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Seminatural Gauge Mediation from Product Group Unification

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Abstract

We propose a focus point gauge mediation model based on the product group unification (PGU), which solves the doublet-triplet splitting problem of the Higgs multiplets. In the focus point gauge mediation, the electroweak symmetry breaking scale can be naturally explained even for multi-TeV stops. It is known that the focus point behavior appears if a ratio of the number of $SU(2)$ doublet messengers to that of $SU(3)$ triplet messengers is close to $5/2$. Importantly, this ratio (effectively) appears in our scenario based on the PGU, if the messenger field is an adjoint representation of $SU(5)$ gauge group. Therefore, our focus point scenario is very predictive. It is also pointed out the gravitino can be dark matter without spoiling the success of the thermal leptogenesis.

1 Introduction

Gauge mediated supersymmetry (SUSY) breaking (GMSB) [1]¹ is very attractive, since it is free from the flavor-changing neutral current problem in the SUSY standard model, which is a serious obstacle for the low-energy SUSY. More interestingly, most of the physical observables are predicted within the framework of renormalizable field theories, once a model of messenger multiplets are specified. In fact, the spectrum of SUSY particles is unambiguously calculated using the number of messengers and their SUSY invariant masses and SUSY breaking B -terms in a minimal GMSB.

Furthermore, it has been pointed out that a focus point behavior [3] is realized in the minimal GMSB if the numbers of $SU(2)_L$ doublet messengers N_D and $SU(3)_c$ triplet messengers N_T have certain values [4–6]² (see also [7] for an earlier work). It turns out that the ratio of N_D to N_T is always close to $5/2$. The focus point enables us to explain the origin of the electroweak symmetry breaking (EWSB) scale even when the masses of the SUSY particles are multi-TeV. Therefore, it is one of the important directions for the SUSY, after the discovery of the Higgs boson with a mass around 125 GeV [8]. In fact, the explanation of the observed Higgs boson mass requires large radiative corrections [9]: it is indicated that stops are at least heavier than 3-4 TeV [10], if their trilinear coupling has a moderate value.

In this paper, we show that the required ratio ($N_D/N_T \sim 5/2$) for the focus point SUSY in the minimal GMSB is indeed fixed by the product group unification [11, 12]. In the product group unification, the doublet-triplet splitting problem and the rapid proton decay problem, which are very severe in the minimal $SU(5)$ grand unified theory (GUT), are easily solved. We show that the messenger numbers are fixed to be $(N_T, N_D) = (2, 5)$ and the widely known fine-tuning measure Δ [13] is small as $\Delta=70$ -130 for the observed Higgs boson mass $m_h \simeq 125$ GeV. Moreover, we point out the gravitino can be a dark matter with a high reheating temperature while avoiding the over-closure of the universe. Thanks to this high reheating temperature, a sufficient baryon number is easily produced via the leptogenesis [14].

2 Focus point gauge mediation from product group unification

In the focus point SUSY, the fine-tuning of the EWSB scale is significantly milder than the naive expectation due to a special relation among soft SUSY breaking masses: even if the mass scale of the SUSY particles is much larger than the EWSB scale $v_{\text{obs}} \simeq 174.1$ GeV, the radiatively generated soft mass for the up-type Higgs is naturally close to the observed EWSB scale. Among the focus point SUSY scenarios, the present focus point GMSB is especially attractive, since the focus point behavior is controlled only by the number of messenger particles.

2.1 Focus point gauge mediation

In the focus point gauge mediation, the required relation among the SUSY breaking masses to relax the fine-tuning is obtained for $N_D/N_T \sim 5/2$, where N_D and N_T are the numbers of the $SU(2)_L$ doublet messengers and $SU(3)_c$ triplet messengers, respectively. To see this, let us show the EWSB conditions and how the relevant mass parameters in the Higgs potential are written in terms of the

¹For early attempts, see also Refs. [2].

²More precisely, in Ref. [6] the focus point SUSY is achieved by fixing a combination of parameters in the superpotential of the messenger sector. Importantly, this combination is RGE invariant. Therefore, the focus point behavior is expected to be robust.

soft mass parameters at the high energy scale. The EWSB scale and the ratio of the Higgs VEVs ($\tan \beta \equiv v_u/v_d$) are determined by the stationary conditions of the Higgs potential:

$$\begin{aligned} \left[\frac{(3/5)g_1^2 + g_2^2}{4} \right] v^2 &\simeq -\mu^2 - \frac{(m_{H_u}^2 + \frac{1}{2v_u} \frac{\partial \Delta V}{\partial v_u}) \tan^2 \beta - (m_{H_d}^2 + \frac{1}{2v_d} \frac{\partial \Delta V}{\partial v_d})}{\tan^2 \beta - 1} \Big|_{M_{IR}}, \\ B_\mu(\tan \beta + \cot \beta) &\simeq m_{H_d}^2 + \frac{1}{2v_d} \frac{\partial \Delta V}{\partial v_d} + m_{H_u}^2 + \frac{1}{2v_u} \frac{\partial \Delta V}{\partial v_u} + 2\mu^2 \Big|_{M_{IR}}, \end{aligned} \quad (1)$$

where M_{IR} is taken to be the stop mass scale, $M_{IR} = (m_{Q_3} m_{\bar{U}_3})^{1/2}$, g_1 and g_2 are gauge coupling constants for $U(1)_Y$ (in the $SU(5)$ GUT normalization) and $SU(2)_L$, respectively, and ΔV is a one-loop effective potential. The soft SUSY breaking mass for the up-type (down-type) Higgs is denoted by m_{H_u} (m_{H_d}). Including ΔV is important to evaluate the fine-tuning for the large stop mass, and it may even be the dominant source of the fine-tuning required to obtain the observed EWSB scale. For large $\tan \beta \gtrsim 10$, which is preferred to enhance the Higgs boson mass, $(m_{H_u}^2 + \frac{1}{2v_u} \frac{\partial \Delta V}{\partial v_u})$ is important to determine the EWSB scale v .

