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Prospects for observing the lowest-lying odd-parity Σ_c and Σ_b baryons

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ABSTRACT

There exist candidates for the negative-parity states $\Lambda_{c,b}(1/2^-, 3/2^-)$ consisting of an isospin-zero, spin-zero light diquark $[ud]$ with one unit of orbital angular momentum with respect to a c, b quark. However, there exists only one candidate for the orbital excitations of the $\Sigma_c(1/2^+)$ and $\Sigma^*_c(3/2^+)$, and none for the orbital excitations of $\Sigma_b(1/2^+)$ or $\Sigma^*_b(3/2^+)$. We extend a previous discussion of odd-parity $\Lambda_{c,b}$ states and explore some patterns of the odd-parity $\Sigma_{c,b}$ baryons consisting of a light isospin-one nonstrange diquark (uu, ud, dd) in a state of $L = 1$ with respect to the spin-1/2 heavy quark (c, b).

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I Introduction

We have used simple quark-model arguments in previous work [1–3] to predict the masses of some of the lowest-lying baryons containing a single c or b quark. While these predictions generally dealt with states with no orbital excitations, the negative-parity states $\Lambda_{c,b}(1/2^-, 3/2^-)$ consisting of an isospin-zero, spin-zero light diquark $[ud]$ with one unit of orbital angular momentum with respect to a c, b quark were discussed briefly. However, the orbital excitations of the $\Sigma_{c,b}(1/2^+)$ and $\Sigma^*_{c,b}(3/2^+)$ were not treated. Here we have labeled states with total angular momentum J; the superscripts denote their parity.

In this paper we extend our previous discussion of odd-parity Λ_Q states, and explore some patterns of those odd-parity $\Sigma_{c,b}$ baryons consisting of a light isospin-one nonstrange diquark (uu, ud, dd) in a state of $L = 1$ with respect to the spin-1/2 heavy quark $Q = (c, b)$. This investigation is timely as a result of the demonstrated capabilities of hadron colliders in studying heavy-hadron spectra (see, e.g., Refs. [4, 5].) A comprehensive study of masses

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Table I: Candidates for Λ_Q having ground-state light diquark and heavy quark Q with relative orbital angular momentum $L = 1$.

J^P	Λ_c		Λ_b	
	Mass (MeV)	Γ (MeV)	Mass (MeV)	Γ (MeV)
$1/2^-$	2592.25±0.28	2.6±0.6	5912.1±0.4	< 0.66
$3/2^-$	2628.11±0.19	< 0.97	5919.73±0.32	< 0.63

of baryons containing heavy quarks was published several years ago [6], but without the detail which might help identify which of several $\Sigma_{c,b}(J^P)$ is being observed.

In Section II we introduce basis notation for the P-wave states of Λ_Q and Σ_Q. We then estimate the energy cost of a P-wave excitation in a heavy-quark baryon by reference to existing data (Sec. III), and work out spin-dependent splitting of states in the limit where the quark Q is much heavier than the diquark (Sec. IV). We make some remarks on production and decay systematics in Sec. V and conclude in Sec. VI. An Appendix contains details of angular momentum calculations.

II Basis states

As in Ref. [6], we limit our discussion to states in which a light diquark remains in its ground state, and is orbitally excited with respect to the heavy quark Q. Internal excitation of the light diquark seems to require more energy. The lowest-lying negative-parity Λ baryons [7], $\Lambda(1405, 1/2^-)$ and $\Lambda(1520, 3/2^-)$, may be regarded as P-wave excitations of an isospin-zero, spin-zero diquark $[ud]$ with respect to the heavier strange quark. The lowest P-wave Λ state in which the $[ud]$ must be internally excited is $\Lambda(1830, 5/2^-)$. Here states are labeled by their masses in MeV.[§]

Quantum chromodynamics (QCD) implies that the light diquark in a Λ_Q ($Q = c, b$) will be $[ud]$ in a state of zero spin and isospin, while that in a Σ_Q will be (uu, ud, dd) in a state of unit spin and isospin. The diquark spin $S_d = 0$ or 1 is then coupled to the heavy quark spin $S_Q = 1/2$ and the orbital angular momentum $L = 1$ to a total angular momentum J.

There are two convenient bases in which to evaluate the product $S_d \otimes S_Q \otimes L$. In the first ($L$–$S$ coupling) we couple S_d and S_Q to a total spin S and then couple S and L to J. In the second (j–j coupling, appropriate in the large-m_Q limit [9]), we couple S_d and L to a total light-quark angular momentum j and then couple j and S_Q to J. We shall tabulate states in both bases and give the transformation between them.

A L–S coupling

For Λ_Q with an isospin-zero diquark with $S_d = 0$, the total quark spin S is necessarily 1/2. Coupling this to the orbital angular momentum $L = 1$, one obtains states of $J^P = 1/2^-$ and $3/2^-$. Candidates for these states in charm and bottom sectors [7] are summarized in Table I.

[§]For an extensive discussion regarding baryons as bound states of a quark and diquark, see Ref. [8] and references therein.

We will anticipate a result of Sec. IV by noting that the fine-structure splitting between the two Λ_b states is less than 1/4 that between the two Λ_c states. This splitting should scale as $m_c/m_b \simeq 1/3$, so the agreement is at best qualitative.

For Σ_Q the only current candidate for a P-wave baryon is $\Sigma_c(2800)$ [7], whose spin and parity have not yet been determined. Ref. [6] assigns it to $J^P = 1/2^-$ or $3/2^-$ (possibly both, overlapping in mass). In L–S coupling, the light $S_d = 1$ diquark and heavy $S_Q = 1/2$ quark can form states with $S = 1/2$ and $3/2$. Coupling $S = 1/2$ to $L = 1$ gives states with $J = 1/2, 3/2$, while coupling $S = 3/2$ to $L = 1$ gives states with $J = 1/2, 3/2, 5/2$. There are thus five Σ_Q states with $L = 1$ and odd parity (under the assumption that the diquark remains in its ground state). We shall introduce the notation $^{2S+1}P_J$ to describe these states as $^2P_{1/2}$, $^2P_{3/2}$, $^4P_{1/2}$, $^4P_{3/2}$, $^4P_{5/2}$, respectively.

