Transformative A_{4} mixing of neutrinos with CP violation

Ernest Ma

Phys. Rev. D **92**, 051301 — Published 4 September 2015

DOI: 10.1103/PhysRevD.92.051301

UCRHEP-T552

Transformative A_4 Mixing of Neutrinos with CP Violation

Ernest Ma

Physics & Astronomy Department and Graduate Division,
University of California, Riverside, California 92521, USA

Abstract

A new theoretical insight into the pattern of neutrino mixing and leptonic CP violation is presented. It leads naturally and uniquely to a specific dark sector of three real neutral scalar singlets, with the radiative implementation of the inverse seesaw mechanism for neutrino mass. The new simple but crucial enabling idea is that a familiar A_4 transformation turns any orthogonal 3×3 matrix into one which predicts $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$ for the neutrino mixing matrix, in good agreement with present data.

In recent years, many theoretical studies have been made regarding the pattern of the 3×3 neutrino mixing matrix. In particular, the use of non-Abelian discrete symmetries is widespread. This came about from the specific example of A_4 [1, 2, 3], where it was shown for the first time how the three very different charged-lepton masses may be incorporated into a symmetry for neutrino mixing, which can explain $\sin^2\theta_{23} = 1/2$. Subsequently, motivated by empirical observation, it was conjectured [4] that the pattern could be tribimaximal, with $\sin^2\theta_{12} = 1/3$ and $\theta_{13} = 0$. It was then shown [5] that A_4 is indeed suitable for obtaining this result. Since 2005, there have been many papers written regarding this possibility.

In 2011 [6] and then more decisively in 2012 [7, 8], θ_{13} was measured to be significantly different from zero, thus falsifying the tribimaximal ansatz. The 2014 Particle Data Group (PDG) values [9] of neutrino parameters are:

$$\sin^2(2\theta_{12}) = 0.846 \pm 0.021, \;\; \Delta m^2_{21} = (7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2, \tag{1}$$

$$\sin^2(2\theta_{23}) = 0.999 \begin{pmatrix} +0.001 \\ -0.018 \end{pmatrix}, \;\; \Delta m^2_{32} = (2.44 \pm 0.06) \times 10^{-3} \text{ eV}^2 \text{ (normal)}, \tag{2}$$

$$\sin^2(2\theta_{23}) = 1.000 \begin{pmatrix} +0.000 \\ -0.017 \end{pmatrix}, \;\; \Delta m^2_{32} = (2.52 \pm 0.07) \times 10^{-3} \text{ eV}^2 \text{ (inverted)}, \tag{3}$$

$$\sin^2(2\theta_{13}) = (9.3 \pm 0.8) \times 10^{-2}, \tag{4}$$

where (normal) refers to the ordering $m_1 < m_2 < m_3$ of neutrino masses, and (inverted) refers to $m_3 < m_1 < m_2$.

More recently [10], combining reactor data, there appears to be a preference for $\delta_{CP} = -\pi/2$ in long-baseline neutrino oscillation data. These new developments are in fact consistent with a special form of the Majorana neutrino mass matrix which first appeared in 2002 [3, 11], i.e.

$$\mathcal{M}_\nu^{(e,\mu,\tau)} = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix}, \tag{5}$$

where A, B are real. This allows $\theta_{13} \neq 0$ and yet $\theta_{23} = \pi/4$ is maintained, together with the prediction that $\delta_{CP} = \pm\pi/2$. Subsequently, this pattern was shown [12] to be protected by a symmetry, i.e. $e \to e$ and $\mu \leftrightarrow \tau$ exchange with CP conjugation. With the knowledge that $\theta_{13} \neq 0$, this extended symmetry is now the subject of many studies, which began with generalized S_4 [13].

