Covariance in models of loop quantum gravity: Spherical symmetry

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Covariance in models of loop quantum gravity: Spherical symmetry

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Spherically symmetric models of loop quantum gravity have been studied recently by different methods that aim to deal with structure functions in the usual constraint algebra of gravitational systems. As noticed by Gambini and Pullin, a linear redefinition of the constraints (with phase-space dependent coefficients) can be used to eliminate structure functions, even Abelianizing the more-difficult part of the constraint algebra. The Abelianized constraints can then easily be quantized or modified by putative quantum effects. As pointed out here, however, the method does not automatically provide a covariant quantization, defined as an anomaly-free quantum theory with a classical limit in which the usual (off-shell) gauge structure of hypersurface deformations in space-time appears. The holonomy-modified vacuum theory based on Abelianization is covariant in this sense, but matter theories with local degrees of freedom are not. Detailed demonstrations of these statements show complete agreement with results of canonical effective methods applied earlier to the same systems (including signature change).

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I. INTRODUCTION

Several suggestions have been made in models of loop quantum gravity which may indicate a potential to provide interesting physical effects. Popular examples are mechanisms to avoid some of the singularities encountered in classical general relativity. Following from a crucial step in the procedure of loop quantization, most of these effects are based on a replacement of polynomial (extrinsic) curvature expressions in the canonical Hamiltonian of the classical theory by bounded (and usually periodic) functions. As can easily be seen by the example of isotropic models, in which the classical Hubble-squared term in the Friedmann equation would be turned into a bounded function, it is then not surprising that upper bounds on curvature or energy densities can be obtained. A more crucial consistency question, also posed in [1], is whether the resulting modified theories can be covariant, or whether the upper bounds on curvature amount to a symmetry-breaking cut-off.

In canonical formulations such as loop quantum gravity, covariance is not manifest but still plays an important role. Instead of using coordinate transformations of space-time tensors, canonical theories refer to gauge transformations which, in geometrical terms, generate deformations of spatial hypersurfaces in space-time [2]. The generator of a deformation normal to a hypersurface is the above-mentioned gravitational Hamiltonian. If it is modified by bounded curvature expressions (or other quantum corrections), it is unclear whether it can still generate gauge transformations. Mathematically, the question is whether modified Hamiltonians can retain closed Poisson brackets or commutators with themselves and with generators of spatial deformations tangential to hypersurfaces. Some information about this question has been gained in recent years using effective [3–8] and operator methods [9–13]. Here we will follow a new but, as we will see, not independent direction toward the same question.

Covariance cannot be addressed in minisuperspace models such as isotropic cosmological ones, because they do not show how temporal and spatial variations of fields are related. The simplest inhomogeneous models are obtained by imposing spherical symmetry, to be considered in this paper. In this setting one has a non-trivial set of hypersurface-deformation generators and brackets or commutators between them. As in the full theory, the bracket of two normal deformations has structure functions instead of structure constants, so that the generators do not form a Lie algebra. The usual quantization methods of gauge theories therefore complicate considerably, and existing quantizations of spherically symmetric models use either reformulations of the generators [14] or quantize the reduced phase space from which the gauge flow has been eliminated [15, 16]. An interesting new proposal of reformulating the generators (and at the same time including some ingredients of a loop quantization) is the Abelianization of normal deformations found recently in [17, 18]. Compared with earlier Abelianizations [19], an important feature mentioned in [18] is that it works even when a scalar field with local physical degrees of freedom is included. There is therefore a chance that the problem of structure functions may be overcome at least in these models.

A question left open in [17, 18] is whether the resulting quantizations are covariant. By quantizing a system in which the brackets of gauge generators have been turned into a Lie algebra, the constructions of [17, 18] certainly provide consistent quantum models. However, it is not clear whether or in what sense they are models of quantum gravity with a consistent space-time picture. This is the question we turn to in the present paper, starting with a discussion of what it means for a canonical theory to be covariant. We will show that the loop-modified vacuum model of [17] is covariant only if the original Hamiltonian, prior to Abelianization, is modified in a restricted way with exactly the same conditions found by effective methods [5]. There is therefore a remarkable convergence between results of Abelianization and the effective framework. We will also show that the modified model of [18] with a scalar field is not covariant, unless a background treatment is used for the scalar on a vaccum solution so that matter and gravity have non-matching versions of covariance. More broadly, we point out that to date no covariant inhomogeneous model with local physical degrees of freedom has been found with holonomy modifications from loop quantum gravity (while such models exist for curvature-independent inverse-triad corrections).

II. COVARIANCE IN CANONICAL TERMS

The canonical formulation of general relativity leads to a phase space given by the spatial metric q_{ab} and momenta related to extrinsic curvature K_{ab}. It is subject to the Hamiltonian constraints $H[N]$, labelled by spatial lapse functions N, and diffeomorphism constraints $D[M^a]$, labelled by spatial shift vector fields M^a. These constraints are first class with closed brackets [20, 21]

$$\{D[M_1^a], D[M_2^a]\} = D[\mathcal{L}_{M_1} M_2^a] \quad (1)$$

$$\{H[N], D[M^a]\} = -H[\mathcal{L}_M N] \quad (2)$$

$$\{H[N_1], H[N_2]\} = D[q^{ab}(N_1\partial_b N_2 - N_2\partial_b N_1)]\,. \quad (3)$$

They generate gauge transformations representing hypersurface deformations [2]. On the space of solutions to the constraints, the same gauge transformations are equivalent to Lie derivatives along space-time vector fields, and therefore represent coordinate freedom. Manifest covariance is replaced by gauge covariance under hypersurface deformations. (For more details on canonical gravity, see for instance [22].)

A. Conditions

This well-known result leads us to two conditions to be realized for a modified or quantized canonical theory to be covariant:

(i) The classical generators $H[N]$ and $D[M^a]$ must be replaced by generators which still have closed brackets, computed either as Poisson brackets in a modified or effective theory, or as commutators of operators in a quantization.

(ii) Brackets of the new generators of gauge transformations must have a classical limit identical with the classical brackets (1)–(3).

When condition (i) is satisfied, one has a consistent gauge theory since the gauge generators eliminate the same number of spurious degrees of freedom as in the classical case. But only when conditions (i) and (ii) are satisfied does one have a consistent *space-time theory*, in which there is a classical regime with the correct space-time structure. Accordingly, we call a modified, effective, or quantum theory *covariant* if and only if conditions (i) and (ii) are satisfied. The constructions in [17, 18] have provided consistent gauge theories obeying (i), but the question of covariance or condition (ii) has not been addressed yet.

An important aspect of condition (ii) is that it is an off-shell statement, for which not only the solution space of constraints $H[N] = 0$ and $D[M^a] = 0$ is relevant but also the behavior of fields not satisfying the constraints. This dependence on off-shell properties is in agreement with the usual understanding of space-time covariance, in which one makes use of line elements or metric tensors not necessarily solving Einstein's (or modified) field equations. It is also an important part of our classical picture of space-time as a stage on which different matter systems can be set up. Even though space-time and matter interact with each other, the covariance conditions commonly posed for matter theories require certain symmetries of the action on *any* background space-time, not necessarily one solving Einstein's equation. The usual covariance statements about (classical or quantum) matter systems on a classical space-time are therefore off-shell. For all we know, there could well be stronger interrelations between space-time and matter if both ingredients are quantum, so that it would no longer be possible to separate a covariant matter theory from an anomaly-free space-time. However, for the combined system to have the correct classical limit, our condition (ii), which is formulated only in this limit, must still hold.

B. Background treatment

In this context, one should therefore avoid taking the viewpoint that on-shell properties are sufficient to decide whether a space-time theory is meaningful. Although all observables computed with a given solution refer to the constraint surface modulo gauge transformations, covariance in the form usually used is a statement about a partial solution space. (For additional reasons, see [23].) Moreover, the full solution space of general relativity or a modified version is too unwieldy and in many cases of interest does not allow manageable on-shell statements in complete terms. Even models such as spherically symmetric gravity with a scalar field remain challenging in this setting. Most evaluations of gravitational theories (including [18]) make use of some kind of background approximation, in which one starts with a simple-enough vacuum solution and then perturbatively includes additional inhomogeneity or matter fields on this background. In practice, the background picture is therefore even more pronounced than the conceptual discussions of the preceding paragraph might indicate.