The spin-dependent potential may be written [6]

$$V_{SD} = a_1 \boldsymbol{L}\cdot\boldsymbol{S}_d + a_2 \boldsymbol{L}\cdot\boldsymbol{S}_Q + b[-\boldsymbol{S}_d\cdot\boldsymbol{S}_Q + (\boldsymbol{S}_d\cdot\boldsymbol{r})(\boldsymbol{S}_Q\cdot\boldsymbol{r})/r^2] + c\boldsymbol{S}_d\cdot\boldsymbol{S}_Q \ , \tag{1}$$

where the first two terms are spin-orbit forces, the third is a tensor force, and the last describes hyperfine splitting. In the L–S basis, the two $J = 1/2$ states and the two $J = 3/2$ states are unmixed only if $a_1 = a_2$. Otherwise they are eigenstates of 2×2 matrices $\mathcal{M}_J$ given in the basis $[^2P_J,\ ^4P_J]$ by

$$\mathcal{M}_{1/2} = \begin{bmatrix} \frac{1}{3}a_2 - \frac{4}{3}a_1 & \frac{\sqrt{2}}{3}(a_2 - a_1) \\ \frac{\sqrt{2}}{3}(a_2 - a_1) & -\frac{5}{3}a_1 - \frac{5}{6}a_2 \end{bmatrix} + b\begin{bmatrix} 0 & 0 \\ 0 & -\frac{3}{2} \end{bmatrix} + c\begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \ , \tag{2}$$

$$\mathcal{M}_{3/2} = \begin{bmatrix} \frac{2}{3}a_1 - \frac{1}{6}a_2 & \frac{\sqrt{5}}{3}(a_2 - a_1) \\ \frac{\sqrt{5}}{3}(a_2 - a_1) & -\frac{2}{3}a_1 - \frac{1}{3}a_2 \end{bmatrix} + b\begin{bmatrix} 0 & 0 \\ 0 & \frac{6}{5} \end{bmatrix} + c\begin{bmatrix} -1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix} \ , \tag{3}$$

$$\mathcal{M}_{5/2} = a_1 + \frac{1}{2}a_2 - \frac{3}{10}b + \frac{1}{2}c \ . \tag{4}$$

Details of this calculation are given in Appendix A.

B j–j coupling

When m_Q is much larger than the diquark mass, the terms a_2, b, and c in (1) all behave as $1/m_Q$ and their expectation values are suppressed in comparison with that of the a_1 term. Thus it makes sense to expand in a basis in which $\boldsymbol{L}\cdot\boldsymbol{S}_d$ is diagonal, treating the other terms in V_{SD} as perturbations. Defining $\boldsymbol{j} = \boldsymbol{L} + \boldsymbol{S}_d$ and squaring, one finds

$$\langle \boldsymbol{L}\cdot\boldsymbol{S}_d \rangle = \frac{1}{2}[j(j+1) - L(L+1) - S_d(S_d+1)] = (-2, -1, 1) \text{ for } j = (0, 1, 2) \ . \tag{5}$$

To lowest order in $1/m_Q$, the five lowest negative-parity Σ_Q states are displaced by $-2a_1$ ($J = 1/2$), $-a_1$ ($J = 1/2$), $-a_1$ ($J = 3/2$), a_1 ($J = 3/2$), and a_1 ($J = 5/2$) from their spin-weighted average. The expansion of these states in terms of L–S eigenstates is

$$|J = \frac{1}{2}, j = 0\rangle = \sqrt{\frac{1}{3}}|^2P_{1/2}\rangle + \sqrt{\frac{2}{3}}|^4P_{1/2}\rangle \ , \tag{6}$$

$$|J = \frac{1}{2}, j = 1\rangle = \sqrt{\frac{2}{3}}|^2P_{1/2}\rangle - \sqrt{\frac{1}{3}}|^4P_{1/2}\rangle \ , \tag{7}$$

$$|J = \frac{3}{2}, j = 1\rangle = \sqrt{\frac{1}{6}}|^2P_{3/2}\rangle + \sqrt{\frac{5}{6}}|^4P_{3/2}\rangle \ , \tag{8}$$

$$|J = \frac{3}{2}, j = 2\rangle = \sqrt{\frac{5}{6}}|^2P_{3/2}\rangle - \sqrt{\frac{1}{6}}|^4P_{3/2}\rangle \ , \tag{9}$$

$$|J = \frac{5}{2}, j = 2\rangle = |^4P_{5/2}\rangle \ . \tag{10}$$

This allows us to evaluate the matrix elements of all the terms in the spin-dependent potential (1) to lowest order in perturbation theory by using the unperturbed eigenfunctions of the first term. The resultant energy shifts are

$$\Delta M(J = \frac{1}{2}, j = 0) = -2a_1 - b \ , \tag{11}$$

$$\Delta M(J = \frac{1}{2}, j = 1) = -a_1 - \frac{1}{2}a_2 - \frac{1}{2}b - \frac{1}{2}c \ , \tag{12}$$

$$\Delta M(J = \frac{3}{2}, j = 1) = -a_1 + \frac{1}{4}a_2 + b + \frac{1}{4}c \ , \tag{13}$$

$$\Delta M(J = \frac{3}{2}, j = 2) = a_1 - \frac{3}{4}a_2 + \frac{1}{5}b - \frac{3}{4}c \ , \tag{14}$$

$$\Delta M(J = \frac{5}{2}, j = 2) = a_1 + \frac{1}{2}a_2 - \frac{3}{10}b + \frac{1}{2}c \ . \tag{15}$$

$$\tag{16}$$

This expresses five mass shifts in terms of four parameters. One linear relation among them is the vanishing of their spin-weighted sum:

$$\sum_J (2J+1)\Delta M(J) = 0 \ . \tag{17}$$

However, a_2 and c always occur in the combination $a_2 + c$, so that the five mass shifts are expressed in terms of the three free parameters a_1, $a_2 + c$, and b. Hence the masses satisfy one additional linear relation besides Eq. (17). This is found to be

$$10M(1/2, 0) - 15M(1/2, 1) + 8M(3/2, 2) - 3M(5/2, 2) = 0 \ , \tag{18}$$

where the first number refers to J and the second to j. The mass $M(3/2, 1)$ does not appear. We shall return to this topic when discussing numerical predictions for fine-structure splittings in Sec. IV.

