lagrange multiplier constraints

Let us look at the term $[\Lambda(X^1)^2]_F$

$$\begin{aligned} e^{-1}\mathcal{L}_\Lambda \;=\;& F^\Lambda (X^1)^2 + \Lambda\left(2X^1F^1 - \bar{\chi}^1 P_L\chi^1\right) - 2\bar{\chi}^\Lambda P_L\chi^1X^1 \\ &+\frac{1}{\sqrt{2}}\bar{\psi}_\mu\gamma^\mu\left(2\Lambda X^1P_L\chi^1 + (X^1)^2P_L\chi^\Lambda\right) \\ &+\tfrac{1}{2}\bar{\psi}_\mu P_R\gamma^{\mu\nu}\psi_\nu\Lambda(X^1)^2 + \text{h.c.} \end{aligned} \tag{A.4}$$

The field equation of Λ is

$$2X^1F^1 - \bar{\chi}^1P_L\chi^1 + \sqrt{2}\bar{\psi}_\mu\gamma^\mu X^1P_L\chi^1 + \tfrac{1}{2}\bar{\psi}_\mu P_R\gamma^{\mu\nu}\psi_\nu(X^1)^2 = 0 . \tag{A.5}$$

This is solved as in the rigid case by

$$X^1 = \frac{\overline{\chi^1}P_L\chi^1}{2F^1} \equiv \frac{\chi^2}{2F}, \qquad F^1\equiv F, \qquad \chi^1\equiv\chi , \tag{A.6}$$

since this kills all components of the chiral multiplet $(X^1)^2$. It follows that the remaining equations for χ^Λ and F^Λ are also satisfied.

A.2 Details of $[\frac{1}{2}\eta_{IJ}X^I\bar{F}^J]_F$ and $[\mathcal{W}(X^I)]_F$

Using [17] as described above, the first two terms of (A.3) can be written as

$$\begin{aligned} e^{-1}\mathcal{L} \;=\;& \tfrac{1}{2}\eta_{IJ}\left(F^I\bar{F}^J + X^I\Box^C\bar{X}^J - \bar{\chi}^IP_L\not{D}\chi^J\right) \\ &+\mathcal{W}_IF^I - \tfrac{1}{2}\mathcal{W}_{IJ}\bar{\chi}^IP_L\chi^J \\ &+\frac{1}{\sqrt{2}}\bar{\psi}_\mu\gamma^\mu\left[\tfrac{1}{2}\eta_{IJ}\left(P_L\chi^I\bar{F}^J + X^I\not{D}P_R\chi^J\right) + \mathcal{W}_IP_L\chi^I\right] \\ &+\tfrac{1}{2}\bar{\psi}_\mu P_R\gamma^{\mu\nu}\psi_\nu\left(\tfrac{1}{2}\eta_{IJ}X^I\bar{F}^J + \mathcal{W}\right) + \text{h.c.} \end{aligned} \tag{A.7}$$

The superpotential and its 1st and 2d derivatives which we need in (A.7) are:

$$\begin{aligned} \mathcal{W} &= a\left(\frac{X^0}{\sqrt{3}}\right)^3 + b\left(\frac{X^0}{\sqrt{3}}\right)^2 X^1 , \\ \mathcal{W}_0 &= 3a\frac{(X^0)^2}{(\sqrt{3})^3} + \frac{2}{3}bX^0X^1 , \qquad \mathcal{W}_1 = \frac{1}{3}b(X^0)^2 , \\ \mathcal{W}_{00} &= 6a\frac{X^0}{(\sqrt{3})^3} + \frac{2}{3}bX^1 , \qquad \mathcal{W}_{01} = \frac{2}{3}bX^0 . \end{aligned} \tag{A.8}$$

This action can be written in the form of eq. (3.2), (3.3) in Sec. 3 (after noting that the Weyl connection b_μ terms cancels).

