One-loop anomalous dimension of top-partner hyperbaryons in a family of composite Higgs models

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One-loop anomalous dimension of top-partner hyperbaryons in a family of composite Higgs models

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An important ingredient of many composite-Higgs scenarios is partial compositeness: a mass for the top quark is generated via mixing with baryonic operators of the same new strong dynamics that produces the composite Higgs particle. These baryonic operators are scale dependent. We construct these operators, and calculate their one-loop anomalous dimension, in a set of ultraviolet completions of the Standard Model proposed by Ferretti and Karateev.

I. INTRODUCTION

A common theme in the phenomenology of the electroweak interaction is that the Standard Model (SM) emerges as a low energy effective theory of some high scale dynamics. That dynamics could be strongly interacting, with the Higgs boson appearing as a composite Nambu-Goldstone boson (NGB). The strongly coupled sector is presumed to have a global symmetry group G which spontaneously breaks to a subgroup H that, in turn, can accommodate the electroweak symmetries $\mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$. Turning on electroweak interactions with the NGBs and, in addition, allowing them to interact with the top quark converts some of the NGBs into pseudo Nambu-Goldstone bosons (pNGBs). The NGB manifold G/H contains a field with the quantum numbers of the SM Higgs, which then develops an expectation value, thereby inducing electroweak symmetry breaking. These models are generically known as "composite Higgs models." For reviews that reflect the evolution of this field see [1–4].

We will call the new strong dynamics hypercolor, to distinguish it from the classic technicolor scenario, and denote by Λ_{HC} its scale. Λ_{HC} must be larger than the electroweak scale of 245 GeV, and phenomenological expectations place it in the range of 1 to few TeV. The SM Higgs also gives masses to fermions. In all composite Higgs extensions of the SM we are familiar with, this occurs through higher dimensional operators, so that the new interactions are (formally) irrelevant. The most important of them are dimension-six, four-fermion operators. They are associated with a yet higher energy scale Λ_{EHC} (for Extended hypercolor), at which, presumably, another gauge symmetry undergoes spontaneous symmetry breaking, much like the exchange of $W^{\pm}$ and Z bosons manifests itself at the hadronic scale through effective four-fermion operators.

According to one possible mechanism for generating fermion masses, called "partial compositeness," an elementary SM fermion couples linearly to strong-sector baryons, which, in effect, allows it to couple to the composite Higgs as well [5]. Mass eigenstates are then linear superpositions of SM fermions and hyperbaryons. Since it is the only fermion whose mass is comparable to the electroweak scale, the top quark plays a special role. Indeed, in the composite-Higgs models of Ref. [6] that we will study in this paper, the top's contribution to the Higgs effective potential is crucial in order to generate a negative curvature at the origin, and, thus, a non-zero expectation value for the Higgs field [7, 8].

Schematically, each four-fermion interaction is given by

$$\mathcal{L}(\mu) = \frac{\lambda(\mu)\overline{q}B + \lambda^*(\mu)\overline{B}q}{\Lambda^2_{EHC}} \ , \tag{1.1}$$

where $q, \overline{q}$ are SM fermions, while $B, \overline{B}$ are interpolating fields for some three-fermion bound states of the hypercolor theory. The coupling $\lambda(\mu)$ is dimensionless, and μ is the scale at which the interaction is probed. We presume that $\mathcal{L}(\mu)$ was induced by the exchanged of superheavy gauge bosons with mass $\sim \Lambda_{EHC} > \Lambda_{HC}$, so that, when probed at that scale, $\lambda(\Lambda_{EHC})$ would be equal to the corresponding gauge coupling squared, up to some $O(1)$ number.

The basic tension confronting such scenarios is that, generically, the EHC interaction will also induce four-fermion operators $\sim \overline{q}q\overline{q}q$, that involve only SM fermions. Flavor physics constraints on such interactions can be quite severe. Depending on exactly what is assumed about the EHC theory, meeting experimental bounds may require the scale Λ_{EHC} to be much larger than Λ_{HC}. Then the question becomes, can the observed fermion masses, and, in particular, the top-quark mass, be generated by interactions such as (1.1)? In order to determine $\lambda(\Lambda_{HC})$, the strength of the interaction (1.1) at the hypercolor scale, we must track the renormalization-group evolution of the hyperbaryon operator B from Λ_{EHC} down to Λ_{HC}. This evolution is driven by the same hypercolor interaction that holds together the constituents of the hyperbaryon. We will assume that all the SM gauge interactions, including QCD, are sufficiently weak at the relevant scales so that their contribution to the renormalization-group flow can be neglected.

So finally, we come to the point of this short note: first, to write down a set of candidate dimension-six operators which couple the top quark to hyperbaryons, and then to compute their anomalous dimensions in the hypercolor theory to one loop.

Which hyperbaryon operators can couple linearly to the top quark, and how they evolve, are questions that can only be addressed within a concrete hypercolor model. In this paper we will study a family of models proposed by Ferretti and Karateev [6]. The requirements underlying their classification include:

- The hypercolor gauge group is simple and asymptotically free.

- There are no gauge anomalies.

- The unbroken global symmetry H is such that

$$H \supset \mathrm{SU}(3)_{\mathrm{color}} \times \mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_X \qquad (1.2)$$
$$\supset \mathrm{SU}(3)_{\mathrm{color}} \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \ ,$$

  with the gauge group of the SM in the last line. The group $\mathrm{SU}(2)_R$ is the familiar custodial symmetry of the SM. Hypercharge is $Y = T_R^3 + X$.

- The coset G/H contains a field with the quantum numbers of the SM Higgs.

- There exist hypercolor baryons with quantum numbers that allow them to couple linearly to SM fermions.

In these models, the Higgs field is identified with pNBGs that originate from the condensation of fermions in a real or pseudoreal irreducible representation (*irrep*) of the hypercolor gauge group. The classification of symmetry breaking patterns was given long ago in Ref. [9, 10]. In the case of a real *irrep*, the minimal number of Weyl (or Majorana) fermions needed to satisfy all requirements is 5. The symmetry-breaking manifold is then SU(5)/SO(5), with the Higgs field taking up 4 out of a total of 14 pNGBs. For a pseudoreal *irrep*, 4 Weyl fermions are needed. The coset is SU(4)/Sp(4), with just 5 pNGBs altogether, where again 4 of them constitute the Higgs field.

In order to have hyperbaryons that can couple linearly to the top quark, fermions in at least one more *irrep* of the hypercolor gauge group are needed. A list of models that satisfy all these requirements was given in Ref. [6]. One model (argued to be the preferred one by Ferretti in Ref. [7]) has an SU(4) gauge group, 5 Majorana fermions in the two-index antisymmetric (as2) representation, and 3 fundamental representation Dirac fermions. Other candidates include SO(d) gauge fields and a mix of vector and spinor representations. (Other proposals for ultraviolet completions which we are aware of include Sp($2N$) gauge groups and two representations of fermions [11] and SU(3) with many fundamentals [12].) Each model is minimal in the sense that, if any fermions are removed, then it would not be possible to satisfy some of the requirements listed above. Of course, additional fermions could be added that do not play any direct role in meeting the above requirements. The main effect of such additional matter would be to slow down the running of the hypercolor coupling.

With only the minimal fermion content listed in Ref. [6], many of the models appear to be QCD-like: their beta function is not small, and they are not expected to exhibit near-conformality.

(This statement can be justified by looking at the two lowest-order coefficients of the beta function. QCD-like behavior is expected when they are both positive, in the conventions of Eqs. (1.4) and (A4). For the SU(4) model, we can go further by invoking large-N scaling as an appropriate way to compare models. We do this by scaling the lowest order coefficients, dividing by N and N^2, b_1/N and b_2/N^2. They would compose the beta function for the 't Hooft coupling g^2N. The SU(4) Ferretti-Karateev model has $b_1/4 = 7/3$, $b_2/16 = 461/192 \simeq 2.4$. In contrast, SU(3) gauge theory with $N_f = 6$ fundamental Dirac fermions has $b_1/3 = 7/3$, $b_2/9 = 26/9 \simeq 2.9$, equal running at one loop and slightly faster running at two loops.)