In this paper, I show how $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$ may be obtained in a very general way, using the familiar unitary 3×3 transformation

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \tag{6}$$

where $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$, which is derivable from A_4 as shown in Ref. [1]. The idea is very simple. Consider the product of $U_\omega \mathcal{O}$, where $\mathcal{O}$ is a real orthogonal 3×3 matrix, i.e.

$$\frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix} \begin{pmatrix} o_{11} & o_{12} & o_{13} \\ o_{21} & o_{22} & o_{23} \\ o_{31} & o_{32} & o_{33} \end{pmatrix} = \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ u_{21} & u_{22} & u_{23} \\ u_{31} & u_{32} & u_{33} \end{pmatrix} = U. \tag{7}$$

It is clear that $u^*_{2i} = u_{3i}$ for $i = 1, 2, 3$. Comparing this with the PDG convention of the neutrino mixing matrix, i.e.

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{8}$$

it is obvious that after rotating the phases of the third column and the second and third rows, the two matrices are identical if and only if $s_{23} = c_{23}$ and $\cos\delta = 0$, i.e. $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$. This was first pointed out in Refs. [14, 15].

To obtain this result, the necessary condition is that the 3×3 Majorana neutrino mass matrix $\mathcal{M}_\nu$ must be diagonalized by an orthogonal matrix in the A_4 basis. Obviously it will be so if $\mathcal{M}_\nu$ is purely real. In that case, in the (e, μ, τ) basis, it is given by

$$\mathcal{M}_\nu^{(e,\mu,\tau)} = U_\omega \begin{pmatrix} a & c & e \\ c & d & b \\ e & b & f \end{pmatrix} U_\omega^T = \begin{pmatrix} A & C & C^* \\ C & D^* & B \\ C^* & B & D \end{pmatrix}, \tag{9}$$

where

$$A = (a + 2b + 2c + d + 2e + f)/3, \tag{10}$$

$$B = (a - b - c + d - e + f)/3, \tag{11}$$

$$C = (a - b - \omega^2 c + \omega d - \omega e + \omega^2 f)/3, \tag{12}$$

$$D = (a + 2b + 2\omega^2 c + \omega d + 2\omega e + \omega^2 f)/3. \tag{13}$$

In other words, the form of Eq. (5) is automatically obtained, as expected.

In the context of A_4, efforts prior to 2011 were concentrated on how to achieve $c = e = 0$ and $d = f$ for tribimaximal mixing without a necessarily real $\mathcal{M}_\nu$, i.e. a residual Z_2 symmetry in the neutrino sector which coexists with the residual Z_3 symmetry implied by U_ω in the charged-lepton sector. This clash or misalignment of residual symmetries is the origin of a basic theoretical problem which has no simple solution. In hindsight, it is a powerful argument against the naive expectation of an exact tribimaximal form of the neutrino mixing matrix. Here A_4 serves simply as a link for a (real) neutrino mass matrix without any symmetry to the charged-lepton sector. The new remarkable result is that the nature of this link, i.e. U_ω, leads to two verifiable specific predictions, i.e. $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$, which agree well with present data. In Ref. [16], $c = e = 0$ is again assumed but d and f are not set equal. In this way $\theta_{13} \neq 0$ is obtained and the further assumption (but without further justification) that a, b, d, f are real leads to $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$.

Here I answer the new important question of how an arbitrary complex neutrino mass matrix can be guaranteed to be purely real, without imposing explicit CP conservation. The key is of course the origin of $\mathcal{O}$ which obviously would be the result of diagonalizing a real 3×3 mass matrix. The only guaranteed such mass matrix is that of three real scalars. Hence the quest for $\mathcal{O}$ leads inexorably to a mechanism by which neutrino masses come from three real scalars. This is the significance of Eq. (7). In the following, I will show that it may be achieved naturally together with the appearance of U_ω in a radiative implementation [17, 18] of neutrino and charged-lepton masses through dark matter (scotogenic), using *only* the one Higgs doublet of the standard model (SM), as suggested by the observation [19, 20] of the 125 GeV particle at the Large Hadron Collider (LHC).