In more technical details, consistency of a matter model as a space-time theory may be formulated by requiring the fields to satisfy the local conservation equation $\nabla^\mu T_{\mu\nu} = 0$ for their stress-energy tensors. Canonically, as shown in [24], this equation follows from the analogs of (1)–(3) for a matter Hamiltonian (assuming, for simplicity, that no curvature couplings are present). In particular, relating stress-energy components to different kinds of derivatives of the matter contributions $\mathcal{H}_{\rm matter}$ and $\mathcal{D}_a^{\rm matter}$ to the local constraints, one can derive the equation

$$\begin{aligned} N\sqrt{\det q}\,\nabla_\mu T^\mu{}_0 \;=\; & -N\frac{\partial\mathcal{H}_{\rm matter}}{\partial t} - N^a\frac{\partial\mathcal{D}_a^{\rm matter}}{\partial t} \\ & +\mathcal{L}_{\vec{N}}C_{\rm matter}[N,N^a] + \frac{\partial q_{ab}}{\partial t}\frac{\delta H_{\rm matter}}{\delta q_{ab}} + \partial_b\left(N^2q^{ab}\mathcal{D}_a^{\rm matter} + 2N^cq^{ba}\frac{\delta H_{\rm matter}}{\delta q^{ac}}\right) . \end{aligned} \tag{4}$$

(The total matter contribution, summing the smeared contributions to the Hamiltonian and diffeomorphism constraints, is denoted by $C_{\rm matter}[N,N^a]$.) The classical off-shell brackets (and not just closed constraint brackets of some form) imply that the two terms $\partial\mathcal{H}_{\rm matter}/\partial t = \{\mathcal{H}_{\rm matter}, H[N,N^a]\}$ and $\partial^a(N^2\mathcal{D}_a^{\rm matter})$ cancel out if (3) holds, and the rest is zero based on other identities. A conservation law therefore follows only if the brackets are not just closed but (in the classical limit) of precisely the form obtained for the classical hypersurface deformations. (In [24], a matter Hamiltonian without curvature coupling has been assumed for simplicity, in which case the matter Hamiltonian and diffeomorphism generators alone have brackets of the form (1)–(3). Again, the importance of off-shell properties is underlined because the matter contributions to the constraints need not vanish separately. Some quantum effects, like those to be studied in the rest of this paper, may introduce additional curvature couplings, but they disappear in the classical limit in which the off-shell condition (ii) is formulated.)

For these independent reasons, off-shell brackets are relevant in the definition of covariance and should be checked before one can claim that a quantized model is a quantum theory of space-time. Even if one uses a background treatment for a matter field on a vacuum solution which latter has been shown to be covariant, there are still conditions to be imposed on the matter model: the existence of a local conservation law. A background treatment makes the construction of models less restrictive, but still such a procedure is far from being arbitrary.

The difference between a background treatment and a background-independent model in standard formulations is that only the latter ensure the existence of solutions to the coupled equations of gravity and matter, such as $G_{\mu\nu} = 8\pi G T_{\mu\nu}$ for general relativity. Compared to a background treatment, coupling gravity to matter in a consistent way implies additional restrictions even if the coupled equations are not actually solved, that is if no back-reaction is considered. Classically, the equation is consistent because the contracted Bianchi identity for $G_{\mu\nu}$ and the local conservation law for $T_{\mu\nu}$ take the same form.

In models of loop quantum gravity, the contracted Bianchi identity, in its canonical form as Poisson brackets of gravitational constraints, is generically modified. Instead of (3), we usually have

$$\{H[N_1], H[N_2]\} = D[\beta q^{ab}(N_1\partial_b N_2 - N_2\partial_b N_1)] \tag{5}$$

with a phase-space function β depending on the spatial metric q_{ab} or extrinsic curvature. A consistent background-independent model then requires the local conservation law, or the Poisson brackets of matter contributions to the constraints, to be modified in a matching way with the same function β. (We emphasize again that this condition is important even if back-reaction of matter on space-time is not considered by solving the coupled equations.) A background treatment, on the other hand, merely requires that the gravitational brackets and matter brackets have consistent but not necessarily matching forms. These contributions would both obey (5), but possibly with different functions β for gravity and matter. As background models, such theories would still be formally consistent, but it would not be clear whether they could be background formulations of covariant background-independent models. The quantization proposed in [18] is an example for a background treatment which, as demonstrated by the derivations that follow, is not known how to be embedded in a covariant background-independent theory of the same symmetry type (setting aside the vastly more complicated question of embedding it in some full quantum theory).

III. ABELIANIZATION OF NORMAL DEFORMATIONS IN SPHERICALLY SYMMETRIC MODELS

Compared with [14, 16], the formulation of spherically symmetric models with real connection variables, given in [25] is most relevant for the inclusion of loop effects as they are currently understood. We first recall these variables for notational purposes, and then discuss features of constraints and possible modifications.

A. Classical theory

Using a radial variable x, not necessarily identical to the area radius r, the spatial metric or line element

$$\mathrm{d}s^2 = \frac{(E^\varphi)^2}{|E^x|}\mathrm{d}x^2 + |E^x|(\mathrm{d}\vartheta^2 + \sin^2\vartheta\mathrm{d}\varphi^2) \tag{6}$$

is expressed by two functions $E^x(x)$ and $E^\varphi(x)$ which are the independent components of a densitized triad reduced to spherical symmetry [26]. (While E^x is a 1-dimensional scalar in the reduced model, E^φ has density weight one; see [25].) The triad components are canonically conjugate to components of extrinsic curvature:

$$\{K_x(x), E^x(y)\} = G\delta(x,y) \tag{7}$$

$$\{K_\varphi(x), E^\varphi(y)\} = \frac{1}{2}G\delta(x,y)\,. \tag{8}$$

1. Vacuum model

The reduced diffeomorphism constraint has only one component,

$$D[M] = \frac{1}{G}\int \mathrm{d}x M(x)\left(-\frac{1}{2}(E^x)'K_x + K_\varphi' E^\varphi\right)\,, \tag{9}$$

and the reduced Hamiltonian constraint is

$$H[N] = -\frac{1}{2G}\int \mathrm{d}x N(x)\left(|E^x|^{-\frac{1}{2}}E^\varphi K_\varphi^2 + 2|E^x|^{\frac{1}{2}}K_\varphi K_x + |E^x|^{-\frac{1}{2}}(1-\Gamma_\varphi^2)E^\varphi + 2\Gamma_\varphi'|E^x|^{\frac{1}{2}}\right) \tag{10}$$

with the spin-connection component $\Gamma_\varphi = -(E^x)'/2E^\varphi$. It is a lengthy but straightforward exercise to confirm that these phase-space functions have the brackets (1)–(3) with the inverse spatial metric q^{ab} replaced by the one component $|E^x|/(E^\varphi)^2$. These brackets control covariance in the reduced model, that is covariance under transformations preserving spherical symmetry.

The reduced model still has structure functions. However, as noted in [17], the linear combination

$$\tilde{\mathcal{C}} := \frac{(E^x)'}{E^\varphi}\mathcal{H} - 2\frac{K_\varphi\sqrt{|E^x|}}{E^\varphi}\mathcal{D} \tag{11}$$

of the original local constraints $\mathcal{H}$ and $\mathcal{D}$ allows one to eliminate K_x from the new constraint $\tilde{\mathcal{C}}$ replacing $\mathcal{H}$ (leaving $\mathcal{D}$ unchanged). Moreover, in the vacuum case, $\tilde{\mathcal{C}} = \mathcal{C}'$ is a total derivative, so that integration by parts removes one derivative at the (small) expense of working with a densitized lapse function $N' =: L$. Since the final smeared constraint

$$\begin{aligned} C[L] &= \int \mathrm{d}x L(x)\mathcal{C}(x) \\ &= -\frac{1}{G}\int \mathrm{d}x L(x)\left(\sqrt{|E^x|}\left(1+K_\varphi^2-\Gamma_\varphi^2\right) + \text{const.}\right)\,, \end{aligned} \tag{12}$$

obtained after integrating by parts $N\tilde{\mathcal{C}} = N\mathcal{C}'$, depends neither on K_x nor on spatial derivatives of K_φ or E^φ, the antisymmetric Poisson bracket of the final constraints C is trivially zero, while

$$\{C[L], D[M]\} = C[(ML)'] \tag{13}$$

as suitable for a constraint with densitized lapse function $L = N'$.

Our Equation (13) corrects a small mistake in Equation (15b) of [27] which has important conceptual ramifications. In (12), an undetermined constant appears because $C[L]$ is derived only for $L = N'$ and boundary terms are ignored in [17]. (The constant can be related to the classical ADM mass if asymptotic flatness is assumed.) The presence of a constant, which does not contribute to the left-hand side of (13), is consistent with (13) because the smearing function $(ML)'$ on the right-hand side is again a total derivative. This smearing function (rather than ML') not only follows from a direct calculation of the bracket, it is also the correct Lie derivative $\mathcal{L}_{M\mathrm{d}/\mathrm{d}x}L = ML' + M'L$ of a scalar L of density weight one. (Recall that L is defined as N', the derivative producing a density weight in the 1-dimensional radial manifold.)

2. Scalar field

With all these features, the original Abelianization of the vacuum constraint might look special and coincidental. However, as noted rather in passing in [18], the same basic idea can be used to Abelianize the bracket of two normal deformations for models with a scalar field, except that the constraint is no longer a total derivative and one does not integrate by parts: The analog of the previous smeared $\tilde{\mathcal{C}}$ is now

$$\begin{aligned} C[N] \;=\; & \frac{1}{G}\int \mathrm{d}x N(x)\Bigg(-\frac{1}{2}\frac{(E^x)'}{\sqrt{|E^x|}}(1+K_\varphi^2) - 2\sqrt{|E^x|}K_\varphi K_\varphi' \\ & +\frac{(E^x)'}{8\sqrt{|E^x|}(E^\varphi)^2}\left(4E^x(E^x)''+((E^x)')^2\right) - \frac{1}{2}\frac{((E^x)')^2\sqrt{|E^x|}(E^\varphi)'}{(E^\varphi)^3} \\ & +2\pi G\frac{(E^x)'}{\sqrt{|E^x|}(E^\varphi)^2}\left(P_\phi^2+(E^x)^2(\phi')^2\right) - 8\pi G\sqrt{|E^x|}\frac{K_\varphi}{E^\varphi}P_\phi\phi'\Bigg)\,. \end{aligned} \tag{14}$$

The Abelianization property is not trivial at all, but by an explicit calculation one can confirm that it is still true. As we shall see, it generalizes to other matter fields as well.

