Polarized gravitational waves from cosmological phase transitions

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Polarized Gravitational Waves from Cosmological Phase Transitions

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We estimate the degree of circular polarization for the gravitational waves generated during the electroweak and QCD phase transitions from the kinetic and magnetic helicity generated by bubble collisions during those cosmological phase transitions.

I.   INTRODUCTION

Gravitational waves (GWs) astronomy opens a new window to study the physical processes in the very early universe: relic GWs propagate almost freely throughout the universe expansion, and thus they retain the information about the physical conditions and physical processes at the moment of their generation (see for reviews, [1–3] and references therein). There are various mechanisms that might generate such GWs. In the present paper we focus on the generation of GWs during cosmological electroweak (EW) and Quantum ChromoDynamic (QCD) phase transitions (PTs) through the turbulent helical sources which can arise and follow the PT bubble collisions. The GW generation mechanism associated with bubble collisions during the first order PTs has been widely discussed in literature, starting from the pioneering works [4–7] and re-addressed later [8–25]

For a cosmological phase transition to produce strong enough turbulent motions and magnetic fields, which will result in the detectable signal of GWs, they must be first order PTs, with bubble formation and bubble collisions. For the EWPT, with the standard EW Lagrangian plus a Stop, the supersymmetric partner of the top quark, called the MSSM EW Lagrangian, the EWPT is first order PT [26]. The QCDPT has been shown to be first order PT by lattice gauge calculations [27, 28].

Both turbulent motions and matic fields can produce relic GWs through their anisotropic stresses, see Refs. [29–39]. It has been pointed that the GWs generated by magnetic fields can be detected through Laser Interferometer Space Antenna (LISA) [40–44]. In difference to the GWs sourced solely by PTs bubble collisions, the presence of turbulent (kinetic and magnetic sources) increases the detection prospects [45] not only from EWPT but from QCDPT too [46].[1] One of main goals of European Space Agency (ESA) - NASA planned join mission LISA [48], was the detection of low frequency GWs (sub-

Hz region). The new development of this program is the European only ESA mission, so called New Gravitational wave Observatory (NGO) - *aka* eLISA (evolved LISA) [49]. One of major parts of its science program consists on the direct detection of GWs from cosmological PTs, see Refs. [50–53] for details.

In the present paper we extend our previous study Ref. [38], and we investigate the degree of polarization of GWs generated via cosmological PTs through helical hydro and magnetized turbulent sources using the formalism given in Ref. [32]. We adjust the previous formalism to determine the polarization degree of GWs from helical kinetic turbulence to a more complex scenario of MHD turbulence present during the cosmological PTs. More precisely, we use the recent results of numerical simulations [54–56] to set the statistical properties of helical MHD turbulence. Another difference from the formalism of Ref. [32] consists in computing the energy density and peak frequency of GWs using the analogy with acoustic waves production by hydrodynamical turbulence [34] (which we can call *aeroacoustic* approach [57–59]).

Charge-conjugation-Parity (CP) violation is necessary for the production of magnetic helicity via bubble collisions, [60]. EWPT and QCDPT bubble collisions result in development of helical (kinetic or/and magnetic) turbulence, due in part to CP violation, which will lead to circularly polarized GWs background. In the case of strong enough helical sources [35, 36], the degree of polarization is potentially detectable [61–67].[2]

In our present study we follow the helical (chiral) magnetic fields generation scenarios (during PTs through bubble collisions) presented in Refs. [73, 74] (EWPT) and Refs. [60, 75] (QCDPT) (see also Ref. [38] for a brief review of these models). Upon generation the magnetic field starts to interact with primordial plasma that leads to development of magnetically dominant MHD and secondary kinetic turbulence, for pioneering studies see Refs. [76–79]. In what follow we adopt the results of numerical

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[1] GWs from QCDPT are potentially detectable through pulsar timing, see Ref. [47] and references therein.

[2] The indirect tool to detect circularly polarized GWs consists on searching parity violating signals on cosmic microwave background maps, see Refs [68–71] for original studies and Ref. [72] for a review.

simulations of Refs. [54–56], [3] and their phenomenological interpretation given in Refs. [80, 81]

The structure of the paper is as follows: In Sec. II we review the GWs generation formalism and define the circular polarization degree of GWs. We discuss the hydro and MHD helical turbulence modeling in Sec. III and compute the GW signal and its polarization in Sec. IV. We give our results for both EWPT and QCDPT generated GWs in Sec. IV, and we conclude in Sec. V. We use natural ($\hbar = 1 = c$) Lorentz-Heaviside units.

II. GRAVITATIONAL WAVES GENERATION OVERVIEW

We assume that GWs are generated through kinetic and MHD turbulence which follow the PT bubble collisions [7, 29–31, 82]. To be general as possible we present the common description for EWPT and QCDPT, defining the PT temperature as $T_\star$ ($T_\star = 100$ GeV for EWPT and $T_\star = 0.15$ Gev for QCDPT, and the typical *proper* length scale through l_0 which can be associated with the PT bubble size l_b (the assumption $l_0 \simeq l_b$ is well justified because in our theory bubble collisions during PT generate a magnetic field at bubble walls, and this initial field starts to interact with primordial plasma resulting in development of kinetic and MHD turbulence with a typical length scale that corresponds to the magnetic field injection scale) [83]. We also define $g_\star$ as a number of relativistic degrees of freedom: for the standard model we have $g_* = 106.75$ as $T \to \infty$. ($g_\star = 100$ for EWPT and $g_\star = 15$ for QCDPT). In our further consideration we assume that the duration of the turbulent sources, τ_T are short compared to the universe expansion time-scale at PTs, i.e. $\tau_T \leq H_\star^{-1}$ with $H_\star^{-1}$ the Hubble radius at PT. This assumption makes possible to neglect the expansion of the universe, although limits our consideration by the GWs background generation *only* from PT, and completely neglects GWs arising from decaying turbulence (which might last log-enough after the end of PTs).