With this in mind, the soft masses for the up-type Higgs at M_{IR} is written in terms of the soft SUSY breaking masses at the messenger scale M_{mess} :

$$\begin{aligned} m_{H_u}^2(4\text{TeV}) &= 0.789m_{H_u}^2 + 0.012m_{H_d}^2 \\ &- 0.236m_Q^2 - 0.140m_{\bar{U}}^2 - 0.032m_{\bar{E}}^2 \\ &+ 0.031m_L^2 - 0.030m_{\bar{D}}^2 \\ &+ 0.011M_{\tilde{b}}^2 + 0.139M_{\tilde{w}}^2 - 0.253M_{\tilde{g}}^2 \\ &- 0.017M_{\tilde{w}}M_{\tilde{g}} - 0.002M_{\tilde{b}}M_{\tilde{g}}, \end{aligned} \quad (2)$$

for $M_{\text{mess}} = 10^9$ GeV and

$$\begin{aligned} m_{H_u}^2(4\text{TeV}) &= 0.744m_{H_u}^2 + 0.016m_{H_d}^2 \\ &- 0.288m_Q^2 - 0.157m_{\bar{U}}^2 - 0.044m_{\bar{E}}^2 \\ &+ 0.043m_L^2 - 0.041m_{\bar{D}}^2 \\ &+ 0.011M_{\tilde{b}}^2 + 0.170M_{\tilde{w}}^2 - 0.444M_{\tilde{g}}^2 \\ &- 0.002M_{\tilde{b}}M_{\tilde{w}} - 0.035M_{\tilde{w}}M_{\tilde{g}} - 0.005M_{\tilde{b}}M_{\tilde{g}}, \end{aligned} \quad (3)$$

for $M_{\text{mess}} = 10^{11}$ GeV, where a coefficient smaller than 10^{-3} is omitted. Soft SUSY breaking masses for $SU(2)$ doublet squarks, $SU(2)$ singlet up squarks, and down squarks are denoted by $m_Q, m_{\bar{U}}$ and $m_{\bar{D}}$, m_L and $m_{\bar{E}}$ are soft SUSY breaking masses for left-handed and right-handed sleptons, and $M_{\tilde{b}}$, $M_{\tilde{w}}$ and $M_{\tilde{g}}$ are the bino, wino and gluino mass, respectively. Here, soft SUSY breaking mass parameters in the right hand side of the above equations are defined at M_{mess} . We take $\tan \beta = 25$, $m_t = 173.34$ GeV and $\alpha_s(m_Z) = 0.1185$.

Let us consider the minimal gauge mediation model with N_T pairs of the $SU(3)_c$ triplet messengers and N_D pairs of the $SU(2)_L$ doublet messengers:

$$W = \lambda_D Z \Psi_D^a \Psi_D^a + \lambda_T Z \Psi_T^I \Psi_T^I, \quad (4)$$

where Ψ_D^a is a $SU(2)_L$ doublet messenger and Ψ_T^I is a $SU(3)_c$ triplet messenger, and $a = 1 \dots N_D$ and $I = 1 \dots N_T$. All soft SUSY breaking masses for the MSSM particles are generated from messenger loops at the messenger scale $M_{\text{mess}} \simeq \lambda_D \langle Z \rangle \simeq \lambda_T \langle Z \rangle$ (see Appendix A.1 for details). Then, $m_{H_u}^2$ can be written as

$$m_{H_u}^2(4\text{TeV}) = [-0.253N_T^2 - 0.011N_D N_T - 1.073N_T$$

$$+ 0.058N_D^2 + 0.380N_D] \left(\frac{\alpha_3}{4\pi} \Lambda \right)^2, \quad (5)$$

for $M_{\text{mess}} = 10^9$ GeV, and

$$\begin{aligned} m_{H_u}^2(4 \text{ TeV}) &= [-0.444N_T^2 - 0.026N_DN_T - 1.286N_T \\ &+ 0.092N_D^2 + 0.431N_D] \left(\frac{\alpha_3}{4\pi} \Lambda \right)^2, \end{aligned} \quad (6)$$

for $M_{\text{mess}} = 10^{11}$ GeV. The contribution to the soft SUSY breaking masses from the messenger loops is parametrized by $\Lambda \equiv \langle F_Z \rangle / \langle Z \rangle$, where $\langle Z \rangle$ and $\langle F_Z \rangle$ originate from the A -term and F -term of the SUSY breaking field Z , respectively. Notice that the relevant parameter Λ is independent of unknown Yukawa couplings λ_T and λ_D [5]. The combination $\alpha_3\Lambda/(4\pi)$ corresponds to the gluino mass scale. If the messenger numbers are taken as $(N_T, N_D) = (2, 5)$, one obtain

$$m_{H_u}^2(4 \text{ TeV}) = 0.020 \left(2 \cdot \frac{\alpha_3}{4\pi} \Lambda \right)^2, \quad (7)$$

for $M_{\text{mess}} = 10^9$ GeV and

$$m_{H_u}^2(4 \text{ TeV}) = -0.035 \left(2 \cdot \frac{\alpha_3}{4\pi} \Lambda \right)^2, \quad (8)$$

for $M_{\text{mess}} = 10^{11}$ GeV: $m_{H_u}^2(4 \text{ TeV})$ is much smaller than the mass scale of the colored SUSY particles. Interestingly, the required numbers of N_D and N_T can be obtained in the product group unified (PGU) theory, which is proposed to solve the doublet-triplet splitting problem and the proton rapid decay problem.

