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Dark Matter at the LHC: EFTs and Gauge Invariance

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Effective field theory (EFT) formulations of dark matter interactions are a convenient and popular way to quantify LHC bounds on dark matter. However, some EFT operators considered do not respect the weak gauge symmetries of the Standard Model. These operators break down at the electroweak scale, rather than the energy scale of new physics, and are invalid at LHC energies. We carefully discuss the circumstances in which such operators can arise, and use the mono-W process to illustrate potential issues in their interpretation and application. We also discuss a simple UV complete model that avoids such difficulties.

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I. INTRODUCTION

Despite overwhelming evidence that dark matter (DM) is the dominant form of matter in the universe, we remain ignorant about its fundamental nature. An appealing class of DM candidates which enjoy considerable theoretical motivation are weakly interacting massive particles (WIMPs): heavy particles with weak-scale mass and interaction strengths. Indeed, the production of WIMPtype particles at the LHC is now one of the foremost goals of the particle physics community.

Given the large number of WIMP-type theories, it is desirable to express the DM interactions in a model-independent manner. This can be achieved with an effective field theory (EFT) framework, in which a set of non-renormalizable effective operators is used to parametrise the interaction of a pair of DM particles with Standard Model (SM) particles. The EFT operators would be obtained as a low energy approximation to a renormalizable theory by integrating out the particle(s) that mediates the interaction. A standard set of operators have been listed in Refs.[1, 2] (see also [3]).

For fermionic dark matter χ interacting with SM fermions f, these operators take the form:

$$\frac{1}{\Lambda^2} \left(\overline{\chi} \Gamma_{\chi} \chi \right) \left(\overline{f} \Gamma_f f \right) , \qquad (1)$$

where Λ has dimensions of mass and is related to the mass M and coupling constants g_i of a heavy mediator as $\Lambda = M/\sqrt{g_1g_2}$, and $\Gamma_{\chi,f}$ are various Gamma matrices.

While the EFT description is very useful at low energies, such as those relevant for direct detection, it now well appreciated that the EFT approach may be unsuitable at LHC energies. Specifically, if the momentum transfer in a process is comparable to or larger than the mass of the mediator, the EFT will not provide an accurate description of the underlying physics. Many recent papers have attempted to quantify the point at which an EFT description is no longer valid [4–8] or have proposed the use of simplified models as an alternative framework for undertaking DM searches at colliders [8–14].

Here we make a more subtle point: if an EFT operator does not respect the weak gauge symmetries of the SM, it may be invalid at energies comparable to the electroweak scale, $v_{\rm EW} \approx 246$ GeV, rather than the energy scale of new physics, Λ . For example, if we attempt to use electroweak gauge symmetry violating operators at LHC energies, serious difficulties can be encountered soon above the EW scale, such as the bad high energy behaviour of cross sections. An example is the well-known unitary violation rising as $s/(4 m_W^2)$ in $W_L W_L$ scattering, due to the longitudinal modes induced by the symmetry breaking of $SU(2)_L$ invariance. In the SM, the violations are removed by an internal Higgs particle, but in the EFT formalism internal fields are "integrated out". Thus, the limit of validity for the operator is the weak scale if any internal W, Z or Higgs particle, or a Higgs vev insertion, is present in the Feynman diagram. More generally, sacred symmetries like the electroweak Ward identity can be violated, which implies a weak-scale cutoff, as we explain later in this paper.

II. EFT OPERATORS AND GAUGE INVARIANCE

The standard list of DM-SM effective operators [2] contains several operators which violate the SM weak gauge symmetries. We argue that if an EFT operator does not respect the weak gauge symmetries of the SM, it necessarily carries a pre-factor of the Higgs vev to some power, a remnant of the $SU(2)_L$ scalar doublet

$$\Phi \equiv \begin{pmatrix} \phi^+ \\ \phi^0 = \frac{1}{\sqrt{2}} (H + v_{\rm EW} + i \Im \phi^0) \end{pmatrix}.$$
 (2)