III Energy cost of a P-wave excitation

One can estimate the cost of a P-wave excitation in baryons with one heavy quark by comparing the masses of ground-state $\Lambda_Q(J^P = 1/2^+)$ baryons with those of the spin-weighted averaged masses $\bar{m}(\Lambda_Q)$ of the P-wave $\Lambda_Q(J^P = 1/2^-, 3/2^-)$ baryons. The mass difference between $\Lambda_Q(J^P = 1/2^-)$ and $\Lambda_Q(J^P = 3/2^-)$ is due to the $\boldsymbol{L} \cdot \boldsymbol{S}_Q$ spin-orbit interaction, which is mathematically analogous to the $\boldsymbol{S}_{(ud)} \cdot \boldsymbol{S}_Q$ color hyperfine interaction responsible for $\Sigma^* - \Sigma$ splitting. Therefore $\bar{m}(\Lambda_Q) = [m(\Lambda_Q, 1/2^-) + 2m(\Lambda_Q, 3/2^-)]/3$. We shall assume that this $\bar{m}(\Lambda_Q) - m(\Lambda_Q)$ splitting, denoted by ΔE_{PS}, is a function only of

Table II: Input masses, values of ΔE_{PS}, and diquark-Q reduced masses for baryons containing a single heavy quark Q.

	Baryon mass		ΔE_{PS}	Reduced mass μ (MeV)	
States	$J^P = 1/2^+$	$\bar{m}(\Lambda_Q)$	(MeV)	$\mu(\Lambda_Q)$	$\mu(\Sigma_Q)$
Λ	1115.7	1481.4	365.7	278.6	318.9
Λ_c	2286.5	2616.2	329.7	431.8	536.8
Λ_b	5619.5	5917.2	297.7	518.3	677.6

the reduced mass of the Q–diquark system. For that purpose we need both diquark and Q masses.

The effective light isoscalar diquark mass in a Λ is

$$m_{[ud]} = m_\Lambda - m_s = 577.7\text{MeV} , \tag{19}$$

where we have taken m_Λ from Ref. [7] and used $m_s = 538$ MeV from a fit to light-quark baryon spectra [3, 11]. (Subsequently we shall use masses from Ref. [7] unless stated otherwise.) The corresponding light isovector diquark mass, ignoring small isospin splittings, is

$$m_{(uu)} = m_{(ud)} = m_{(dd)} = \frac{m_\Sigma + 2m_{\Sigma^*}}{3} - m_s = 782.8 \text{ MeV} . \tag{20}$$

The spin-averaged diquark mass is $[m_{[ud]} + 3m_{(uu)}]/4 = 731.5$ MeV, close to $2m_u = 2m_d = 726$ MeV in Refs. [3, 11]. The difference of a few MeV may be regarded as an estimate of systematic error in our approach.

The heavy quark masses may be estimated using the difference between Λ_Q and isoscalar diquark masses:

$$m_c = m_{\Lambda_c} - m_\Lambda + m_s = 1708.8 \text{ MeV} , \quad m_b = m_{\Lambda_b} - m_\Lambda + m_s = 5041.8 \text{ MeV} . \tag{21}$$

These are within a couple of MeV of values estimated in Ref. [3] by a slightly different method.

We summarize results in Table II, and plot values of ΔE_{PS} as a function of diquark-Q reduced mass μ in Fig. 1. Also shown are reduced masses for the corresponding Σ_Q states. We shall use these to estimate values of ΔE_{PS} for Σ_Q states by linear extrapolation from those for Λ_c and Λ_b states.¶

The dashed line in Fig. 1 shows just one possible extrapolation of the sparse information provided by the Λ_Q states. The predictions for the values of ΔE_{PS} for the Σ_Q states must thus be taken with some care. Using these, however, we obtain the results shown in Table III.

¶We assume here that the state $\Lambda(J^P = 1/2^+)$ is $\Lambda(1405)$, which probably has some distortion of the mass due to coupled-channel effects to $\bar{K}N$. This could influence the Λ point in Fig. 1. The J^P values of $\Lambda_b(5912)$ and $\Lambda_b(5920)$ are assumed to be $1/2^-$ and $3/2^-$, respectively.

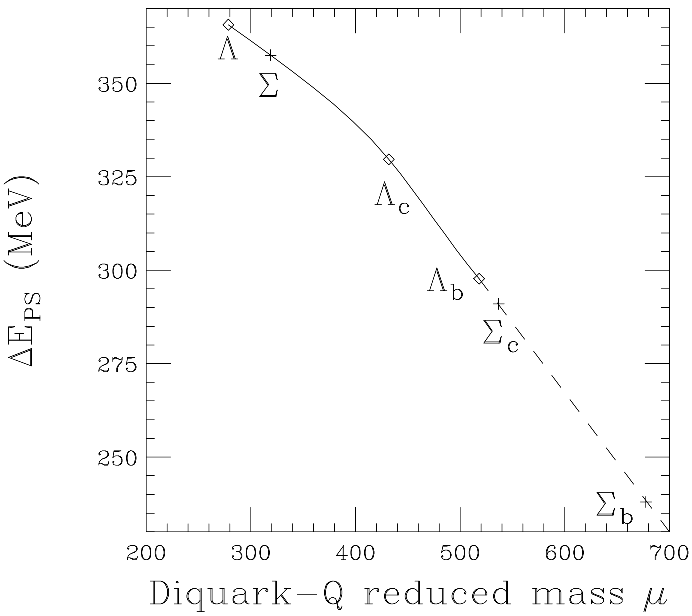

Figure 1: Dependence of P–S splitting parameter ΔE_{PS} on diquark-Q reduced mass μ. Diamonds denote experimental Λ_Q points; crosses denote interpolated or extrapolated Σ_Q points.

Table III: Parameters leading to estimates of spin-averaged masses $\bar{M}$ for P-wave excitations of a light $I = S = 1$ diquark with respect to a heavy quark Q. Here $M_{0Q} \equiv [M(\Sigma_Q) + 2M(\Sigma_Q^*)/3$.

Heavy quark Q	$M(\Sigma_Q)$ (MeV)	$M(\Sigma_Q^*)$ (MeV)	M_{0Q} (MeV)	ΔE_{PS} (MeV)	$\bar{M}$ (MeV)
c	2453.4	2518.1	2496.5	290.9	2787.4
b	5814.3	5833.8	5827.3	238.8	6066.1

Table IV: Masses of lowest-lying negative-parity Σ_c and Σ_b baryons in the model of Ref. [6].