The next major step is the elimination of auxiliary fields, but it is useful to first make some simplifications in our superconformal action. This will facilitate the derivation of (3.4). The simplifications are possible because we know that all the gauge connections recombine in covariant derivatives in order to make field equations supercovariant. It saves a lot of work to recognize this structure. In particular, the equation of motion for X^1 should be a conformally covariant equation modulo other field equations. We start by writing the $\bar{X}^1$ field equation, before imposing the constraint:

$$
\begin{aligned}
e^{-1}\frac{\delta\mathcal{L}_1}{\delta\bar{X}^1} &= \square^C X^1 + \overline{\mathcal{W}}_{01}\bar{F}^0 - \tfrac{1}{2}\overline{\mathcal{W}}_{001}\bar{\chi}^0 P_R\chi^0 \\
&\quad + \frac{1}{\sqrt{2}}\bar{\psi}_\mu\gamma^\mu\left[\not{\mathcal{D}}P_L\chi^1 + \overline{\mathcal{W}}_{01}P_R\chi^0 + \frac{1}{\sqrt{2}}(F^1+\overline{\mathcal{W}}_1)P_R\gamma^\nu\psi_\nu\right] \\
&\quad - \tfrac{1}{2}\bar{\psi}_\mu P_L\psi^\mu\left[F^1+\overline{\mathcal{W}}_1\right] .
\end{aligned} \tag{A.9}
$$

Note that the expression in square brackets in the second line is the field equation of $P_R\chi^1$, while the one in the third line is the field equation of $\bar{F}^1$. Writing out some covariant derivatives leads to further simplifications. One of these is that terms with F^1 all cancel. These simplification lead to :

$$
\begin{aligned}
e^{-1}\frac{\delta\mathcal{L}_1}{\delta\bar{X}^1} &= \square'^C X^1 + \overline{\mathcal{W}_{01}}\bar{F}^0 - \tfrac{1}{2}\overline{\mathcal{W}_{001}}\bar{\chi}^0 P_R\chi^0 \\
&\quad + \frac{1}{\sqrt{2}}\bar{\psi}_\mu\gamma^\mu\overline{\mathcal{W}}_{01}P_R\chi^0 + \frac{1}{\sqrt{2}}\bar{\psi}_\mu\gamma^{\mu\nu}P_L\mathcal{D}'_\nu\chi^1 + \frac{1}{2}\bar{\psi}_\mu\gamma^{\mu\nu}P_L\psi_\nu\overline{\mathcal{W}}_1 .
\end{aligned} \tag{A.10}
$$

One can see that this allows us to identify the terms $A_c X^1 + B_c$ in (3.4). The modified conformal derivatives that appear in (A.10) are given by

$$
\begin{aligned}
\square'^C X &= e^{a\mu}\left(\partial_\mu\mathcal{D}_a X - 2b_\mu\mathcal{D}_a X + \chi_{\mu\,ab}\mathcal{D}^b X + 2f_{\mu a}X + \mathrm{i}A_\mu\mathcal{D}_a X + \frac{1}{\sqrt{2}}\bar{\phi}_\mu\gamma_a P_L\chi\right) , \\
\mathcal{D}_a X &= e_a^\mu\left(\partial_\mu X - b_\mu X - \mathrm{i}\,A_\mu X - \frac{1}{\sqrt{2}}\bar{\psi}_\mu P_L\chi\right) , \\
P_L\mathcal{D}'_\mu\chi &= P_L\left[\left(\partial_\mu + \frac{1}{4}\omega_\mu{}^{bc}\gamma_{bc} - \tfrac{3}{2}b_\mu + \tfrac{1}{2}\mathrm{i}A_\mu\right)\chi - \frac{1}{\sqrt{2}}\left(\not{\mathcal{D}}X\right)\psi_\mu - \sqrt{2}X\phi_\mu\right] .
\end{aligned} \tag{A.11}
$$

As stated above the explicit b_μ terms cancel with those in the spin connection $\omega_\mu{}^{ab} = \omega_\mu{}^{ab}(e,b,\psi)$ and in f_μ^μ (given in [17, (16.26)].

A.3 Gaussian integration of auxiliary field F^0

Since F^1 occurs in the expression for X^1, we cannot use its field equation immediately. But the other auxiliary fields: F^0 and A_μ eliminated quite simply. We start with F^0; its elimination preserves the general structure of (3.4).