In the one-loop approximation, the enhancement (or suppression) factor resulting from running from a high scale μ down to a smaller scale μ' is then

$$\frac{\lambda(\mu')}{\lambda(\mu)} = \left(\frac{g^2(\mu')}{g^2(\mu)}\right)^{\frac{\gamma_1}{2b_1}} = \left(\frac{\log(\mu/\Lambda_{HC})}{\log(\mu'/\Lambda_{HC})}\right)^{\frac{\gamma_1}{2b_1}} , \tag{1.3}$$

where now Λ_{HC} is the scale occurring the solution of the beta function[1]

$$\beta(g^2) = \mu\frac{\partial g^2}{\partial \mu} = -\frac{g^4}{16\pi^2}\, b_1 - \frac{g^6}{(16\pi^2)^2}\, b_2 + \cdots , \tag{1.4}$$

within the one-loop approximation, and the anomalous dimension is

$$\gamma(g) = \frac{\mu}{Z_B}\frac{\partial Z_B}{\partial \mu} = -\frac{g^2}{16\pi^2}\,\gamma_1 + \cdots , \tag{1.5}$$

where Z_B is the wave-function renormalization factor for the hyperbaryon B. We use similar conventions for the β and γ functions. A negative value for γ (or a positive one-loop coefficient γ_1), means that the coupling $\lambda(\mu')$ grows when μ' is decreased. For the running from Λ_{EHC} down to Λ_{HC} we take $\mu \approx \Lambda_{EHC}$ and $\mu' \approx \Lambda_{HC}$, leading to

$$\frac{\lambda(\Lambda_{HC})}{\lambda(\Lambda_{EHC})} \approx \log^{\frac{\gamma_1}{2b_1}}(\Lambda_{EHC}/\Lambda_{HC}) . \tag{1.6}$$

As mentioned above, additional matter could be added to the model to slow down the running of the hypercolor coupling. Suppose that, all the way from Λ_{EHC} to Λ_{HC}, the hypercolor coupling was approximately constant and equal to g_*. Then, instead of Eq. (1.6), running would give a power law enhancement

$$\frac{\lambda(\Lambda_{HC})}{\lambda(\Lambda_{EHC})} \approx \left(\frac{\Lambda_{EHC}}{\Lambda_{HC}}\right)^{-\gamma(g_*)} , \tag{1.7}$$

[1] See Eq. (A4) for the general form of the one- and two-loop coefficients.

or, if the one loop formula were valid,

$$\frac{\lambda(\Lambda_{HC})}{\lambda(\Lambda_{EHC})} \approx \left(\frac{\Lambda_{EHC}}{\Lambda_{HC}}\right)^{\gamma_1 g_*^2/(16\pi^2)} , \tag{1.8}$$

potentially producing a much bigger effect.

In this paper we construct the top-partner hyperbaryon operators for some of the models of Ref. [6], and compute their one-loop anomalous dimension. In Sec. II we discuss the SU(4) model, and in Sec. III we extend the discussion to models based on an SO(d) gauge group. We discuss our results in Sec. IV. Technical details are relegated to appendices.

The technology needed for this calculation was originally developed for the study of proton decay in Grand Unified theories [13]. A comprehensive review may be found in Ref. [14].

II. RESULTS FOR THE SU(4) MODEL

We review the construction of top-partner hyperbaryon operators, following Refs. [7, 8]. The hypercolor gauge group is SU(4). The fermion content is the following: We have 5 Majorana fermions χ_i, $i = 1, \ldots, 5$, in the real, two-index antisymmetric *irrep*, and three fundamental representation Dirac fermions.

The Majorana field χ can be written in terms of a right-handed Weyl fermion Υ as

$$\chi_{ABi} = \begin{pmatrix} \Upsilon_{ABi} \\ \frac{1}{2}\epsilon_{ABCD}\,\epsilon\,\bar{\Upsilon}^T_{CDi} \end{pmatrix} , \tag{2.1a}$$

$$\overline{\chi}_{ABi} = \frac{1}{2}\epsilon_{ABCD}\chi^T_{CDi}C = \left(-\tfrac{1}{2}\epsilon_{ABCD}\Upsilon^T_{CDi}\epsilon \;\; \bar{\Upsilon}_{ABi}\right) . \tag{2.1b}$$

We use capital letters for the SU(4) hypercolor indices. Multiple indices of a single object will always be fully antisymmetrized. We have suppressed spinor indices. C is the charge-conjugation matrix, $\epsilon = i\sigma_2$ is the two-dimensional ϵ-tensor acting on the Weyl spinor index, and the superscript T denotes the transpose in spinor space.

The three Dirac fermions ψ_a, $a = 1, 2, 3$, are in the fundamental representation. ψ_a can be written in terms of two right-handed Weyl fermions, Ψ_a in the fundamental *irrep* and $\tilde{\Psi}_a$ in the anti-fundamental, as

$$\psi_{Aa} = \begin{pmatrix} \Psi_{Aa} \\ \epsilon\,\bar{\tilde{\Psi}}^T_{Aa} \end{pmatrix} , \qquad \overline{\psi}_{Aa} = \left(-\tilde{\Psi}^T_{Aa}\,\epsilon\;\bar{\Psi}_{Aa}\right) . \tag{2.2}$$

	$SU(5)$	$SU(3)\times SU(3)'$	$SU(3)_c$	$U(1)_X$	$U(1)'$	
$\Upsilon(\Psi\Psi)$	$\mathbf{5}$	$(\overline{\mathbf{3}},\mathbf{1})\times(\overline{\mathbf{3}},\mathbf{1})\to(\mathbf{3},\mathbf{1})$	$\mathbf{3}$	2/3	7/3	B_R
$\Upsilon(\bar{\tilde{\bar{\Psi}}}\bar{\tilde{\bar{\Psi}}})$	$\mathbf{5}$	$(\mathbf{1},\overline{\mathbf{3}})\times(\mathbf{1},\overline{\mathbf{3}})\to(\mathbf{1},\mathbf{3})$	$\mathbf{3}$	2/3	-13/3	B'_R
$\bar{\Upsilon}(\bar{\Psi}\bar{\Psi})$	$\overline{\mathbf{5}}$	$(\mathbf{3},\mathbf{1})\times(\mathbf{3},\mathbf{1})\to(\overline{\mathbf{3}},\mathbf{1})$	$\overline{\mathbf{3}}$	-2/3	-7/3	$\overline{B}_R$
$\bar{\Upsilon}(\tilde{\bar{\Psi}}\tilde{\bar{\Psi}})$	$\overline{\mathbf{5}}$	$(\mathbf{1},\mathbf{3})\times(\mathbf{1},\mathbf{3})\to(\mathbf{1},\overline{\mathbf{3}})$	$\overline{\mathbf{3}}$	-2/3	13/3	$\overline{B}'_R$
$\bar{\Upsilon}(\Psi\Psi)$	$\overline{\mathbf{5}}$	$(\overline{\mathbf{3}},\mathbf{1})\times(\overline{\mathbf{3}},\mathbf{1})\to(\mathbf{3},\mathbf{1})$	$\mathbf{3}$	2/3	13/3	B_L
$\bar{\Upsilon}(\bar{\tilde{\bar{\Psi}}}\bar{\tilde{\bar{\Psi}}})$	$\overline{\mathbf{5}}$	$(\mathbf{1},\overline{\mathbf{3}})\times(\mathbf{1},\overline{\mathbf{3}})\to(\mathbf{1},\mathbf{3})$	$\mathbf{3}$	2/3	-7/3	B'_L
$\Upsilon(\bar{\Psi}\bar{\Psi})$	$\mathbf{5}$	$(\mathbf{3},\mathbf{1})\times(\mathbf{3},\mathbf{1})\to(\overline{\mathbf{3}},\mathbf{1})$	$\overline{\mathbf{3}}$	-2/3	-13/3	$\overline{B}_L$
$\Upsilon(\tilde{\bar{\Psi}}\tilde{\bar{\Psi}})$	$\mathbf{5}$	$(\mathbf{1},\mathbf{3})\times(\mathbf{1},\mathbf{3})\to(\mathbf{1},\overline{\mathbf{3}})$	$\overline{\mathbf{3}}$	-2/3	7/3	$\overline{B}'_L$

TABLE 1: Local hyperbaryon operators of the SU(4) model. The leftmost column gives the Weyl-fermion content, and the rightmost column the notation used for the operator. The remaining columns list the quantum numbers.

The global symmetry is

$$G = \mathrm{SU}(5)\times\mathrm{SU}(3)\times\mathrm{SU}(3)'\times\mathrm{U}(1)_X\times\mathrm{U}(1)'\ , \tag{2.3}$$

with quantum numbers $(\mathbf{5},\mathbf{1},\mathbf{1})_{(0,-1)}$ for Υ; $(\mathbf{1},\overline{\mathbf{3}},\mathbf{1})_{(1/3,5/3)}$ for Ψ; and $(\mathbf{1},\mathbf{1},\mathbf{3})_{(-1/3,5/3)}$ for $\tilde{\Psi}$. The symmetry-breaking pattern is $G\to H$ with

$$H \;=\; \mathrm{SU}(3)_{\mathrm{color}}\times\mathrm{SO}(5)\times\mathrm{U}(1)_X\ .$$

The unbroken group H satisfies Eq. (1.2), with $\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\subset\mathrm{SO}(5)$. The symmetry breaking $\mathrm{SU}(5)\to\mathrm{SO}(5)$ is induced by the Majorana-fermion condensate $\langle\overline{\chi}_i\chi_j\rangle\propto\delta_{ij}$. Like in QCD, the breaking $\mathrm{SU}(3)\times\mathrm{SU}(3)'\to\mathrm{SU}(3)_{\mathrm{color}}$, where $\mathrm{SU}(3)_{\mathrm{color}}$ is the diagonal subgroup, is induced by the condensate $\langle\overline{\psi}_a\psi_b\rangle\propto\delta_{ab}$. Both condensates also break $\mathrm{U}(1)'$.