2.2 Product group unification

We consider $SU(5) \times U(3)_H$ model, where $U(3)_H \simeq SU(3)_H \times U(1)_H$. In the PGU, the superpotential is given by

$$W = \mu H^i \bar{H}_i + H^i \bar{B}_i^\alpha \bar{T}_\alpha + \bar{H}_i B_\alpha^i T^\alpha, \quad (9)$$

where B and \bar{B} are bi-fundamental fields, transforming as $B = (\mathbf{5}, \bar{\mathbf{3}})$ and $\bar{B} = (\bar{\mathbf{5}}, \mathbf{3})$ under $SU(5) \times SU(3)_H$ gauge group, and T and \bar{T} are $SU(3)_H$ triplets, $T = (\mathbf{1}, \mathbf{3})$ and $\bar{T} = (\mathbf{1}, \bar{\mathbf{3}})$. The $SU(5) \times U(3)_H$ breaks down to $SU(3)_c \times SU(2)_L \times U(1)_Y$ by the VEV of B and \bar{B} . The relevant superpotential is given by

$$W = Y(\bar{B}_i^\alpha B_\alpha^i - 3v_B^2) + B_\alpha^i A_\beta^\alpha \bar{B}_i^\beta \quad (10)$$

where Y and A are singlet and adjoint of $SU(3)_H$, and v_B is of the order of the GUT scale. The charge assignment which is consistent with the seesaw mechanism [15] is shown in Table 1. Here, N denotes a right-handed neutrino, Σ_{24} and Σ'_8 are a messenger superfield and its $SU(3)_H$ counterpart, respectively, and Z is a SUSY breaking field. Although the $U(1)_R$ is anomalous and broken at the quantum level, we use it for constraining the classical Lagrangian.³ The superpotential of the messenger sector will be discussed later.

³Note that the $U(1)_R$ charge assignment in Table 1 does not forbids dimension-five operators $\mathbf{101010\bar{5}}$, which can lead to dangerously large proton decay rates even when suppressed by the Planck scale [16, 17]. However, the dimension-five operators are easily suppressed by changing the R charge assignment [12] without spoiling the main conclusions in this paper.

Table 1: The charge assignment.

	$\bar{\mathbf{5}}$	$\mathbf{10}$	N	H	\bar{H}	B	\bar{B}	T	\bar{T}	Y	A	Σ_{24}	Σ'_8	Z
$SU(5)_{\text{GUT}}$	$\bar{\mathbf{5}}$	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{5}$	$\bar{\mathbf{5}}$	$\mathbf{5}$	$\bar{\mathbf{5}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{24}$	$\mathbf{1}$	$\mathbf{1}$
$SU(3)_H$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\bar{\mathbf{3}}$	$\mathbf{3}$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\mathbf{1}$	$\mathbf{8}$	$\mathbf{1}$	$\mathbf{8}$	$\mathbf{1}$
$U(1)_H$	0	0	0	0	0	1	-1	-1	1	0	0	0	0	0
$U(1)_R$	1/5	3/5	1	4/5	6/5	0	0	4/5	6/5	2	2	-1	3	4

With the superpotential above, the bi-fundamental fields B and \bar{B} get VEVs:

$$\langle B_\alpha^i \rangle = v_B \delta_\alpha^i, \quad \langle \bar{B}_i^a \rangle = v_B \delta_i^a. \quad (11)$$

As a result $SU(5) \times U(3)_H$ breaks down to $SU(3)_c \times SU(2)_L \times U(1)_Y$. The gauge couplings are unified approximately at the GUT scale as

$$g_1^2/g_2^2 = \left(1 + \frac{(1/15)g_5^2}{g_{1H}^2}\right)^{-1}, \quad g_3^2/g_2^2 = \left(1 + \frac{g_5^2}{g_{3H}^2}\right)^{-1}, \quad g_2^2 = g_5^2 \quad (12)$$

where $g_{1H}, g_{3H} \gg g_5$ is assumed such that $g_1^2/g_2^2 \approx 1$ and $g_3^2/g_2^2 \approx 1$ are satisfied. The gauge couplings of $U(1)_Y$ and $SU(3)_c$ are predicted to be slightly smaller than that of $SU(2)_L$ at the GUT scale.

2.3 Messenger sector

Now let us consider the messenger sector. Since MSSM particles get soft SUSY breaking masses through messenger loops with the SM gauge interactions, the messenger superfields belong to representations of $SU(5)_{\text{GUT}}$. From a small representation, following possibilities are listed:

- (a) $\mathbf{5} + \bar{\mathbf{5}}$ case, where $\mathbf{5} = (\mathbf{3}, \mathbf{1}) + (\mathbf{1}, \mathbf{2})$
- (b) $\mathbf{10} + \bar{\mathbf{10}}$ case, where $\mathbf{10} = (\mathbf{3}, \mathbf{2}) + (\bar{\mathbf{3}}, \mathbf{1}) + (\mathbf{1}, \mathbf{1})$
- (c) $\mathbf{15} + \bar{\mathbf{15}}$ case, where $\mathbf{15} = (\mathbf{1}, \mathbf{3}) + (\bar{\mathbf{3}}, \mathbf{2}) + (\mathbf{6}, \mathbf{1})$
- (d) $\mathbf{24}$ case, where $\mathbf{24} = (\mathbf{1}, \mathbf{3}) + (\mathbf{8}, \mathbf{1}) + (\bar{\mathbf{3}}, \mathbf{2}) + (\mathbf{3}, \mathbf{2}) + (\mathbf{1}, \mathbf{1})$,

where (n_1, n_2) corresponds to n_1 and n_2 dimensional representations of $SU(3)_c$ and $SU(2)_L$. Larger representations are not attractive since they lead to the Landau pole below the GUT scale, unless the messenger scale is sufficiently high.

If all of the messengers participate in generation of the soft SUSY breaking masses for the MSSM particles, $N_D/N_T = 1$ in all the cases. However, $SU(2)_L$ singlets can be made heavy by the mechanism described below in the PGU, so that we effectively get $N_D/N_T \gg 1$. Then, the case (a) corresponds to $(N_T, N_D) = (0, 1)$. For (b) we can have $(N_T, N_D) = (2, 3)$ if the messenger in the representation $(\bar{\mathbf{3}}, \mathbf{1})$ (and $(\mathbf{3}, \mathbf{1})$) is heavy. However, $N_D/N_T = 1.5$ is too small to realize the focus point. For (c), one can obtain $(N_T, N_D) = (2, 7)$ if the messenger in $(\mathbf{6}, \mathbf{1})$ and its vector-like partner are heavy. In this case, N_D/N_T is too large. Finally in the case (d), we obtain the sparticle mass spectrum corresponding to $(N_T, N_D) = (2, 5)$, if the octet messenger in $(\mathbf{8}, \mathbf{1})$ representation is heavy. Based on these considerations, we choose the messenger superfields in the adjoint representation of $SU(5)_{\text{GUT}}$. This is a clear advantage over the models in Refs. [4–6], where the required numbers of (N_T, N_D) are chosen by hand.