Acting as an $SU(2)_L$ doublet, enough powers of Φ are required to form an $SU(2)_L$ -invariant operator. The fields ϕ^{\pm} and $\Im \phi$ are gauged away to become, in unitary gauge, the longitudinal modes of the W^{\pm} and Z. So, in fact, it is the real, neutral field $\frac{1}{\sqrt{2}}(H+v_{\rm EW})$ whose n^{th} power appears in the operator. Commonly, the H part of the

expression is omitted, leaving just an implicit $v_{\rm EW}^n$ in the coefficient. Of course, the $v_{\rm EW}^n$ must come with a Λ^{-n} . Omission of the H part in the operator may ignore some interesting phenomenology. In this paper, we will also ignore the H contributions to operators, and focus on the operators proportional to $(v_{\rm EW}/\Lambda)^n$. Such terms in the coefficients of SU(2)-violating operators clearly satisfy the criterion that as SU(2) symmetry is restored, $v_{\rm EW} \to 0$, the operator's coefficient vanishes, and the operator decouples. ¹

A. Scalar operator

Consider the scalar (or pseudo scalar) operators

$$\frac{m_q}{\Lambda^3} \left(\overline{\chi} \chi \right) \left(\overline{q} q \right) = \frac{m_q}{\Lambda^3} \left(\overline{\chi} \chi \right) \left(\overline{q}_L q_R + h.c. \right). \tag{3}$$

This operator is clearly not $SU(2)_L$ invariant, as χ and q_R are $SU(2)_L$ singlets, while q_L is a component (either u_L or d_L) of the usual left-handed SM doublet, Q_L . A coupling to the Higgs boson has been anticipated by the factor of m_q in the coefficient. Most authors invoke minimal flavor violation to motivate this choice of normalization. Although this $SU(2)_L$ violating effective operator can be a good low energy description of new physics, notice that its coefficient cannot be arbitrarily large as it is controlled by the Higgs vev. Although formally a dimension 6 operator, it is competitive only with dimension 7 operators, given its $1/\Lambda^3$ normalization.

B. Vector operator:

Now consider vector (or axial vector) operators of the form $\,$

$$\frac{1}{\Lambda^2} \left(\overline{\chi} \gamma^{\mu} \chi \right) \left(\overline{q} \gamma_{\mu} q \right) = \frac{1}{\Lambda^2} \left(\overline{\chi} \gamma^{\mu} \chi \right) \left(\overline{q}_L \gamma_{\mu} q_L + \overline{q}_R \gamma_{\mu} q_R \right) . \tag{4}$$

These operators respect $SU(2)_L$ provided that the coefficients of the u_L and d_L operators are equal². Any $(\overline{u}_L\gamma_\mu u_L)$ operator that does not have a matching d_L term should be suppressed by two powers of $v_{\rm EW}/\Lambda$ (one for each unmatched u_L):

$$\frac{v_{\rm EW}^2}{\Lambda^4} (\overline{\chi} \gamma^\mu \chi) (\overline{u_L} \gamma_\mu u_L). \tag{5}$$

² Isospin violating operators, such as those invoked in [15, 16], can obviously be crafted from the right handed quark fields. Including the suppressed coefficient, this SU(2)-violating operator competes with dimension 8 operators, i.e., while the SU(2) conserving (axial)vector operators are dimension 6, SU(2) violating (axial)vector operators compete with subdominant, higher-order, dimension 8 operators.

III. MONO-W AND $SU(2)_L$ INVARIANCE

Issues arise if one tries to use gauge symmetry violating operators at LHC energies. For particular processes, the lack of gauge invariance can manifest as a violation of unitarity in high energy scattering. As an example of a problem encountered with an $SU(2)_L$ violating EFT, consider the following operator:

$$\frac{1}{\Lambda^2} \left(\overline{\chi} \gamma^{\mu} \chi \right) \left(\overline{u} \gamma_{\mu} u + \xi \overline{d} \gamma_{\mu} d \right). \tag{6}$$

This Lagrangian violates $SU(2)_L$, unless $\xi=1$. The case of unequal u and d couplings was considered in Ref. [17], where a very strong constructive(destructive) "interference effect" was found for $\xi=-1(+1)$, the degree of which depends on the energy scale. The analysis of Ref. [17] was subsequently repeated by the LHC experimental collaborations ATLAS [18, 19] and CMS [20, 21]. We shall demonstrate that the large cross section enhancement for $\xi\neq+1$ is in fact due the production of longitudinally polarized W's as a result of breaking gauge invariance.