Let us now discuss the hyperbaryon operators. We first require that the only source of explicit breaking of $\mathrm{SU}(3)\times\mathrm{SU}(3)'$ to $\mathrm{SU}(3)_{\mathrm{color}}$ will be the QCD interactions. This implies that the hyperbaryon must contain two same-type fundamental fermions, e.g., $\Psi\Psi$ or $\bar{\tilde{\bar{\Psi}}}\bar{\tilde{\bar{\Psi}}}$, but not $\Psi\bar{\tilde{\bar{\Psi}}}$. The quantum numbers of the allowed hyperbaryon operators are listed in

Table 1. The explicit form of the unprimed operators in Table 1 is

$$
\begin{aligned}
B_{Ria} &= -\frac{1}{2}\epsilon_{ABCD}\epsilon_{abc}\, P_R\, \chi_{ABi}\, \left(\psi^T_{Cb}\, C P_R\, \psi_{Dc}\right) && \text{(2.4a)}\\
&= \frac{1}{2}\epsilon_{ABCD}\epsilon_{abc}\, \Upsilon_{ABi}\, \left(\Psi^T_{Cb}\, \epsilon\, \Psi_{Dc}\right) \ ,\\
\overline{B}_{Ria} &= \frac{1}{2}\epsilon_{ABCD}\epsilon_{abc}\, \overline{\chi}_{ABi} P_L\, \left(\overline{\psi}_{Cb}\, C P_L\, \overline{\psi}^T_{Dc}\right) && \text{(2.4b)}\\
&= \frac{1}{2}\epsilon_{ABCD}\epsilon_{abc}\, \bar{\Upsilon}_{ABi}\, \left(\bar{\Psi}_{Cb}\, \epsilon\, \bar{\Psi}^T_{Dc}\right) \ ,\\
B_{Lia} &= -\frac{1}{2}\epsilon_{ABCD}\epsilon_{abc}\, P_L \chi_{ABi}\, \left(\psi^T_{Cb}\, C P_R\, \psi_{Dc}\right) && \text{(2.4c)}\\
&= \epsilon_{abc}\, \epsilon\, \bar{\Upsilon}^T_{ABi}\, \left(\Psi^T_{Ab}\, \epsilon\, \Psi_{Bc}\right) \ ,\\
\overline{B}_{Lia} &= \frac{1}{2}\epsilon_{ABCD}\epsilon_{abc}\, \overline{\chi}_{ABi} P_R\, \left(\overline{\psi}_{Cb}\, C P_L\, \overline{\psi}^T_{Dc}\right) && \text{(2.4d)}\\
&= \epsilon_{abc}\, \Upsilon^T_{ABi}\, \epsilon\, \left(\bar{\Psi}_{Ab}\, \epsilon\, \bar{\Psi}^T_{Bc}\right) \ . && \text{(2.4e)}
\end{aligned}
$$

The chiral projectors are $P_R = (1+\gamma_5)/2$ and $P_L = (1-\gamma_5)/2$. The primed operators are obtained from Eq. (2.4) by interchanging $P_R \leftrightarrow P_L$ inside the $\psi\psi$ and $\overline{\psi\psi}$ bilinears. The CP transformation acts as

$$
\psi \to \gamma_2 \overline{\psi}^T \ , \qquad \overline{\psi} \to \psi^T \gamma_2 \ , \tag{2.5}
$$

for both Dirac and Majorana fermions. The sign choices of Eqs. (2.4a)- (2.4d) imply that the hyperbaryon operators will transform in the same way, and eventually, that the 4-Fermi lagrangian that couples the SM fermions to the hypercolor baryons will be CP-invariant.

At the one-loop level we encounter the three one-particle irreducible diagrams shown in Fig. 1. The double line represents the sextet Majorana fermion, while the single lines represent the two fundamental Dirac fermions. In all cases studied in this paper the matrix of wave-function renormalization factors is diagonal. The Z-factor of a given hyperbaryon operator can be expressed as

$$
Z = 1 + \frac{g^2}{64\pi^2\epsilon}\Big(\mathcal{G}(a)\mathcal{D}(a) + \mathcal{G}(b,c)\mathcal{D}(b+c) - \sum_{j=1}^{3} C_2(j)\Big) \ , \tag{2.6}
$$

where $\epsilon = 2 - d/2$. With Eq. (1.5) and $\beta(g^2,\epsilon) = -2\epsilon g^2 + \beta(g^2)$, the one-loop anomalous dimension γ_1 is half the expression inside the parentheses on the right-hand side.

In Eq. (2.6), the summation in last term accounts for the wave-function renormalization factors of the external legs, here given by $2C_2(\text{fund}) + C_2(\text{sextet})$. After separating out a common factor arising from the logarithmically divergent momentum integral, the contribution of each one-particle irreducible graph can be expressed as the product of a group

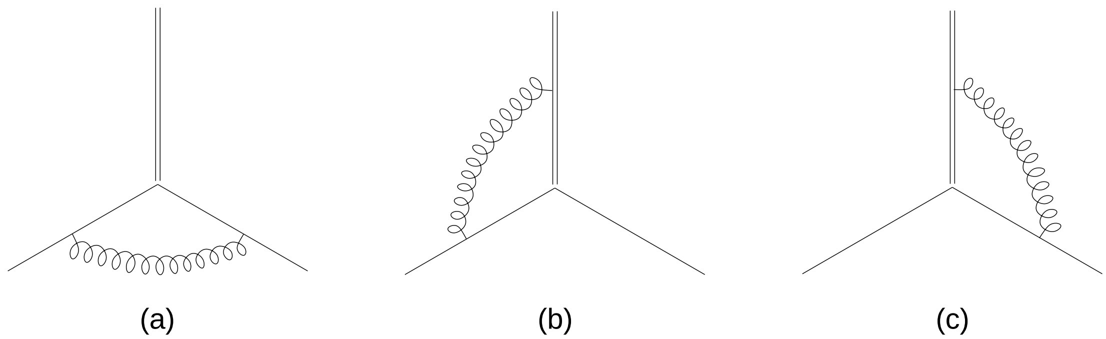

FIG. 1: One-loop diagrams. For the SU(4) model, the double line represents a sextet (as2) fermion, which carries the open spin index of the hyperbaryon. The other two lines are fundamental-*irrep* fermions. For SO(d) models, the double line belongs to the vector *irrep*, while the other fermion lines belong to a (chiral) spinor *irrep*.

theoretical factor $\mathcal{G}$ and a Dirac-algebra factor $\mathcal{D}$. Fig. 1(a) gives rise to the term $\mathcal{G}(a)\mathcal{D}(a)$. Figs. 1(b) and (c) share a common group theoretical factor that we denote as $\mathcal{G}(b,c)$, and their combined contribution is given by $\mathcal{G}(b,c)\mathcal{D}(b+c)$, where $\mathcal{D}(b+c)$ sums up the Dirac factors from both graphs. The necessary SU(N) group theory is summarized in App. A. Details on the calculation of the Dirac factors are listed in App. C. The resulting factors are summarized in Tables 2 and 3.

The operators in Table 1 all share the Dirac tensor structure $P_1 \otimes P_2$, where P_1 and P_2 can each be P_L or P_R. The first projector P_1 is sandwiched between the two fundamental fermions: either between $\psi^T C$ and ψ, or between $\overline{\psi}$ and $C\overline{\psi}^T$, while P_2 multiplies the open fermion index of the Majorana fermion χ. The Dirac factors $\mathcal{D}(a)$ and $\mathcal{D}(b+c)$ do not depend on the choice of the chiral projectors. Thus, for all the operators in Table 1 we obtain

$$\gamma_1(P_1 \otimes P_2) = \frac{15}{4} , \tag{2.7}$$

where we have indicated the common Dirac tensor structure of the hyperbaryons.

If we allow $\mathrm{SU}(3) \times \mathrm{SU}(3)'$ to be broken explicitly to $\mathrm{SU}(3)_{\mathrm{color}}$ by the SM–hypercolor lagrangian, we may also consider hyperbaryons where the two fundamental fermions are mixed, *e.g.*, $\bar{\bar{\Psi}}\Psi$. This allows for two more operators

$$B^{mix}_{L,R} \propto \epsilon_{ABCD}\epsilon_{abc}\, P_{L,R}\gamma_\mu\, \chi_{ABi}\, \left(\psi^T_{Cb}\, C\gamma_\mu\gamma_5\, \psi_{Dc}\right) . \tag{2.8}$$

	$P_1 \otimes P_2$	$P_1\gamma_\mu \otimes P_2\gamma_\mu$	$\sigma_{\mu\nu} \otimes \sigma_{\mu\nu}$
(a)	16	4	0
(b+c)	8	20	24

TABLE 2: Dirac factors arising from diagram (a) and from the sum of diagrams (b) and (c), for the various possible tensor structures that form the hyperbaryon. P_1 and P_2 can each be one of the chiral projectors $P_{L,R}$.