The superpotential in the messenger sector is given by

$$W = \lambda_{24} Z (\Sigma_{24})_j^i (\Sigma_{24})_i^j + \frac{c_0}{M_P} \bar{B}_i^\alpha (\Sigma_{24})_j^i (\Sigma'_8)_\alpha^\beta B_\beta^j, \quad (13)$$

where Z is a SUSY breaking field, and the charge assignment of the fields is shown in Table 1. The first term gives

$$\lambda_{24} (\langle Z \rangle + \langle F_Z \rangle \theta^2) \left(X \bar{X} + \text{Tr}(\Sigma_3^2) + \text{Tr}(\Sigma_8^2) \right), \quad (14)$$

where X , \bar{X} , Σ_3 and Σ_8 correspond to $(\mathbf{3}, \mathbf{2})$, $(\bar{\mathbf{3}}, \mathbf{2})$, $(\mathbf{1}, \mathbf{3})$ and $(\mathbf{8}, \mathbf{1})$, and $U(1)_Y$ charges of X and \bar{X} are $-5/6$ and $5/6$. From the second term in Eq.(13), the octet messenger Σ_8 has as a SUSY-invariant Dirac mass with Σ'_8 , $W \ni \mu_8 \text{Tr}(\Sigma_8 \Sigma'_8)$. This dirac mass is estimated as $\mu_8 = 10^{12}-10^{14}$ GeV, depending on v_B and c_0 . The mass eigenvalues are

$$M_\pm^2 = \mu_8^2 \left[1 + \frac{1}{2} k^2 \pm \frac{1}{2} k \sqrt{4 + k^2} \right], \quad (15)$$

where $k = \lambda_{24} \langle Z \rangle / \mu_8$. Provided that $\mu_8 \gg \lambda_{24} \langle Z \rangle$, the mass spectrum of the MSSM particles is determined by the light messengers, X , \bar{X} and Σ_3 . The relevant part of the messenger sector is given by

$$W = \lambda_X (\langle Z \rangle + \langle F_Z \rangle \theta^2) X \bar{X} + \lambda_3 (\langle Z \rangle + \langle F_Z \rangle \theta^2) \text{Tr}(\Sigma_3^2), \quad (16)$$

where $\lambda_X \sim \lambda_3$. Then, we have

$$\begin{aligned} M_{\tilde{w}} &\simeq \frac{\alpha_2}{4\pi} (5\Lambda), & M_{\tilde{g}} &\simeq \frac{\alpha_3}{4\pi} (2\Lambda), \\ m_Q^2 &\simeq \frac{8}{3} \left(\frac{\alpha_3}{4\pi} \right)^2 (2\Lambda^2) + \frac{3}{2} \left(\frac{\alpha_2}{4\pi} \right)^2 (5\Lambda^2), \\ m_U^2 &= m_D^2 \simeq \frac{8}{3} \left(\frac{\alpha_3}{4\pi} \right)^2 (2\Lambda^2), \\ m_{H_u}^2 &= m_{H_d}^2 = m_L^2 \simeq \frac{3}{2} \left(\frac{\alpha_2}{4\pi} \right)^2 (5\Lambda^2), \end{aligned} \quad (17)$$

where we have neglected the contributions proportional to α_1 and α_1^2 . Apart from the $U(1)_Y$ contributions, the generated mass spectrum is essentially the same as that in the minimal gauge mediation model defined by Eq.(4) with $(N_T, N_D) = (2, 5)$. Therefore, as shown in Eq.(7) and (8), this gauge mediation model using adjoint messengers significantly reduces the fine-tuning of the EWSB scale.

With the messenger multiplets Σ_{24} and the additional multiplet Σ'_8 , the evolution of the gauge couplings are shown in Fig. 1, neglecting small mass splitting of M_\pm , i.e. $k = 0$. We use two-loop renormalization group equations [18]. The gauge coupling unification holds in a non-trivial way.

3 The fine-tuning and mass spectrum

Let us numerically estimate the fine-tuning Δ in the region where the observed Higgs boson mass is explained. We also show the mass spectra of the MSSM particles in the relevant region. In numerical calculation, we use `softsusy 3.6.1` [19] to evaluate the SUSY mass spectra. The Higgs boson mass is calculated using `FeynHiggs 2.11.2` [20].

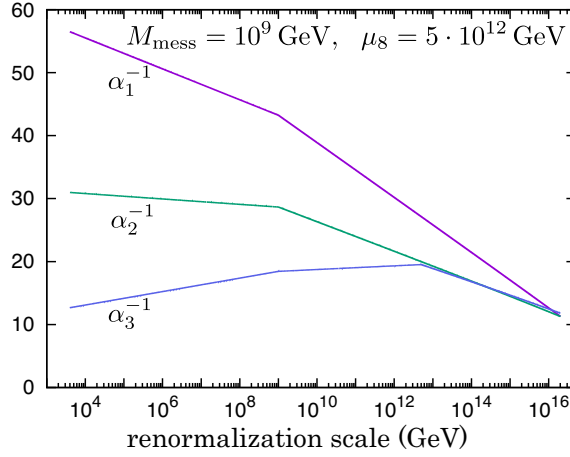


Figure 1: The running of the gauge coupling with adjoint messengers. We use two-loop renormalization group equations and take $m_{\text{SUSY}} = 4 \text{ TeV}$.