Under A_4, let the three families of leptons transform as

$$(\nu_i, l_i)_L \sim \underline{3}, \quad l_{iR} \sim \underline{1}, \underline{1}', \underline{1}''. \tag{14}$$

Add the following new particles, all assumed odd under an exactly conserved discrete Z_2 (dark) symmetry, whereas all SM particles are even:

$$(E^0, E^-)_{L,R} \sim \underline{1}, \quad N_{L,R} \sim \underline{1}, \quad s_i \sim \underline{3}, \tag{15}$$

where (E^0, E^-) is a fermion doublet, N a neutral fermion singlet, and $s_{1,2,3}$ are *real* neutral scalar singlets. Together with the one Higgs doublet (ϕ^+, ϕ^0) of the SM, one-loop radiative inverse seesaw neutrino masses are generated [21, 22] as shown in Fig. 1.

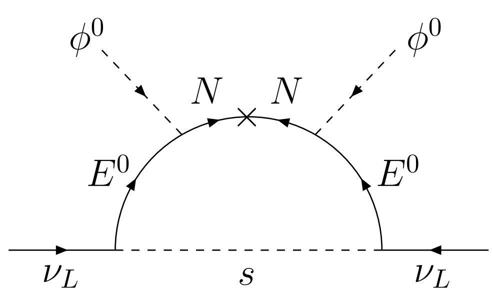

Figure 1: One-loop generation of inverse seesaw neutrino mass.

These new terms in the Lagrangian are given by

$$\begin{aligned}\mathcal{L}' &= -m_N \bar{N} N - m_E(\bar{E}^0 E^0 + \bar{E}^- E^-) - \frac{1}{2} m_L N_L N_L - \frac{1}{2} m_R N_R N_R + \frac{1}{2}(m_s^2)_{ij} s_i s_j \\ &+ f_D \bar{N}_L (E_R^0 \phi^0 - E_R^- \phi^+) + f_F \bar{N}_R (E_L^0 \phi^0 - E_L^- \phi^+) + f s_i (\bar{E}_R^0 \nu_{iL} + \bar{E}_R^- l_{iL}) + H.c. \end{aligned} \tag{16}$$

The mass matrix linking $(\bar{N}_L, \bar{E}^0_L)$ to (N_R, E^0_R) is then

$$\mathcal{M}_{N,E} = \begin{pmatrix} m_N & m_D \\ m_F & m_E \end{pmatrix}, \tag{17}$$

where $m_D = f_D\langle\phi^0\rangle$, and $m_F = f_F\langle\phi^0\rangle$. As a result, N and E^0 mix to form two Dirac fermions of masses $m_{1,2}$, with mixing angles

$$\begin{aligned} m_D m_E + m_F m_N &= \sin\theta_L \cos\theta_L (m_1^2 - m_2^2), & (18)\\ m_D m_N + m_F m_E &= \sin\theta_R \cos\theta_R (m_1^2 - m_2^2). & (19) \end{aligned}$$

To connect the loop, Majorana mass terms m_L and m_R are necessary. Since both E and N may be defined to carry lepton number, these terms violate lepton number softly and may be naturally small, thus realizing the mechanism of inverse seesaw [23, 24, 25]. The one-loop Majorana neutrino mass is given by

$$\begin{aligned} m_\nu &= f^2 m_R \sin^2\theta_R \cos^2\theta_R (m_1^2 - m_2^2)^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{(k^2 - m_s^2)} \frac{1}{(k^2 - m_1^2)^2} \frac{1}{(k^2 - m_2^2)^2} \\ &+ f^2 m_L m_1^2 \sin^2\theta_R \cos^2\theta_L \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_s^2)} \frac{1}{(k^2 - m_1^2)^2} \\ &+ f^2 m_L m_2^2 \sin^2\theta_L \cos^2\theta_R \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_s^2)} \frac{1}{(k^2 - m_2^2)^2} \\ &- 2 f^2 m_L m_1 m_2 \sin\theta_L \sin\theta_R \cos\theta_L \cos\theta_R \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_s^2)} \frac{1}{(k^2 - m_1^2)} \frac{1}{(k^2 - m_2^2)}. \end{aligned} \tag{20}$$