The color factors are the same as before, while the Dirac factors are shown in the $\gamma_\mu \otimes \gamma_\mu$ column of Table 2. The resulting one-loop anomalous dimension is

$$\gamma^{(0)}(P_1\gamma_\mu \otimes P_2\gamma_\mu) = \frac{15}{2} \, , \tag{2.9}$$

which is twice as big as that of Eq. (2.7).

Since the spin wave function of the fundamental fermions must be antisymmetric, the symmetry properties under charge conjugation (Eq. (C1)) imply that we cannot use $\psi^T C\gamma_\mu \psi$ nor $\psi^T C\sigma_{\mu\nu} \psi$.

III. RESULTS FOR SO(d) MODELS

In this section we study models whose gauge symmetry is the orthogonal group SO(d). We will mainly focus on even values of d. These models, which are discussed in Sec. III A, correspond to the $p = 3$ case of Ref. [6]. We briefly discuss the $p = 2$ models [6] where the gauge group is SO(d) with d odd in Sec. III B, and "flipped" models where the role of the vector and spinor *irreps* is interchanged in Sec. III C. The calculation of the one-loop anomalous dimension for all these models is summarized in Sec. III D. For convenience, we review in App. B the needed SO(d) group theory, focusing mainly on the spinor *irreps*. This Appendix is a bit long, but it turns out that determining the charge conjugation properties of our states is not completely straightforward until we look at the explicit form of the Dirac matrices.

	SU(4)	SO(d)
(a)	$-\frac{5}{8}$	$-\frac{(d-2)^2-d}{8}$
(b,c)	$-\frac{5}{4}$	$-\frac{d-1}{2}$

TABLE 3: Hypercolor factors arising from diagram (a) and from diagram (b) or (c). For some more details on the SU(4) calculation, see App. A. For the SO(d) case, see App. B 3.

A. SO($d = 2n$)

Here we consider SO(d) models with $d = 6, 8, 10, 12, 14$. The $d = 6$, 8 and 10 models are QCD-like in that b_1 and b_2 are both positive. For $d \geq 16$ the hypercolor theory loses asymptotic freedom. The $d = 12$ and 14 models have Banks-Zaks fixed points. Notice, however, that the number of additional triplets of ordinary color grows with d, and that QCD loses asymptotic freedom already for $d \geq 10$.

In all models, the fermion content includes 5 Majorana fermions χ_i in the vector *irrep*, which, like the sextet fermions of SU(4), produce the coset SU(5)/SO(5) after chiral symmetry breaking. Instead of the SU(4) fundamental *irrep* fermions we will now have two color triplets of Weyl fermions each belonging to some chiral spinor *irrep*. The smallest gauge group SO(6) is isomorphic to SU(4). Hence, this case provides a cross check of the calculation.

The gauge singlets that form the hyperbaryons are always constructed from one SO(d) vector, and from two SO(d) spinors (that also carry the ordinary color). As we will see, the details of how a vector is formed from two spinors vary. Our SO(d) notation is explained in App. B, and the results pertaining to the Dirac algebra and charge conjugation symmetry in $d = 2n$ dimensions are collected in App. B 1. We are guided by Table 7, where we have tabulated the sign factors that occur in some key relations, which, in turn, will influence the construction. We will have to distinguish several cases. First, for $d = 4n + 2$ the chiral spinor *irreps* are complex conjugate. We denote the chiral *irrep* projected by $\mathcal{P}_+$ and $\mathcal{P}_-$ (Eq. (B12)) as $\mathbf{s}$ and $\overline{\mathbf{s}}$ respectively. We will have to further distinguish between $d = 8n + 2$ and $d = 8n+6$. The last case is $d = 4n$ where the chiral spinor *irreps* are real (for $d = 8n+4$) or pseudoreal (for $d = 8n$). The *irrep* projected by $\mathcal{P}_\pm$ is denoted $\mathbf{s}_\pm$ in this case.

1. $\boldsymbol{d = 8n + 6}$

We start with the $d = 8n + 6$ case, remembering that SO(6) should reproduce the SU(4) model of Sec. II. The same construction that works for SO(6) will work also for SO(14). The complex-*irrep* Weyl fermions are $\Psi_{a\alpha}$, transforming as $(\mathbf{s}, \overline{\mathbf{3}}, \mathbf{1})$ under SO$(d)\times$SU(3)$\times$SU(3)$'$, and $\tilde{\Psi}_{a\alpha}$, transforming as $(\overline{\mathbf{s}}, \mathbf{1}, \mathbf{3})$. We use $\alpha, \beta, \ldots = 1, 2$, for the 4-dimensional Weyl index, and, as before, $a, b, \ldots = 1, 2, 3$, for triplets of SU(3) or SU(3)$'$.

Performing the SO(d) contractions requires some care, because the spinor conventions which are customary when SO(d) is the spacetime symmetry are different from those we will prefer when SO(d) is an internal (gauge) symmetry. We start by grouping the chiral spinors into SO(d) Dirac spinors,

$$\zeta = \begin{pmatrix} \Psi \\ \tilde{\Psi} \end{pmatrix} , \qquad \overline{\zeta} = \begin{pmatrix} \bar{\Psi} & \bar{\tilde{\Psi}} \end{pmatrix} . \tag{3.1}$$

These objects transform as (see Eq. (B10))

$$\delta\zeta = \Sigma_{ij}\zeta \ , \qquad \delta\overline{\zeta} = -\overline{\zeta}\,\Sigma_{ij} \ . \tag{3.2}$$

Of course, this implies that $\zeta, \overline{\zeta}$ span a *reducible* representation of $SO(d)$. In terms of the irreducible chiral representations, this is equivalent to

$$\delta\Psi = (\Sigma_{ij})_{++}\Psi \ , \qquad \delta\bar{\Psi} = -\bar{\Psi}(\Sigma_{ij})_{++} \ , \tag{3.3}$$

$$\delta\tilde{\Psi} = (\Sigma_{ij})_{--}\tilde{\Psi} \ , \qquad \delta\bar{\tilde{\Psi}} = -\bar{\tilde{\Psi}}(\Sigma_{ij})_{--} \ , \tag{3.4}$$

where we are using the block notation of Eq. (B13). These look like conventional gauge transformation rules, with generators $(\Sigma_{ij})_{++}$ and $(\Sigma_{ij})_{--}$ for the $\mathbf{s}$ and $\overline{\mathbf{s}}$ *irreps* respectively. We furthermore observe that, thanks to the d-dimensional charge conjugation symmetry (B16), $\mathcal{C}\overline{\zeta}^T$ and ζ have the same SO(d) transformation properties, and the same is true for $\overline{\zeta}$ and $\zeta^T\mathcal{C}$.

With this information at hand, we may now construct bilinears that transform as an SO(d) vector, and which are constructed from two copies of the same chiral *irrep*. They are

$$\begin{aligned}
\zeta^T(\mathcal{C}\Gamma_i)_{++}\zeta \ &\Rightarrow \ \Psi^T_{b\alpha}(\mathcal{C}\Gamma_i)_{++}\Psi_{c\beta} \ , \\
\overline{\zeta}(\Gamma_i\mathcal{C})_{++}\overline{\zeta}^T \ &\Rightarrow \ \bar{\Psi}_{b\alpha}(\Gamma_i\mathcal{C})_{++}\bar{\Psi}^T_{c\beta} \ , \\
\zeta^T(\mathcal{C}\Gamma_i)_{--}\zeta \ &\Rightarrow \ \tilde{\Psi}^T_{b\alpha}(\mathcal{C}\Gamma_i)_{--}\tilde{\Psi}_{c\beta} \ , \\
\overline{\zeta}(\Gamma_i\mathcal{C})_{--}\overline{\zeta}^T \ &\Rightarrow \ \bar{\tilde{\Psi}}_{b\alpha}(\Gamma_i\mathcal{C})_{--}\bar{\tilde{\Psi}}^T_{c\beta} \ ,
\end{aligned} \tag{3.5}$$

where on the right-hand side we have exposed the chiral *irrep* content, and reintroduced the 4-dimensional Weyl index, as well as the triplet index of SU(3) or SU(3)′. According to Table 7, these products are antisymmetric on their SO(6) spinor indices, consistent with the contractions of the fundamental-*irrep* fermions in the SU(4) language.