GWs (the tensor metric perturbations above the standard Friedmann-Lemaître-Robertson-Walker homogeneous and isotropic background) are generated from turbulence (including kinetic and magnetic fluctuations) through the presence of anisotropic stresses as

$$\nabla^2 h_{ij}(\mathbf{x}, t) - \frac{\partial^2}{\partial t^2} h_{ij}(\mathbf{x}, t) = -16\pi G \Pi_{ij}^{(T)}(\mathbf{x}, t), \quad (1)$$

where $h_{ij}(\mathbf{x}, t)$ is the tensor metric perturbation, t is physical time, i and j are spatial indices (repeated indices are summed), and G is the gravitational constant. We have neglected the term $\propto \partial h_{ij}(\mathbf{x}, t)/\partial t$ due to our assumption of the short duration of turbulence. $P_{ij}^{(T)}$ (the script "T" indicates that we are interested on the tensor part of the turbulent source) is the traceless part of the stress-energy tensor $T_{ij}(\mathbf{x}, t)$, which is constructed from kinetic (K) or magnetic (M) turbulence normalized vector fields[4] (as we will show below the equipartition is established between kinetic and magnetic turbulent motions which simply doubles the source term, i.e. $\Pi_{ij}^{(K)} + \Pi_{ij}^{(M)} \simeq 2\Pi_{ij}^{(T)}$) given by [84]:

$$\Pi_{ij}^{(T)}(\mathbf{x}, t) = T_{ij}(\mathbf{x}, t) - \frac{1}{3} \delta_{ij} T(\mathbf{x}, t), \quad (4)$$

here $T \equiv [T]_k^k$ is the trace of the T_{ij} tensor.

As we can expect the kinetic and magnetic turbulent fluctuations generate stochastic GWs, which can be characterized by the wave number-space two-point function as,

$$\langle h_{ij}^\star(\mathbf{k}, t) h_{lm}(\mathbf{k}', t + \tau) \rangle = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{k}') \times$$
$$\times \left[\mathcal{M}_{ijlm}(\hat{\mathbf{k}}) H(k, \tau) + i\mathcal{A}_{ijlm}(\hat{\mathbf{k}}) \mathcal{H}(k, \tau) \right]. \quad (5)$$

Here we use the Fourier transform pair of the tensor perturbation as: $h_{ij}(\mathbf{k}, t) = \int d^3x\, e^{i\mathbf{k} \cdot \mathbf{x}} h_{ij}(\mathbf{x}, t)$ and $h_{ij}(\mathbf{x}, t) = \int d^3k\, e^{-i\mathbf{k} \cdot \mathbf{x}} h_{ij}(\mathbf{k}, t)/(2\pi)^3$. The brackets $\langle ... \rangle$ denote an ensemble average over realization of the stochastic source. The spectral functions $H(k, t)$ and $\mathcal{H}(k, t)$ determine the GW amplitude and polarization, $4\mathcal{M}_{ijlm}(\hat{\mathbf{k}}) \equiv P_{il}(\hat{\mathbf{k}})P_{jm}(\hat{\mathbf{k}}) + P_{im}(\hat{\mathbf{k}})P_{jl}(\hat{\mathbf{k}}) - P_{ij}(\hat{\mathbf{k}})P_{lm}(\hat{\mathbf{k}})$, and $8\mathcal{A}_{ijlm}(\hat{\mathbf{k}}) \equiv \hat{\mathbf{k}}_q [P_{jm}(\hat{\mathbf{k}})\epsilon_{ilq} + P_{il}(\hat{\mathbf{k}})\epsilon_{jmq} + P_{im}(\hat{\mathbf{k}})\epsilon_{jlq} + P_{jl}(\hat{\mathbf{k}})\epsilon_{imq}]$ are tensors, with the projection tensor $P_{ij}(\hat{\mathbf{k}}) = \delta_{ij} - \hat{k}_i \hat{k}_j$ (with δ_{ij} - the Kronecker delta, $\hat{k}_i = k_i/k$ and $k = |\mathbf{k}|$), ϵ_{ijl} is the totally antisymmetric symbol. We choose GW propagation direction pointing the unit vector $\hat{\mathbf{e}}_3$, and we use the usual circular polarization basis tensors $e_{ij}^\pm = -(\mathbf{e}_1 \pm i\mathbf{e}_2)_i \times (\mathbf{e}_1 \pm i\mathbf{e}_2)_j/\sqrt{2}$. We define two states h^+ and h^- corresponding to right- and left-handed circularly

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[3] We underline the nature of the secondary character of fluid motions, because the bubble collision itself might lead to development of purely hydrodynamical turbulence during PTs, see Refs. [31], while here we note that bubble collisions result in generation of magnetic fields [60, 73–75].