J	$M(\Sigma_c)$ (MeV)	$M(\Sigma_b)$ (MeV)
1/2	2713, 2799	6095, 6101
3/2	2773, 2798	6087, 6096
5/2	2789	6084

IV Fine and hyperfine structure

We return to the question of fine and hyperfine structure based on Eqs. (11–18). First of all, the hyperfine term c is expected to be negligible, as it results from a short-distance interaction whose matrix elements between P-wave states should be very small. Second, we have some idea of the magnitude of a_2 as a term $a_2\langle \boldsymbol{L}\cdot\boldsymbol{S_Q}\rangle$ is responsible for the splittings between $\Lambda_c(2592, 1/2^-)$ and $\Lambda_c(2628, 3/2^-)$, and between $\Lambda_b(5912, 1/2^-)$ and $\Lambda_b(5920, 3/2^1)$ (see Table I):

$$a_2 = \frac{2}{3}[M(\Lambda_Q, 3/2^-) - M(\Lambda_Q, 1/2^-)] = \left\{ \begin{array}{c} 76.3\ (\Lambda) \\ 23.9\ (\Lambda_c) \\ 5.1\ (\Lambda_b) \end{array} \right\} . \tag{22}$$

These quantities scale roughly as inverse heavy quark mass, though a_2 for b baryons as evaluated using the masses in Table I is a bit smaller than m_c^b/m_b^b times a_2 for charmed baryons.

There are many ways to make use of Eqs. (11–18), but we shall mention just two. First, we can take the difference of two masses of states with the same j to obtain two linear combinations of a_2 and b:

$$M(3/2,1) - M(1/2,1) = \frac{3}{4}a_2 + \frac{3}{2}b , \tag{23}$$

$$M(5/2,2) - M(3/2,2) = \frac{5}{4}a_2 - \frac{1}{2}b. \tag{24}$$

This allows one to extract a_2 and b, given splittings of the levels with the same j. Given the masses for $j = 1$ and $j = 2$, one can use the sum rule (18) to solve for $M(1/2, 0)$ and compare with observation. If the entire parameter space is mapped out for Σ_c states, one can use the scaling relations

$$a_1(b) = a_1(c) , \quad a_2(b) = (m_c^b/m_b^b)a_2(c) , \quad b(b) = (m_c^b/m_b^b)b(c) , \tag{25}$$

where the superscript refers to an effective quark mass in a baryon, to predict the splittings for Σ_b states.

A second use of Eqs. (11-18) is to check the consistency of masses quoted in Ref. [6] with the perturbative expressions. Two masses are quoted for $J = 1/2$ and two for $J = 3/2$, as summarized in Table IV.

In Ref. [6] it is not clear to which j a state with given $J = 1/2$ or $J = 3/2$ corresponds. However, one can use the sum rule (18) to predict $M(3/2, 2)$ given either choice

Table V: Masses of P-wave $c\bar{s}$ mesons.

State	(J,j)	Mass (MeV)
$D^*_{s0}(2317)$	(0,1/2)	2317.7
$D_{s1}(2460)$	(1,1/2)	2459.5
$D_{s1}(2536)$	(1,3/2)	2535.1
$D^*_{s2}(2573)$	(2,3/2)	2571.9

for $[M(1/2,0), M(1/2,1)]$. For Σ_c states, choosing [2713,2799] MeV one finds $M(3/2,2) = 2903$ MeV, while [2799,2713] MeV yields $M(3/2,2) = 2634$ MeV. Neither mass corresponds to one of the two choices in Table IV. A similar exercise for Σ_b states predicts $M(3/2,2) = 6102$ or 6083 MeV, closer to the predictions [6096,6087] MeV of Ref. [6].

We can progress further if we use an estimate of the parameter a_1 from the charmed-strange meson sector. One must view this estimate with some caution as it involves the mesons called by the Particle Data Group [7] $D^*_{s0}(2317)$ and $D_{s1}(2460)$, which turned out to be lighter than expected. (For a discussion of this point with further references, see [12]. The masses of these states may be governed in part by chiral dynamics lying outside the predictive power of a spin-dependent potential [13]). We shall assume, as we did for the Σ_Q baryons, that the mass eigenstates are those with definite j composed of the orbital angular momentum L and the light-quark degrees of freedom (here, the strange quark spin). The masses of states with definite total angular momentum J and light-quark total angular momentum j are summarized in Table V.

Repeating the steps in which one expands around eigenstates of j (details are given in the Appendix), one finds the following expressions for the meson masses $M(J,j)$:

$$M(0,1/2) = \bar{M} - a_1 - a_2 - b + \frac{1}{4}c \ , \quad (26)$$

$$M(1,1/2) = \bar{M} - a_1 + \frac{1}{3}a_2 + \frac{1}{3}b - \frac{1}{12}c \ , \quad (27)$$

$$M(1,3/2) = \bar{M} + \frac{1}{2}a_1 - \frac{5}{6}a_2 + \frac{1}{6}b - \frac{5}{12}c \ , \quad (28)$$

$$M(2,3/2) = \bar{M} + \frac{1}{2}a_1 + \frac{1}{2}a_2 - \frac{1}{10}b + \frac{1}{4}c \ . \quad (29)$$

The parameters which reproduce the masses in Table V are

$$\bar{M}(c\bar{s}) = 2513.4 \text{ MeV} \ , \quad a_1(c\bar{s}) = 89.4 \text{ MeV} \ , \quad a_2(c\bar{s}) = 40.7 \text{ MeV} \ , \quad b(c\bar{s}) = 65.6 \text{ MeV} \ , \quad (30)$$

where we have neglected the hyperfine term for reasons mentioned earlier.

We can now relate $a_1(c\bar{s})$ to the terms a_1 describing the coefficients of $\boldsymbol{L}\cdot\boldsymbol{S}_d$ in Σ_c and Σ_b, which we assume to be equal:

$$a_1 = (m_s^m/m_{(uu)})a_1(c\bar{s}) = (483 \text{ MeV}/783 \text{ MeV})89.4 \text{ MeV} = 55.1 \text{ MeV} \ . \quad (31)$$

Here we have used the effective mass $m_s^m = 483$ MeV of a strange quark in a meson [3], and the mass of the $I = S = 1$ light diquark calculated in Eq. (20).