Next, we deal with the 4-dimensional Dirac structure. To this end, we will group the Weyl fermions into ordinary Dirac fermions that belong to a specific *irrep*, **s**, of the SO(d) gauge group.[2] Once again using the SO(d) charge-conjugation symmetry, the Dirac fermions are (compare Eq. (2.2))

$$\psi = \begin{pmatrix} \Psi \\ \epsilon\, \mathcal{C}_{+-}\, \bar{\tilde{\Psi}}^T \end{pmatrix} , \qquad \overline{\psi} = \begin{pmatrix} -\tilde{\Psi}^T \epsilon\, \mathcal{C}_{-+} \;\; \bar{\Psi} \end{pmatrix} . \tag{3.6}$$

To be explicit, the SO(d) transformation rules are $\delta\psi = (\Sigma_{ij})_{++}\psi$, $\delta\overline{\psi} = -\overline{\psi}(\Sigma_{ij})_{++}$. Contracting the 4-dimensional fermion index into an as-yet unspecified Dirac matrix X, the bilinears of Eq. (3.5) become

$$\begin{aligned} &\epsilon_{abc}\, \psi_b^T (\mathcal{C}\Gamma_i)_{++} C P_R X P_R \psi_c \ , \\ &\epsilon_{abc}\, \overline{\psi}_b (\Gamma_i \mathcal{C})_{++} P_L X P_L C \overline{\psi}_c^T \ , \\ &\epsilon_{abc}\, \overline{\psi}_b (\Gamma_i \mathcal{C})_{++} P_R X P_R C \overline{\psi}_c^T \ , \\ &\epsilon_{abc}\, \psi_b^T (\mathcal{C}\Gamma_i)_{++} C P_L X P_L \psi_c \ , \end{aligned} \tag{3.7}$$

where we have performed the color contraction as well. The 4-dimensional Dirac matrix X must commute with γ_5. Moreover, the 4-dimensional spinor wave function must be antisymmetric, and consulting Eq. (C1), the only choice is $X = I$. As expected, after multiplying by χ_i to form an SO(d) singlet, this reproduces the hyperbaryons of Table 1 and Eq. (2.4).

As in Sec. II we may decide to relax the assumption that SU(3) and SU(3)′ are good symmetries separately, and insist only on SU(3)$_{\rm color}$. This allows us to replace $P_L X P_L$ or $P_R X P_R$ by $\gamma_\mu \gamma_5$ in Eq. (3.7).

[2] The 4-dimensional Dirac fermions are obviously different from the objects introduced in Eq. (3.1), which have components transforming as both **s** and $\bar{\mathbf{s}}$ under SO(d), and are Weyl fermions as far as their spacetime transformation is concerned.

2. $\boldsymbol{d = 8n + 2}$

This case is represented only by SO(10). The construction is similar, except that the products in Eq. (3.5) are now symmetric on the SO(10) spinor index. Hence, the 4-dimensional spin wave function must be symmetric as well. If we want $SU(3)$ and $SU(3)'$ to be good symmetries separately, the Dirac fermion bilinears we can use are again given by Eq. (3.7), but now we must take $X = \sigma_{\mu\nu}$. As before, relaxing this condition will allow for $X = \gamma_\mu$ as well. [This time, both $CP_R\gamma_\mu$ and $CP_L\gamma_\mu$ will be projected to $C\gamma_\mu$, which is the symmetric spin wave function.]

3. $\boldsymbol{d = 4n}$

As mentioned already, for SO($4n$) the chiral *irreps* are real or pseudoreal. This means that there would be no hypercolor gauge anomalies even if we use Weyl fermions that belong only to one of the chiral *irreps*. But, because now $\mathcal{C}$ commutes with $\mathcal{P}_\mp$, we have $(\mathcal{C}\Gamma_i)_{++} = (\mathcal{C}\Gamma_i)_{--} = 0$, as well as, as always, $(\Gamma_i)_{++} = (\Gamma_i)_{--} = 0$. A bilinear transforming as an SO($4n$) vector must therefore be constructed from one *irrep* of each chirality. It also follows that we cannot construct hyperbaryons that will respect $SU(3)$ and $SU(3)'$ separately. Insisting on $\mathrm{SU}(3)_{\mathrm{color}}$ only, we take $\Psi_{a\alpha}$ to be in $(\mathbf{s}_+, \overline{\mathbf{3}})$ of $\mathrm{SO}(d) \times \mathrm{SU}(3)_{\mathrm{color}}$, while $\tilde{\Psi}_{a\alpha}$ is in $(\mathbf{s}_-, \mathbf{3})$. The products that transform as an SO(d) vector and $\mathrm{SU}(3)_{\mathrm{color}}$ triplets are

$$\begin{aligned} &\epsilon_{abc}\bar{\tilde{\Psi}}_{b\alpha}(\Gamma_i)_{-+}\Psi_{c\beta} \ , \\ &\epsilon_{abc}\bar{\Psi}_{b\alpha}(\Gamma_i)_{+-}\tilde{\Psi}_{c\beta} \ . \end{aligned} \tag{3.8}$$

Just in order to be able to use the 4-dimensional Dirac matrices, we can trivially embed the Weyl fermions into 4-component fermions with one vanishing chirality,

$$\Psi \to \eta = P_R\eta \ , \quad \bar{\Psi} \to \overline{\eta} = \overline{\eta}P_L \ , \quad \tilde{\Psi} \to \tilde{\eta} = P_R\tilde{\eta} \ , \quad \bar{\tilde{\Psi}} \to \overline{\tilde{\eta}} = \overline{\tilde{\eta}}P_L \ , \tag{3.9}$$

so that Eq. (3.8) leads to

$$\begin{aligned} &\epsilon_{abc}\,\overline{\tilde{\eta}}_b(\Gamma_i)_{-+}P_L X P_R \eta_c \ , \\ &\epsilon_{abc}\,\overline{\eta}_b(\Gamma_i)_{+-}P_L X P_R \tilde{\eta}_c \ . \end{aligned} \tag{3.10}$$

Since we have two different chiral *irreps* there is no definite symmetry. Given the four-dimensional chiral projectors we take $X = \gamma_\mu$.

	d	$P_1 \otimes P_2$	$P_1\gamma_\mu \otimes P_2\gamma_\mu$	$\sigma_{\mu\nu} \otimes \sigma_{\mu\nu}$
$4n$	8,12	–	B	–
$8n+2$	10	–	B	A
$8n+6$	6,14	A	B	–

TABLE 4: Summary of SO($2n$) hyperbaryon properties. The first column shows the mod-8 family, and the next the relevant values of d. For each of the three Dirac tensor structures, the symbols have the following meaning: A or B indicate that the tensor structure is allowed for the given family, whereas a $-$ sign indicates that it is disallowed. For A, both SU(3) and SU(3)$'$ are good symmetries, whereas for B, only SU(3)$_{\text{color}}$ is a good symmetry. The disallowed cases refer to the minimal fermion content as described in the main text.

This completes the construction of the hyperbaryon operators for SO($2n$). For convenience, we summarize our findings in Table 4. As a final comment, we note that our analysis applies to the minimal fermion content of these models, and that adding more fermions will in general lead to more possibilities. For example, in the $d = 4n$ case, adding new fermions $\Psi'_{a\alpha}$ in $(\mathbf{s}_+, \mathbf{3})$ and $\tilde{\Psi}'_{a\alpha}$ in $(\mathbf{s}_-, \overline{\mathbf{3}})$ would allow for bilinears such as $\epsilon_{abc}\tilde{\Psi}^T_{b\alpha}(\mathcal{C}\Gamma_i)_{-+}\Psi'_{c\beta}$ and $\epsilon_{abc}\Psi^T_{b\alpha}(\mathcal{C}\Gamma_i)_{+-}\tilde{\Psi}'_{c\beta}$.

B. SO($d = 2n + 1$)

Some of the $p = 2$ cases of Ref. [6] make use of an SO(d) gauge group with d odd. In this case there are no chiral spinor *irreps*, and the Dirac-like *irrep* is real or pseudoreal. We take 6 Weyl fermions in the Dirac *irrep*, so that their global symmetry is SU(6). We embed SU(3)$_{\text{color}} \subset$ SU(3) $\times$ SU(3)$' \subset$ SU(6) by declaring that the first three copies, denoted $\Psi_{a\alpha}$, transform as $\mathbf{3}$ of SU(3), while the last three, denoted $\tilde{\Psi}_{a\alpha}$, transform as $\overline{\mathbf{3}}$ of SU(3)$'$. As before, we have the 5 Majorana fermions in the vector *irrep*.

When constructing the spinor bilinears we have to distinguish between $d = 4n + 1$ and $d = 4n + 3$. In the latter case, the bilinear $\epsilon_{abc}\Psi^T_{b\alpha}\mathcal{C}\Gamma_i\Psi_{c\beta}$ transforms as a d-dimensional vector, whereas in the former case this is true for $\epsilon_{abc}\Psi^T_{b\alpha}\Gamma_h\mathcal{C}\Gamma_i\Psi_{c\beta}$ where Γ_h is γ_5 or its generalization (see App. B 2). The hyperbaryon–SM coupling that uses these bilinears will break SU(6) explicitly to SU(3) × SU(3)′. In addition, we may use the bilinear $\epsilon_{abc}\bar{\bar{\Psi}}_{b\alpha}\Gamma_i\Psi_{c\beta}$ for any odd d, in which case only $SU(3)_{color}$ is preserved. We leave it to the reader to figure out which 4-dimensional Dirac structures are allowed by the symmetry properties in each case.