In order to eliminate the auxiliary field F^0, we first collect the terms in the action with F^I. We write $\mathcal{L}_{1,F}$ as

$$\mathcal{L}_{1,F} = \eta_{IJ}\left(F^I + \eta^{IK}\overline{\mathcal{W}}_{\bar{K}}\right)\left(\bar{F}^J + \eta^{JL}\mathcal{W}_L\right) - \mathcal{W}_I\eta^{IJ}\overline{\mathcal{W}}_{\bar{J}}\,. \tag{A.12}$$

We eliminate F^0 and thus remain with

$$\mathcal{L}_{1,F} \approx \left(F^1 + \overline{\mathcal{W}}_1\right)\left(\bar{F}^1 + \mathcal{W}_1\right) - \mathcal{W}_1\overline{\mathcal{W}}_{\bar{1}} + \mathcal{W}_0\overline{\mathcal{W}}_{\bar{0}}\,. \tag{A.13}$$

Note that the term quadratic in $\mathcal{W}_0$ adds an additional term to the A_c term, so that now after elimination of F^0 we have the following entries for (3.4)

$$\begin{aligned}
A_c &= \left[\partial_\mu\sqrt{g}g^{\mu\nu}\partial_\nu + \mathrm{i}\,e\,t_c^\mu\partial_\mu + \tfrac{1}{2}\mathrm{i}\,\partial_\mu(e\,t_c^\mu) + e\,r_c\right]\ , && \text{(A.14)}\\
r_c &= r_0^c + \mathcal{W}_{01}\overline{\mathcal{W}}_{01}\\
&= -\tfrac{1}{6}R(\omega(e)) + \tfrac{1}{6}\bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu^{(0)}\psi_\rho - A^aA_a - \tfrac{1}{6}\mathcal{L}_{\mathrm{SG,torsion}} + \tfrac{4}{9}|bX^0|^2\,, && \text{(A.15)}\\
B_c &= B^1 + \overline{W}_{01}\left[\mathcal{W}_0\right]_{X^1=0}\\
&- \tfrac{1}{2}\overline{\mathcal{W}}_{IJ1}\bar{\chi}^IP_R\chi^J + \frac{1}{\sqrt{2}}\overline{\mathcal{W}}_{I1}\bar{\psi}_\mu\gamma^\mu P_R\chi^I + \tfrac{1}{2}\overline{\mathcal{W}}_1\bar{\psi}_\mu P_L\gamma^{\mu\nu}\psi_\nu\\
&= \frac{1}{\sqrt{2}}\left[-e^{-1}\partial_\mu\left(e\bar{\psi}_\nu\gamma^\mu\gamma^\nu P_L\chi^1\right) - \tfrac{2}{3}\bar{\chi}^1P_L\gamma^{\mu\nu}D_\mu\psi_\nu + \mathrm{i}A^\mu\bar{\psi}_\mu P_L\chi^1\right]\\
&+ \frac{\bar{b}}{3}\Big(2\frac{1}{\sqrt{3}}a(X^0)^2\bar{X}^0 - \bar{\chi}^0P_R\chi^0 + \sqrt{2}\bar{\psi}\cdot\gamma P_R\chi^0\bar{X}^0 + \frac{1}{2}(\bar{X}^0)^2\bar{\psi}_\mu\gamma^{\mu\nu}P_L\psi_\nu\Big). && \text{(A.16)}
\end{aligned}$$

$$\begin{aligned}
C_c &= -e^{-1}\bar{X}^0\left[\partial_\mu\sqrt{g}g^{\mu\nu}\partial_\nu + \mathrm{i}\,e\,t_c^\mu\partial_\mu + \tfrac{1}{2}\mathrm{i}\,\partial_\mu(e\,t_c^\mu) + e\,r_0^c\right]X^0\\
&- \bar{X}^0B^0 - X^0\bar{B}^0 + C_0 + \left[\mathcal{W}_0\right]_{X^1=0}\left[\overline{\mathcal{W}}_0\right]_{\bar{X}^1=0}\\
&+ \left[\left(-\tfrac{1}{2}\mathcal{W}_{IJ}\bar{\chi}^IP_L\chi^J + \frac{1}{\sqrt{2}}\bar{\psi}_\mu\gamma^\mu\mathcal{W}_IP_L\chi^I + \tfrac{1}{2}\bar{\psi}_\mu P_R\gamma^{\mu\nu}\psi_\nu\,\mathcal{W}\right)_{X^1=0} + \text{h.c.}\right] && \text{(A.17)}
\end{aligned}$$

A.4 Gaussian integration of auxiliary field A_μ.