At parton level, the mono-W process is $u(p_1)\overline{d}(p_2) \to \chi(k_1)\overline{\chi}(k_2)W^+(q)$. The relevant diagrams are given in Fig.1, and the corresponding contributions to the amplitude $\mathcal{M} \equiv \mathcal{M}^{\alpha} \epsilon_{\alpha}^{\lambda}(q) \equiv (\mathcal{M}_{1}^{\alpha} + \mathcal{M}_{2}^{\alpha})\epsilon_{\alpha}^{\lambda}(q)$ are

$$\mathcal{M}_{1}^{\alpha} = \frac{1}{\Lambda^{2}} \left[\bar{v}(p_{2}) \gamma^{\alpha} \frac{-g_{W}}{p_{2}^{\prime} - \not q} \gamma^{\mu} \frac{P_{L}}{\sqrt{2}} u(p_{1}) \right] \left[\bar{u}(k_{1}) \gamma_{\mu} v(k_{2}) \right],$$

$$\mathcal{M}_{2}^{\alpha} = \frac{\xi}{\Lambda^{2}} \left[\bar{v}(p_{2}) \gamma^{\mu} \frac{g_{W}}{p_{1}^{\prime} - \not q} \gamma^{\alpha} \frac{P_{L}}{\sqrt{2}} u(p_{1}) \right] \left[\bar{u}(k_{1}) \gamma_{\mu} v(k_{2}) \right], (7)$$

where g_W is the weak coupling constant, and $\epsilon_{\alpha}^{\lambda}$ is the polarization vector of the W. We note that the W longitudinal polarization vector at high energy is

$$\epsilon_{\alpha}^{L} = \frac{q_{\alpha}}{m_{W}} + \mathcal{O}\left(\frac{m_{W}}{E}\right) \sim \frac{\sqrt{s}}{m_{W}}.$$
 (8)

Thus the high energy W_L contribution to the usual polarization sum, $\sum_{\lambda} \epsilon_{\alpha}^{\lambda} \epsilon_{\beta}^{\lambda *} = -g_{\alpha\beta} + \frac{q_{\alpha}q_{\beta}}{m_W^2}$, is $\epsilon_{\alpha}^L \epsilon_{\beta}^L * \approx q_{\alpha}q_{\beta}/m_W^2 \sim s/m_W^2$.

We can verify that the sum of the two amplitudes of Fig.1 is not gauge invariant unless $\xi=1$, by observing that the relevant Ward identity is not satisfied. At high energy, the Goldstone boson equivalence theorem requires that the amplitude for emission of a longitudinally polarized W_L is equivalent to that for the emission of the corresponding Goldstone boson. Since the Goldstone boson couples to quarks with strength proportional to their mass, these terms are close to zero. (See Ref.[22] for a

¹ In what follows, we will assume that there is but a single vev, $v_{\rm EW}$. If there were further vevs, the good relation $m_W = \cos\theta_W \, m_Z$ requires the additional vevs to come from additional doublet fields, or to be be small if coming from non-doublet fields. The vevs then add in quadrature to give $(2 \, m_W/g_2)^2$. Thus, any individual vev will offer an energy-scale below the SM vev. In the sense that we will argue against larger energy-scales for effective operators, our assumption of a single EW vev is conservative.

similar discussion about the related process $\chi\chi\to\nu eW$.) The Ward identity for the longitudinal W at high energy therefore takes the form

$$\mathcal{M}^{\alpha} \epsilon_{\alpha}^{L} \approx \frac{q_{\alpha}}{m_{W}} \mathcal{M}^{\alpha}(q, ...) = i \mathcal{M}(\phi^{+}(q)) \simeq 0.$$
 (9)

For the sum of the mono-W amplitudes of Fig.1 we find

$$q_{\alpha}\mathcal{M}^{\alpha} = \frac{g_W}{\Lambda^2} \left[\bar{v}(p_2) \left(1 - \xi \right) \gamma^{\mu} \frac{P_L}{\sqrt{2}} u(p_1) \right] \left[\bar{u}(k_1) \gamma_{\mu} v(k_2) \right], \tag{10}$$

which clearly vanishes only for $\xi = 1$.

The "interference effect" seen in the mono-W process is not truly due to constructive/destructive interference as previously claimed, but is just a manifestation of the fact that the breaking of electroweak gauge-invariance has given rise to a W_L component. The increased cross section for $\xi \neq 1$ is in fact due to unphysical terms that grow like s/m_W^2 , which originate from the $+q_\alpha q_\beta/m_W^2$ term in the polarization sum. At high energy, these terms would grow large enough to violate unitarity. But even at lower energy, their presence may be problematic.