This formula holds for s as a mass eigenstate. If A_4 is unbroken, then $s_{1,2,3}$ all have the same mass and $\mathcal{M}_\nu$ is proportional to the identity matrix. However, if A_4 is softly broken by the necessarily real $s_i s_j$ mass terms, then the neutrino mass matrix is given by

$$\mathcal{M}_\nu = \mathcal{O} \begin{pmatrix} m_{\nu 1} & 0 & 0 \\ 0 & m_{\nu 2} & 0 \\ 0 & 0 & m_{\nu 3} \end{pmatrix} \mathcal{O}^T, \tag{21}$$

where $\mathcal{O}$ is an orthogonal matrix. Now each $m_{\nu i}$ may be complex because f, m_L, m_R may be complex in Eq. (20), but a common unphysical phase, say for ν_1, may be rotated away, leaving just two relative Majorana phases for ν_2 and ν_3, owing to the relative phase between m_L and m_R with different $s_{1,2,3}$ masses in Eq. (20). Hence $\mathcal{M}_\nu$ is diagonalized by $\mathcal{O}$, which is all that is required to obtain $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$, once U_ω is applied. This shows that the neutrino mass matrix does not have to be real. It only has to be diagonalized by an orthogonal matrix.

To derive U_ω, the simplest way is to copy Ref. [1] and add three Higgs doublets $\Phi_i \sim \underline{3}$. This leads to the charged-lepton mass matrix

$$\begin{aligned} \mathcal{M}_l &= \begin{pmatrix} f_e v_1^* & f_\mu v_1^* & f_\tau v_1^* \\ f_e v_2^* & f_\mu \omega^2 v_2^* & f_\tau \omega v_2^* \\ f_e v_3^* & f_\mu \omega v_3^* & f_\tau \omega^2 v_3^* \end{pmatrix} \\ &= \begin{pmatrix} v_1^* & 0 & 0 \\ 0 & v_2^* & 0 \\ 0 & 0 & v_3^* \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix} \begin{pmatrix} f_e & 0 & 0 \\ 0 & f_\mu & 0 \\ 0 & 0 & f_\tau \end{pmatrix}. \end{aligned} \tag{22}$$

For $v_1 = v_2 = v_3$, a residual Z_3 symmetry exists with $m_e = \sqrt{3} f_e v$, etc. and U_ω becomes the transformation linking $\mathcal{M}_l$ to $\mathcal{M}_\nu$. However, this scenario requires four Higgs doublets. It is thus somewhat problematic in the face of present data regarding the observed [19, 20] 125 GeV particle, which is entirely consistent with being the one Higgs boson h of the SM.

To obtain charged-lepton masses in the context of A_4 with just the SM Higgs doublet, the general radiative framework of Ref. [18] is adopted. The specific scenario here requires the addition of two sets of charged scalars odd under dark Z_2:

$$x_i^- \sim \underline{3}, \quad y_i^- \sim \underline{1}, \underline{1}', \underline{1}''. \tag{23}$$

The one-loop diagram is given in Fig. 2. To connect x with y, A_4 must be broken, either softly so that the link is again U_ω to obtain the desired residual Z_3 symmetry, or spontaneously using three singlet scalar fields $\chi_i \sim \underline{3}$ with equal vacuum expectation values. In this way, the three Higgs doublets of the original A_4 model are replaced in a renormalizable theory for obtaining charged-lepton masses. Note that the latter may be considered as the

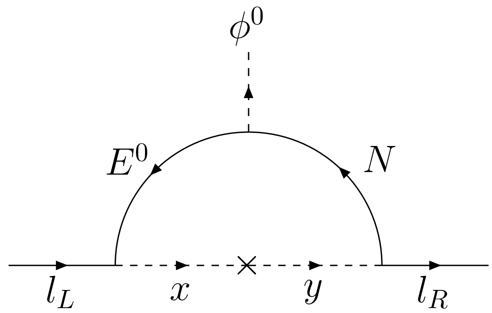

Figure 2: One-loop generation of charged-lepton mass.

ultraviolet completion of the common practice of using the nonrenormalizable dimension-five term $\bar{l}_L l_R \bar{\phi}^0 \chi$ for such a purpose.