C. Flipped models

When the spinor *irrep* is real or pseudoreal, we may interchange the role of the vector and spinor *irreps*. For example, when the spinor *irrep* is real, we may use 5 copies of this *irrep* to generate the SU(5)/SO(5) coset. In this case, 6 copies of the vector *irrep* will be used, and the identification of ordinary color with a subgroup of SU(6) will be as in the previous subsection. When the spinor *irrep* is pseudoreal, the minimal number is 4 copies, which trades the SU(5)/SO(5) coset with SU(4)/Sp(4). Once again we leave the detailed construction of the hyperbaryons to the reader.

D. Calculation of γ_1

The one-loop calculation is very similar to the SU(4) case. The SO(d) Feynman rules are given in App. B 4. Again the Z-factor of the hyperbaryon takes the form of Eq. (2.6). This means that the anomalous dimension depends on which 4-dimensional Dirac tensor structure is used (see Table 2), but not on further details of the construction. The SO(d) group theoretic factors are of course different. For some details on their calculation, see App. B 3. The results for the one-particle irreducible graphs are given in the rightmost column of Table 3, and for the wave functions of the external legs in Table 6.

The usual conventions for $SU(N)$ and SO(d) (see Appendices A and B) are such that, for SO(6), all group theoretic factors are bigger by a factor 2 than those of SU(4). To account for this, the coupling constants are to be related according to $g[\mathrm{SU}(4)] = \sqrt{2}g[\mathrm{SO}(6)]$.

Using Eq. (2.6) and Tables 6, 2 and 3 we finally obtain the following one-loop anomalous dimension

$$\gamma_1 = \begin{cases} 3(d-1)(d/4-1)\ , & P_1 \otimes P_2\ , \\ 3(d-1)\ , & P_1\gamma_\mu \otimes P_2\gamma_\mu\ , \\ (d-1)(5-d/4)\ , & \sigma_{\mu\nu} \otimes \sigma_{\mu\nu}\ . \end{cases} \tag{3.11}$$

This result is valid for all the SO(d) hyperbaryons we have discussed.

IV. DISCUSSION

The results we have presented are entirely straightforward. Yet, to our knowledge, this is the first calculation of the scaling dimension of any partial compositeness operator, in a four dimensional, field theoretic completion of a Composite-Higgs scenario.

By far, the literature on Composite Higgs makes use of effective field theories. (Compare the discussion in [15–17] and in the review articles [1–4].) These can be higher-dimensional theories, which are not renormalizable; or they can be four-dimensional non-linear sigma models, generalizing the chiral lagrangian of QCD. The vast scope of this body of work shows that much can be learned from the effective field theory approach. Still, there are many questions that can only be answered within an ultraviolet completion. The values of anomalous dimensions are one example. Following Ref. [6, 7], we have taken the notion of an ultraviolet completion to mean an asymptotically-free gauge theory with fermionic matter. These hypercolor models contain a candidate composite Higgs field arising as a (pseudo) NGB from chiral symmetry breaking. In addition, they contain hyperbaryons that can serve as top partners, thereby allowing for the scenario of a partially composite top quark.

A key question for the viability of partial compositeness is whether the ultraviolet scale Λ_{EHC} introduced in Eq. (1.1) can be taken much higher than the hypercolor scale Λ_{HC} so that flavor constraints are satisfied, while, at the same time, the top-quark coupling to the hyperbaryon is sufficiently enhanced relative to its naive magnitude, so that a realistic top-quark mass emerges. Apart from noting that a large anomalous dimension is not in conflict with rigorous bounds, the effective field theory approach has nothing concrete to say about this question.

In order to establish scaling properties, a study of the dynamics of a concrete model is required. As a first step, we have constructed in this paper the candidate top-partner hyperbaryon operators in various hypercolor models, and calculated their one-loop anomalous dimension. Our main results are Eqs. (2.7), (2.9) and (3.11). The encouraging aspect of these results is that in essentially all cases that we studied, we find that quantum effects enhance, rather than suppress, the coupling between the top quark and the hyperbaryon.

However, quite clearly, many of the models of Ref. [6], with their minimal fermion content, would not give rise to nearly as much enhancement as one needs. (This includes the SU(4) (equivalently SO(6)) model, and the SO(8) model, the ones which maintain asymptotic freedom for QCD.) The reason is that it is almost certain that the running gauge coupling of these theories is QCD-like. In QCD, we enter the perturbative regime at about 0.1 Fermi. The running on any shorter distance scale is then dominated by one loop. Experience with QCD is that any further non-perturbative running from 0.1 Fermi to 1 Fermi would produce at most some $O(1)$ additional enhancement. In the hypercolor theory, the combined effect of one-loop running from Λ_{EHC} to, say, $10\Lambda_{HC}$, for which the enhancement factor (1.6) is logarithmic, plus some $O(1)$ additional enhancement for the range of $10\Lambda_{HC}$ to Λ_{HC}, would not come anywhere close to the total enhancement needed to overcome the power suppression arising from naive dimensional counting.

Additional fermions could be added to the minimal fermion content of the models of Ref. [6] for the purpose of slowing down the running. As long as the coupling is sufficiently weak, the relevant one-loop formula, Eq. (1.8), is expected to provide a reasonable approximation. Of course, when the coupling gets bigger, eventually perturbative calculations cannot be trusted any more, and only a full-fledged non-perturbative lattice calculation can determine the anomalous dimension.

Finally, we would like to comment that if it is assumed that the scale Λ_{EHC} is only involved in the mass generation for the top (or top and bottom) quarks, while some other physics is responsible for mass generation for the four lighter quarks and for the leptons, then the flavor constraints are significantly weaker. In this case the ratio $\Lambda_{EHC}/\Lambda_{HC}$ can be taken much smaller than is usually assumed. An important phenomenological constraint, which appears to be satisfied quite comfortably by the hypercolor models, comes from the $Z \to b\bar{b}$ decay [7, 18]. To our knowledge, a more systematic study of the flavor constraints of this scenario is not available to date.

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	D	T	C_2
fund	N	$\frac{1}{2}$	$\frac{N^2-1}{2N}$
adj	N^2-1	N	N
sym2	$\frac{N(N+1)}{2}$	$\frac{N+2}{2}$	$\frac{(N+2)(N-1)}{N}$
as2	$\frac{N(N-1)}{2}$	$\frac{N-2}{2}$	$\frac{(N-2)(N+1)}{N}$

TABLE 5: Dimensionality $D(r)$, quadratic Casimir $C_2(r)$, and trace $T(r)$ for some SU(N) representations: fundamental (fund), adjoint (adj), two-index symmetric (sym2), and two-index anti-symmetric (as2).

Appendix A: SU(N) groups

The group generators are hermitian, and satisfy the commutation relations $[T_a, T_b] = if_{abc}T_c$, where the structure constants f_{abc} are fully antisymmetric. Given an *irrep* r, the quadratic Casimir $C_2(r)$ is defined by $T_aT_a = C_2 I$ where I is the identity matrix, and the trace $T(r)$ by $\mathrm{tr}(T_aT_b) = T(r)\,\delta_{ab}$. These invariants are related by

$$\frac{C_2(r)}{T(r)} = \frac{D(\mathrm{adj})}{D(r)} \ , \tag{A1}$$

where $D(r)$ is the dimension of the representation. See Table 5 for their values for some *irreps*.

For the one-loop calculation we need the explicit form of the group generators. For the sextet (as2) *irrep*, the infinitesimal transformation is given by $\delta_a \chi_{AB} = i(T_a)_{AB}^{CD}\,\chi_{CD}$, with

$$(T_a)_{AB}^{CD} = \frac{1}{2}\left(\delta_A^C (T_a)_B^D - \delta_B^C (T_a)_A^D - \delta_A^D (T_a)_B^C + \delta_B^D (T_a)_A^C\right) \ . \tag{A2}$$

where $(T_a)_A^B$ are the usual generators in the fundamental *irrep*. We also need the closure relation satisfied by these generators,

$$(T_a)_A^B (T_a)_C^D = \frac{1}{2}\left(\delta_C^B \delta_A^D - \frac{1}{N}\delta_A^B \delta_C^D\right) \ . \tag{A3}$$

For completeness, we also record the general form of the one- and two-loop coefficients

	D	T	C_2
vec	d	2	$d-1$
adj	$\frac{d(d-1)}{2}$	$2(d-2)$	$2(d-2)$
chir	$2^{d/2-1}$	$2^{d/2-3}$	$\frac{d(d-1)}{8}$

TABLE 6: Dimensionality $D(r)$, quadratic Casimir $C_2(r)$, and trace $T(r)$ for some SO(d) representations: vector (vec), adjoint (adj), and chiral spinor (chir). The latter applies only when d is even.

of the beta function (for any gauge group),

$$b_1 = \frac{11}{3} C_2(\text{adj}) - \frac{4}{3} \sum_r N_f(r) T(r) , \tag{A4}$$
$$b_2 = \frac{34}{3} C_2^2(\text{adj}) - \sum_r N_f(r) T(r) \left(\frac{20}{3} C_2(\text{adj}) + 4C_2(r) \right) ,$$

where the sum is over the fermion representations, and $N_f(r)$ is the number of Dirac fermions in the *irrep* r (a Majorana or Weyl fermion counts as one half Dirac fermion).