Knowing $\bar{M}$, a_1, and a_2 for both Σ_c and Σ_b P-wave excitations, all we are missing is the tensor coefficient b. We can plot predicted masses as functions of b and see if one or

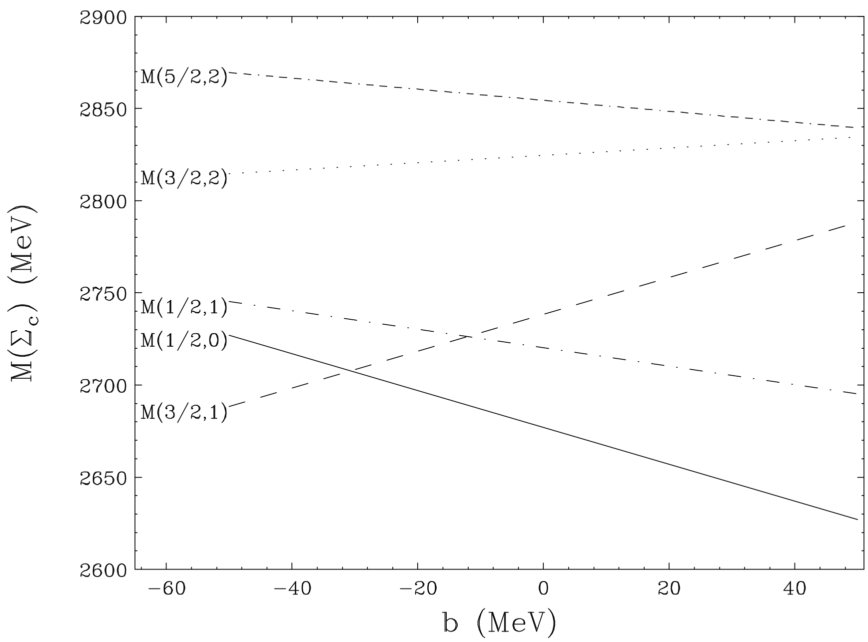

Figure 2: Masses of P-wave Σ_c states as functions of tensor force parameter b.

more solutions exist with b scaling as the inverse of the heavy quark mass. The relevant expressions, for masses $M(J,j)$ in MeV, are

$$\begin{aligned}
\Sigma_c : M(1/2,0) &= 2677.1 - b \ , && (32)\\
M(1/2,1) &= 2720.3 - \frac{1}{2}b \ , && (33)\\
M(3/2,1) &= 2738.2 + b \ , && (34)\\
M(3/2,2) &= 2824.6 + \frac{1}{5}b \ , && (35)\\
M(5/2,2) &= 2854.5 - \frac{3}{10}b \ ; && (36)
\end{aligned}$$

$$\begin{aligned}
\Sigma_b : M(1/2,0) &= 5955.8 - b \ , && (37)\\
M(1/2,1) &= 6008.4 - \frac{1}{2}b \ , && (38)\\
M(3/2,1) &= 6012.2 + b \ , && (39)\\
M(3/2,2) &= 6117.4 + \frac{1}{5}b \ , && (40)\\
M(5/2,2) &= 6123.8 - \frac{3}{10}b \ . && (41)
\end{aligned}$$

The results are plotted as a function of the tensor parameter b in Figs. 2 and 3. For moderate values of b, there is a clear separation between the three lowest masses $M(1/2,0)$, $M(1/2,1)$, $M(3/2,1)$ and the two highest masses $M(3/2,2)$ and $M(5/2,2)$. These are, coincidentally, the states most likely to be relatively narrow and hence easier to observe, as we shall see in the next Section.

Calculations of excited Σ_c masses have been made in lattice QCD. Predictions were presented in [14,15], based on operators that obey an SU(3) symmetry of u, d, c quarks and

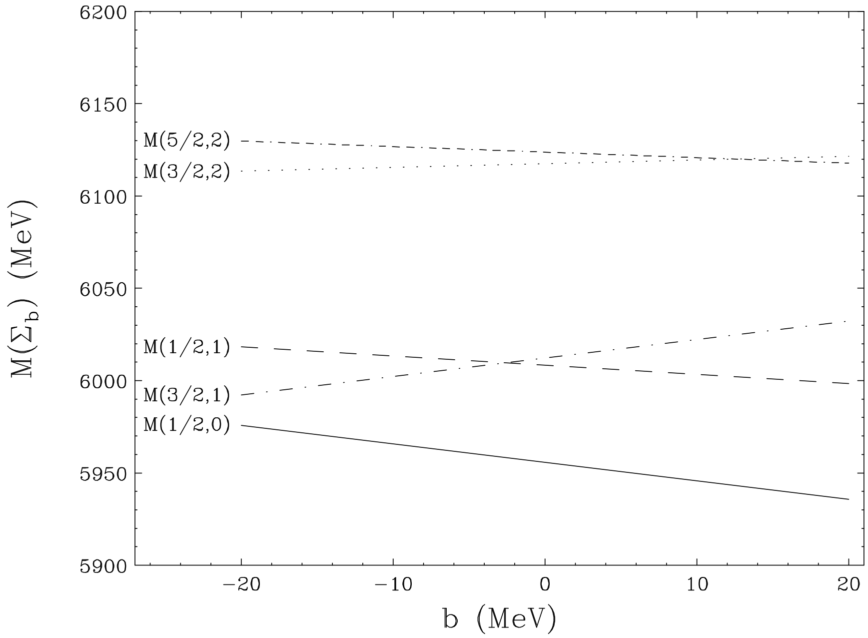

Figure 3: Masses of P-wave Σ_b states as functions of tensor force parameter b.

an SU(2) of spin, combined into an SU(6) of which the $L = 1$ baryons belong to a 70-plet with SU(3) $\times$ SU(2) decomposition (1,2)+(8,2)+(8,4)+(10,2). The additional $J = 1/2$ and $J = 3/2$ levels in [14, 15] are related to spin-zero light-diquark excitations, which have not been considered in our case.

The lattice calculations were performed at a pion mass of 400 MeV (so no chiral extrapolation) and a single lattice spacing (not extrapolated to continuum), yielding a calculated Λ_c mass 149 MeV above its experimental value. Subtracting 149 MeV from mass values of Ref. [14] (Fig. 3), one obtains the values, labeled by mass in MeV and total spin [16]:

Octet: (2794,1/2), (2799,1/2), (2875,3/2), (2884,3/2), and (2873,5/2),

Decuplet: (2964,1/2) and (2978,3/2),

where the octet and decuplet assignments are only approximate.