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[4] The kinetic and magnetic perturbation stress-energy tensor are

$$T_{ij}^{(K)}(\mathbf{x}, t) = \mathrm{w} u_i(\mathbf{x}, t) u_j(\mathbf{x}, t), \quad (2)$$

$$T_{ij}^{(M)}(\mathbf{x}, t) = \mathrm{w} b_i(\mathbf{x}, t) b_j(\mathbf{x}, t), \quad (3)$$

where $\mathrm{w} = \rho + p$ is enthalpy of the fluid with density energy, ρ, and pressure p, $\mathbf{u}(\mathbf{x}, t)$ is the kinetic motion velocity field and $\mathbf{b}(\mathbf{x}, t)$ is normalized magnetic field, $\mathbf{b} = \mathbf{B}/\sqrt{4\pi\mathrm{w}}$, that represents the Alfvén velocity, v_A of the magnetic field. The normalized energy of the magnetic field is then $\mathcal{E}_M(\eta) = \langle \mathbf{b}^2(t) \rangle/2$, while the normalized kinetic energy density is given through $\mathcal{E}_K(t) = \langle \mathbf{u}^2(t) \rangle/2$. The advantage of such representation consists on eliminating the expansion of the universe, since physical and comoving values of the normalized magnetic field amplitude are the same.

polarized GWs, $h_{ij} = h^+ e^+_{ij} + h^- e^-_{ij}$. Through above notations the circular polarization degree was derived for GWs from Gamma Ray Bursts by [85] and in the context of cosmological GWs is reproduced [32],

$$\mathcal{P}^{\mathrm{GW}}(k) = \frac{\langle h^{+\star}(\mathbf{k})h^+(\mathbf{k}') - h^{-\star}(\mathbf{k},)h^-(\mathbf{k}')\rangle}{\langle h^{+\star}(\mathbf{k})h^+(\mathbf{k}') + h^{-\star}(\mathbf{k})h^-(\mathbf{k}')\rangle} = \frac{\mathcal{H}(k)}{H(k)}. \quad (6)$$

Here we omit the time dependence of the polarization degree $\mathcal{P}^{\mathrm{GW}}(k)$.

As we already underlined we are interested on GWs generation only from a short duration sources acting during PTs. After generation the GWs propagate almost freely, and we account for the expansion of the universe by a simple re-scaling of the frequency and the amplitude by a factor equal to

$$\frac{a_\star}{a_0} \simeq 8 \times 10^{-16} \left(\frac{100\,\mathrm{GeV}}{T_\star}\right)\left(\frac{100}{g_\star}\right)^{1/3}, \quad (7)$$

This factor is safely canceled when computing $\mathcal{P}^{\mathrm{GW}}(k)$, although the more complex consideration of decaying turbulence (long lasting sources) will make $\mathcal{P}^{\mathrm{GW}}(k)$ time dependent function.

To estimate the polarization degree of GWs from PT generated helical fields we need to compute two spectral functions $\mathcal{H}(k)$ and $H(k)$ at the moment of PT, which are determined by the helical anisotropic sources $\Pi^{(K)}_{ij}$ and $\Pi^{(M)}_{ij}$. In our previous work we have computed the typical amplitude and frequency of GWs generated during cosmological PTs [38]. In Ref. [32] the polarization degree of GWs from kinetic (hydro) turbulence has been estimated. It has been shown that fully helical turbulence leads to $\mathcal{P}^{\mathrm{GW}}(k) \to 1$. In the present work we follow the GWs generation formalism from helical magnetized sources presented in Refs. [35, 36], and apply it to the EWPT and QCDPT cases.

III. KINETIC AND MHD TURBULENCE MODELING

The magnetic field amplitude (i.e. total magnetic field energy density) is strongly limited by the big bang nucleosynthesis (BBN) bound requesting that the total magnetic field energy density cannot exceed the 10% of the radiation density at the moment of the magnetic field generation. In terms of the *effective* comoving magnetic field value, $B_{\mathrm{eff}} \simeq 8.4 \cdot 10^{-7}(100/g_\star)^{1/6}$ Gauss (G) or in the terms of Alfvén velocity $v_A \equiv |\mathbf{b}| \leq 0.4$ [83].

As we noted above the magnetic field generated at one scale (the magnetic field initial spectrum can be approximated being peaked at the typical wavenumber $k_0 = 2\pi/l_0$, so in Fourier $\mathbf{k}$-space described by $\delta^{(3)}(\mathbf{k}-\mathbf{k}_0)$ function) after interactions with plasma leads to devel-

opment of turbulence,[5] and the sharply peaked initial spectrum is redistributed respectively. For isotropic *stationary* turbulence the normalized magnetic vector field two-point correlation function is:

$$\langle b_i^*(\mathbf{k})b_j(\mathbf{k}')\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k} - \mathbf{k}')\,F^M_{ij}(\mathbf{k}) \quad (8)$$

where

$$F^M_{ij}(\mathbf{k}) = P_{ij}(\hat{\mathbf{k}})S_M(k) + i\epsilon_{ijl}\hat{k}_l A_M(k). \quad (9)$$

The power-law spectral function $S_M(k) = S_0 k^{n_S}$ and $A_M(k) = A_0 k^{n_S - n_A} k^{n_A}$ determine the energy density and current helicity of the magnetic field[6] and n_S and n_A are magnetic field and helicity spectral indices which determine the spatial distribution of the magnetic field and its helicity. The establishment of stationary turbulence with the stationary (time- independent) two-point correlation function given through Eqs. (8) - (9) requires the presence of long-lasting sources. To account for the short acting PT turbulent source (the turbulence duration time τ_{T} is short enough compared to the universe expansion time-scale $H_\star^{-1}$) we have to modify the spectra $S_M(k)$ and $A_M(k)$ making them time dependent, see below.