To explicitly demonstrate this behaviour, we now derive an analytic expression for the parton-level mono-W

process $\overline{d}u\to \overline{\chi}\chi W^+.$ We work in the center-of-mass frame, and follow the phase space parametrization described in Section V of Ref.[23]. We define θ to be the angle of the W w.r.t. the beam line and $x=2E_W/\sqrt{s},$ where \sqrt{s} is the total invariant mass. For simplicity we take $m_\chi=0$ (the cross section will be approximately independent of m_χ for $m_\chi^2\ll s$). We include a factor of 1/3 from averaging over initial state quark colors.

For $\xi = 1$ the differential cross section is well behaved and is given by

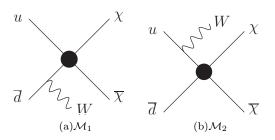


FIG. 1. Contributions to the mono-W process $u(p_1)\overline{d}(p_2) \to \chi(k_1)\overline{\chi}(k_2)W^+(q)$, in the effective field theory framework.

$$\left. \frac{d^2 \sigma}{dx d \cos \theta} \right|_{\xi=1} = \frac{A}{3^2 2^8 \pi^3 \Lambda^4 \left(s^2 x^2 \sin^2 \theta + 2 s m_W^2 \left(\cos \left(2 \theta \right) - 2 x + 1 \right) + 4 m_W^4 \right)^2},\tag{11}$$

where

$$A = s^{2}g_{W}^{2}\sqrt{x^{2} - \frac{4m_{W}^{2}}{s}}\left(1 - x + \frac{m_{W}^{2}}{s}\right)\left[s^{3}x^{2}\sin^{2}\theta\left(\cos\left(2\theta\right)x^{2} + 3x^{2} - 8x + 8\right) + 2s^{2}m_{W}^{2}\left(\cos\left(4\theta\right)x^{2} + 2\cos\left(2\theta\right)\left(x^{3} - x^{2} - 4x + 4\right) - 2x^{3} + 17x^{2} - 24x + 8\right) - 4sm_{W}^{4}\left(\cos\left(4\theta\right) + \cos\left(2\theta\right)\left(x^{2} + 4x - 8\right) - x^{2} + 20x - 17\right) + 16m_{W}^{6}\left(\cos\left(2\theta\right) + 3\right)\right],$$

If we take the limit $m_W \to 0$, remove the color factor 1/3, and replace $g_W/\sqrt{2}$ with the electron charge e, we find Eq. 11 reproduces the cross section for the $e^+e^- \to \overline{\chi}\chi\gamma$ monophoton process reported by Ref.[23, 24] for $m_\chi = 0$ and unpolarized e^+e^- beams. This provides a useful check for our more complicated mono-W calculation.

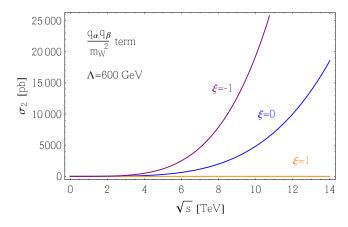
For $\xi \neq 1$, however, the cross section is not well behaved at high energy. The $+q_{\alpha}q_{\beta}/m_W^2$ term in the polarization sum contributes to the cross section a term

$$\frac{d^2\sigma}{dxd\cos\theta}\Big|_{q_{s},q_{s}/m_{W}^{2}} = \frac{(\xi-1)^2s^2g_W^2\sqrt{x^2 - \frac{4m_W^2}{s}}\left(2x^2\sin^2\theta - 16x + 16 + \frac{4m_W^2}{s}\left(\cos\left(2\theta\right) + 3\right)\right)}{3^22^{13}\pi^3\Lambda^4m_W^2}, \tag{12}$$

which violates unitarity when $s \gg m_W^2$.

The total cross sections, for $m_{\chi}=0$, are plotted in Figs(2,3) as a function of \sqrt{s} . We also calculate the cross sections in MadGraph [25], and find the results agree. For brevity of notation, we have defined σ_1 and σ_2 to be the contributions to the cross section from the $-g_{\alpha\beta}$ and $+q_{\alpha}q_{\beta}/m_W^2$ terms in the polarization sum, respectively. The $\xi=0,-1$ cross sections grow faster with \sqrt{s}

than for $\xi=1$. At LHC energies the cross sections are already dominated by the unphysical terms arising from the longitudinal polarization, unless $\xi \simeq 1$.