There is therefore a chance that Abelianizations of normal deformations can give rise to generic results at least in midi-superspace models. (Indeed, normal deformations in polarized Gowdy models can be Abelianized in a very similar way [28, 29].) Since the method relies on eliminating one component of extrinsic curvature from the Hamiltonian constraint, it is not clear how useful it could be in the full theory where no component is distinguished. It is also important that H, like D, is linear in the extrinsic-curvature component to be eliminated, which again is not true for any component in the full theory.

B. Modifications

Loop quantization of spherically symmetric models [25, 30] proceeds by turning E^x and E^φ into derivative operators on spin-network states, while K_x and K_φ are not directly represented. Instead, these degrees of freedom are realized via holonomy operators quantizing $h_{[x_1,x_2]} := \exp(i\int_{x_1}^{x_2} K_x(x)\mathrm{d}x)$ and $h_{\{x\}} := \exp(iK_\varphi(x))$. (We label "extended holonomies" of K_x by intervals $[x_1, x_2]$ and "point holonomies" of K_φ by points $\{x\}$.) The first expression is a gauge-invariant version of the U(1)-holonomy of the x-component of a connection, while the second expression models the same exponential behavior for the angular component.

In order to proceed to a quantization of the constraints, one has to make sure that all ingredients can be expressed by holonomies instead of curvature (or connection) components. Since the classical constraints are at most quadratic in the latter, they require modifications (often viewed as regularizations) before they can be turned into operators. (One can avoid modifications of the diffeomorphism constraint by representing the finite flow it generates instead of the infinitesimal generator [31]. We comment on this step and possible problems in App. A.) As mentioned in the introduction, unbounded functions of the classical curvature components are then replaced by bounded functions such as $h_{\{x\}}$ for K_φ^2. Applied to the Hamiltonian constraint, this process amounts to a modification which may break covariance.

In [17, 18], consistent gauge theories have been found even with a modification of the K_φ-dependence, making use of Abelianization results. However, the covariance question remains to be addressed. We now answer this question (with two different outcomes) in the two cases of the vacuum model and the scalar model. After this, we extend Abelianization results to general spherically symmetric matter systems, with the same outcome as for a scalar field.

1. Vacuum model

It is clear that a modified constraint $C[L]$ obtained after replacing K_φ^2 in (12) by $\delta^{-2}\sin^2(\delta K_\varphi)$ (or any other function of K_φ) preserves the Abelian nature of the vacuum constraint. Condition (i) for a consistent gauge theory is therefore respected by the modification. The question whether condition (ii) for a space-time model is respected is less trivial to answer. Without the modification, we know that the Abelian constraint comes from a system which obeys the classical hypersurface-deformation brackets. However, this observation does not guarantee that there is a formulation of the modified constrained system which (i) is closed for all values of δ and (ii) has brackets in agreement with classical generators of hypersurface deformations in the classical limit $\delta \to 0$.

Let us begin by modifying the first two terms of the usual classical Hamiltonian constraint with arbitrary functions of the extrinsic-curvature component K_φ. This procedure is equivalent to including only point-wise holonomy corrections for the angular component of the connection coefficient:

$$H[N] = -\frac{1}{2G}\int \mathrm{d}x N(x)\Big(|E^x|^{-\frac{1}{2}}E^\varphi f_1(K_\varphi)+2|E^x|^{\frac{1}{2}}f_2(K_\varphi)K_x + |E^x|^{-\frac{1}{2}}(1-\Gamma_\varphi^2)E^\varphi+2\Gamma_\varphi'|E^x|^{\frac{1}{2}}\Big)\,. \tag{15}$$

We first define a new linear combination of the modified Hamiltonian constraint and the usual diffeomorphism constraint, just as in the classical case, to eliminate K_x from the new constraint while leaving the diffeomorphism constraint unchanged

$$\tilde{\mathcal{C}} := \frac{(E^x)'}{E^\varphi}\mathcal{H} - 2\frac{f_2(K_\varphi)\sqrt{|E^x|}}{E^\varphi}\mathcal{D}. \tag{16}$$

The new constraint has the form

$$\begin{aligned}\tilde{C}[N] &= -\frac{1}{G}\int \mathrm{d}x N(x)\tilde{\mathcal{C}}(x) \\ &= -\frac{1}{G}\int \mathrm{d}x N(x)\left\{\frac{\mathrm{d}}{\mathrm{d}x}\left[\sqrt{|E^x|}\left(1-\Gamma_\varphi^2\right)\right]+\frac{1}{2}|E^x|^{-1/2}(E^x)'f_1+2|E^x|^{1/2}f_2K_\varphi'\right\}\,.\end{aligned} \tag{17}$$

It is straightforward to see that the condition for $\tilde{\mathcal{C}}$ to be a total derivative is

$$2f_2 = \frac{\mathrm{d}f_1}{\mathrm{d}K_\varphi}\,. \tag{18}$$

If this equation is true, we obtain a Lie algebra for the system of constraints as in the classical case. A more-general analysis of consistent modifications of the Abelianized constraints is given in Sec. III B 4.

Alternatively, we could have started from the classical version of the new constraint $C[N]$ in (12), after Abelianization, introduced the modification function f_1 as in [17], and then asked whether the modified constraint can be redefined as part of a constrained system with hypersurface deformations as the classical limit. To do so, we should find out how we can go from the (modified) Abelianized system of constraints to a new system of constraints $\mathcal{H}$ and $\mathcal{D}$ which, in the classical limit, are the generators of hypersurface deformations. After modifying the Abelianized constraint, we go back to a system of Hamiltonian and diffeomorphism constraints by a linear combination of $\mathcal{D}$ with the new constraint, which can only be the inverse of (16), with f_2 obeying (18) for the correct hypersurface-deformation brackets to be realized in the classical limit (after integrating by parts the modified (12)). The new system has the classical diffeomorphism constraint and a modified Hamiltonian constraint with the first two terms proportional to functions of K_φ which automatically obey the relation (18) as a consequence of integrating by parts.

2. *Equivalence with effective methods and deformed constraint brackets*

This outcome, including the precise form of the relation (18), is just what happens when one tries to close the algebra of constraints without Abelianization, starting with holonomy modifications directly in the Hamiltonian constraint [5, 8]. Thus, the Abelianized system of constraints in [17, 18] is equivalent to the system of modified constraints with deformed structure functions from effective models, provided one makes sure that the modified system is still covariant. In particular, the hypersurface-deformation brackets are closed but deformed for $\delta \neq 0$.

Although Abelianization of normal deformations allows one to remove structure functions from the brackets of constraints, for covariant versions the same modifications of brackets of hypersurface deformations are obtained as found in direct treatments of structure functions [5, 8]: For holonomy-modified spherically symmetric models, we have brackets (5) with

$$\beta = \frac{\partial f_2}{\partial K_\varphi} = \frac{1}{2}\frac{\partial^2 f_1}{\partial K_\varphi^2} \tag{19}$$

using (18). This function is negative near a local maximum of f_1, indicating signature change [32, 33]. This important consequence and related implications of holonomy modifications cannot be avoided by reformulating the constraint algebra because covariance conditions still require one to check the brackets of hypersurface deformations even if their generators are not used directly as constraints. Realizing these relationships, there is complete agreement between the modified models based on Abelianizations, presented in [17, 18], and the earlier constructions of anomaly-free effective models in [5, 8].

3. *Scalar field*

It is easy to see that the Hamiltonian constraint of a spherically symmetric gravity theory coupled to matter *cannot* be modified according to holonomy corrections as incorporated previously. If we look back at the classical form of the Hamiltonian (14), we realise that Abelization works due to some subtle cancellations. The bracket between the second term from the gravitational part in (14) (proportional to K'_φ) and the first term from the scalar part (proportional to P_ϕ) is cancelled by the bracket between the first term and the third term (proportional to $P_\phi \phi'$), both from the scalar part. Similarly, the bracket between the same (second) term from the gravitational part and the second term from the scalar part (proportional to ϕ') is cancelled by the term arising from the bracket between the second and third term of the scalar part. However, the most interesting cancellation happens between the brackets of the first and second terms of the scalar part and the bracket of the fourth term of the gravitational part (proportional to $(E^\varphi)'$) and the third term of the scalar part.

If we now replace the extrinsic-curvature components by some arbitrary functions of this variable, the resulting bracket of constraints can never be made to close into a combination of constraints, let alone made zero for an Abelian bracket. If we replace K_φ^2 in the gravitational part by some function $f(K_\varphi)$, then the K_φ in the third term of the scalar part has to be replaced by $\mathrm{d}f/\mathrm{d}K_\varphi$, such that the first two pairs of cancellations are still valid just as in the classical case. However, with this modification, the bracket between the first and second terms of the scalar part (which do not contain K_φ to be modified) is *not* cancelled by the bracket coming from the term proportional to $(E^\varphi)'$ from the gravitational part and the third term from the scalar part, the latter now having been modified. (Section III B 4 contains a more-explicit demonstration.)