Following the description of hydro- and MHD turbulence generated during PTs, we distinguish three spatial spectral sub-regimes of turbulent fluctuations: (i) the large scale decay range $k_{H_\star} < k < k_0$ (where physical wavenumbers $k_{H_\star} = 2\pi/H_\star^{-1}$ and k_0 correspond to the PT Hubble length scale and the largest PT length size); the minimal wavenumber corresponds to the Hubble scale $H_\star^{-1}$ beyond which causally generated magnetic field is *frozen-in* and any interactions are forbidden due to causality requirement; (ii) the turbulent (or so called *inertial*) range $k_0 < k < k_D$ (where k_D the damping scale of turbulence through viscous dissipation and magnetic resistivity, which is determined by plasma properties); (iii) the damping range $k > k_D$. All these typical wavenumbers ($k_{H_\star}$, k_0, and k_D) are time-dependent due to interactions of magnetic field with plasma and the expansion of the universe. Magnetic helicity presence plays here a crucial role leading to re-arrangement of the helical structure at large scales [87]. The expansion of the universe lead to additional effects: namely, the PT bubble size determined length scale l_0 is strengthened by a factor $a(t)/a_\star$ (being $\propto t^{1/2}$ during the radiation-dominated

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[5] Primordial plasma is a perfect conductor with extremely high values of kinetic and magnetic Reynolds numbers, and current numerical simulations are still behind to approach necessary resolutions and timescales to describe adequately physical conditions and processes in the early universe.

[6] Magnetic helicity defined as $\langle \mathbf{A}(\mathbf{x}) \cdot \mathbf{B}(\mathbf{x})\rangle$ is a gauge-dependent quantity, while normalized (or regular expressed through $\mathbf{B}(\mathbf{x})$) current helicity $\langle \mathbf{b}(\mathbf{x}) \cdot [\nabla \times \mathbf{b}(\mathbf{x})]\rangle$ is gauge independent, see Ref. [86] for details, also it allows the direct analogy with the kinetic helicity $\langle \mathbf{u}(\mathbf{x}) \cdot [\nabla \times \mathbf{u}(\mathbf{x})]\rangle$, and thus to consider both helical sources in a common formalism.

epoch and $\propto t^{2/3}$ during the matter-dominated epoch), while the Hubble length scale $H_\star^{-1} \propto t$. As a result a perturbation with $k_{H_0} < k < k_{H_\star}$ (with $k_{H_0} = 2\pi/H_0$ and H_0 is the today Hubble radius) will enter the Horizon at some point [88]. Since we are focused on the short duration sources, we will completely neglect the GWs signal for the large-scale decay range $k < k_0$ (where the spectral shape of the field is given through the causal Batchelor spectrum with $n_S = 2$ [89]). Obviously the GWs signal from the viscous damping range $k > k_D$ is also negligibly small.

The realizability condition implies that $|A_M(k)| \leq S_M(k)$ (the modulus sign reflects a possibility of having positive or negative helicity). The spectral indices values n_S and n_A strongly depend on the turbulence model. In the inertial range ($k_0 < k < k_D$), for non-helical turbulence the Kolmogoroff model implies $n_S = -11/3$. Some models lead to the different spectral shapes such as $n_S = -7/2$ - Iroshnikov-Kraichnan model for magnetized turbulence [91, 92], $n_S = -4$ - the weak turbulence model [93] or magnetically dominant turbulence [90]. In the presence of helicity the consideration is even more complex, and requires careful investigation through numerical simulations that it is beyond the scope of the present paper. Based on the phenomenological dimensionless description, if the process is driven by the magnetic energy dissipation at small scales, it is assumed $n_S = -11/3$ and $n_A = -14/3$ (so called *helical* Kolmogoroff model) [94], while if the process is determined by helicity transfer (inverse cascade) and helicity dissipation at small scales it is adopted $n_S = -13/3 = n_A$ [95].

To account the short-duration turbulence (not enough to establish the stationary turbulent motions) we have to consider turbulent fluctuations time-de-correlation which can be accounted for via introducing the characteristic function $f(\eta(k), \tau)$ (with $\eta(k)$ the autocorrelation function), [96]:

$$f(\eta(k), \tau) = exp\Big[-\frac{2\pi^2}{9}\Big(\frac{\tau}{\tau_0}\Big)K^{4/3}\Big] \qquad (10)$$

with τ_0 - the largest turbulent eddy turn-over time, $K \equiv k/k_0$. In this case, to determine the magnetic field two point correlation function in real ($\mathbf{x}$) space, we have to account for magnetic field fluctuations at *different* time moments and at *different* positions, i.e. $\langle b_i(\mathbf{x}, t)b_j(\mathbf{x} + \mathbf{R}, t + \tau)\rangle$. Accordingly, in Fourier space, the two-point correlation function will be determined by the $\bar{F}^M_{ij}(\mathbf{k}, t)$ with the time dependent spectral functions $S_M(k, t)$ and $A_M(k, t)$:

$$\begin{aligned}\bar{F}^M_{ij}(\mathbf{k}, t) &= \Big[P_{ij}(\hat{\mathbf{k}})S_M(k, t) + i\epsilon_{ijl}\hat{k}_l A_M(k, t)\Big] \\ &\times f(\eta(k), t)\end{aligned} \qquad (11)$$

Comparing with the stationary spectrum, see Eq. (8), we see that formally we replace $F^M_{ij}(\mathbf{k}) \equiv F^M_{ij}(\mathbf{k}, t)$, by $\bar{F}^M_{ij}(\mathbf{k}, t) = F^M_{ij}(\mathbf{k}, t)f(\eta(k), \tau)$, To avoid a complex description of accounting time dependence of $S(k, t)$ and

$A(k, t)$, we use the Proudman argument for kinetic turbulence [57], according which *the description of decaying turbulence lasting for τ_T can be replaced by the description of stationary turbulence with time duration of $\tau_T/2$.* Below we briefly discuss our approach.