Appendix B: SO(d) groups

We label the generators as M_{ij} where $i, j = 1, 2, \dots, d$, and $M_{ji} = -M_{ij}$. They are conventionally antihermitian, and satisfy the commutation relations

$$[M_{ij}, M_{k\ell}] = \delta_{i\ell} M_{jk} - \delta_{ik} M_{j\ell} - \delta_{j\ell} M_{ik} + \delta_{jk} M_{i\ell} . \tag{B1}$$

In the vector *irrep*, the SO(d) generators are

$$(M_{ij}^{\text{vec}})_{k\ell} = \delta_{ik}\delta_{j\ell} - \delta_{jk}\delta_{i\ell} . \tag{B2}$$

The definitions of the invariants $C_2(r)$ and $T(r)$ given in App. A may be applied to iM_{ij}, the hermitian version of the generators. See Table 6 for the invariants of some *irreps*.

The groups SU(4) and SO(6) are isomorphic. However, the conventional normalizations of SU(N) and SO(d) generators are such that every SO(6) group invariant is twice as big compared to the corresponding SU(4) invariant. As explained in Sec. III D, we make up for this by postulating that $g[\text{SU}(4)] = \sqrt{2} g[\text{SO}(6)]$.

1. Dirac algebra in $d = 2n$ dimensions

The (euclidean) Dirac algebra

$$\{\Gamma_i, \Gamma_j\} = 2\delta_{ij} \ , \qquad i, j = 1, 2, \ldots, d \ , \tag{B3}$$

is realized on $2^n \times 2^n$ matrices, where $n = d/2$. The generalization of γ_5 is defined as

$$\Gamma_h = \eta_h \Gamma_1 \Gamma_2 \cdots \Gamma_{2d} \tag{B4}$$

(the subscript h stands for "handedness"). The phase factor η_h is chosen such that Γ_h is hermitian and $\Gamma_h^2 = 1$.

Explicit construction. In order to verify various properties we will need, it is useful to have an explicit construction of the Dirac matrices as tensor products of the three Pauli matrices σ_a and the 2×2 identity matrix I. The iterative construction works slightly differently in $d = 4n$ and $d = 4n + 2$ dimensions. Starting with the $d = 4n$ sequence we have, for $d = 4$,

$$\begin{aligned}
\Gamma_1 &= \sigma_1 \otimes \sigma_2 \, , \\
\Gamma_2 &= \sigma_2 \otimes \sigma_2 \, , \\
\Gamma_3 &= \sigma_3 \otimes \sigma_2 \, , \\
\Gamma_4 &= I \otimes \sigma_1 \, , \\
\Gamma_h = \Gamma_5 &= I \otimes \sigma_3 \, .
\end{aligned} \tag{B5}$$

For $d = 8$,

$$\begin{aligned}
\Gamma_1 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_2 \, , \\
\Gamma_2 &= \sigma_2 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_2 \, , \\
\Gamma_3 &= \sigma_3 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_2 \, , \\
\Gamma_4 &= I \otimes \sigma_2 \otimes \sigma_1 \otimes \sigma_2 \, , \\
\Gamma_5 &= I \otimes \sigma_3 \otimes \sigma_1 \otimes \sigma_2 \, , \\
\Gamma_6 &= I \otimes I \otimes \sigma_2 \otimes \sigma_2 \, , \\
\Gamma_7 &= I \otimes I \otimes \sigma_3 \otimes \sigma_2 \, , \\
\Gamma_8 &= I \otimes I \otimes I \otimes \sigma_1 \, , \\
\Gamma_h = \Gamma_9 &= I \otimes I \otimes I \otimes \sigma_3 \, .
\end{aligned} \tag{B6}$$

For the $d = 4n + 2$ sequence we start with $d = 2$,

$$\begin{aligned} \Gamma_1 &= \sigma_1 \,, \\ \Gamma_2 &= \sigma_2 \,, \\ \Gamma_h = \Gamma_3 &= \sigma_3 \,. \end{aligned} \tag{B7}$$

Next, $d = 6$,

$$\begin{aligned} \Gamma_1 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \,, \\ \Gamma_2 &= \sigma_2 \otimes \sigma_1 \otimes \sigma_1 \,, \\ \Gamma_3 &= \sigma_3 \otimes \sigma_1 \otimes \sigma_1 \,, \\ \Gamma_4 &= I \otimes \sigma_2 \otimes \sigma_1 \,, \\ \Gamma_5 &= I \otimes \sigma_3 \otimes \sigma_1 \,, \\ \Gamma_6 &= I \otimes I \otimes \sigma_2 \,, \\ \Gamma_h = \Gamma_7 &= I \otimes I \otimes \sigma_3 \,, \end{aligned} \tag{B8}$$

and $d = 10$,

$$\begin{aligned} \Gamma_1 &= \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \,, \\ \Gamma_2 &= \sigma_2 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \,, \\ \Gamma_3 &= \sigma_3 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \,, \\ \Gamma_4 &= I \otimes \sigma_2 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \,, \\ \Gamma_5 &= I \otimes \sigma_3 \otimes \sigma_1 \otimes \sigma_1 \otimes \sigma_1 \,, \\ \Gamma_6 &= I \otimes I \otimes \sigma_2 \otimes \sigma_1 \otimes \sigma_1 \,, \\ \Gamma_7 &= I \otimes I \otimes \sigma_3 \otimes \sigma_1 \otimes \sigma_1 \,, \\ \Gamma_8 &= I \otimes I \otimes I \otimes \sigma_2 \otimes \sigma_1 \,, \\ \Gamma_9 &= I \otimes I \otimes I \otimes \sigma_3 \otimes \sigma_1 \,, \\ \Gamma_{10} &= I \otimes I \otimes I \otimes I \otimes \sigma_2 \,, \\ \Gamma_h = \Gamma_{11} &= I \otimes I \otimes I \otimes I \otimes \sigma_3 \,. \end{aligned} \tag{B9}$$

From these examples it should be clear how to proceed for each sequence.

Spinor representations. The antisymmetrized product of two Dirac matrices

$$\Sigma_{ij} = \frac{1}{4} \left[\Gamma_i, \Gamma_j \right] \,. \tag{B10}$$

d	$C^T = \eta_1 C$	$C\Gamma_h = \eta_2 \Gamma_h C$	$(C\Gamma_i)^T = \eta_3 C\Gamma_i$	$(C\Gamma_i\Gamma_h)^T = \eta_4 C\Gamma_i\Gamma_h$
2	$-$	$-$	$+$	$+$
4	$-$	$+$	$+$	$-$
6	$+$	$-$	$-$	$-$
8	$+$	$+$	$-$	$+$

TABLE 7: Sign factors $\eta_1, \ldots, \eta_4$ occurring in $d = 2n$ dimensions in the relations defined at the top of each column. Notice that $\eta_3 = -\eta_1$ and that $\eta_2\eta_3\eta_4 = -1$. In calculating η_4 we use $\Gamma_h^T = \Gamma_h$. The periodicity is 8, so that the signs for $2n > 8$ may be found on the line that corresponds to $2n$ mod 8.

satisfies the SO(d) commutation relations (B1). We may use these matrices to reexpress the Dirac algebra (B3) as

$$\Gamma_i \Gamma_j = \delta_{ij} + 2\Sigma_{ij} \ , \tag{B11}$$

The 2^n dimensional space on which they act spans a Dirac-like reducible representation. Introducing d-dimensional chiral projectors:

$$\mathcal{P}_\pm = \frac{1}{2}(1 \pm \Gamma_h) \ , \tag{B12}$$

and noting that $[\Sigma_{ij}, \Gamma_h] = 0$, we may split $\Sigma_{ij} = \Sigma_{ij}^+ + \Sigma_{ij}^-$, where $\Sigma_{ij}^\pm = \mathcal{P}_\pm \Sigma_{ij} = \Sigma_{ij}\mathcal{P}_\pm$. The generators Σ_{ij}^+ and Σ_{ij}^- act on the irreducible chiral representations of SO($2n$).

We introduce block notation, where every $2^n \times 2^n$ Dirac-space matrix X is written in 2×2 block form corresponding to the projectors $\mathcal{P}_\pm$,

$$X = \begin{pmatrix} X_{++} & X_{+-} \\ X_{-+} & X_{--} \end{pmatrix} \ . \tag{B13}$$

where each block is a $2^{n-1} \times 2^{n-1}$ matrix.

