Running of the U(1) coupling in the dark sector

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Running of the $U(1)$ coupling in the dark sector

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The "dark photon" γ_d of a gauged $U(1)_d$ can become practically invisible if it primarily decays into light states from a dark sector. We point out that, in such scenarios, the running of the $U(1)_d$ "fine structure constant" α_d, with momentum transfer q^2, can be significant and potentially measurable. The γ_d kinetic mixing parameter ε^2 is also expected to run with q^2, through its dependence on α_d. We show how the combined running of $\varepsilon^2\alpha_d$ may provide a probe of the spectrum of dark particles and, for $\alpha_d \gtrsim$ few $\times\, 0.1$, substantially modify predictions for "beam dump" or other intense source experiments. These features are demonstrated in simple models that contain light dark matter and a scalar that breaks $U(1)_d$. We also discuss theoretic considerations, regarding the $U(1)_d$ model in the ultraviolet regime, that may suggest the infrared upper bound $\alpha_d \lesssim 0.1$.

In recent years, various phenomenological considerations have motivated the introduction of a light vector boson, the "dark photon" γ_d, with a mass $m_{\gamma_d} \lesssim 1$ GeV - associated with a spontaneously broken gauged $U(1)_d$. The dark photon couples to the Standard Model (SM) only through kinetic mixing [1] of dark charge with hypercharge, parameterized by $\varepsilon \ll 1$. At low energies, γ_d kinetically mixes with the photon and couples primarily to the SM electromagnetic current with a strength εe, where e is the electromagnetic coupling. A great deal of theoretical and experimental effort has been directed towards dark photon physics [2, 3]. Important motivation for these efforts have been provided by potential astrophysical signals of dark matter [4], as well as the 3.6σ deviation from the SM prediction of the muon anomalous magnetic dipole moment $g_\mu - 2$ [5] which can be explained by a light γ_d and $\varepsilon \approx 2\times 10^{-3}$ [6]. Such small values of ε most naturally arise from loop effects [1] of particles charged under both the dark and SM hypercharge $U(1)$ interactions. For example, a typical 1-loop value might be $\varepsilon \sim e g_d/(16\pi^2)$.

If γ_d is the lightest state in the dark sector then its decay is mainly to charged SM states - typically leptons $\ell = e, \mu$ - which could be used to detect it. However, in the presence of light dark states, in particular dark matter, γ_d would decay mainly into those particles, since in typical scenarios $g_d \gg \varepsilon e$, where g_d is the coupling constant of the $U(1)_d$ gauge interactions. The different phenomenology of such a nearly "invisible" γ_d provides new possibilities to explain various anomalies, such as the aforementioned $g_\mu - 2$, but is subject to different sets of experimental constraints [7–10]. In addition, the invisible γ_d scenario offers an interesting opportunity for producing and detecting sub-GeV dark matter in accelerator based experiments. Boosted dark photons - produced in high intensity fixed target experiments - decay in flight and lead to a "dark matter beam" which can be detected downstream [7, 8, 11–13]. The detection rate depends on γ_d couplings to the dark and visible sectors, g_d and εe, respectively; see also Ref. [14]. The signal will look like electron scattering, modulo radiative corrections and scaled by $\varepsilon^2 g_d^2/e^2$.

The coupling of light dark matter to the dark photon can provide a mechanism for obtaining its observed abundance as a thermal relic density. Here, the annihilation of dark matter via γ_d into SM states is mediated by kinetic mixing which allows one to derive a rough relation among the model parameters. Using the results of Refs. [12, 15], one finds

$$\alpha_d \sim 0.02\, w \left(\frac{10^{-3}}{\varepsilon}\right)^2 \left(\frac{m_{\gamma_d}}{100 \text{ MeV}}\right)^4 \left(\frac{10 \text{ MeV}}{m_d}\right)^2, \quad (1)$$

where $\alpha_d \equiv g_d^2/(4\pi)$, m_d is the mass of a dark matter state, $w \sim 10$ for a complex scalar [12], and $w \sim 1$ for a fermion [15]. Here, and elsewhere in this work, α_d and ε are defined by their values at low momentum transfer $q^2 \sim m_{\gamma_d}^2$. Given that some of the existing bounds [10] and proposed experiments [15] probe values of ε as low as 10^{-4}, Eq. (1) would then require $\alpha_d \gtrsim 1$, keeping other parameters at their above reference values, to get the correct dark matter relic abundance. Note that if dark matter density is set by an asymmetry, efficient annihilation of its symmetric population would require somewhat larger annihilation cross sections, compared to the $\sim$ pb implied by Eq. (1), and hence even larger α_d values. Thus, generally speaking, values of $\alpha_d \gtrsim 0.01 - 0.1$ can be motivated if the dark photon is assumed to decay primarily into dark matter states.