3.1 The fine-tuning

To estimate the fine-tuning, we employ the following fine-tuning measure [13]:⁴

$$\Delta = \max\{\Delta_a\}, \quad \Delta_a = \left\{ \frac{\partial \ln v}{\partial \ln \mu}, \frac{\partial \ln v}{\partial \ln |F_Z|}, \frac{\partial \ln v}{\partial \ln B_{\text{mess}}} \right\}_{v=v_{\text{obs}}}, \quad (19)$$

where $v_{\text{obs}} \simeq 174.1 \text{ GeV}$, and B_{mess} is the Higgs B -term at the messenger scale, $B_\mu/\mu|_{M_{\text{mess}}}$. Here, $V \ni B_\mu H_u H_d + h.c.$ and $W \ni \mu H_u H_d$.

Now, let us show the required Δ to explain the observed Higgs boson mass around 125 GeV is significantly reduced. In Fig. 2, we show the contours of the calculate Higgs boson mass m_h and Δ . The top pole mass is taken to be $m_t(\text{pole}) = 174.34 \text{ GeV}$ and $\alpha_s(m_Z) = 0.1185$. The observed Higgs boson mass is explained with a mild fine-tuning $\Delta = 70\text{-}130$. Here, we comment on the consistent range of m_h with the measured value. The combined measurement of the Higgs boson mass by the ATLAS and CMS has an uncertainty $\pm 0.5 \text{ GeV}$ at 2σ level [8], and the experimental uncertainty in the top mass measurement is $\pm 1.5 \text{ GeV}$ at 2σ level [21]. Moreover, the theoretical uncertainty estimated by FeynHiggs 2.11.2 is about $\pm 1 \text{ GeV}$ for the Higgs boson mass. In total, we consider the region of $m_h \gtrsim 123 \text{ GeV}$ to be consistent with the measured Higgs boson mass.

For comparison, we also show that the results in the minimal gauge mediation model with $N_5 = N_T = N_D = 2$ in Fig. 3. In the region $m_h = 123\text{-}125 \text{ GeV}$, $\Delta = 750\text{-}1500$ for $M_{\text{mess}} = 10^{8-9} \text{ GeV}$. We see that the fine-tuning in our focus point gauge mediation model is about ten times better than the minimal gauge mediation model with the complete $SU(5)$ multiplets.

3.2 Mass spectra of the SUSY particles

Here, we list Δ and mass spectra of the SUSY particles for different model points in Table. 2. The model point **P1** (**P2**) is shown as the blue (red) star in Fig. 2. It can be seen that the observed Higgs

⁴The result of the derivative with respect to the messenger scale is similar to the derivative with respect to $|F_Z|$. This is because

$$\frac{\partial}{\partial \ln M_{\text{mess}}} \ln v = \left(-\frac{\partial}{\partial \ln \Lambda} + \frac{\partial}{\partial \ln M_{\text{mess}}} \Big|_{\text{fixed } \Lambda} \right) \ln v. \quad (18)$$

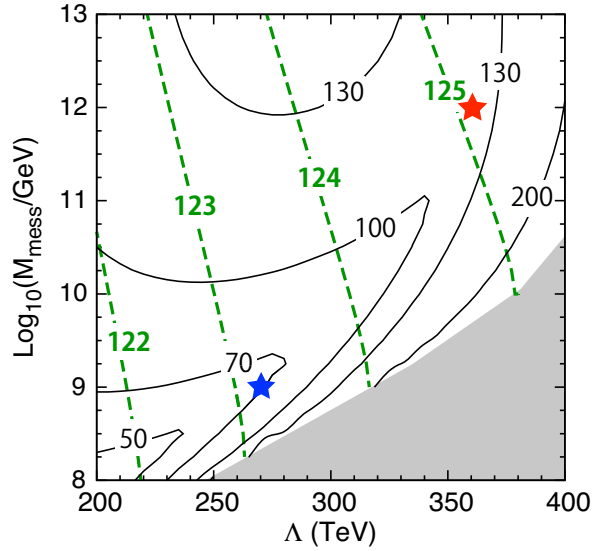


Figure 2: The Higgs boson mass in the unit of GeV (green) and Δ (black) in the gauge mediation model with adjoint messengers. We take $\tan \beta = 25$, $m_t(\text{pole}) = 173.34$ GeV and $\alpha_s(m_Z) = 0.1185$.

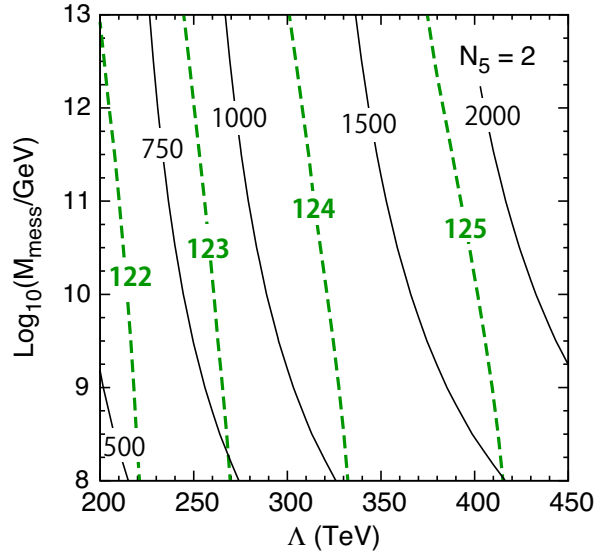


Figure 3: The Higgs boson mass and Δ in the minimal gauge mediation with $N_5 = 2$. The other parameters are the same in Fig. 2.

boson mass is explained for $m_{\text{stop}} = 3\text{-}5$ TeV. In this case, the gluino mass is as heavy as 3.5-5 TeV. Still, the fine-tuning Δ is as mild as 70-120.

4 The gravitino dark matter

Finally, the gravitino dark matter is discussed. We show that the gravitino can be dark matter without spoiling the success of the thermal leptogenesis.

In gauge mediation models, the gravitino is the lightest SUSY particle and a dark matter candidate. The abundance of the gravitino can be estimated for different two cases: $T_R > T_f$ and $T_R < T_f$, where

Table 2: Mass spectra and Δ for different model points.