Turbulence during PTs generated through magnetic helicity can described through two major stages [36]: during the first stage the main process is determined by the magnetic energy direct cascade that last few largest eddy turnover times $\tau_0 = 2\pi/(k_0 v_0)$, where $v_0 < 1$ is the turbulent eddy velocity ($v_0 \simeq M$ for kinetic turbulence where M is the Mach number and $v_0 \simeq v_A$ for magnetic turbulence) determined by PT and magnetogenesis model parameters, see Ref. [31, 97], i.e. $\tau_T = s_0\tau_0$ (with $s_0 = 3 - 5$). The magnetic field induces vorticity fluctuations, and at the end of the first (semi)equipartition between kinetic and magnetic energies is reached, that results in doubling the value of the source for GWs. The magnetic energy density power spectra are then determined by the proper dissipation rate per unit enthalpy ε_M as: $S_0 = \pi^2 C_K \varepsilon^{2/3}$ with C_K constant order of unity, and $\varepsilon = k_0 v_0^3$. Note that the autocorrelation function $\eta(k) = \varepsilon^{1/3}k^{2/3}/\sqrt{2\pi}$ [98]. Although the Kolmogoroff model is valid *only* for non-relativistic turbulence, while during PTs we might deal with $v_0 \simeq 1$ (relativistic turbulence), our estimates for the amplitude and polarization degrees of GWs signal are qualitatively justified, see [29]. The second stage consists on helicity transfer (inverse cascade). The scaling laws for this stage are still under debate. Based on our previous consideration [36], we assume that (i) the details of the scaling laws during this stage will not affect substantially our estimates; (ii) instead of considering decay-turbulence we will again consider the stationary turbulence with scale-dependent duration time. Then we obtain for the helical Kolmogoroff model, $A_0 = \pi^2 C_K \sigma/(k_0\varepsilon^{1/3})$, where σ is the magnetic helicity dissipation rate per unit enthalpy, leading to $A_0/S_0 = \sigma/(\varepsilon k_0)$. The helical Kolmogoroff model is mainly relevant for weakly helical fields, $|A(k)| \ll S(k)$, which is a case of magnetic fields generated during PTs.[7] According to results of recent numerical simulations, see Ref. [56], the weakly helical turbulence even accounting for the free decay of turbulence, show an establishment of the spectra in a good agreement with the helical Kolmogoroff model, as well as equipartition between magnetic and kinetic energy densities. Thus we adopt $n_S = -11/3$ and $n_H = -14/3$ for the inertial range, with $S_0 = \pi^2 k_0^{2/3}v_0^2$ and $A_0 = hS_0$ with h which determines the fraction of helicity dissipation, $h \equiv \sigma/(\varepsilon k_0)$. The turbulence fluctuation velocity $v_0 = v_A$ is determined by the magnetogenesis mechanism and for the model of our interest is given by $v_0 \simeq 0.2$ ($B_{\text{eff,in}} \simeq 5 \cdot 10^{-7}$ G)

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[7] Note that the substantially helical case is usually determined by the helicity transfer (inverse cascade), $S_0 = C_S\sigma^{2/3}$ and $A_0 = C_A\sigma^{2/3}$ [95].

[38] for EWPT model of Refs. [73, 74] and $v_0 \simeq 0.01$ ($B_{\mathrm{eff,in}} \simeq 2 \cdot 10^{-8}$ G) [54] for QCDPT model of Refs. [60, 75].

IV. GRAVITATIONAL WAVES SIGNAL AMPLITUDE AND POLARIZATION

In this section we compute the GW signal (stain amplitude) and polarization degree from the hydro and MHD turbulence generated during the first order cosmological PTs. To determine the amplitude of GWs we proceed as it is described in Ref. [36]. We derive the energy density spectrum of the GWs at the end of PT (in our approximation the end of turbulence). The energy density of GWs is given through the ensemble average as

$$\rho_{GW}(\mathbf{x},t) = \frac{1}{32\pi G}\langle \partial_t h_{ij}(\mathbf{x},t)\partial_t h_{ij}(\mathbf{x},t)\rangle, \quad (12)$$

As we noted the rescaling of the GWs amplitude and frequency given through Eq. (7) is irrelevant when computing the polarization degree, while it is crucial for estimation of GWs energy density.

A. Gravitational Wave Signal

Assuming the homogeneous and isotropic turbulent source lasting for τ_T, and using far field approximation, see [34], the total energy density of GWs at a given spatial point and a given time can be obtained by integrating over all sources within a spherical shell centered at that observer, with a shell thickness corresponding to the duration of the turbulent source (in our case the duration of PT), and a radius equal to the proper distance along any light-like path from the observer to the source (causality requirement), and then

$$\rho_{GW}(\omega) = \frac{d\rho_{GW}}{d\ln\omega} = 16\pi^3\omega^3 G\,\mathrm{w}^2\tau_T H_{ijij}(\omega,\omega), \quad (13)$$

where ω is the angular frequency measured at the moment of generation of GWs, and $H_{ijij}(\omega,\omega)$ is a complicated function of ω (which is computed through using of aero-acoustic approximation and Millionshchikov quasi-normality [94]), given as

$$H_{ijij}(\mathbf{k},\omega) \simeq H_{ijij}(0,\omega) = \frac{7C_K^2\varepsilon}{6\pi^{3/2}}\int_{k_0}^{k_D}\frac{dk}{k^6}\times$$
$$\exp\left(-\frac{\omega^2}{\varepsilon^{2/3}k^{4/3}}\right)\mathrm{erfc}\left(-\frac{\omega}{\varepsilon^{1/3}k^{2/3}}\right) \quad (14)$$