Charge conjugation. In any even dimension $d = 2n$ one can find a charge conjugation matrix $\mathcal{C}$ that satisfies

$$\mathcal{C}\Gamma_i = -\Gamma_i^T \mathcal{C} \ , \qquad i = 1, 2, \ldots, 2n \ , \tag{B14}$$

and $\mathcal{C}^{-1} = \mathcal{C}^\dagger = \mathcal{C}^T$. For the Dirac matrices we have constructed explicitly, the charge conjugation matrices follow a simple pattern ($\epsilon = i\sigma_2$)

$$\mathcal{C}(2) = \epsilon \; , \tag{B15a}$$

$$\mathcal{C}(4) = \epsilon \otimes \sigma_3 \; , \tag{B15b}$$

$$\mathcal{C}(6) = \epsilon \otimes \sigma_3 \otimes \epsilon \; , \tag{B15c}$$

$$\mathcal{C}(8) = \epsilon \otimes \sigma_3 \otimes \epsilon \otimes \sigma_3 \; , \tag{B15d}$$

$$\mathcal{C}(10) = \epsilon \otimes \sigma_3 \otimes \epsilon \otimes \sigma_3 \otimes \epsilon \; , \tag{B15e}$$

and so on. $\mathcal{C}$ is antisymmetric for $d = 2, 4 \bmod 8$, and symmetric for $d = 0, 6 \bmod 8$. Also, $[\mathcal{C}, \Gamma_h] = 0$ for $d = 4n$, whereas $\{\mathcal{C}, \Gamma_h\} = 0$ for $d = 4n + 2$. We have tabulated the signs that occur in some basic relations in Table 7.

It follows from Eq. (B14) that

$$\mathcal{C}\Sigma_{ij} = -\Sigma_{ij}^T \mathcal{C} \; . \tag{B16}$$

In $d = 4n + 2$ dimensions we have $\{\mathcal{C}, \Gamma_h\} = 0$, which implies that the two chiral *irreps* projected by $\mathcal{P}_\pm$ are complex conjugates. For $d = 4n$, $\mathcal{C}$ commutes with Γ_h. Taking into account the symmetry properties of $\mathcal{C}$ itself (see the first column of Table 7) the chiral *irreps* are real for $d = 8n$ and pseudoreal for $d = 8n + 4$.

2. Dirac algebra in $d = 2n + 1$ dimensions

When d is odd we take the last Dirac matrix to be $\Gamma_{2n+1} = \Gamma_h$. We have to distinguish between two families. For $d = 4n + 3$, $\mathcal{C}$ anticommutes with Γ_h, and Eq. (B14) applies for $i = 1, 2, \ldots, 2n + 1$. As a result, the construction of an SO(d) vector from a spinor bilinear works as for even d (except of course that the spinor *irrep* is not chiral any more). For $d = 4n + 1$, Eq. (B14) does not generalize to the last Dirac matrix. Instead, we may use

$$(\mathcal{C}\Gamma_h)\Gamma_i = +\Gamma_i^T(\mathcal{C}\Gamma_h) \; , \qquad\qquad i = 1, 2, \ldots, 2n + 1 \; . \tag{B17}$$

3. Some identities

Here we collect some useful identities. First,

$$\Gamma_i \Gamma_j \Gamma_i = (2 - d)\Gamma_j \; . \tag{B18}$$

Next, using the above identity and Eq. (B11) we have

$$-\Sigma_{ij}\Gamma_k\Sigma_{ij} = \frac{(d-2)^2 - d}{4}\,\Gamma_k \ , \tag{B19}$$

which is used for the color factor $\mathcal{G}(a)$ in Table 3. Last, for the color factor $\mathcal{G}(b,c)$, we use an identity involving the generators of both the vector and spinor *irreps*,

$$\Gamma_\ell \Sigma_{ij} (M^{\rm vec}_{ij})_{\ell k} = -\Sigma_{ij}\Gamma_\ell (M^{\rm vec}_{ij})_{\ell k} = (d-1)\Gamma_k \ . \tag{B20}$$

where the vector *irrep*'s generators $(M^{\rm vec}_{ij})_{k\ell}$ are given by Eq. (B2). In order to arrive at the group theoretic factors in the rightmost column of Table 3 we have to divide these results by 2, because the unconstrained summation on the left-hand side amounts to summing over each SO(d) generator twice.

4. SO(d) Feynman rules

In order to avoid double counting, it is convenient to label the SO(d) generators by an ordered pair $[ij]$ where $1 \le i < j \le d$. The tree-level gluon propagator is then

$$\left\langle A_{\mu[ij]} A_{\nu[kl]} \right\rangle = \frac{\delta_{\mu\nu}\delta_{ik}\delta_{jl}}{p^2} \ , \tag{B21}$$

where we have used the Feynman gauge for definiteness. The (massless) fermion lagrangian is

$$\mathcal{L}^\pm_F \ = \ \overline{\psi}_{A\alpha}\,(\gamma_\mu)_{\alpha\beta} (D_\mu)^\pm_{AB}\,\psi_{B\beta} \ , \tag{B22}$$

$$(D_\mu)^\pm_{AB} \ = \ \delta_{AB}\partial_\mu + \sum_{i<j} A_{\mu[ij]} (\Sigma^\pm_{[ij]})_{AB} \ , \tag{B23}$$

where the $\pm$ superscript refers to the chirality of the *irrep*. Notice the absence of a factor of i in front of the gauge field's term in Eq. (B23), because the generators are already antihermitian.

When we contract a gluon propagator between two fermion vertices with generators $M^{(1)}_{[ij]}$ and $M^{(2)}_{[ij]}$ we will get, schematically,

$$\sum_{i<j} M^{(1)}_{[ij]} \otimes M^{(2)}_{[ij]} = \frac{1}{2}\sum_{ij} M^{(1)}_{[ij]} \otimes M^{(2)}_{[ij]} \ , \tag{B24}$$

where, in the unrestricted summation on the right-hand side, we have used that $M_{ij} = -M_{ji}$ by definition.

Appendix C: 4-dimensional Dirac matrices

Denoting by C the charge conjugation matrix in four dimensions, Eq. (B15b), the behavior under charge conjugation is

$$\begin{aligned} (C\,I)^T &= -(C\,I) \\ (C\,\gamma_5)^T &= -(C\,\gamma_5) \\ (C\,\gamma_\mu)^T &= +(C\,\gamma_\mu) \\ (C\,\gamma_5\gamma_\mu)^T &= -(C\,\gamma_5\gamma_\mu) \\ (C\,\sigma_{\mu\nu})^T &= +(C\,\sigma_{\mu\nu})\ , \end{aligned} \tag{C1}$$

where in this appendix, I is the identity matrix in Dirac space. As usual (compare Eq. (B10))

$$\sigma_{\mu\nu} = (i/2)[\gamma_\mu, \gamma_\nu]\ . \tag{C2}$$

When working out the one-loop diagrams, thanks to the common color factor of the graphs in Figs. 1(b) and (c), summing them together always simplifies the Dirac algebra. For the derivation of the results in the second row of Table 2 we have use the following identities. First,

$$\gamma_\mu\gamma_\nu \otimes (\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu) = 8I \otimes I\ , \tag{C3}$$

which follows immediately from the Dirac algebra. The next identity

$$\gamma_\mu\gamma_\nu\gamma_\rho \otimes (\gamma_\mu\gamma_\nu\gamma_\rho + \gamma_\rho\gamma_\nu\gamma_\mu) = 20\gamma_\rho \otimes \gamma_\rho\ , \tag{C4}$$

can be proved using

$$\gamma_\mu\gamma_\nu\gamma_\rho = \delta_{\mu\nu}\gamma_\rho - \delta_{\mu\rho}\gamma_\nu + \delta_{\nu\nu}\gamma_\mu + \epsilon_{\mu\nu\rho\tau}\gamma_5\gamma_\tau\ . \tag{C5}$$

Finally,

$$\begin{aligned} \gamma_\mu\gamma_\nu\sigma_{\lambda\rho} \otimes (\gamma_\mu\gamma_\nu\sigma_{\lambda\rho} + \sigma_{\lambda\rho}\gamma_\nu\gamma_\mu) \ &= \\ &= (\delta_{\mu\nu} - i\sigma_{\mu\nu})\sigma_{\lambda\rho} \otimes [(\delta_{\mu\nu} - i\sigma_{\mu\nu})\sigma_{\lambda\rho} + \sigma_{\lambda\rho}(\delta_{\mu\nu} + i\sigma_{\mu\nu})] \\ &= 8\sigma_{\lambda\rho} \otimes \sigma_{\lambda\rho} - \frac{1}{2}[\sigma_{\mu\nu}, \sigma_{\lambda\rho}] \otimes [\sigma_{\mu\nu}, \sigma_{\lambda\rho}] \\ &= 24\sigma_{\lambda\rho} \otimes \sigma_{\lambda\rho}\ , \end{aligned} \tag{C6}$$

where we have used $\sigma_{\mu\nu} = 2i\Sigma_{\mu\nu}$ and Eqs. (B1) and (B11).

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