P1		P2	
M_{mess}	10^9 GeV	M_{mess}	10^{12} GeV
Λ	270 TeV	Λ	360 TeV
$\tan \beta$	25	$\tan \beta$	25
m_h	123.2 GeV	m_h	125.1 GeV
Δ	67	Δ	109
$ \Delta_\mu $	56	$ \Delta_\mu $	108
μ	492 GeV	μ	685 GeV
$m_{3/2}$	$64 \text{ keV}/\lambda_{24}$	$m_{3/2}$	$86 \text{ MeV}/\lambda_{24}$
m_{gluino}	3.6 TeV	m_{gluino}	4.7 TeV
m_{squark}	$3.7\text{--}4.5 \text{ TeV}$	m_{squark}	$4.7\text{--}6.1 \text{ TeV}$
m_{stop}	$3.1, 4.2 \text{ TeV}$	m_{stop}	$3.6, 5.5 \text{ TeV}$
m_A	2.5 TeV	m_A	3.7 TeV
$m_{\tilde{e}_L}(m_{\tilde{\mu}_L})$	2.7 TeV	$m_{\tilde{e}_L}(m_{\tilde{\mu}_L})$	4.1 TeV
$m_{\tilde{e}_R}(m_{\tilde{\mu}_R})$	1.4 TeV	$m_{\tilde{e}_R}(m_{\tilde{\mu}_R})$	2.4 TeV
$m_{\tilde{\tau}_1}$	1.3 TeV	$m_{\tilde{\tau}_1}$	2.2 TeV
$m_{\chi_1^0}$	503 GeV	$m_{\chi_1^0}$	701 GeV
$m_{\chi_1^\pm}$	505 GeV	$m_{\chi_1^\pm}$	703 GeV
$m_{\chi_2^\pm}$	3.4 TeV	$m_{\chi_2^\pm}$	4.6 TeV

T_R is the reheating temperature and T_f is the freeze-out temperature of the gravitino,

$$T_f \simeq 9.3 \text{ TeV} \left(\frac{g_*(T_f)}{230} \right)^{1/2} \left(\frac{m_{3/2}}{100 \text{ keV}} \right)^2 \left(\frac{5 \text{ TeV}}{m_{\tilde{g}}} \right)^2. \quad (20)$$

If the reheating temperature is smaller than T_f , the gravitino is produced mainly from the thermal scattering and the decay of superparticles [22]. In this case, numerical analysis shows us that the reheating temperature should be lower than about 100 GeV – 1 TeV in order that gravitino does not overclose the Universe [23].

The second case, $T_R > T_f$, may be more attractive since the observed baryon number is explained by the thermal leptogenesis [14]. With this high reheating temperature, the gravitino is thermalized and the relic abundance is estimated as

$$\Omega_{3/2}^{th} h^2 \simeq 50 \cdot \left(\frac{m_{3/2}}{100 \text{ keV}} \right) \left(\frac{230}{g_*(T_f)} \right), \quad (21)$$

which does not depend on T_R . Therefore, for $m_{3/2} \simeq 100 \text{ keV}$, if we obtain a dilution factor, $D \simeq 490$, from the extra entropy production, the observed dark matter abundance is explained. This dilutes the baryon asymmetry as well, but that generated by the thermal leptogenesis can be large enough as discussed later.

The significant entropy production may occur for $T_R > M_{24} (\equiv \lambda_{24} \langle Z \rangle)$ [24, 25]. Provided that the decay temperature of the messenger T_d is sufficiently small,⁵ the energy density of the messenger

⁵To be more precise, the messenger field dominates the energy density of the universe at the temperature [24]

$$T_c \simeq \frac{4}{3} M_{\text{mess}} Y_{\text{mess}} \simeq 490 \text{ GeV} \left(\frac{M_{\text{mess}}}{10^9 \text{ GeV}} \right)^2, \quad (22)$$

once dominates the energy density of the universe after the gravitino is freeze-out. Then, it decays via the superpotential:⁶

$$W_{\text{decay}} = \frac{k_0}{M_P} \bar{H}^2 \mathbf{10} \Sigma_{24}. \quad (23)$$

The decay of the octet messenger Σ_8 is suppressed by the colored Higgs mass, and, we thus assume it is heavier than the reheating temperature and not created, so that we hereafter consider the decays of the $SU(2)$ charged messengers, X , \bar{X} and Σ_3 . In the following, we calculate the scalar messenger decay. Note that we have to take into account not only the interaction with fermions arising solely from W_{decay} , but also the F -term potential involving the effective mass term of Σ_{24} . The total decay width is

$$\Gamma_{\text{mess}} \simeq 10^{-4} k_0^2 \frac{M_{\text{mess}}^3}{M_P^2}. \quad (24)$$

The dilution factor is estimated as

$$D \left(\equiv \frac{s_{\text{after}}}{s_{\text{before}}} \right) \simeq \frac{4}{3} \frac{M_{\text{mess}} Y_{\text{mess}}}{T_d} \simeq 490 \left(\frac{M_{\text{mess}}}{10^9 \text{ GeV}} \right)^2 \left(\frac{1 \text{ GeV}}{T_d} \right), \quad (25)$$

where s_{after} and s_{before} is the entropy density after or before the messenger decay, respectively, and T_d is the decay temperature:

$$T_d \simeq \left(\frac{45}{2\pi^2 g_*(T_d)} \right)^{1/4} \sqrt{M_P \Gamma_{\text{mess}}} \sim 90 k_0 \left(\frac{61.75}{g_*(T_d)} \right)^{1/4} \left(\frac{M_{\text{mess}}}{10^9 \text{ GeV}} \right)^{3/2} \text{ GeV}. \quad (26)$$

Thus, D can be written as follows.

$$D \simeq \frac{490}{90 k_0} \left(\frac{M_{\text{mess}}}{10^9 \text{ GeV}} \right)^{1/2} \left(\frac{61.75}{g_*(T_d)} \right)^{-1/4} \quad (27)$$

The gravitino abundance is the following:

$$\Omega_{3/2} h^2 = (1/D) \Omega_{3/2}^{th} h^2 \simeq 0.1 \cdot \left(\frac{490}{D} \right) \left(\frac{m_{3/2}}{100 \text{ keV}} \right) \left(\frac{230}{g_*(T_f)} \right). \quad (28)$$

Here, one can see that the observed relic abundance $\Omega_{3/2} h^2 \simeq 0.12$ is naturally achieved without any significant tunings. It is emphasized that the abundance of the gravitino no longer depends on the reheating temperature as long as the messengers were once in the thermal bath. The sufficient baryon number can be produced thorough the leptogenesis for $M_N \gtrsim 10^{11-12} \text{ GeV}$ [24, 26] and $T_R \gtrsim M_N$, where M_N is the mass of the right-handed neutrino.