Here, erfc(x) is the complementary error function defined as $\mathrm{erfc}(x) = 1 - \mathrm{erf}(x)$, where $\mathrm{erf}(x) = \int_0^x dy\exp(-y^2)$ is the error function [99]. The integral in Eq. (14) is dominated by the large scale ($k \simeq k_0$) contribution so, for direct-cascade turbulence during the first stage (direct cascade, see Sec. III), the peak frequency is

$$\omega_{\max}^{(I)} \simeq k_0 M. \quad (15)$$

where M is Mach number. To compute the GWs signal arising from the inverse cascade stage we have consider two models separately: Model A assumes that the correlation length during the inverse cascade scales as $\xi_M \propto t^{1/2}$ and Model B corresponds to the correlation length scaling as $\xi_M \propto t^{2/3}$. We obtain that in both models peak frequency during the second stage are equal and are determined by the Hubble frequency as [36]:

$$\omega_{\max}^{(II)} \simeq 2\pi H_\star \quad (16)$$

while the GW amplitude are slightly different in Model A and B as

$$H_{ijij}^{(A)}(\mathbf{k},\omega) \simeq H_{ijij}(0,\omega) = \frac{7C_1^2 M^3\zeta_\star^{3/2}}{12\pi^{3/2}k_0}\int_{k_S}^{k_0}\frac{dk}{k^4}$$
$$\exp\left(-\frac{\omega^2 k_0^2}{\zeta_\star M^2 k^4}\right)\mathrm{erfc}\left(-\frac{\omega k_0}{\zeta_\star^{1/2}Mk^2}\right), \quad (17)$$

and

$$H_{ijij}(\mathbf{k},\omega)^{(A)} \simeq H_{ijij}(0,\omega) = \frac{7C_1^2 M^3\zeta_\star^{3/2}}{6\pi^{3/2}k_0^{3/2}}\int_{k_S}^{k_0}\frac{dk}{k^{7/2}}$$
$$\exp\left(-\frac{\omega^2 k_0}{\zeta_\star M^2 k^3}\right)\mathrm{erfc}\left(-\frac{\omega k_0^{1/2}}{\zeta_\star^{1/2}Mk^{3/2}}\right). \quad (18)$$

Here $\zeta_\star$ determines the amount of initial magnetic helicity and is equal to $\zeta_\star = \langle \mathbf{a}(\mathbf{x})\cdot\mathbf{b}(\mathbf{x})\rangle/(\xi_M\mathcal{E}_M)$ (with $\mathbf{a}(\mathbf{x}) = \mathbf{A}(\mathbf{x})/\mathrm{w}$ normalized vector potential), and $k_S = 2\pi/l_S$ is the typical scale at which the inverse cascade stops: either because the cascade time τ_{cas} reaches the expansion time scale $H_\star^{-1}$ or because the characteristic length scale $l_S \simeq \xi_M$ reaches the Hubble radius $H_\star^{-1}$. The value of k_S can be found by using above conditions, being equal to $k_S = k_0\zeta_\star^{-1/4}(\gamma/M)^{1/2}$. Note that the integrals in expressions Eqs. (17) - (18) are In this dominated by the large scale $k \simeq k_S$ contributions and are maximal at $\omega_{\max}^{(II)}$.

B. Gravitational Wave Polarization

To compute the polarization degree of GWs we need to estimate the tensor perturbations source two point correlation function $\mathcal{F}_{ijlm}(k,\tau) \equiv \langle\Pi_{ij}^{(T)\star}(\mathbf{k},t)\Pi_{lm}^{(T)}(\mathbf{k}',t'+\tau)\rangle$, which can be expressed through the forms $\mathcal{M}_{ijlm}$ and $\mathcal{A}_{ijlm}$ (which are defined below Eq. (5)), as

$$\mathcal{F}_{ijlm}(k,\tau) = (2\pi)^3\delta^{(3)}(\mathbf{k}-\mathbf{k}')$$
$$\times\left[\mathcal{M}_{ijlm}\mathcal{S}(k,\tau) + i\mathcal{A}_{ijlm}\mathcal{Q}(k,\tau)\right]. \quad (19)$$

As we discussed above for the non-stationary turbulence $S(k,\tau)$ and $A(k,\tau)$ are complex functions of τ and k (see Eqs. (10) - (11) for the time de-correlation function and the magnetic field spectrum). Following Ref. [32] we split the spatial and temporal dependence as