Even considering that the lattice predictions are expected to be more reliable for mass splittings than for absolute masses, this pattern is very far from that in Fig. 2 for any value of the parameter b. It will be interesting to compare experimental values with these two sets of predictions.

V Production and decay systematics

In the absence of further dynamical guidance, we may assume that the cross section for production of a state with total angular momentum J is proportional to its statistical weight $(2J + 1)$. The highest-spin states are thus most likely to be produced under such a hypothesis. To progress further one might have to take account of how the $I = J = 1$ diquark is excited with respect to the heavy quark Q. However, there is an additional circumstance favoring Σ_Q states with higher spin.

For the P-wave D mesons [9], the D^{**} states with light-quark total angular momentum

Table VI: Values of final light-quark spin accessible in decays of P-wave charmed mesons.

J	j	$L=0$	$L=2$
0	1/2	1/2	3/2, 5/2
1	1/2	1/2	3/2, 5/2
1	3/2	3/2	1/2, 3/2, 5/2, 7/2
2	3/2	3/2	1/2, 3/2, 5/2, 7/2

Table VII: Values of final light-quark spin accessible in decays of P-wave baryons with one heavy quark.

J	j	$L=0$	$L=2$
1/2	0	0	2
1/2	1	1	1, 2, 3
3/2	1	1	1, 2, 3
3/2	2	2	0, 1, 2, 3, 4
5/2	2	2	0, 1, 2, 3, 4

$j=1/2$ are allowed to decay to $D\pi$ and/or $D^*\pi$ only via S waves, while those with $j=3/2$ can decay to $D\pi$ and/or $D^*\pi$ only via D waves, and hence are relatively narrow. This situation is illustrated in Table VI, which lists the final light-quark spins accessible when combining j with the orbital angular momentum L of pseudoscalar meson emission. In decays to $D\pi$ or $D^*\pi$ the final light-quark total spin is 1/2, so $j=1/2$ states decay to $D\pi$ or $D^*\pi$ only via S waves, while $j=3/2$ states decay to $D\pi$ or $D^*\pi$ only via D waves. The D^{**} states with $j=3/2$ $D_1(2420)$ and $D_2^*(2460)$ [7] are thus the ones which have been firmly identified, while those with $j=1/2$, being relatively broader, still have not been pinned down. In the $c\bar{s}$ case, the $j=1/2$ states would have been broad except that they lie below the corresponding open charm thresholds and thus must decay via isospin violation or electromagnetically.

In the case of P-wave Σ_Q states, a similar hierarchy holds, illustrated in Table VII. The light-quark spin in a Λ_Q is zero, so the state $(J,j)=(1/2,0)$ can decay to $\Lambda_Q\pi$ in an S wave, while $(3/2,2)$ and $(5/2,2)$ can decay to $\Lambda_Q\pi$ in a D wave. The light-quark spin in a Σ_Q or Σ_Q^* is 1, so (1/2,1) and (3/2,1) can decay to $\Sigma^{(*)}\pi$ in both partial waves, while (3/2,2) and (5/2,2) can decay to $\Sigma^{(*)}\pi$ only in a D wave. Consequently, as the main decay modes of (3/2,2) and (5/2,2) need to be in D waves, these will be the narrow states, and hence the more easily observed of the five predicted ones. We thus expect that the Σ_c state by the BaBar Collaboration at $2846 \pm 8 \pm 10$ MeV [17] is a candidate for the (5/2,2) or (3/2,2) state. However, it is not seen by Belle [18], who see an isotriplet of states near 2800 MeV decaying to $\Lambda_c\pi$.

VI Conclusions

We have suggested some ways to look for P-wave excitations of Σ_c and Σ_b baryons, concentrating on those levels in which the light diquark in the ground state baryons with $I = J = 1$ acquires one unit of orbital angular momentum with respect to the heavy quark $Q = (c, b)$. In this limit there are five expected P-wave Σ_Q states: two with total spin $J = 1/2$, two with $J = 3/2$, and one with $J = 5/2$. In the heavy-quark limit one can treat Q as a spectator and discuss the total light-quark angular momentum $\boldsymbol{j} = \boldsymbol{L} + \boldsymbol{S}_d$, where $S_d = 1$ is the spin of the diquark, so $j = 0, 1, 2$. We have estimated masses as a function of one unknown parameter b describing tensor forces. For modest values of b, the states with $(J, j) = (3/2, 2)$ and (5/2,2) lie highest, and these are also the ones we expect are most likely to be detected first. They lie somewhat above 2800 MeV for charm and 6100 MeV for beauty.

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Appendix: Angular momentum state construction

L–S coupling

In the L–S basis, the terms proportional to b and c in the spin-dependent potential (1) are diagonal. We start by showing that the tensor force term can be written in terms of the total spin $\boldsymbol{S} = \boldsymbol{S}_d + \boldsymbol{S}_Q$:

$$\frac{S_{12}}{2} \equiv \langle (3\boldsymbol{S}\cdot\boldsymbol{r})(3\boldsymbol{S}\cdot\boldsymbol{r})/r^2 - \boldsymbol{S}^2 \rangle = \langle 6(\boldsymbol{S}_d\cdot\boldsymbol{r})(\boldsymbol{S}_Q\cdot\boldsymbol{r})/r^2 - 2\boldsymbol{S}_d\cdot\boldsymbol{S}_Q \rangle \ , \tag{42}$$

which can be evaluated using an identity from Ref. [10]:

$$\langle n_i n_j \rangle - \frac{1}{3}\delta_{ij} = a[L_iL_j + L_jL_i - \frac{2}{3}\delta_{ij}L(L+1)] \ , \quad a = -1/[(2L-1)(2L+3)] \ . \tag{43}$$

In the present case for $L = 1$, we have

$$\langle S_{12} \rangle = -\frac{6}{5}\langle [L_iL_j + L_jL_i - \frac{4}{3}\delta_{ij}]S_iS_j \rangle \ . \tag{44}$$