Although the result is negative in the sense that a simple Abelianization does not lead to a covariant modified theory, there is again agreement with effective methods. Attempts to include scalar fields in spherically symmetric models within an effective approach, along the lines of [5, 8] for vacuum models, have failed to provide closed brackets of constraints including holonomy modifications. The reason for this lack of closure is the appearance of precisely the same terms that do not cancel out in an attempted Abelianization. At present, it is not known whether holonomy-modified spherically symmetric models with a scalar field can be anomaly-free, or whether their normal deformations can be Abelianized. We will demonstrate the equivalence of these negative results based on effective methods and partial Abelianizations after introducing more-general matter systems.

4. *General matter model*

We now consider generic (spherically symmetric) matter systems with non-derivative couplings to gravity. We assume a consistent or first-class gravity-matter system of this kind, which has been obtained by inserting modification functions in a classical matter system without curvature coupling and higher spatial derivatives. The classical matter Hamiltonian therefore obeys the bracket (3) on its own, without including gravitational terms. We assume same property to be true for a modified Hamiltonian obtained in this way even if modification functions are allowed to depend on curvature components (but not on spatial derivatives). In fact, it turns out to be difficult to find consistent modified theories violating this assumption because cross-terms of the gravitational and matter parts of constraints in the $\{H,H\}$-bracket would lead to higher spatial derivatives in the bracket which, if nonzero, could not be absorbed in a constraint to produce a first-class system One can also confirm this property explicitly for the matter Hamiltonians given below, where correction functions are allowed to depend on K_φ.

The form of modifications assumed here therefore implies that the matter parts of the diffeomorphism and Hamiltonian constraints, $H_{\rm matter}[N] = \int \mathrm{d}x N \mathcal{H}_{\rm matter}$ and $D_{\rm matter}[M] = \int \mathrm{d}x M \mathcal{D}_{\rm matter}$, satisfy

$$\{D_{\rm matter}[M], D_{\rm matter}[N]\} = D_{\rm matter}[MN' - NM'] \tag{20}$$

$$\{H_{\rm matter}[M], D_{\rm T}[N]\} = -H_{\rm matter}[NM'] \tag{21}$$

$$\{H_{\rm matter}[M], H_{\rm matter}[N]\} = D_{\rm matter}\left[\bar{\beta}|E^x|(E^\varphi)^{-2}(MN' - NM')\right] \tag{22}$$

where $D_{\rm T}[N] := D[N] + D_{\rm matter}[N]$ is the total diffeomorphism constraint, including the gravitational part. Classically one would have $\bar{\beta} = 1$ (and $\mathcal{H}_{\rm matter}$ would only depend on the triad fields), but here we are allowing for a correction function $\bar{\beta}(K_\varphi, E^x)$ to take into account possible deformations of the matter part as in (5). Therefore, to compute brackets we assume that $\mathcal{H}_{\rm matter}$ may also depend on K_φ (but not on K_x, nor on derivatives of K_φ or the triad). The brackets of total Hamiltonian constraints, combining gravity and matter contributions, then do not decouple from each other, and cross-terms will have to be considered below. (Cross-terms must vanish in this case according to the argument given at the beginning of this subsection, but will do so only with additional restrictions on the modification functions.)

Examples of such deformed matter systems include the scalar field, dust and null dust: In the first case, we have a canonical pair

$$\{\phi(x), P_\phi(y)\} = \frac{1}{4\pi}\delta(x,y), \tag{23}$$

and corresponding constraints

$$D_{\rm matter}[N] = 4\pi \int {\rm d}x\, N P_\phi\, \phi'\,, \tag{24}$$

and

$$H_{\rm matter}[M] = 4\pi \int {\rm d}x M \left(\nu \frac{P_\phi^2}{2|E^x|^{1/2}E^\varphi} + \sigma \frac{|E^x|^{3/2}\phi'^{\,2}}{2E^\varphi} + |E^x|^{1/2}E^\varphi \frac{U(\phi)}{2} \right), \tag{25}$$

with correction functions $\nu(K_\varphi, E^x)$ and $\sigma(K_\varphi, E^x)$ such that $\bar\beta = \nu\sigma$. For dust fields [34], we have two canonical pairs with

$$\{\tau(x), P_\tau(y)\} = \{\Phi(x), P_\Phi(y)\} = \frac{1}{4\pi}\delta(x,y)\,, \tag{26}$$

and a contribution

$$D_{\rm matter}[N] = 4\pi \int {\rm d}x\, N\,(P_\tau \tau' + P_\Phi \Phi') \tag{27}$$

to the diffeomorphism constraint, while the matter part of the Hamiltonian constraint is

$$H_{\rm matter}[M] = 4\pi \int {\rm d}x\, M \sqrt{P_\tau^2 + \bar\beta \frac{|E^x|}{(E^\varphi)^2}(P_\tau\tau' + P_\Phi\Phi')^2}\,. \tag{28}$$

For null dust fields [35], only the the second canonical pair in (26) survives and

$$H_{\rm matter}[M] = 4\pi \int {\rm d}x\, M \sqrt{|\bar\beta|}\frac{\sqrt{|E^x|}}{E^\varphi}|P_\Phi \Phi'|\,. \tag{29}$$

Starting from the classical linear combination of constraints

$$\tilde{\mathcal{C}}_{\rm T} = \frac{(E^x)'}{E^\varphi}(\mathcal{H} + \mathcal{H}_{\rm matter}) - 2\frac{K_\varphi\sqrt{|E^x|}}{E^\varphi}(\mathcal{D} + \mathcal{D}_{\rm matter})\,,$$

one may replace K_φ^2, $K_\varphi K_\varphi'$ and K_φ multiplying the matter part of the diffeomorphism constraint by three different functions f_1, F_2 and $F_{\rm matter}$. We therefore define

$$\tilde{\mathcal{C}}_{\rm matter} := \frac{(E^x)'}{E^\varphi}\mathcal{H}_{\rm matter} - 2\frac{F_{\rm matter}(K_\varphi, E^x)\sqrt{|E^x|}}{E^\varphi}\mathcal{D}_{\rm matter} \tag{30}$$

and

$$\tilde{\mathcal{C}}_{\rm T} := \tilde{\mathcal{C}} + \tilde{\mathcal{C}}_{\rm matter}\,. \tag{31}$$

In the first term of gravitational contributions to the constraint, we now consider a more-general modified expression:

$$\begin{aligned} \tilde{C}[M] \;=\; & -\frac{1}{2G}\int {\rm d}x\, M \Bigg(|E^x|^{-1/2}(E^x)'(1 + f_1(K_\varphi, E^x)) + 2|E^x|^{1/2}F_2(K_\varphi, K_\varphi', E^x) \\ & - \frac{(E^x)'}{4(E^\varphi)^2}\left(4|E^x|^{1/2}(E^x)'' + |E^x|^{-1/2}((E^x)')^2\right) + \frac{|E^x|^{1/2}((E^x)')^2(E^\varphi)'}{(E^\varphi)^3} \Bigg) \end{aligned} \tag{32}$$

where K_φ^2 has been replaced by a function f_1 of K_φ (and possibly E^x), and $K_\varphi K_\varphi'$ by a function F_2 of these same variables. We will also assume the orientation $E^x > 0$.

Using the equivalent expression

$$\tilde{C}[M] = -\frac{1}{2G}\int {\rm d}x\, M \left\{ \frac{\rm d}{{\rm d}x}\left[2\,|E^x|^{1/2}\left(1 - \Gamma_\varphi^2\right)\right] + 2(|E^x|^{1/2})' f_1 + 2|E^x|^{1/2}F_2 \right\}\,, \tag{33}$$

it is straight-forward to see that requiring the term inside the parenthesis to be a total derivative restricts f_1 and F_2 to be independent of E^x, and F_2 to be linear in K'_φ:

$$F_2(K_\varphi, K'_\varphi) = 2f_2(K_\varphi)K'_\varphi \qquad \text{with} \qquad 2f_2 = \frac{\mathrm{d}f_1}{\mathrm{d}K_\varphi}\,. \tag{34}$$

(Substituting the first condition back in (33) we recover (17) and the second condition is again the same as the one obtained from effective models for the closure of the modified Hamiltonian and diffeomorphism constraints.)

Using (20), (21) and (22), we compute the bracket

$$\begin{aligned}\{\tilde{C}_{\mathrm{T}}[M], \tilde{C}_{\mathrm{T}}[N]\} \;=\; & \{\tilde{C}[M], \tilde{C}[N]\} + \int \mathrm{d}x(MN' - NM') \\ & \times\Bigg\{ \frac{|E^x|}{(E^\varphi)^2}\left[\frac{((E^x)')^2}{(E^\varphi)^2}\left(\bar{\beta} - \frac{\partial F_{\mathrm{matter}}}{\partial K_\varphi}\right) + 2F_{\mathrm{matter}}\left(2F_{\mathrm{matter}} - \frac{\partial F_2}{\partial K'_\varphi}\right)\right]\mathcal{D}_{\mathrm{matter}} \\ & - \frac{|E^x|^{1/2}(E^x)'}{(E^\varphi)^2}\left(2F_{\mathrm{matter}} - \frac{\partial F_2}{\partial K'_\varphi}\right)\left(\mathcal{H}_{\mathrm{matter}} - E^\varphi\,\frac{\partial \mathcal{H}_{\mathrm{matter}}}{\partial E^\varphi}\right) \\ & + \frac{|E^x|^{1/2}((E^x)')^3}{2(E^\varphi)^4}\,\frac{\partial \mathcal{H}_{\mathrm{matter}}}{\partial K_\varphi}\Bigg\}\,. \end{aligned} \tag{35}$$