As a result, the charged-lepton mass matrix is given by

$$\mathcal{M}_l = U_\omega^\dagger \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \tag{24}$$

with

$$m_e = f' f_e \mu_e u \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2 - m_{1e}^2)(k^2 - m_{2e}^2)} \left[\frac{m_1 \cos\theta_R \sin\theta_L}{k^2 - m_1^2} - \frac{m_2 \cos\theta_L \sin\theta_R}{k^2 - m_2^2} \right], \tag{25}$$

where f' is the $E_L^0 l_L x^*$ Yukawa coupling, f_e is the $N_R e_R y_1^*$ Yukawa coupling, μ_e is the scalar trilinear $x y_1^* \chi$ coupling, u is the vacuum expectation value of χ, and $m_{1e,2e}$ are the mass eigenvalues of the 2×2 mass-squared matrix

$$\mathcal{M}_{xy_1}^2 = \begin{pmatrix} m_x^2 & \mu_e u \\ \mu_e u & m_{y_1}^2 \end{pmatrix}, \tag{26}$$

with $\mu_e u = \sin\theta_e \cos\theta_e (m_{1e}^2 - m_{2e}^2)$, and similarly for m_μ and m_τ. One immediate consequence of a radiative charged-lepton mass is that the Higgs Yukawa coupling $h\bar{l}l$ is no longer exactly m_l/v as in the SM. Its deviation is not suppressed by the usual one-loop factor of $16\pi^2$ and may be large enough to be observable [26].

There is a one-to-one correlation of the neutrino mass eigenstates to the $s_{1,2,3}$ mass eigenstates, the lightest of which is dark matter [27, 28]. It is also clear from Eq. (20) that all three neutrino masses are expected to be of the same order of magnitude, and their mass-squared differences are related to the scalar mass differences. The most recent cosmological data [29] imply

$$\sum m_\nu < 0.23 \text{ eV}. \tag{27}$$

This would mean that the effective neutrino mass m_{ee} in neutrinoless double beta decay is bounded below 0.07 eV for normal ordering and 0.08 eV for inverted ordering.

Due to the presence of the A_4 symmetry, the dark matter parity of this model is also derivable from lepton parity [30]. Under lepton parity, let the new particles $(E^0, E^-), N$ be even and s, x, y be odd, then the same Lagrangian is obtained. As a result, dark parity is simply given by $(-1)^{L+2j}$, which is odd for all the new particles and even for all the SM particles. Note that the tree-level Yukawa coupling $\bar{l}_L l_R \phi^0$ would be allowed by lepton parity alone, but is forbidden here because of the A_4 symmetry. The lightest s is dark matter. If its relic density as well as direct-detection cross section are determined only by the $\lambda s^2(\Phi^\dagger \Phi)$ interaction, then its allowed parameter space is limited to a small region just below $m_h/2$ [31]. In the above model, if the interaction of s with χ is also taken into account, it adds to the ss annihilation cross section but not to the elastic scattering of s off nuclei. The former may then satisfy the relic abundance requirement and yet the latter will evade the direct-detection constraint. Details will be presented elsewhere.