Then we turn to the elimination of A_μ. We write as in [17, (17.21)]

$$\begin{aligned}
e^{-1}\frac{\delta\mathcal{L}_1}{\delta A^\mu} &= \mathrm{i}\left[(\mathcal{D}_\mu X^I)\eta_{IJ}\bar{X}^{\bar{J}} - \text{h.c.}\right] + \frac{1}{2}\mathrm{i}\eta_{IJ}\bar{\chi}^IP_L\gamma_\mu\chi^{\bar{J}}\\
&= 2A_\mu X^I\eta_{IJ}\bar{X}^J + \mathrm{i}\left[\left(\partial_\mu X^I + \frac{1}{\sqrt{2}}\bar{\psi}_\mu P_L\chi^I\right)\eta_{IJ}\bar{X}^{\bar{J}} - \text{h.c.}\right]\\
&\quad + \frac{1}{2}\mathrm{i}\eta_{IJ}\bar{\chi}^IP_L\gamma_\mu\chi^{\bar{J}}\,. && \text{(A.18)}
\end{aligned}$$

where $\frac{1}{N} = -\frac{1}{X^0\bar{X}^0} - \frac{X^1\bar{X}^1}{(X^0\bar{X}^0)^2}$. The solution for A_μ is

$$\begin{aligned}
A_\mu &= \mathcal{A}_\mu + \mathcal{A}_\mu^{\rm F}\,,\\
&\qquad \mathcal{A}_\mu = {\rm i}\frac{1}{2N}\eta_{IJ}(X^I\partial_\mu\bar{X}^J - \bar{X}^I\partial_\mu X^J) = \mathcal{A}_\mu^0 + \mathcal{A}_\mu^1\,,\\
&\qquad\quad \mathcal{A}_\mu^0 = {\rm i}\frac{1}{2N}(-X^0\partial_\mu\bar{X}^0 + \bar{X}^0\partial_\mu X^0)\,,\\
&\qquad\quad \mathcal{A}_\mu^1 = {\rm i}\frac{1}{2X^0\bar{X}^0}(-X^1\partial_\mu\bar{X}^1 + \bar{X}^1\partial_\mu X^1)\,,\\
&\qquad \mathcal{A}_\mu^{\rm F} = \frac{1}{4N}{\rm i}\eta_{IJ}\left[\sqrt{2}\bar{\psi}_\mu(P_L\chi^J\bar{X}^I - P_R\chi^J X^I) + \bar{\chi}^I P_L\gamma_\mu\chi^J\right] = \mathcal{A}_\mu^{\rm F0} + \mathcal{A}_\mu^{\rm F1}\,,\\
&\qquad\quad \mathcal{A}_\mu^{\rm F0} = -\frac{1}{4N}{\rm i}\left[\sqrt{2}\bar{\psi}_\mu(P_L\chi^0\bar{X}^0 - P_R\chi^0 X^0) + \bar{\chi}^0 P_L\gamma_\mu\chi^0\right]\,,\\
&\qquad\quad \mathcal{A}_\mu^{\rm F1} = -\frac{1}{4X^0\bar{X}^0}{\rm i}\left[\sqrt{2}\bar{\psi}_\mu(P_L\chi^1\bar{X}^1 - P_R\chi^1 X^1) + \bar{\chi}^1 P_L\gamma_\mu\chi^1\right]\,.
\end{aligned} \tag{A.19}$$

The terms $\mathcal{A}_\mu^0$ and $\mathcal{A}_\mu^{\rm F0}$ vanish after gauge-fixing (3.6), and the on-shell value of $\mathcal{A}_\mu$ simplifies to

$$A_\mu = {\rm i}\frac{\kappa^2}{6}[(\bar{X}\partial_\mu X - X\partial_\mu\bar{X}) - \frac{1}{2}\left[\sqrt{2}\bar{\psi}_\mu(P_L\chi\bar{X} - P_R\chi X) + \bar{\chi}P_L\gamma_\mu\chi\right]]\,. \tag{A.20}$$

After elimination, the entire effect of A_μ resides in a contribution to the Lagrangian

$$\mathcal{L}_A = e\,NA^\mu A_\mu = \frac{\kappa^2}{24}\chi^2\bar{\chi}^2\,. \tag{A.21}$$