(For details, see App. B.) We first note that the bracket

$$\begin{aligned}\{\tilde{C}[M], \tilde{C}[N]\} \;=\; & -\frac{1}{2G}\int \mathrm{d}x\,(MN' - NM')\frac{|E^x|\,((E^x)')^2}{(E^\varphi)^3}\Bigg[\frac{\partial F_2}{\partial K_\varphi} - \frac{\partial^2 F_2}{\partial K_\varphi \partial K'_\varphi}K'_\varphi \\ & + \left(\frac{1}{2|E^x|}\left(\frac{\partial f_1}{\partial K_\varphi} - \frac{\partial F_2}{\partial K'_\varphi}\right) - \frac{\partial^2 F_2}{\partial E^x \partial K'_\varphi}\right)(E^x)' - \frac{\partial^2 F_2}{(\partial K'_\varphi)^2}\,K''_\varphi\Bigg] \end{aligned} \tag{36}$$

by itself may form a closed system only if it vanishes identically: since (36) does not depend on K_x and $(E^x)''$, $\{\tilde{C}[M], \tilde{C}[N]\} = \mathcal{F}_{\tilde{\mathcal{C}}}\tilde{\mathcal{C}} + \mathcal{F}_{\mathcal{D}}\mathcal{D}$ implies $\mathcal{F}_{\tilde{\mathcal{C}}} = \mathcal{F}_{\mathcal{D}} = 0$. This is the Abelianization condition in the vacuum case.

The vanishing of the K''_φ term again implies that F_2 must depend linearly on K'_φ. Using this condition, all terms proportional to K'_φ cancel out. The vanishing of the first term and the $(E^x)'$-term imply that F_2 has the form $F_2 = 2f_2(K_\varphi, E^x)K'_\varphi + f_3(E^x)$, for a general function f_3 of the triad component E^x, as well as

$$\frac{\partial f_1}{\partial K_\varphi} - 2f_2 = 4|E^x|\frac{\partial f_2}{\partial E^x}\,. \tag{37}$$

This requirement matches (34) in the case of correction functions independent of E^x.

With these conditions, we can now look at the additional contributions to (35) in the presence of matter. Using the expression obtained for F_2 and requiring the total bracket (35) to be zero, we must have

$$F_{\mathrm{matter}} = f_2 \quad , \quad \bar{\beta} = \frac{\partial f_2}{\partial K_\varphi} \quad \text{and} \quad \frac{\partial \mathcal{H}_{\mathrm{matter}}}{\partial K_\varphi} = 0\,. \tag{38}$$

The last condition in (38) tells us that no deformation of the matter part depending on K_φ is consistent with Abelianization (or a closed system). Furthermore, in the case of deformations of the matter Hamiltonian independent of curvature, $\bar{\beta}(E^x)$ can only be a function of the triad. Thus also in this case, the second condition in (38) implies that the only possible dependence on K_φ of the whole system is the classical one. If a deformation consistent with Abelianization exists, it must contain other derivatives of the fields. Remarkably, however, in the classical case with $\bar{\beta} = 1$ Abelianization of the constraint $\tilde{C}_{\mathrm{T}}[M]$ follows for general matter systems satisfying (20)–(22), not just for a scalar field.

5. *Maxwell field*

To arrive at the negative conclusions above, it was crucial that the matter contribution to the diffeomorphism constraint is assumed to be non-zero. It is well-known, however, that substiting a spherically symmetric ansatz in the canonical action for a Maxwell field leads to a consistent reduced system with a vanishing contribution to the diffeomorphism (or vector) constraint [36]. This property leaves the possibility open for a consistent Abelianization of the Einstein-Maxwell system (which, however, does not have local degrees of freedom in spherical symmetry).

Indeed, in this case the canical pairs are

$$\{\mathcal{A}_x(x), P(y)\} = \frac{1}{4\pi}\delta(x,y)\,, \tag{39}$$

with $\mathcal{A}_x(x)$ the sole spatial radial component of the vector potential and $\mathcal{E}^x := P\sin\vartheta$ the only non-zero radial component of the (densitized) electric field. The contribution to the Hamiltonian constraint is

$$H_{\rm matter}[M] = 4\pi\int {\rm d}x\, M\frac{E^\varphi P^2}{2\,|E^x|^{3/2}}\,, \qquad (40)$$

and there is the additional (Maxwell) Gauss constraint:

$$G_{\rm Maxwell}[\Lambda] = 4\pi\int {\rm d}x\,\Lambda\, P'\,. \qquad (41)$$

There is no contribution to the vector constraint obtained from the $\{H,H\}$-bracket, so the system does not satisfy (20)–(22) but instead

$$D_{\rm T}[N] \;=\; D[N] \qquad (42)$$

$$\begin{aligned}\{H_{\rm matter}[M], D_{\rm T}[N]\} \;&=\; -H_{\rm matter}[NM'] \qquad (43)\\ &\quad -G_{\rm Maxwell}\left[MNE^\varphi|E^x|^{-3/2}P\right]\\ \{H_{\rm matter}[M], H_{\rm matter}[N]\} \;&=\; 0\,. \qquad (44)\end{aligned}$$

As before, one may also consider a deformed system (which satisfies the same bracket relations) with

$$H_{\rm matter}[M] = 4\pi\int {\rm d}x\, M\frac{\nu\, E^\varphi P^2}{2\,|E^x|^{3/2}}\,, \qquad (45)$$

and correction function $\nu(K_\varphi, E^x)$. The combined constraint $\tilde{C}_{\rm T}[M]$ results from taking $\mathcal{D}_{\rm matter}=0$ in expression (30), so that now $\{\tilde{C}_{\rm matter}[M],\tilde{C}_{\rm matter}[N]\}=0$ and only the last term in (35) survives:

$$\{\tilde{C}_{\rm T}[M],\tilde{C}_{\rm T}[N]\} = \{\tilde{C}[M],\tilde{C}[N]\} + \int {\rm d}x (MN'-NM')\frac{|E^x|^{1/2}((E^x)')^3}{2(E^\varphi)^4}\,\frac{\partial\mathcal{H}_{\rm matter}}{\partial K_\varphi}\,. \qquad (46)$$

Thus, a consistent Abelian deformation is always possible, but again only as long as the correction function ν is independent of curvature. The gravitational contribution to the Hamiltonian constraint, however, can be modified in a curvature-dependent way. Nevertheless, this model is not a counter-example to our statements that no covariant holonomy-modified models with local degrees of freedom are known, because there are no local degrees of freedom in the spherically symmetric Einstein–Maxwell system. As we shall recall in the next section, these properties are again fully compatible with results [37] using effective methods and the requirement of anomaly-freedom.

We can interpret this system as further circumstantial evidence that local degrees of freedom seem to be responsible for making it more difficult (if not impossible) to find covariant models with holonomy modifications. The spherically symmetric Einstein–Maxwell system can obey the required consistency conditions, but only because the Gauss constraint allows one to eliminate the new kinematical degree of freedom, given by the Maxwell fields, from the diffeomorphism constraint. The same constraint, $P'=0$ in its local version, removes the new kinematical degree of freedom from the reduced phase space. In contrast to the scalar or dust examples, the non-gravitational local kinematical degree of freedom therefore does not lead to local physical degrees of freedom, which then do not seem to present an obstacle to a consistent holonomy-modified model.

Looking back at these calculations, the modified Einstein–Maxwell system can be consistent despite the fact that there is no contribution to the diffeomorphism constraint because in this case the matter contribution $\int {\rm d}x\,\delta N\mathcal{A}_x P'$ to the infinitesimal generator of radial diffeomorphisms is a multiple of the Gauss constraint. Again, this is a special property of reduced models and unlikely to extend to general configurations. One may consistently define the Einstein–Maxwell constrained system as in [37], with non-zero contribution

$$D_{\rm matter}[N] = -4\pi\int {\rm d}x\, N\mathcal{A}_x P'\,. \qquad (47)$$

However, this alternative set of constraints satisfies (20)–(22) with $\bar\beta=0$ and hence does not lead to an Abelian deformation. (In fact, even classically, the corresponding system of constraints $C_{\rm T}[M]$ and $D_{\rm T}[N]$ is not closed unless the Gauss constraint is included.) Even though the two initial systems of constraints with different contributions to $D_{\rm T}[N]$ are equivalent, the two systems derived from them by substituting the Hamiltonian constraint with $C_{\rm T}[M]$ are not.

The generator of spatial diffeomorphisms, (47), has also been used in [38] in the context of Abelianization. While Abelianization of normal hypersurface deformations could be achieved in this case, it was possible only by fixing the U(1)-gauge of the Maxwell contribution. Our construction leads to a more general result, showing Abelianization even if no partial gauge fixing is used.

C. Impossible modifications

We will now verify explicitly that the impossibility of obtaining a (partially) Abelian algebra from deformations of the classical $\tilde{\mathcal{C}}_{\rm T}$ constraint is consistent with negative results for an anomaly-free deformed constraint algebra.