In conclusion, it has been pointed out that the phenomenologically successful values of $\theta_{23} = \pi/4$ and $\delta_{CP} = \pm\pi/2$ for the neutrino mixing matrix is derivable from the familiar A_4 transformation of Eq. (6) if it is multiplied by an orthogonal matrix. This leads to the specific notion that a desirable neutrino mass matrix should come from three real scalars in the context of A_4. To obtain the latter naturally, a specific scotogenic one-loop radiative model of neutrino and charged-lepton masses is proposed, where the particles appearing in the loop have odd dark matter parity. These predicted new particles should have masses at the scale of weakly interacting dark matter, i.e. 1 TeV or less, and be potentially observable at the LHC, which has just resumed operation at CERN.

This work is supported in part by the U. S. Department of Energy under Grant No. DE-SC0008541.

References

[1] E. Ma and G. Rajasekaran, Phys. Rev. **D64**, 113012 (2001).

[2] E. Ma, Mod. Phys. Lett. **A17**, 2361 (2002).

[3] K. S. Babu, E. Ma, and J. W. F. Valle, Phys. Lett. **B552**, 207 (2003).

[4] P. F. Harrison, D. H. Perkins, and W. G. Scott, Phys. Lett. **B530**, 167 (2002).

[5] E. Ma, Phys. Rev. **D70**, 031901 (2004).

[6] K. Abe *et al.*, (T2K Collaboration), Phys. Rev. Lett. **107**, 041801 (2011).

[7] F. P. An *et al.* (Daya Bay Collaboration), Phys. Rev. Lett. **108**, 171803 (2012).

[8] S.-B. Kim *et al.* (RENO Collaboration), Phys. Rev. Lett. **108**, 191802 (2012).

[9] Particle Data Group, K. A. Olive *et al.*, Chin. Phys. **C38**, 090001 (2014).

[10] K. Abe *et al.*, (T2K Collaboration), Phys. Rev. **D91**, 072010 (2015).

[11] E. Ma, Phys. Rev. **D66**, 117301 (2002).

[12] W. Grimus and L. Lavoura, Phys. Lett. **B579**, 113 (2004).

[13] R. N. Mohapatra and C. C. Nishi, Phys. Rev. **D86**, 073007 (2012).

[14] K. Fukuura, T. Miura, E. Takasugi, and M. Yoshimura, Phys. Rev. **D61**, 073002 (2000).

[15] T. Miura, E. Takasugi, and M. Yoshimura, Phys. Rev. **D63**, 013001 (2001).

[16] X.-G. He, arXiv:1504.01560 [hep-ph].

[17] E. Ma, Phys. Rev. **D73**, 077301 (2006).

[18] E. Ma, Phys. Rev. Lett. **112**, 091801 (2014).

[19] G. Aad *et al.* (ATLAS Collaboration), Phys. Lett. **B716**, 1 (2012).

[20] S. Chatrchyan *et al.* (CMS Collaboration), Phys. Lett. **B716**, 30 (2012).

[21] S. Fraser, E. Ma, and O. Popov, Phys. Lett. **B737**, 280 (2014).

[22] E. Ma, A. Natale, and O. Popov, Phys. Lett. **B746**, 114 (2015).

[23] D. Wyler and L. Wolfenstein, Nucl. Phys. **B218**, 205 (1983).

[24] R. N. Mohapatra and J. W. F. Valle, Phys. Rev. **D34**, 1642 (1986).

[25] E. Ma, Phys. Lett. **B191**, 287 (1987).

[26] S. Fraser and E. Ma, Europhys. Lett. **108**, 11002 (2014).

[27] V. Silveira and A. Zee, Phys. Lett. **B161**, 136 (1985).

[28] J. M. Cline, P. Scott, K. Kainulainen, and C. Weniger, Phys. Rev. **D88**, 055025 (2013).

[29] P. A. R. Ade *et al.* (PLANCK Collaboration), arXiv:1502.01589 [astro-ph.CO].

[30] E. Ma, arXiv:1502.02200 [hep-ph].

[31] L. Feng, S. Profumo, and L. Ubaldi, JHEP **1503**, 045 (2015).