$S(k,\tau) = S(k)D_S(\tau)$ and $A(k,\tau) = A(k)D_A(\tau)$ which is a valid approximation for $k \simeq k_0$ (the range which mostly contributes to the GWs signal). Generalizing the stationary case [69] through accounting for the time-dependent functions $D_S(\tau)$ and $D_A(\tau)$ the forms for $\mathcal{S}(k,\tau)$ and $\mathcal{Q}(k,\tau)$ are given as,

$$
\begin{aligned}
\mathcal{S}(k,\tau) &= \frac{\mathrm{w}^2}{(2\pi)^6} \int d^3p_1 \int d^3p_2 \delta^{(3)}(\mathbf{k} - \mathbf{p_1} - \mathbf{p_2}) \\
&\times \left[(1+\alpha^2)(1+\beta^2)D_S^2(\tau)S(p_1)S(p_2) + \right. \\
&\qquad \left. + 4\alpha\beta D_A^2(\tau)A(p_1)A(p_2) \right],
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
\mathcal{Q}(k,\tau) &= \frac{\mathrm{w}^2 D_S(\tau)D_A(\tau)}{128\pi^6} \int d^3p_1 \int d^3p_2 \\
&\times \delta^{(3)}(\mathbf{k} - \mathbf{p_1} - \mathbf{p_2}) \left[(1+\alpha^2)\beta S(p_1)S(p_2) \right. \\
&\qquad \left. + (1+\beta^2)\alpha A(p_1)A(p_2) \right],
\end{aligned}
\tag{21}
$$

where $\alpha = \hat{\mathbf{k}} \cdot \hat{\mathbf{p}}_1$ and $\beta = \hat{\mathbf{k}} \cdot \hat{\mathbf{p}}_2$ with $\hat{\mathbf{p}}_1 = \mathbf{p}_1/p_1$ ($p_1 = |\mathbf{p}_1|$) and $\hat{\mathbf{p}}_2 = \mathbf{p}_2/p_2$ ($p_1 = |\mathbf{p}_1|$). The helical source term $\mathcal{Q}(k,\tau)$ vanishes for turbulence without helicity. Since in the helical Kolmogoroff model the time de-correlation is mostly determined by the energy density dissipation, thus at first order approximation we can assume that $D_S(\tau) \simeq D_A(\tau)$ (both functions are monotonically decreasing functions). By next we should connect $\mathcal{S}$ and $\mathcal{Q}$ with the $H(k,t)$ and $\mathcal{H}(k,t)$ functions, which determine the GWs polarization degree, see. for details Ref. [32].

Magnetic helicity generated via bubble wall collisions during the first order PTs is determined by the corresponding energy scales Λ_{PT} ($\Lambda_{\mathrm{PT}} \simeq 100$ GeV and 0.15 GeV for EWPT and QCDPT respectively); and the PT bubble lengths (l_b). In addition, the bubble wall velocity substantially affects the turbulent motions development [82].

In the present paper we focus on magnetic helicity generation mechanisms following Refs. [60, 75]. In the framework of these magnetogenesis scenarios magnetic helicity during PTs with the magnetic wall in the $x - y$ plane is given by

$$
\mathcal{H}_M = A_z B_z \quad \text{or} \quad \mathcal{H}_M = \frac{(B_z)^2}{\Lambda_{\mathrm{PT}}},
\tag{22}
$$

We note that the EWPT energy (mass) scale is approximately equal to Higgs mass, $\Lambda_{\mathrm{EWPT}} \simeq M_{\mathrm{Higgs}}$. The model parameters such as bubble and wall sizes and wall velocity depend on PT modeling, and are determined in Refs. [60, 73, 74]. We quote also the physical values of magnetic field amplitudes as $B_\star^{(\mathrm{EW})} \simeq 6.45 \times 10^4 \mathrm{GeV}^2$ [73, 74] and $B_\star^{(\mathrm{QCD})} \simeq 1.5 \times 10^{-3} \mathrm{GeV}^2$ [60].

The fraction of initial magnetic helicity ($\zeta_\star$) can be expressed in terms of the magnetic field correlation length (which can be taken to be equal to l_b) and the maximal allowed length scale $H_\star^{-1}$, as $\zeta_\star \simeq l_b/H_\star^{-1}$. Following Refs. [73, 74] the normalized magnetic field generated during the first order EWPT (at the time moment $\simeq$

10^{-11} sec) is equal to $v_A \simeq 0.2$, and assuming around 100 bubble within Hubble length scale, fractional helicity is $\zeta_\star^{(\mathrm{EW})} \simeq 0.01$. Comoving magnetic helicity itself is expressed:

$$
\mathcal{H}_{M,\star}^{(\mathrm{EW})} \simeq \frac{(B^{(\mathrm{EW})})^2}{125 \mathrm{GeV}},
\tag{23}
$$

Assuming that the magnetic field (with $v_A \simeq 0.01$) is correlated over the wall thickness (the QCD momentum 0.15 Gev) [60], results in extremely small magnetic helicity generated during the first order QCDPT (at time moment $\simeq 10^{-5}$ sec),

$$
\mathcal{H}_M^{(\mathrm{QCD})}(t_\star) \simeq \frac{(B^{(\mathrm{QCD})})^2}{0.15 \mathrm{GeV}},
\tag{24}
$$

which corresponds $\zeta_\star^{(\mathrm{QCD})} \ll 1$. On the other hand, making the field correlated over the bubble length scale, leads to the fractional helicity $\zeta_\star^{(\mathrm{QCD})} \simeq 0.2$