Again, consider a classical spherically symmetric matter system with non-derivative couplings, and such that correction functions in the Hamiltonian $H_{\rm T}[M] = H[M] + H_{\rm matter}[M]$ do not contain derivatives of the gravitational fields K and E. It is easy to see that if the deformed vacuum algebra satisfies

$$\{H[M], H[N]\} = D[\beta|E^x|(E^\varphi)^{-2}(MN' - NM')] \quad (48)$$

with a correction function β depending on the connection or extrinsic curvature, and the matter contribution to the diffeomorphism constraint is non-trivial, then the matter Hamiltonian must be deformed with correction functions also depending on extrinsic curvature. (This is at least true if we assume no second or higher derivatives of the matter fields.) Indeed, if we assume $\mathcal{H}_{\rm matter}$ to be independent of K_x and K_φ it follows that

$$S := \{H[M], H_{\rm matter}[N]\} - (M \leftrightarrow N) = 0\,,$$

and therefore the 'crossed' or 'mixed' brackets vanish and we have

$$\{H_{\rm T}[M], H_{\rm T}[N]\} = \{H[M], H[N]\} + \{H_{\rm matter}[M], H_{\rm matter}[N]\}\,.$$

For a first-class algebra we must have

$$\{H_{\rm matter}[M], H_{\rm matter}[N]\} = D_{\rm matter}[\beta|E^x|(E^\varphi)^{-2}(MN' - NM')]\,.$$

There cannot be additional multiples of the (total) Hamiltonian constraint since the latter contains second derivatives of E^x. However, the right-hand side of the above expression for the bracket depends on curvature, so the left hand side, that is $H_{\rm matter}[M]$, must depend on curvature after all.

Motivated by the previous observations and by consistent deformations with inverse-triad corrections obtained in [5–7], we will consider matter systems which additionally satisfy (20), (21) and (22) with a correction function $\bar{\beta}(K_x, K_\varphi, E^x)$ depending on both extrinsic curvature components and E^x. (The scalar and dust fields above with deformation functions also depending on K_x satisfy these conditions.) For these systems or any other model with matter Hamiltonians depending on connection or extrinsic-curvature components, we have

$$\begin{aligned} S \;=\; & \int {\rm d}x\,(MN' - NM')\left[\left(\frac{|E^x|^{-\frac{1}{2}}(E^x)'}{2E^\varphi} - \frac{|E^x|^{\frac{1}{2}}(E^\varphi)'}{(E^\varphi)^2}\right)\frac{\partial\mathcal{H}_{\rm matter}}{\partial K_x} + \frac{|E^x|^{\frac{1}{2}}(E^x)'}{2(E^\varphi)^2}\frac{\partial\mathcal{H}_{\rm matter}}{\partial K_\varphi}\right] \\ & + \int {\rm d}x\,(MN'' - NM'')\frac{|E^x|^{\frac{1}{2}}}{E^\varphi}\frac{\partial\mathcal{H}_{\rm matter}}{\partial K_x}\,. \end{aligned} \quad (49)$$

As shown in [8], variations by $MN' - NM'$ and $MN'' - NM''$ are independent, so that $\partial\mathcal{H}_{\rm matter}/\partial K_x = 0$. Therefore, restricting now to K_x-independent corrections,

$$\begin{aligned} \{H_{\rm T}[M], H_{\rm T}[N]\} \;=\; & \int {\rm d}x\,(MN' - NM')\left(\frac{|E^x|}{(E^\varphi)^2}\left(\beta\mathcal{D} + \bar{\beta}\mathcal{D}_{\rm matter}\right)\right. \\ & \left. + \frac{|E^x|^{\frac{1}{2}}(E^x)'}{2(E^\varphi)^2}\frac{\partial\mathcal{H}_{\rm matter}}{\partial K_\varphi}\right)\,. \end{aligned} \quad (50)$$

It is now easy to see that the last term cannot be a linear combination containing the total Hamiltonian because the latter contains second-order derivatives of E^x in its gravitational part while the former does not. Since $\mathcal{H}_{\rm matter}$ must be independent of K_x, this last term cannot contain a multiple of the gravitational part of the diffeomorphism constraint $\mathcal{D}$ either. Hence the only possibilities left for a closed algebra are that the last term vanishes or that it is a multiple of $\mathcal{D}_{\rm matter}$. Since we are assuming $\mathcal{D}_{\rm matter} \neq 0$, this last possibility is, however, inconsistent since it would require β to depend on $(E^x)'$. It then follows again that deformations of the matter

Hamiltonian must be independent of K_φ:

$$\frac{\partial \mathcal{H}_{\rm matter}}{\partial K_\varphi} = 0 \quad \text{and} \quad \beta = \bar{\beta}\,.$$

If $\mathcal{H}_{\rm matter}$ is independent of both curvature components, then $\bar{\beta}$ is necessarily independent of curvature too and the last condition above precludes any vacuum deformation such as (19) depending on curvature. We have come full circle, in this case the only consistent deformations of the combined gravity-matter system independent of K_x must also be independent of K_φ. Only triad-dependent deformations of the type found in [5–7] are allowed.

For Maxwell fields, there is no contribution from the $\{H_{\rm matter}[M], H_{\rm matter}[N]\}$-bracket, and therefore $\bar{\beta}\mathcal{D}_{\rm matter} = 0$ in (50). Deformations of the gravitational and matter parts of the Hamiltonian effectively 'decouple' and we see that consistent or anomaly-free deformations are possible with K_φ-dependent deformations (19) of the gravitational part and an undeformed or deformed but curvature-independent matter Hamiltonian.

IV. CONCLUSIONS

Abelianization of normal hypersurface deformations can eliminate structure functions from constraint brackets and thereby open up access to standard quantization methods applied to gravitational models. However, by itself, this result leaves the question of covariance unaddressed, which is important for gravitational theories. As shown here, covariance of modified theories is indeed non-trivial in this setting, and it is not always realized: A standard holonomy modification of the Abelianized constraint does not lead to hypersurface-deformation generators with the correct classical limit if a scalar field or other matter with local physical degrees of freedom are coupled to gravity.

In our general discussion of covariance in canonical systems, we have highlighted the important distinction between background treatments and background-independent theories. Even if back-reaction is not considered, there is a difference between these two cases as regards covariance in non-classical systems. Hypersurface-deformation generators may then be deformed in different ways as one departs from the classical limit, but a consistent gravity-matter system requires the same deformation of both ingredients. A background treatment in which covariance is required separately for gravity and matter, on the other hand, may formally give rise to more options. As an example, the holonomy-modified scalar model of [18] does not correspond to a covariant gravity-matter system, as shown here, but the actual constructions of [18] make use of a gravitational background and may be formally consistent. (We note that two different kinds of modifications appear in [18], holonomy modifications and a discretization of the scalar Hamiltonian. While the latter is in the foreground in [18], we have tested only the former in the present paper. Covariance conditions on discretized scalar Hamiltonians remain to be explored, but possible discrete versions of hypersurface-deformation brackets are known [39].)

An interesting result is also the fact that there seems to be complete agreement on this question, addressed with different methods: Abelianization and anomaly freedom implemented with effective techniques as introduced in [3] in the context of cosmological perturbations. This convergence of results obtained by different methods gives further support to the phenomenon of signature change discovered by an analysis of canonical effective models [32]. At first sight, it might seem that the constructions of [17, 18] do not lead to modified space-time structures in spherically symmetric models, unlike what effective calculations have shown in the same models [5, 8]. However, if one actually poses the question of covariance and space-time structure in the constructions of [17, 18], one finds, as shown here, that covariance requires the Hamiltonian constraints to be modified with the same restriction (18) as found in [5, 8] for anomaly-free effective models. If K_φ^2 is replaced by some function $f(K_\varphi)$, in effective and Abelianized models the same modified brackets

$$\{H[N_1], H[N_2]\} = -D[\beta(K_\varphi)(|E^x|/(E^\varphi)^2)(N_1N_2'-N_2N_1')] \tag{51}$$

are realized for generators of hypersurface deformations, with a non-trivial function

$$\beta(K_\varphi) = \frac{1}{2}\frac{\partial^2 f}{\partial K_\varphi^2}\,. \tag{52}$$

(Signature change is indicated by β changing sign, which always happens if $f(K_\varphi)$ has a local maximum. For the popular modification $f(K_\varphi) = \delta^{-2}\sin^2(\delta K_\varphi)$, for instance, $\beta(K_\varphi) = \cos(2\delta K_\varphi)$.) The agreement of results is promising, but at the same time one then has to take seriously the resulting modified space-time structures at high curvature, which can lead to problems of indeterminism and Cauchy horizons for black holes [40] or global issues for cosmological perturbation equations [33].

In this light, the language used in [17, 18], speaking about quantum systems *on quantum space-time* does not seem justified because covariance conditions, which are usually understood as being crucial for space-time theories, have not been checked. (This language goes back to cosmological constructions in [41, 42], where it seems equally unjustified because the background minisuperspace models used in these examples do not even allow one to test covariance and the consistency of quantum space-time structures. Instead, metric structures are merely postulated.) In the scalar model, no consistent space-time structure of the holonomy-modified theory is known, so that it seems unclear how to use formal solutions of these systems for an analysis of Hawking radiation, the stated aim of [18].

At present, it is not known whether covariance can always be realized in the presence of holonomy modifications from loop quantum gravity, even if one restricts oneself to the rather tractable spherically symmetric models. Especially the presence of local physical degrees of freedom seems to pose a challenge, as indicated by the general matter models considered here (as well as the spherically symmetric Einstein–Maxwell system as discussed in Sec. III B 5) and the polarized Gowdy models of [28]. This result of our paper might pose a challenge to loop quantum gravity. We certainly did not discuss full quantizations of the models considered, but if the theory is to have the correct semiclassical limit, brackets of the form analyzed here will be encountered in some way.