The product $L_iL_jS_iS_j$ is just $(\boldsymbol{L}\cdot\boldsymbol{S})^2$, while use of the commutation relations $[L_i, L_j] = i\epsilon_{ijk}L_k$ and $[S_i, S_j] = i\epsilon_{ijk}S_k$ yields $L_jL_iS_iS_j = (\boldsymbol{L}\cdot\boldsymbol{S})^2 + \boldsymbol{L}\cdot\boldsymbol{S}$. Consequently, we have

$$\langle S_{12} \rangle = -\frac{6}{5}[\langle 2(\boldsymbol{L}\cdot\boldsymbol{S})^2 + \boldsymbol{L}\cdot\boldsymbol{S} \rangle - \frac{4}{3}S(S+1)] \ . \tag{45}$$

The expectation values of $\boldsymbol{L}\cdot\boldsymbol{S}$ may be evaluated, of course, by squaring the identity $\boldsymbol{J} = \boldsymbol{L} + \boldsymbol{S}$:

$$\langle \boldsymbol{L}\cdot\boldsymbol{S} \rangle = \frac{1}{2}[J(J+1) - L(L+1) - S(S+1)] \ . \tag{46}$$

Table VIII: Expectation values of $\boldsymbol{L}\cdot\boldsymbol{S}$ and tensor term S_{12} for P-wave Σ_Q baryons in the L–S-coupling basis states.

State	$\langle \boldsymbol{L}\cdot\boldsymbol{S}\rangle$	$\langle S_{12}\rangle$
$^2P_{1/2}$	-1	0
$^2P_{3/2}$	$\frac{1}{2}$	0
$^4P_{1/2}$	$-\frac{5}{2}$	-6
$^4P_{3/2}$	-1	$\frac{24}{5}$
$^4P_{5/2}$	$\frac{3}{2}$	$-\frac{6}{5}$

The values of $\langle \boldsymbol{L}\cdot\boldsymbol{S}\rangle$ and $\langle S_{12}\rangle$ are shown in Table VIII.

The matrix elements of $\boldsymbol{L}\cdot\boldsymbol{S}_d$ and $\boldsymbol{L}\cdot\boldsymbol{S}_Q$ may be evaluated by explicit construction of states with a given J_3 as linear combinations of states $|S_{d3}, S_{Q3}, L_3\rangle$ where $S_{d3}+S_{Q3}+L_3 = J_3$. By angular momentum invariance, it suffices to use a single J_3 for each matrix element. An operator $\boldsymbol{L}\cdot\boldsymbol{S}_i$, where $i = d, Q$, may be expressed as

$$\boldsymbol{L}\cdot\boldsymbol{S}_i = L_3 S_{i3} + \frac{1}{2}[L_+ S_{i-} + L_- S_{i+}] \ , \tag{47}$$

and the usual rules for raising and lowering third components of angular momenta apply. The relevant basis states are

$$|^2P_{1/2}, J_3 = \frac{1}{2}\rangle = \frac{\sqrt{2}}{3}|1,-\frac{1}{2},0\rangle - \frac{1}{3}|0,\frac{1}{2},0\rangle - \frac{\sqrt{2}}{3}|0,-\frac{1}{2},1\rangle + \frac{2}{3}|-1,\frac{1}{2},1\rangle \ , \tag{48}$$

$$|^4P_{1/2}, J_3 = \frac{1}{2}\rangle = \frac{1}{\sqrt{2}}|1,\frac{1}{2},-1\rangle - \frac{1}{3}|1,-\frac{1}{2},0\rangle - \frac{\sqrt{2}}{3}|0,\frac{1}{2},0\rangle + \frac{1}{3}|0,-\frac{1}{2},1\rangle + \frac{1}{3\sqrt{2}}|-1,\frac{1}{2},1\rangle, \tag{49}$$

$$|^2P_{3/2}, J_3 = \frac{3}{2}\rangle = \sqrt{\frac{2}{3}}|1,-\frac{1}{2},1\rangle - \sqrt{\frac{1}{3}}|0,\frac{1}{2},1\rangle \ , \tag{50}$$

$$|^4P_{3/2}, J_3 = \frac{3}{2}\rangle = \sqrt{\frac{3}{5}}|1,\frac{1}{2},0\rangle - \sqrt{\frac{2}{15}}|1,-\frac{1}{2},1\rangle - \frac{2}{\sqrt{15}}|0,\frac{1}{2},1\rangle \ , \tag{51}$$

$$|^4P_{5/2}, J_3 = \frac{5}{2}\rangle = |1,\frac{1}{2},1\rangle \ . \tag{52}$$

The matrix elements of $\boldsymbol{L}\cdot\boldsymbol{S}_i$ $(i = d, Q)$ in the basis $[^2P_J, ^4P_J]$ are then found to be

$$\langle \boldsymbol{L}\cdot\boldsymbol{S}_d\rangle_{J=1/2} = \begin{bmatrix} -\frac{4}{3} & -\frac{\sqrt{2}}{3} \\ -\frac{\sqrt{2}}{3} & -\frac{5}{3} \end{bmatrix} \ , \quad \langle \boldsymbol{L}\cdot\boldsymbol{S}_Q\rangle_{J=1/2} = \begin{bmatrix} \frac{1}{3} & \frac{\sqrt{2}}{3} \\ \frac{\sqrt{2}}{3} & -\frac{5}{6} \end{bmatrix} \ , \tag{53}$$

$$\langle \boldsymbol{L}\cdot\boldsymbol{S}_d\rangle_{J=3/2} = \begin{bmatrix} \frac{2}{3} & -\frac{\sqrt{5}}{3} \\ -\frac{\sqrt{5}}{3} & -\frac{2}{3} \end{bmatrix} \ , \quad \langle \boldsymbol{L}\cdot\boldsymbol{S}_Q\rangle_{J=3/2} = \begin{bmatrix} -\frac{1}{6} & \frac{\sqrt{5}}{3} \\ \frac{\sqrt{5}}{3} & -\frac{1}{3} \end{bmatrix} \ , \tag{54}$$

while the matrix elements for $J = 5/2$ are

$$\langle \boldsymbol{L}\cdot\boldsymbol{S}_d\rangle_{J=5/2} = 1 \ , \quad \langle \boldsymbol{L}\cdot\boldsymbol{S}_Q\rangle_{J=5/2} = \frac{1}{2} \ . \tag{55}$$

The sums of the matrix elements for $\boldsymbol{L}\cdot\boldsymbol{S}_d$ and $\boldsymbol{L}\cdot\boldsymbol{S}_Q$ reproduce those of $\boldsymbol{L}\cdot\boldsymbol{S}$ quoted in Table VIII.