5 Conclusion and discussion

We have proposed a focus point gauge mediation model based on the product group unification. It has been shown that the observed Higgs boson mass around 125 GeV is explained with a mild fine-tuning $\Delta \sim 70$ -130, an order of magnitude improvement over the minimal gauge mediation models. In addition, the focus point naturally appears from in the PGU if the messenger field is an adjoint

where Y_{mess} is the number density of the messengers divided by the entropy density. Hence, $T_c > T_d$ is required.

⁶There also exists $K = (1/M_P) H \Sigma_{24} \bar{\mathbf{5}} + h.c.$ and $K = (1/M_P) \bar{H} \Sigma_{24} (\bar{\mathbf{5}}_i)^\dagger + h.c..$ However, the former becomes $W \sim (m_{3/2}/M_P) H \Sigma_{24} \bar{\mathbf{5}}$ after the Kähler transformation, which leads to a very small decay width and, because of the equation of the motion, the latter is equivalent to Eq. (23) with the down-type Yukawa suppression.

representation of $SU(5)$ gauge group; unlike other focus point gauge mediation models, we do not need to choose the continuous parameters, which control the focus point behavior. This enables our scenario to be very predictive. On the cosmological aspect, the gravitino can be dark matter without spoiling the success of the thermal leptogenesis. The observed baryon number of the universe is easily explained without the overproduction of the gravitino.

An unfortunate consequence of our model is that the masses of the SUSY particles are rather heavy, and it may be challenging to test it at the LHC,⁷ even with the high luminosity running. However, the relevant region with a mild fine-tuning is expected to be covered by the high energy upgrade of the LHC to 33 TeV [27]. The higgsino is always the next-lightest supersymmetric particle beyond the gravitino, and can be lighter than 500 GeV. Therefore, it may be a target at the future e^+e^- linear collider experiments, such as ILC and CLIC.

Finally, let us comment on the proton decay, which can be a probe of our scenario. In the PGU, the colored Higgs multiplets are heavy without a difficulty, and hence, induced dimension five proton decay can be suppressed. On the other hand, for the dimension six proton decay induced by X and Y gauge bosons, the decay rate of the proton can be ten times larger than that in the MSSM without any extra-particles [28]. This is because we have the messenger field with a large representation of $SU(5)$, contributing the beta-functions of the standard model gauge couplings. Therefore, the $SU(5)$ gauge coupling constant is larger than that of the standard SUSY GUT. Thus, the dimension six proton decay, in particular the $p \rightarrow e^+\pi^0$ mode, is a good target at the proposed Hyper-Kamiokande experiment [29].

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A The soft SUSY breaking masses in gauge mediated SUSY breaking

Here, we list the formulae for the gaugino and scalar masses in GMSB models. We consider two models: the one is the model with messenger multiplets, which are fundamental representations of $SU(2)_L$ and $SU(3)_c$, and the other is a model with messenger multiplet in the adjoint representation of $SU(5)$.

⁷In our scenario, the next-to-lightest SUSY particle is stable at the collider time scale, because of the gravitino mass larger than about 100 keV.

A.1 Messenger in fundamental representation

We consider a gauge mediation model with N_D and N_T pairs of the messenger multiplets transforming in the fundamental representation of $SU(2)_L$ and $SU(3)_c$. The relevant superpotential is given by

$$W = (\lambda_D Z + M_D) \Psi_D^a \Psi_D^a + (\lambda_T Z + M_T) \Psi_T^I \Psi_T^I, \quad (29)$$

where Ψ_D and Ψ_T are $SU(2)_L$ doublet and $SU(3)_c$ triplet, respectively, and $U(1)_Y$ charges of Ψ_D and Ψ_T are taken as $(-1/2)$ and $(1/3)$. The index a (I) runs 1 to N_D (N_T).

Then, gaugino masses are given by

$$M_{\tilde{b}} \simeq \frac{g_1^2}{16\pi^2} \left(\frac{3}{5} N_D + \frac{2}{5} N_T \right) \Lambda, \quad M_{\tilde{w}} \simeq \frac{g_2^2}{16\pi^2} N_D \Lambda, \quad M_{\tilde{g}} \simeq \frac{g_3^2}{16\pi^2} N_T \Lambda, \quad (30)$$

where $\Lambda = \lambda_D F_Z / M_D = \lambda_T F_Z / M_T$, provided that $\lambda_T = \lambda_D$ and $M_T = M_D$ hold at the GUT scale. The SM gauge couplings of $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ are denoted by g_3 , g_2 and g_1 . Here, $\lambda_{(D,T)} \langle Z \rangle \ll M_{(D,T)}$ is assumed. Scalar masses are