This simple form arises only from the last term of (A.20) after Fierz rearrangement. It contributes to the expression C in (3.7), which is

$$\begin{aligned}
C &= \frac{1}{2\kappa^2}\left[R(\omega(e)) - \bar{\psi}_\mu\gamma^{\mu\nu\rho}D_\nu^{(0)}\psi_\rho + \mathcal{L}_{\rm SG,torsion}\right] + 3\frac{m^2}{\kappa^2} - f^2\\
&\quad + \frac{1}{\sqrt{2}}f\bar{\psi}_\mu\gamma^\mu\chi + \frac{m}{2\kappa^2}\bar{\psi}_\mu\gamma^{\mu\nu}\psi_\nu + \frac{\kappa^2}{24}\chi^2\bar{\chi}^2\\
&\quad - \tfrac{1}{2}\bar{\chi}\not{D}^{(0)}\chi - \tfrac{1}{32}{\rm i}\,e^{-1}\varepsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_\nu\psi_\rho\bar{\chi}\gamma_*\gamma_\sigma\chi - \tfrac{1}{2}\bar{\psi}_\mu P_R\chi\bar{\psi}^\mu P_L\chi\,.
\end{aligned} \tag{A.22}$$

A.5 Non-Gaussian integration of auxiliary field F

Here we give the detailed derivation of results in the last part of Sec 3. We start with the action (3.7) where $X = \frac{\chi^2}{2F}$ and $\bar{X} = \frac{\bar{\chi}^2}{2\bar{F}}$. Then we solve for the fields F and $\bar{F}$ using their algebraic equations of motion. The field equation for F is

$$\frac{\delta\mathcal{L}(X,F)}{\partial\bar{F}} - \frac{\bar{X}}{\bar{F}}\frac{\delta\mathcal{L}(X,F)}{\partial\bar{X}} = 0 \quad\longrightarrow\quad F + f - \frac{\bar{X}}{\bar{F}}(A\,X + B) = 0\,. \tag{A.23}$$

This implies that

$$F = -f + \mathcal{O}(\bar{\chi}^2)\,,\qquad \bar{F} = -\bar{f} + \mathcal{O}(\chi^2)\,, \tag{A.24}$$

where e.g. $\mathcal{O}(\bar{\chi}^2)$ means that the correction terms are proportional to an undifferentiated $\bar{\chi}^2$. The complete expression is

$$F = -f\left[1 - \frac{\bar{X}}{f\bar{F}}\,(A\,X + B)\right], \tag{A.25}$$

Since $\bar{X}$ is nilpotent, we have also

$$F^{-1} = -\frac{1}{f}\left[1 + \frac{\bar{X}}{f\bar{F}}\,(A\,X + B)\right], \tag{A.26}$$

This allows to write the following expression for X

$$\begin{aligned} X &= \frac{\chi^2}{2}F^{-1} = -\frac{\chi^2}{2f}\left[1 + \frac{\bar{X}}{f\bar{F}}\,(A\,X + B)\right] \\ &= -\frac{\chi^2}{2f}\left[1 + \frac{\bar{\chi}^2}{2f\bar{f}^2}\left(A\,\frac{\chi^2}{2F} + B\right)\right] \\ &= -\frac{\chi^2}{2f}\left[1 - \frac{\bar{\chi}^2}{2f\bar{f}^2}\left(A\,\frac{\chi^2}{2f} - B\right)\right], \end{aligned} \tag{A.27}$$

where the second line is obtained using (A.24) and for the third line we observe that the two derivatives in A must both act on χ^2 in order not to be killed by the overall factor χ^2.[9] We define now for convenience

$$\mathcal{A} = \frac{\bar{\chi}^2}{2f\bar{f}^2}\left(A\,\frac{\chi^2}{2f} - B\right). \tag{A.28}$$

The quantity $\mathcal{A}$ is thus fully determined by the functions A, B and f that appear in the action and the fermionic composite scalar χ^2 (and their complex conjugates). The dependent field X is $X = -\frac{\chi^2}{2f}(1-\mathcal{A})$. In order to find F we have to consider

$$\begin{aligned} \bar{X}[AX + B] &= (1-\bar{\mathcal{A}})\frac{\bar{\chi}^2}{2\bar{f}}\left[A\left(\frac{\chi^2}{2f}(1-\mathcal{A})\right) - B\right] \\ &= (1-\bar{\mathcal{A}})\left[f\bar{f}\mathcal{A} - \frac{\bar{\chi}^2}{2\bar{f}}A\left(\frac{\chi^2}{2f}\mathcal{A}\right)\right]. \end{aligned} \tag{A.29}$$