Table IX: Expectation values of $\boldsymbol{L}\cdot\boldsymbol{S}$ and tensor term S_{12} for P-wave $c\bar{s}$ mesons in the L–S-coupling basis states.

State	$\langle\boldsymbol{L}\cdot\boldsymbol{S}\rangle$	$\langle S_{12}\rangle$
$^3P_0$	–2	–4
$^1P_1$	0	0
$^3P_1$	–1	2
$^3P_2$	1	$-\frac{2}{5}$

j–j coupling

In the limit of very large m_Q, the term proportional to a_1 in the spin-dependent potential (1) dominates over the others, which may be treated perturbatively. Thus one can either work directly with j–j coupling (as in Sec. II or Ref. [9]) or use eigenstates of (53) and (54) to denote matrix elements of $\boldsymbol{L}\cdot\boldsymbol{S}_d$. For $J=1/2$ the eigenvalues λ and corresponding eigenvectors are

$$\lambda=-2:\ |J=1/2,j=0\rangle=\sqrt{\frac{1}{3}}|^2P_{1/2}\rangle+\sqrt{\frac{2}{3}}|^4P_{1/2}\rangle\ , \tag{56}$$

$$\lambda=-1:\ |J=1/2,j=1\rangle=\sqrt{\frac{2}{3}}|^2P_{1/2}\rangle-\sqrt{\frac{1}{3}}|^4P_{1/2}\rangle\ , \tag{57}$$

while for $J=3/2$ they are

$$\lambda=-1:\ |J=3/2,j=1\rangle=\sqrt{\frac{1}{6}}|^2P_{3/2}\rangle+\sqrt{\frac{5}{6}}|^4P_{3/2}\rangle\ , \tag{58}$$

$$\lambda=+1:\ |J=3/2,j=2\rangle=\sqrt{\frac{5}{6}}|^2P_{3/2}\rangle-\sqrt{\frac{1}{6}}|^4P_{3/2}\rangle\ . \tag{59}$$

Note that these states correspond to definite values of j, where $\boldsymbol{j}=\boldsymbol{L}+\boldsymbol{S}_d$. The coefficient of a_1 in the spin-dependent potential for the $J^P=\frac{5}{2}^+$ state is 1, and it has $j=2$.

Details of calculation for P-wave $c\bar{s}$ mesons

A calculation similar to that in the previous two subsections may be performed for mesons with one heavy quark. We chose the $c\bar{s}$ system because there exist candidates for all four expected levels. As in the case of Σ_Q baryons, we find it convenient to work in the j–j basis in which the analogue of the first term in Eq. (1), namely $a_1(c\bar{s})\boldsymbol{L}\cdot\boldsymbol{S}_s$, is diagonal. We first calculate the expectation values of $\boldsymbol{L}\cdot\boldsymbol{S}$ and the tensor operator S_{12} in the basis states $^{2S+1}P_J$, with S_d in Eq. (1) everywhere replaced by the spin S_s of the strange quark. The results are shown in Table IX. The expectation value of the hyperfine term $\boldsymbol{S}_s\cdot\boldsymbol{S}_Q$ is 1/4 for the ^3P states and –3/4 for the $^1P_1$ state.

Now we construct L–S basis states so as to evaluate the expectation values of $\boldsymbol{L}\cdot\boldsymbol{S}_i$ ($i=s,Q$). We label the states by $|S_{s3},S_{Q3},L_3\rangle$. We do not need to exhibit the $^3P_0$ state as it is pure $j = 1/2$. The results are

$$|{}^3P_2, J_3 = 2\rangle \;=\; |\frac{1}{2}, \frac{1}{2}, 1\rangle \;, \tag{60}$$

$$|{}^3P_1, J_3 = 1\rangle \;=\; \frac{1}{2}|-\frac{1}{2}, \frac{1}{2}, 1\rangle + \frac{1}{2}|\frac{1}{2}, -\frac{1}{2}, 1\rangle - \frac{1}{\sqrt{2}}|\frac{1}{2}, \frac{1}{2}, 0\rangle \;, \tag{61}$$

$$|{}^1P_1, J_3 = 1\rangle \;=\; \frac{1}{\sqrt{2}}|\frac{1}{2}, -\frac{1}{2}, 1\rangle - \frac{1}{\sqrt{2}}|-\frac{1}{2}, \frac{1}{2}, 1\rangle \;. \tag{62}$$

Using Eq. (47), in the basis $({}^1P_1, {}^3P_1)$, the matrices describing mixing of states are

$$\langle \boldsymbol{L} \cdot \boldsymbol{S}_s \rangle_{J=1} = \begin{bmatrix} 0 & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/2 \end{bmatrix} \;, \quad \langle \boldsymbol{L} \cdot \boldsymbol{S}_Q \rangle_{J=1} = \begin{bmatrix} 0 & -1/\sqrt{2} \\ -1/\sqrt{2} & -1/2 \end{bmatrix} \tag{63}$$

The eigenstates $[\alpha, \beta]^T$ of the first matrix satisfy $\beta/\sqrt{2} = -\alpha$ for eigenvalue -1 and $\beta = \alpha/\sqrt{2}$ for eigenvalue $1/2$. The relation between L–S eigenstates and (J, j) eigenstates is then

$$|J = 0, j = 1/2\rangle \;=\; |{}^3P_0\rangle \;, \tag{64}$$

$$|J = 1, j = 1/2\rangle \;=\; \sqrt{\frac{1}{3}}|{}^1P_1\rangle - \sqrt{\frac{2}{3}}|{}^3P_1\rangle \;, \tag{65}$$

$$|J = 1, j = 3/2\rangle \;=\; \sqrt{\frac{2}{3}}|{}^1P_1\rangle + \sqrt{\frac{1}{2}}|{}^3P_1\rangle \;, \tag{66}$$

$$|J = 2, j = 3/2\rangle \;=\; |{}^3P_2\rangle \;. \tag{67}$$

Using these expressions one can calculate the expectation values of all the operators contributing to the masses of the P-wave $c\bar{s}$ mesons, with the results shown in the text.

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