$$\begin{aligned} m_{\tilde{Q}}^2 &\simeq \frac{2}{(16\pi^2)^2} \left[\frac{4}{3} g_3^4 (N_T \Lambda^2) + \frac{3}{4} g_2^4 (N_D \Lambda^2) + \frac{3}{5} g_1^4 (\tilde{\Lambda}_1^2) \frac{1}{6^2} \right], \\ m_{\tilde{U}}^2 &\simeq \frac{2}{(16\pi^2)^2} \left[\frac{4}{3} g_3^4 (N_T \Lambda^2) + \frac{3}{5} g_1^4 (\tilde{\Lambda}_1^2) \left(\frac{2}{3} \right)^2 \right], \\ m_{\tilde{D}}^2 &\simeq \frac{2}{(16\pi^2)^2} \left[\frac{4}{3} g_3^4 (N_T \Lambda^2) + \frac{3}{5} g_1^4 (\tilde{\Lambda}_1^2) \frac{1}{3^2} \right], \\ m_{\tilde{L}}^2 &\simeq \frac{2}{(16\pi^2)^2} \left[\frac{3}{4} g_2^4 (N_D \Lambda^2) + \frac{3}{5} g_1^4 (\tilde{\Lambda}_1^2) \frac{1}{2^2} \right], \\ m_{\tilde{E}}^2 &\simeq \frac{2}{(16\pi^2)^2} \left[\frac{3}{5} g_1^4 (\tilde{\Lambda}_1^2) \right], \\ m_{H_u}^2 &= m_{H_d}^2 = m_L^2, \end{aligned} \quad (31)$$

where $\tilde{\Lambda}_1^2 \equiv [(3/5)N_D + (2/5)N_T]\Lambda^2$.

A.2 Messenger in adjoint representation

The messenger multiplet in the **24** representation of $SU(5)$ is considered. After $SU(5)$ is broken down to the SM gauge group, the superpotential in the messenger sector is written as

$$W = (\lambda_8 Z + M_8) \text{Tr}(\Sigma_8^2) + (\lambda_3 Z + M_3) \text{Tr}(\Sigma_3^2) + (\lambda_X Z + M_X) \Psi_X \Psi_{\bar{X}}, \quad (32)$$

where Σ_8 and Σ_3 are adjoint representations of $SU(3)_c$ and $SU(2)_L$, respectively, and Ψ_X ($\Psi_{\bar{X}}$) is the bi-fundamental (anti-bi-fundamental) representation of those SM gauge group with a $U(1)_Y$ charge of $-5/6$ ($5/6$). Then, gaugino masses are given by

$$M_{\tilde{b}} \simeq \frac{g_1^2}{16\pi^2} (5\Lambda_X), \quad M_{\tilde{w}} \simeq \frac{g_2^2}{16\pi^2} (2\Lambda_3 + 3\Lambda_X), \quad M_{\tilde{g}} \simeq \frac{g_3^2}{16\pi^2} (3\Lambda_8 + 2\Lambda_X), \quad (33)$$

where $\Lambda_8 = \lambda_8 F_Z / M_8$, $\Lambda_3 = \lambda_3 F_Z / M_3$ and $\Lambda_X = \lambda_X F_Z / M_X$. Scalar masses are

$$\begin{aligned} m_{\tilde{Q}}^2 &\simeq \frac{2}{(16\pi^2)^2} \left[\frac{4}{3} g_3^4 (3\Lambda_8^2 + 2\Lambda_X^2) + \frac{3}{4} g_2^4 (2\Lambda_3^2 + 3\Lambda_X^2) + \frac{3}{5} g_1^4 (5\Lambda_X^2) \frac{1}{6^2} \right], \\ m_{\tilde{U}}^2 &\simeq \frac{2}{(16\pi^2)^2} \left[\frac{4}{3} g_3^4 (3\Lambda_8^2 + 2\Lambda_X^2) + \frac{3}{5} g_1^4 (5\Lambda_X^2) \left(\frac{2}{3} \right)^2 \right], \end{aligned}$$

$$\begin{aligned}
m_D^2 &\simeq \frac{2}{(16\pi^2)^2} \left[\frac{4}{3} g_3^4 (3\Lambda_8^2 + 2\Lambda_X^2) + \frac{3}{5} g_1^4 (5\Lambda_X^2) \frac{1}{3^2} \right], \\
m_L^2 &\simeq \frac{2}{(16\pi^2)^2} \left[\frac{3}{4} g_2^4 (2\Lambda_3^2 + 3\Lambda_X^2) + \frac{3}{5} g_1^4 (5\Lambda_X^2) \frac{1}{2^2} \right], \\
m_E^2 &\simeq \frac{2}{(16\pi^2)^2} \left[\frac{3}{5} g_1^4 (5\Lambda_X^2) \right], \\
m_{H_u}^2 &= m_{H_d}^2 = m_L^2.
\end{aligned} \tag{34}$$

B Two-loop beta-functions for the gauge couplings

Following Ref. [18], we show the two-loop beta-functions of the SM gauge couplings with X , \bar{X} , Σ_3 and Σ_8 . The contributions to the beta-functions from Σ_3 and Σ_8 are

$$\frac{dg_i}{dt} = \left(\frac{dg_i}{dt} \right)_{\text{MSSM}} + \frac{\Delta b_i^{(1)}}{16\pi^2} g_i^3 + \frac{\Delta b_i^{(2)}}{(16\pi^2)^2} g_i^5, \tag{35}$$

where $(\Delta b_1^{(1)}, \Delta b_2^{(1)}, \Delta b_3^{(1)}) = (0, 2, 3)$ and $(\Delta b_1^{(2)}, \Delta b_2^{(2)}, \Delta b_3^{(2)}) = (0, 24, 54)$. The contributions from X and \bar{X} are

$$\frac{dg_i}{dt} = \left(\frac{dg_i}{dt} \right)_{\text{MSSM}} + \frac{\Delta' b_i^{(1)}}{16\pi^2} g_i^3 + \frac{\Delta' b_{ij}^{(2)}}{(16\pi^2)^2} g_i^3 g_j^2, \tag{36}$$

where $(\Delta' b_1^{(1)}, \Delta' b_2^{(1)}, \Delta' b_3^{(1)}) = (5, 3, 2)$ and

$$\Delta' b_{ij}^{(2)} = \begin{pmatrix} \frac{25}{3} & 15 & \frac{80}{3} \\ 5 & 21 & 16 \\ \frac{10}{3} & 6 & \frac{68}{3} \end{pmatrix}. \tag{37}$$

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