In the last term, the A should fully act on the leading factor of $\mathcal{A}$ in (A.28) in order that this factor does not clash with the leading $\bar{\chi}^2$. It should also fully act as the $\Box$ factor, which means that we can write A also as $\bar{A}$ in order to get the following elegant equation:

$$\begin{aligned} \frac{\bar{\chi}^2}{2\bar{f}}A\left(\frac{\chi^2}{2f}\mathcal{A}\right) &= \frac{\bar{\chi}^2\chi^2}{4f\bar{f}}\left(\bar{A}\frac{\bar{\chi}^2}{2\bar{f}f^2}\right)\left(A\,\frac{\chi^2}{2f} - B\right) \\ &= \mathcal{A}\frac{\chi^2}{2f}\bar{A}\frac{\bar{\chi}^2}{2\bar{f}} = \mathcal{A}\left(f\bar{f}\bar{\mathcal{A}} + \frac{\chi^2}{2f}\bar{B}\right). \end{aligned} \tag{A.30}$$

[9]In fact, we could move f outside of the A operator, and even replace the A by only its part $\Box$, but this is not convenient for what follows below.

Introducing this in (A.29) and using the nilpotency of $\mathcal{A}$, and $\bar{\mathcal{A}} = \mathcal{O}(\chi^2)$ gives

$$\bar{X}[AX + B] = \mathcal{A} f \bar{f} \left[1 - 2\bar{\mathcal{A}} - \frac{\chi^2}{2f^2\bar{f}} \bar{B} \right] . \tag{A.31}$$

We find therefore with (A.25)

$$F = -f \left[1 - \frac{\bar{f}}{\bar{F}} \mathcal{A} \left(1 - 2\bar{\mathcal{A}} - \frac{\chi^2}{2f^2\bar{f}} \bar{B} \right) \right] . \tag{A.32}$$

This implies e.g.

$$-\frac{\bar{f}}{\bar{F}} = 1 - \bar{\mathcal{A}} + \mathcal{O}(\chi^2 \bar{\chi}^2) , \tag{A.33}$$

which gives as final expression for F:

$$F = -f \left[1 + \mathcal{A} \left(1 - 3\bar{\mathcal{A}} - \frac{\chi^2}{2f^2\bar{f}} \bar{B} \right) \right] . \tag{A.34}$$

Due to the orders of nilpotent quantities, we also obtain

$$(F + f)(\bar{F} + \bar{f}) = f \bar{f} \mathcal{A} \bar{\mathcal{A}} . \tag{A.35}$$

Also, the other quantity that appears in the action simplifies:

$$\bar{X}(AX + B) + X\bar{B} = \mathcal{A} f \bar{f} \left[1 - 2\bar{\mathcal{A}} \right] - \frac{\chi^2}{2f} \bar{B} . \tag{A.36}$$

Observe that

$$f \bar{f} \mathcal{A} = \frac{\bar{\chi}^2}{2\bar{f}} A \frac{\chi^2}{2f} - \frac{\bar{\chi}^2}{2\bar{f}} B , \tag{A.37}$$

where the first term is real up to a total derivative, such that the expression (A.36) leads to a real action. We can write the whole Lagrangian (3.7) as

$$\begin{aligned} e^{-1}\mathcal{L} &= f\bar{f} \left(-1 + \mathcal{A} - \mathcal{A}\bar{\mathcal{A}} \right) - \frac{\chi^2}{2f} \bar{B} + C \\ &= -f\bar{f} + \frac{\bar{\chi}^2}{2\bar{f}} A \frac{\chi^2}{2f} - \left(\frac{\chi^2}{2f} \bar{B} + \frac{\bar{\chi}^2}{2\bar{f}} B \right) + C \\ &\quad - \frac{\chi^2 \bar{\chi}^2}{16(f\bar{f})^2} \left(f^{-1} \Box \chi^2 - 2B \right) \left(\bar{f}^{-1} \Box \bar{\chi}^2 - 2\bar{B} \right) . \end{aligned} \tag{A.38}$$

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