The partial nature of our no-go results [53] can be used to suggest how covariant holonomy-modified models with local degrees of freedom could possibly be realized. One way to avoid the negative conclusions would be to include higher spatial derivatives of the matter field. Such terms are expected in continuum effective models of loop quantum gravity because matter fields and their standard derivative terms in the Hamiltonian have to be discretized for an operator acting on spin-network states [43]. For anomalies to cancel out, holonomy modifications in the gravitational contribution to the constraint would have to be carefully adjusted to matter discretizations. So far, these two quantization steps have been considered as independent, but off-shell anomaly-freedom may force one to combine them. If a consistent version then becomes possible, it would have several unexpected features, in addition to making consistent models rather tightly constrained. First, for the covariance conditions of the gravitational background and the matter system to match, the matter discretization would have to depend on extrinsic curvature because the modified structure function (52) of a covariant holonomy-modified background has such a dependence. Secondly, holonomy modifications in one direction (here, an angular direction in spherically symmetric models which gives rise to point holonomies of K_φ) would have to be closely related to the matter discretization in another direction (here, the radial one so as to have higher spatial derivatives). It is not clear whether covariant models can be found by implementing these features, evading our no-go results. (For radial holonomy modifications in vacuum spherically symmetric models, higher spatial derivatives do not seem to help much [8].) Nevertheless, there is a chance that it would be fruitful to match covariance conditions of gravitational terms with holonomy modifications, as studied in [5, 8] and in the present paper, with methods to obtain consistent discretizations as studied for instance in [39, 44, 45].

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Appendix A: Diffeomorphism constraint

The diffeomorphism constraint in loop quantum gravity is usually not constructed by writing the classical expression in terms of holonomies and inserting basic operators, but rather by lifting the spatial flow generated by the constraint to the state space [31]. (See [46] for an alternative.)

In spherically symmetric models [25], one can represent states by referring to an orthonormal basis

$$\psi_{\{(x_1,k_1,\mu_1),\ldots,(x_n,k_n,\mu_n)\}}[K_x,K_\varphi] = \prod_{j=1}^{n} \exp\left(ik_j \int_{x_j}^{x_j+1} K_x \mathrm{d}x\right) \exp\left(i\mu_j K_\varphi(x_j)\right) \tag{A1}$$

with integer k_j, real numbers μ_j, and x_j in the radial manifold. (For simplicity, we assume the radial manifold to be compact. In the notation used to write states, we set $k_{n+1}=0$.) Spatial diffeomorphisms Φ can easily be represented unitarily by

$$\hat{\Phi}\psi_{\{(x_1,k_1,\mu_1),\ldots,(x_n,k_n,\mu_n)\}} := \psi_{\{(\Phi(x_1),k_1,\mu_1),\ldots,(\Phi(x_n),k_n,\mu_n)\}}\,. \tag{A2}$$

This action can be used to factor out spatial diffeomorphisms by group averaging, but it does not define a diffeomorphism constraint: States with different $\{x_1,\ldots,x_n\}$ are orthogonal to each other, so that one cannot take a t-derivative of the quantized flow of a 1-parameter family $\Phi_t=\exp(tv)$ with a spatial vector field v as an infinitesimal generator.

1. Effective constraints

In a continuum effective theory, on the other hand, there should be a well-defined version of the diffeomorphism constraint, possibly with quantum corrections. For instance, in the canonical framework of [47–49], the effective constraint would be computed as the expectation value of $\hat{\Phi}$ in a suitable class of semiclassical states obtained by superpositions of the basis states. For a local effective theory (and therefore the classical limit) to exist, these superposed states must be such that expectation values $\langle\hat{E}^x\rangle$ and so on are differentiable functions of x in some coarse-graining approximation. A derivative expansion of these or more-complicated expectation values (such as the Hamiltonian constraint) then gives rise to a theory with gauge transformations of infinitesimal diffeomorphisms acting on effective fields.

In order to compute an effective constraint, one need not construct explicit semiclassical states, which would be challenging in models of loop quantum gravity. Instead, one parameterizes states by expectation values and moments of basic operators, so that a semiclassical regime can be specified more easily by a certain hierarchy of the moments by powers of $\hbar$. By the same condition, the derivative expansion can be combined with a semiclassical expansion, in which the classical diffeomorphism constraint is extended by moment terms. Not only expectation values of the basic operators but also their fluctuations and higher moments are then subject to gauge transformations.

In addition to expectation values of basic operators quantizing E^x, K_x, E^φ and K_φ in the case of spherically symmetric models, the moments are defined as

$$\Delta\left[(E^\varphi)^{n_1}(E^x)^{n_2}(K_\varphi)^{n_3}(K_x)^{n_4}\right] := \left\langle\left(\widehat{\Delta E^\varphi}\right)^{n_1}\left(\widehat{\Delta E^x}\right)^{n_2}\left(\widehat{\Delta K_\varphi}\right)^{n_3}\left(\widehat{\Delta K_x}\right)^{n_4}\right\rangle_{\rm symm} \tag{A3}$$

in totally symmetric ordering, where $\widehat{\Delta\zeta} := \hat{\zeta} - \langle\hat{\zeta}\rangle$ if ζ represents a generic phase space variable. In a loop quantization, one would use holonomy operators instead of quantized components of extrinsic curvature. These variables form a phase space, with a Poisson bracket based on an extension of

$$\{\langle\hat{A}\rangle, \langle\hat{B}\rangle\} = \frac{\langle[\hat{A},\hat{B}]\rangle}{i\hbar} \tag{A4}$$

to moments by the Leibnitz rule.

For expectation values of basic operators, (A4) reduces to the classical bracket. The bracket (A4) applied to moments is not the only extension of the classical bracket one could think of, but it is distinguished by the fact that a closed commutator algebra of some set of operators, such as some first-class constraint operators $\hat{C}_I$, implies a closed algebra of effective constraints, defined as $\langle\widehat{\rm pol}\hat{C}_I\rangle$ with polynomials $\widehat{\rm pol}$ in basic operators, under Poisson brackets. One can therefore analyze the possibility of first-class quantizations by computing Poisson brackets of effective constraints, which in most cases is much more feasible than analyzing the possibility of closed commutators. The effective constraints can be computed in terms of the moments by Taylor expanding the expectation value in $\langle\widehat{\Delta\zeta}\rangle$ [47–49].

In models with local kinematical degrees of freedom, we proceed formally in order to illustrate the main features. (But see [50] for a demonstration that canonical effective methods can also be applied to quantum field theories.) For the diffeomorphism constraint of the spherically symmetric vacuum model, given in (9), we have an infinite family of effective constraints $D[N]_{\rm pol} := \left\langle\widehat{\rm pol}\,\hat{D}[N]\right\rangle$ where $\widehat{\rm pol}$ now stands for arbitrary polynomials in the $\widehat{\Delta\zeta}$ of spherically symmetric variables. We assume that we have selected a consistent factor-ordering choice for the operator $\hat{D}[N]$, which in this case is known to exist [13]. For semi-classical states, we have

$$\Delta\left[(E^\varphi)^{n_1}(E^x)^{n_2}(K_\varphi)^{n_3}(K_x)^{n_4}\right] \equiv \mathcal{O}\left(\hbar^{(n_1+n_2+n_3+n_4)}\right) .$$

This hierarchy allows us to consider a closed system of finitely many local effective constraints to any fixed order in $\hbar$, after expanding each of these constraints (starting with the diffeomorphism constraint for pol $=1$) in terms of basic expectation values $\langle\hat{\zeta}\rangle$ and the moments.

As follows from general considerations of effective constrained systems [48, 49, 51], no new observables arise in this way, but quantum corrections to the classical reduced phase space appear. For every new quantum variable given by a moment, there is a higher-order effective constraint with $\widehat{\rm pol} \neq 1$ which fixes the moment or removes it by the gauge flow. So far, this property has been demonstrated for finite-dimensional models, but such a result is sufficient for the usual counting of local degrees of freedom in which one subtracts the number of constraints plus gauge flows from the number of kinematical degrees of freedom.

2. Local observables?

The statement in our last paragraph is in conflict with an observation made in [17, 18], pointing out a large class of new *local* observables in loop-quantized spherically symmetric models. However, on closer inspection, these observables have the following, problematic origin: In loop quantizations such as the one sketched above, one constructs a state space using auxiliary ingredients in addition to the classical phase-space variables (or corresponding quantum numbers): While k_j and μ_j in (A1) give eigenvalues of the quantized E^x and E^φ, respectively, the vertex positions x_j have no classical correspondence. By group averaging (A2), the diffeomorphism constraint is then solved by factoring out the vertex positions, that is the non-classical ingredients. In the classical theory, however, the diffeomorphism constraint and its flow provide non-trivial relationships between E^x, E^φ and their momenta, which do not follow from the group-averaging construction. By ignoring these relationships, the loop-quantized theory has additional local observables, but their meaning is obscure because their origin is the auxiliary vertex positions introduced for kinematical states. Indeed, [17, 18] explicitly state that their local observables parameterize the sequence of successive k_j, which depends on how the spurious vertex positions are injected in states. As our discussion of effective constraints shows, these observables, while they may look like local degrees of freedom, cannot be part of a local effective theory. And even though coordinate-dependent vertex positions are averaged over, they leave a trace in the resulting theory by the missing relationships between kinematical phase-space variables.