In this work, we point out that the running of α_d - as a function of momentum transfer q, due to quantum loops of light dark sector states - can have important phenomenological implications.[1] In dark photon models it is generally assumed that the kinetic mixing parameter ε corresponds to $q^2 \approx 0$ and remains constant with increasing q. However, as mentioned earlier, kinetic mixing is naturally loop-induced and hence $\varepsilon^2 \propto \alpha\alpha_d$, with

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[1] See, *e.g.*, Refs. [16, 17] for possible cosmological effects of running dark sector couplings, within different frameworks.

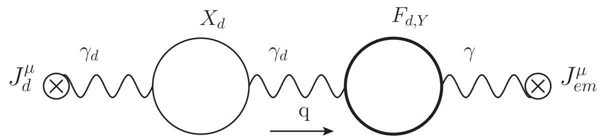

FIG. 1: Vacuum polarization correction to kinetic mixing. The $U(1)_d$ current J_d^μ couples to the SM electromagnetic current J_{em}^μ, via kinetic mixing, mediated by quantum effects of heavy particles $F_{d,Y}$ (thick loop) that carry both dark charge and hypercharge. The vacuum polarization correction from light dark states X_d (thin loop) leads to the running of $\alpha_d(q)$ with momentum transfer q which induces a similar running in the kinetic mixing parameter $\varepsilon^2(q)$

$\alpha \equiv e^2/(4\pi)$. Thus, the running of α_d, due to the effect of light dark states, induces a similar running in ε^2 that would lead to its growth with increasing q and the combination $\alpha_d\, \varepsilon^2$ actually grows like α_d^2.

The above effect is illustrated in Fig.1, where the vacuum polarization correction to the interaction of a $U(1)_d$ current J_d^μ with the SM electromagnetic current J_{em}^μ, mediated by kinetic mixing, is illustrated. In Fig.1, the thick loop, denoted by $F_{d,Y}$, represents heavy states - charged under both the SM hypercharge and $U(1)_d$ - that generate kinetic mixing at the quantum level. The thin loop, denoted by X_d, represents light dark states whose vacuum polarization contributions lead to the running of $\alpha_d(q)$. The loop-induced $\varepsilon^2(q) \propto \alpha(q)\alpha_d(q)$ inherits its running from $\alpha_d(q)$. The well-known running of $\alpha(q)$ due to vacuum polarization effects in quantum electrodynamics is relatively small and, hence, ignored in our discussions.

Given the discussion following Eq. (1), values of α_d not far below unity are well motivated if light dark matter communicates with the SM through a heavier invisible γ_d mediator. We show that the growth of α_d with momentum transfer q can constrain the regime of validity of calculations, depending on the choice of model parameters, and may potentially lead to significant observable effects.

Although the existence of a Landau pole signals strong coupling behavior, the onset of strong coupling, where perturbation theory starts to break down, is probably closer to $\alpha_d \sim 1$. From a theoretical point of view, the approach of a Landau pole for α_d can be interpreted as the onset of a new non-Abelian interaction, or some other ultraviolet completion, that would supplant the low energy $U(1)_d$ and avoid an ever-growing interaction strength with larger q. Such considerations can have interesting implications for the underlying model at energy scales well above m_{γ_d} [16, 17], but will not be studied here, except to mention their use as a potential constraint on α_d.

Before going further, we would like to remark that light vector models with direct coupling to the SM have also been invoked in various contexts [18]. Such direct couplings have to be tiny, or else one would observe severe deviations from standard physics. Much of the phenomenological discussion in our work would also apply to these models, as long as one assumes $\mathcal{O}(1)$ values for α_d and dark sector charges $Q_d^{\rm DS} \sim 1$, but small charges for SM fields $Q_d^{\rm SM} \lesssim 10^{-3}$, under the new Abelian gauge interactions [19]. We will assume the alternative kinetic mixing picture in our work, as it naturally yields suppressed interactions between the visible and the dark sectors (in principle, γ_d could also have mass mixing with the SM Z, leading to additional phenomenology [20]).

As our basic model, we will assume that the dark sector contains a dark matter state, a fermion ψ or a scalar ϕ, as well as a dark Higgs particle with non-zero vacuum expectation value $\langle \Phi_d \rangle$ that is responsible for the breaking of $U(1)_d$; for simplicity all these particles are assumed to have unit charges $|Q_d| = 1$ under $U(1)_d$. We will focus on the regime of momentum transfer $q > m_{\gamma_d}$, where symmetry breaking effects are negligible and the running of α_d is significant. In typical proposed fixed target experiments, 10 MeV $\lesssim q \lesssim$ GeV, where $m_{\gamma_d} \gtrsim 10$ MeV is probed. As we are focusing on "invisible" dark photon models, the dark matter state (ψ or ϕ) will be assumed lighter than γ_d. Assuming that Φ_d is not strongly self coupled, it is quite natural to expect that its mass parameter $m_{\Phi_d} \sim m_{\gamma_d}$ and hence typically less than q. Hence, we will include both the dark matter state and Φ_d contributions to the running of α_d until $q \lesssim m_{\gamma_d}$. The infrared value of the $U(1)_d$ coupling can thus be defined by its value at $q^2 = m_{\gamma_d}^2$, denoted by $\alpha_d(m_{\gamma_d})$.