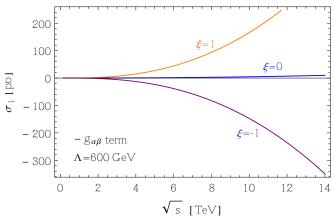


FIG. 2. Total parton-level cross sections as a function of energy, for $\Lambda=600~{\rm GeV}$ and particular choices of ξ . Upper: contribution from the $+q_{\alpha}q_{\beta}/m_W^2$ term in the polarization sum. The cross section scales simply as $(1-\xi)^2$. Lower: contribution of the $-g_{\alpha\beta}$ term in the polarization sum. At LHC energies the $q_{\alpha}q_{\beta}$ terms dominates unless $\xi\simeq 1$. (Notice the differing vertical scales between the upper and lower panels.)

IV. RENORMALIZABLE MODELS AND EFTS

We now consider a renormalizable, gauge invariant, model of DM interactions, and examine the way in which unequal couplings to u and d quarks can be obtained. Consider the case where $\overline{q}q \to \overline{\chi}\chi$ is mediated by the exchange of a t-channel scalar. The Lagrangian is given by

$$\mathcal{L}_{\text{int}} = f \overline{Q_L} \eta \chi_R + h.c$$

= $f_{ud} \left(\eta_u \overline{u}_L + \eta_d \overline{d}_L \right) \chi_R + h.c.,$ (13)

where $Q_L = (u_L, d_L)^T$ is the quark doublet, $\eta = (\eta_u, \eta_d)^T \sim (3, 2, 1/3)$ is a scalar field that transforms under the SM gauge group like Q_L , and f is a coupling constant. Such couplings are present in supersymmetric (SUSY) models, with χ identified as a neutralino and η a squark doublet, and have been considered as a simplified model for DM interactions in Refs. [26–29].

If we take the EFT limit, assuming the η are very

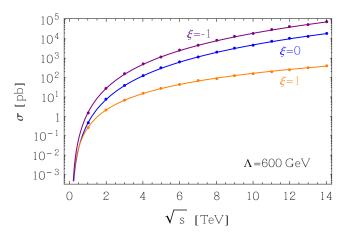


FIG. 3. Total parton-level cross sections for $\Lambda = 600$ GeV, for particular choices of ξ . Solid lines are the analytic calculation and dots are the MadGraph calculation.

heavy, the lowest order operators are of dimension 6:

$$\frac{1}{\Lambda_u^2} (\overline{u} \Gamma u) (\overline{\chi} \Gamma \chi) \quad \text{and} \quad \frac{1}{\Lambda_d^2} (\overline{d} \Gamma d) (\overline{\chi} \Gamma \chi), \quad (14)$$

where the suppression scales are $\Lambda_{u,d} \propto m_{\eta_{u,d}}/f$. The relevant Lorentz structure Γ is a sum of vector and axial vector terms as can be seen by Fierz transforming the t-channel matrix elements obtained from Eq.13 to s-channel form [30].

The strength of DM interactions with u and d quarks can differ if the masses of η_u and η_d are non-degenerate. However, given that (η_u, η_d) form an electroweak doublet, their mass splitting must be controlled by $v_{\rm EW}$. The relevant terms in the scalar potential are [31]

$$V = m_1^2(\Phi^{\dagger}\Phi) + \frac{1}{2}\lambda_1(\Phi^{\dagger}\Phi)^2 + m_2^2(\eta^{\dagger}\eta) + \frac{1}{2}\lambda_2(\eta^{\dagger}\eta)^2 + \lambda_3(\Phi^{\dagger}\Phi)(\eta^{\dagger}\eta) + \lambda_4(\Phi^{\dagger}\eta)(\eta^{\dagger}\Phi).$$
(15)

If $m_1^2 < 0$ and $m_2^2 > 0$, the SM Higgs doublet obtains a non-zero vev, while the η does not. The presence of λ_4 splits the η masses as

$$m_{\eta_d}^2 = m_2^2 + \lambda_3 v_{\rm EW}^2 + \lambda_4 v_{\rm EW}^2,$$
 (16)

$$m_{\eta_u}^2 = m_2^2 + \lambda_3 v_{\rm EW}^2 \,,$$
 (17)

implying that $\delta m_{\eta}^2 \equiv m_{\eta_d}^2 - m_{\eta_u}^2 = \lambda_4 \, v_{\rm EW}^2$. However, although we have engineered unequal scalar masses and thus unequal DM couplings to u and d quarks, the scenario of Ref. [17] can not be realised, as we shall explain below