In loop-quantized spherically symmetric models, the implementation of the diffeomorphism constraint directly follows the full theory [31]. Although the diffeomorphism constraint is usually considered well-understood in loop quantum gravity, several problems of the theory related to its solutions remain and indicate difficulties both with coordinate independence (vertex positions affecting observables even after spatial diffeomorphisms have been factored out) and the classical limit (observables without a place in local effective theories).

Appendix B: Constraint bracket for matter models

We can compute the bracket (35) by splitting the gravity and matter parts and by exploiting the anti-symmetry property:

$$\begin{aligned}\{\tilde{C}_{\rm T}[M],\tilde{C}_{\rm T}[N]\} &= \{\tilde{C}[M]+\tilde{C}_{\rm matter}[M],\tilde{C}[N]+\tilde{C}_{\rm matter}[N]\}\\ &= \{\tilde{C}[M],\tilde{C}[N]\}+\{\tilde{C}_{\rm matter}[M],\tilde{C}_{\rm matter}[N]\}\\ &\quad +\{\tilde{C}[M],\tilde{C}_{\rm matter}[N]\}-\{\tilde{C}[N],\tilde{C}_{\rm matter}[M]\}\,.\end{aligned}\tag{B1}$$

The gravity part

$$\{\tilde{C}[M],\tilde{C}[N]\}=2G\int {\rm d}x\,\frac{1}{2}\frac{\delta\tilde{C}[M]}{\delta K_\varphi}\frac{\delta\tilde{C}[N]}{\delta E^\varphi}-(M\leftrightarrow N)$$

is simple and results in expression (36). The 'mixed' brackets are also straight-forward:

$$\begin{aligned}\{\tilde{C}[M]D &\;,\;\tilde{C}_{\rm matter}[N]\}-\{\tilde{C}[N],\tilde{C}_{\rm matter}[M]\}\\ &= 2G\int{\rm d}x\,\frac{1}{2}\left(\frac{\delta\tilde{C}[M]}{\delta K_\varphi}\frac{\delta\tilde{C}_{\rm matter}[N]}{\delta E^\varphi}-\frac{\delta\tilde{C}[M]}{\delta E^\varphi}\frac{\delta\tilde{C}_{\rm matter}[N]}{\delta K_\varphi}\right)-(M\leftrightarrow N)\\ &= \int{\rm d}x\,(MN'-NM')|E^x|^{1/2}\left(\frac{((E^x)')^2}{2(E^\varphi)^3}\frac{\partial\tilde{\mathcal{C}}_{\rm matter}}{\partial K_\varphi}-\frac{\partial F_2}{\partial K'_\varphi}\frac{\partial\tilde{\mathcal{C}}_{\rm matter}}{\partial E^\varphi}\right)\\ &= \int{\rm d}x\,(MN'-NM')\frac{|E^x|^{1/2}(E^x)'}{E^\varphi}\left(\frac{((E^x)')^2}{2(E^\varphi)^3}\frac{\partial\mathcal{H}_{\rm matter}}{\partial K_\varphi}-\frac{\partial F_2}{\partial K'_\varphi}\frac{\partial\mathcal{H}_{\rm matter}}{\partial E^\varphi}\right)\\ &\quad -\frac{|E^x|}{(E^\varphi)^2}\left(\frac{((E^x)')^2}{(E^\varphi)^2}\frac{\partial F_{\rm matter}}{\partial K_\varphi}+2F_{\rm matter}\frac{\partial F_2}{\partial K'_\varphi}\right)\mathcal{D}_{\rm matter}+\frac{|E^x|^{1/2}(E^x)'}{(E^\varphi)^2}\frac{\partial F_2}{\partial K'_\varphi}\mathcal{H}_{\rm matter}\,.\end{aligned}\tag{B2}$$

For the matter part we use

$$\begin{aligned}\{\tilde{C}_{\rm matter}[M],\tilde{C}_{\rm matter}[N]\} &= \{H_{\rm matter}[\tilde{M}]-D_{\rm matter}[\hat{M}],H_{\rm matter}[\tilde{N}]-D_{\rm matter}[\hat{N}]\}\\ &= \{H_{\rm matter}[\tilde{M}],H_{\rm matter}[\tilde{N}]\}+\{D_{\rm matter}[\hat{M}],D_{\rm matter}[\hat{N}]\}\\ &\quad -\Big(\{H_{\rm matter}[\tilde{M}],D_{\rm matter}[\hat{N}]\}-\{H_{\rm matter}[\tilde{N}],D_{\rm matter}[\hat{M}]\}\Big)\,,\end{aligned}\tag{B3}$$

with

$$\tilde{M}:=\frac{(E^x)'}{E^\varphi}M\,,\qquad \hat{M}:=\frac{2F_{\rm matter}\sqrt{|E^x|}}{E^\varphi}M\tag{B4}$$

and similarly for $\tilde{N}$ and $\hat{N}$. Since $\mathcal{H}_{\rm matter}$ does not depend on K_x and because of anti-symmetry of the bracket we may use (22) directly:

$$\begin{aligned} \{H_{\rm matter}[\tilde{M}], H_{\rm matter}[\tilde{N}]\} &= \{H_{\rm matter}[\tilde{M}], H_{\rm matter}[\tilde{N}]\}\Big|_{\tilde{M},\hat{N}} \\ &= D_{\rm matter}[\bar{\beta}|E^x|(E^\varphi)^{-2}(\tilde{M}\tilde{N}' - \tilde{N}\tilde{M}')] \\ &= D_{\rm matter}[\bar{\beta}|E^x|((E^x)')^2(E^\varphi)^{-4}(MN' - NM')]\,, \end{aligned} \tag{B5}$$

where the notation $|_{\tilde{M},\hat{N}}$ indicates that in the bracket $\tilde{M}$ and $\hat{N}$ are taken as constant on phase space. Similarly, since $\mathcal{D}_{\rm matter}$ does not depend on gravitational variables, we can use (20):

$$\begin{aligned} \{D_{\rm matter}[\hat{M}], D_{\rm matter}[\hat{N}]\} &= D_{\rm matter}[\hat{M}\hat{N}' - \hat{N}\hat{N}'] \\ &= D_{\rm matter}[4F^2_{\rm matter}|E^x|(E^\varphi)^{-2}(MN' - NM')]. \end{aligned} \tag{B6}$$

Computing the last line in (B3) is more subtle. First we write

$$\{H_{\rm matter}[\tilde{M}], D_{\rm matter}[\hat{N}]\} = \{H_{\rm matter}[\tilde{M}], D_{\rm T}[\hat{N}]\} - \{H_{\rm matter}[\tilde{M}], D[\hat{N}]\}\,. \tag{B7}$$

One now may check that

$$\{H_{\rm matter}[\tilde{M}], D_{\rm T}[\hat{N}]\} - (M \leftrightarrow N) = \{H_{\rm matter}[\tilde{M}], D_{\rm T}[\hat{N}]\}\Big|_{\tilde{M},\hat{N}} - (M \leftrightarrow N)\,. \tag{B8}$$

There are two additional terms (proportional to $MN' - NM'$) arising from the phase-space dependence of the smearing fields which could add to the bracket: one coming from the integration by parts of $(E^x)'$ in $(\delta\tilde{M}/\delta E^x)\mathcal{H}_{\rm matter}\delta D_{\rm T}[\hat{N}]/\delta K_x - (M \leftrightarrow N)$ and the other from the integration by parts of K'_φ in the gravitational part of the diffeomorphism constraint in $(\delta H_{\rm matter}[\tilde{M}]/\delta E^\varphi)(\delta D_{\rm T}[\hat{N}]/\delta K_\varphi) - (M \leftrightarrow N)$. However, these two terms exactly cancel, and hence we may use (21):

$$\begin{aligned} \{H_{\rm matter}[\tilde{M}], D_{\rm T}[\hat{N}]\} - (M \leftrightarrow N) &= -H_{\rm matter}[\tilde{M}'\hat{N}] - (M \leftrightarrow N) \\ &= H_{\rm matter}[2F_m|E^x|^{1/2}(E^x)'(E^\varphi)^{-2}(MN' - NM')]\,. \end{aligned} \tag{B9}$$

Finally, it is straight forward to check that

$$\{H_{\rm matter}[\tilde{M}], D[\hat{N}]\} - (M \leftrightarrow N) = \int {\rm d}x\,(MN' - NM')\frac{2F_{\rm matter}|E^x|^{1/2}(E^x)'}{E^\varphi}\frac{\partial\mathcal{H}_{\rm matter}}{\partial E^\varphi}\,. \tag{B10}$$

Putting everything back in (B3),

$$\begin{aligned} \{\tilde{C}_{\rm matter}[M], \tilde{C}_{\rm matter}[N]\} = \int {\rm d}x\,(MN' - NM')\Bigg[&\frac{|E^x|}{(E^\varphi)^2}\left(\frac{((E^x)')^2}{(E^\varphi)^2}\bar{\beta} + 4F^2_{\rm matter}\right)\mathcal{D}_{\rm matter} \\ &-\frac{2F_{\rm matter}|E^x|^{1/2}(E^x)'}{(E^\varphi)^2}\left(\mathcal{H}_{\rm matter} - E^\varphi\frac{\partial\mathcal{H}_{\rm matter}}{\partial E^\varphi}\right)\Bigg]\,. \end{aligned} \tag{B11}$$

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