Using the approximation given above, the polarization degree of GWs, $\mathcal{P}^{\mathrm{GW}}(k)$, for the Kolmogoroff helical turbulence model can be estimated through (in our simplified description the time dependence is canceled because of $D_S(\tau) \simeq D_A(\tau)$):

$$
\mathcal{P}^{\mathrm{GW}}(k) = \frac{\mathcal{H}(k)}{H(k)} = \frac{\mathcal{I}_A(K)}{\mathcal{I}_S(K)}
\tag{25}
$$

where $K \equiv k/k_0$ is a normalized wavenumber, and

$$
\begin{aligned}
\mathcal{I}_S(K) &\simeq \int dP_1\, P_1 \int dP_2\, P_2 \bar{\Theta} \left[(1+\alpha_p^2)(1+\beta_p^2)P_1^{n_S} P_2^{n_S} \right. \\
&\qquad \left. + 4h^2 \alpha_p \beta_p P_1^{n_A} P_2^{n_A} \right],
\end{aligned}
\tag{26}
$$

$$
\begin{aligned}
\mathcal{I}_A(K) &\simeq 2h \int dP_1\, P_1 \int dP_2\, P_2 \bar{\Theta} \left[(1+\alpha_p^2)\beta_p P_1^{n_S} P_2^{n_A} \right. \\
&\qquad \left. + (1+\beta_p^2)\alpha_p P_1^{n_A} P_2^{n_S} \right].
\end{aligned}
\tag{27}
$$

Here h is the model parameter (which is related to the helicity fraction, see below, and we assume it to be equal to 1, 0.5, 0.1. $P_1 = p_1/k_0$, $P_2 = p_2/k_0$, $\alpha_p = (K^2 + P_1^2 - P_2^2)/(2KP_1)$, $\beta_p = (P^2 + P_2^2 - P_1^2)/(2KP_2)$, $\bar{\Theta} \equiv \theta(P_1 + P_2 - K)\theta(P_1 + K - P_2)\theta(P_2 + K - P_1)$, and θ is the Heaviside step function which is zero (unity) for negative (positive) argument. The integration limits ranges from 1 (we discard the source existence for the wavenumbers below k_0) to k_D/k_0.

We emphasize that the fractional helicity parameter $\zeta_\star$ discussed above is defined through normalized magnetic helicity (the integral quantity), while the parameter $h \equiv \sigma/(k_0\varepsilon)$ is defined through the normalized magnetic energy density and normalized magnetic helicity (e.g. it is determined by the power spectra for magnetic energy density and helicity at small length scales). Under the model adopted here (the Kolmogoroff helical turbulence with $n_S = -11/3$ and $n_A = -14//3$) these both quantities coincide $\zeta_\star \simeq h$.

To keep our description as general as possible we present our results for the GWs polarization degree
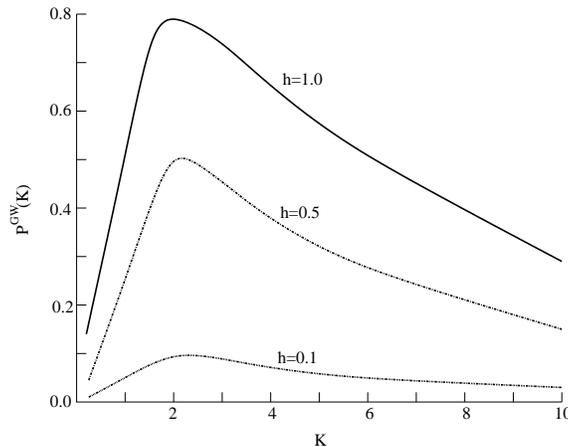
FIG. 1: The GW polarization degree $\mathcal{P}^{\mathrm{GW}}(K)$ ($K = k/k_0$) in terms of the model parameter $h = 1.0, 0.5, 0.1$

$\mathcal{P}^{\mathrm{GW}}(k)$ in terms of the normalized wavenumber K. As we discussed above the typical wavenumber k_0 is determined by the turbulent eddy length scale l_0 ($k_0 = 2\pi/l_0$) and is significantly different for EWPT and QCDPT. The model parameter h, which determines the helicity fraction, varies depending on the magnetogenesis model. The results for $\mathcal{P}^{\mathrm{GW}}(k)$ with $h = 0.1, 0.5$, and 1 are shown in Figure 1.

V. CONCLUSIONS

We computed the GWs signal produced during first order cosmological PTs through hydro and MHD turbulence. We also derive the polarization degree of GWs assuming the validity of the helical Kolmogoroff model, shown in Figure 1. The GWs polarization is present at the background level, and for maximally helical sources the polarization degree approaches unity at its maximum, around $k \sim 2k_0$, and decreases fast at small scales $k \gg k_0$. The formalism presented in this paper might be used to estimate the polarization degree of GWs from helical hydro and MHD turbulence in the differential rotating neutron stars [100] or stellar convection [101]. Note from Figure 1 that the detectability of the polarization degree is determined by the helicity fraction parameter h. We plan in our future research to make estimates of h values depending on magnetogenesis models during cosmological PTs.

Previously we have estimated the GWs amplitude, $h_C(f)$, from the first order EWPT and QCDPT [38], through assumptions of non-helical magnetic fields [60, 73, 74]. We have shown that EWPT generated GWs are potentially detectable through LISA-like missions [48] in the case for strong enough EWPT [44] (for QCDPT generated GWs detection prospects see ReF. [46]). In the present paper we expand our previous results by considering GWs from helical magnetic and hydro turbulence. Probing the circular polarization of GWs background is a challenging task [66], and it is quite difficult at the monopole mode. To detect the circular polarization at the dipole or/and the octopule mode requires at least the system of two unaligned detectors, and LISA was designed ideally to provide detection of these anisotropic components whose magnitudes are small as 1% of the detector noise [61–63]. The similar detection prospects are expected from eLISA data. The planned eLISA mission [49] is originally design to detect un-polarized GWs backgrounds (including GWs from cosmological PTs) [102] with the sensitivity given on Fig. 1 of Ref. [51] (see also comparison with LISA's sensitivity). Although the GWs polarization detection is beyond the currently discussed eLISA science, our study should help further developments.

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