If we attempt to naively connect our renormalizable model with an EFT like that of Eq.6, we find that the parameter ξ is given by $\xi=1/(1+\delta m_\eta^2/\Lambda^2)=1/(1+\lambda_4\,v_{EW}^2/\Lambda^2)$. For $\Lambda\gtrsim 1$ TeV and a perturbative value for λ_4 , ξ will not deviate far from 1. (Negative ξ can not be obtained from this renormalizable model.) Furthermore, it is clear that $SU(2)_L$ violating effects enter the EFT at order v_{EW}^2/Λ^4 , i.e., the same order in Λ as a dimension

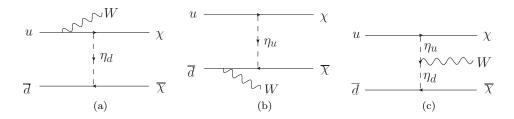


FIG. 4. Contributions to the mono-W process $u(p_1)\overline{d}(p_2) \to \chi(k_1)\overline{\chi}(k_2)W^+(q)$, in a UV complete model.

8 operator. If we consider effects of this order we must, for self consistency, consider other dimension 8 operators that arise from the model. Inclusion of these operators, with appropriate coefficients, will be essential for gauge invariance.

In the renormalizable model, the mono-W process proceeds via the gauge invariant set of diagrams in Fig.4 3 4 [32–34]. In the EFT limit, the diagrams in Fig.4(a) and (b) map onto those in Fig.1(a) and (b) respectively. The diagram in Fig.4(c), in which the W is radiated from the η propagator, is suppressed by an additional heavy scalar propagator and is hence subdominant to the ISR diagrams. It enters the EFT as a dimension 8 operator [35], contributes on an equal footing with the SU(2) violating contributions of diagrams (a) and (b), and enforces the cancellation of unphysical W_L contributions to cross sections.

Although we have framed the discussion in terms of a particular renormalizable model, the general features will hold for all UV completions of Eq.6 in which $\xi \neq 1$. The violation of the Ward Identity in Eq.10 indicates that the two diagrams of Fig.1 are not a gauge invariant set, hence there must be an additional diagram. While the form of the missing diagram(s) will depend on the particular UV completion, their presence is dictated by gauge invariance. No matter the size of ξ , these extra diagrams prevent large unphysical W_L enhancements.

V. CONCLUSION

An important observation of Ref. [17] is that, of the mono-X processes, the mono-W is unique in its ability to probe different DM couplings to u and d quarks. This important insight is correct. However, we have argued that the size of any $SU(2)_L$ violating difference of the u and d quark couplings must be protected by the EW scale, and therefore cannot be arbitrarily large. $SU(2)_L$ violating operators can be obtained by integrating out the SM Higgs or by including Higgs vev insertions. Therefore, they should have coefficients suppressed by powers of $(v_{\rm EW}/\Lambda)$ or $(m_{\rm fermion}/\Lambda)$ and thus are of higher order in $1/\Lambda$ than they would naively appear. To include SU(2) violating effects in a way that is self consistent and properly respects the EW Ward identity, one should use a renormalizable, gauge invariant, model rather than an EFT, to avoid spurious \mathcal{W}_L contributions. These observations will be an important guide to the LHC collaborations in the interpretation of their current [18–21] and forthcoming mono-W dark matter search results, and to theorists constructing dark matter models.

VI. ACKNOWLEDGMENTS

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³ In the good EW SU(2) limit, the η_u and η_d are mass degenerate, and the massless W^{\pm} emitted in diagram (c) establishes the validity of the EW Ward identity [32, 33]. When EW SU(2) is broken, the η_u and η_d masses are split, and the new massive-W longitudinal mode must restore the EW Ward identity by coupling to the η proportional to δm_{η}^2 [31]. This argument provides an interpretation of the result found earlier in [31] that the internal longitudinal mode couples proportional to δm_{η}^2 . In fact, in [31] it was shown that this longitudinal W mode will dominate the W emission probability for some range of model parameters.

⁴ Note that in the renormalizable theory, in the high energy limit, W_L production arises solely from the amplitude of Fig.4(c), and only when $\delta m_\eta^2 \neq 0$.

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