Since we are focused on the kinematic regime where $q > m_{\gamma_d}$, we will ignore the mass of the vector boson, capturing the leading behavior as a function of m_{γ_d}/q. This suffices for the purposes of our discussions and to highlight the key features of generic invisible γ_d scenarios. Detailed calculations for specific experiments and model parameters lie outside the scope of this work.

In determining the regime where $\alpha_d \gtrsim 1$, higher order effects can become important and we will therefore perform a 2-loop analysis. The 2-loop beta function of $U(1)_d$, with n_F fermions and n_S scalars of unit charge, is given by (see, for example, Refs. [21–24])

$$\beta(\alpha_d) = \frac{\alpha_d^2}{2\pi}\left[\frac{4}{3}\left(n_F + \frac{n_S}{4}\right) + \frac{\alpha_d}{\pi}(n_F + n_S)\right] , \qquad (2)$$

where $\beta(\alpha_d) \equiv \mu\, d\alpha_d/d\mu$. Here, μ is the renormalization scale, which we will later take to be set by the momentum transfer q characterizing the interactions of γ_d.

In Fig.2, we have plotted the running of α_d in the basic model with one dark matter state. Throughout our analysis, a dark Higgs scalar Φ_d, assumed to break $U(1)_d$, is included in the running and hence $n_S \geq 1$ for all of our results. We will consider a minimum momentum transfer $q_0 = 100$ MeV, which is used to set the value of α_d at the lower kinematic range of typical experiments. The solid and dashed curves correspond to fermionic and scalar dark matter $[(n_F, n_S) = (1, 1)$ and $(0, 2)$ in Eq. (2)], respectively; $\alpha_d(q_0) = 0.6$ (0.9) is represented by the thin (thick) curve. Based on the form

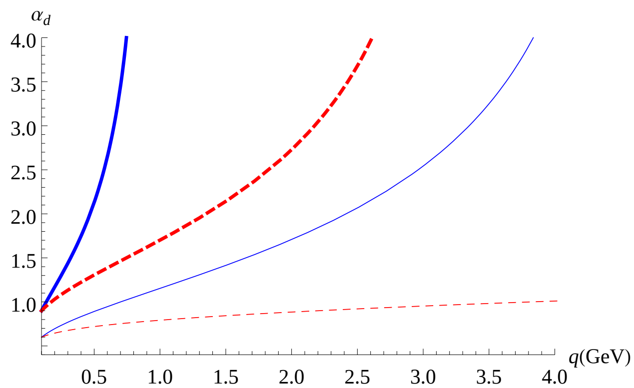

FIG. 2: Running of $\alpha_d(q)$ in the basic model with one dark matter state and one dark Higgs boson. The solid (dashed) curves correspond to a fermion (scalar) dark matter state and the thin (thick) curves correspond to $\alpha_d(q_0) = 0.6$ (0.9), where $q_0 = 0.1$ GeV. We have implicitly assumed $m_{\gamma_d} \lesssim q_0$.

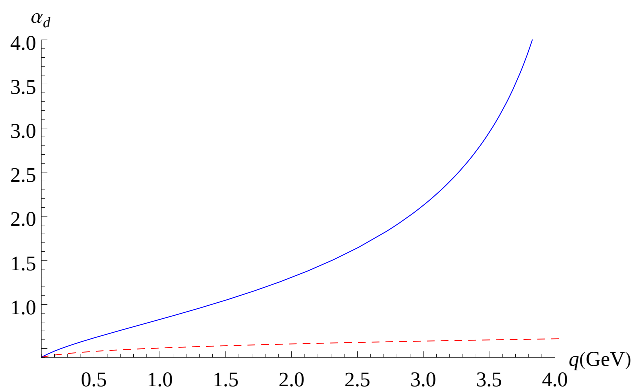

FIG. 3: Running of $\alpha_d(q)$ with two dark matter states and one dark Higgs boson. The solid (dashed) curve corresponds to fermionic (scalar) dark matter and $\alpha_d(q_0) = 0.4$, where $q_0 = 0.1$ GeV, again assuming $m_{\gamma_d} \lesssim q_0$.

of Eq. (2), for $\alpha_d \gtrsim \pi$ the perturbative analysis becomes unreliable, due to strong coupling.

As can be seen from the plot in Fig.2, in all cases, except for the one with $\alpha_d(q_0) = 0.6$ and $(n_F, n_S) = (0, 2)$, the value of α_d grows large, *i.e.* $\alpha_d \gtrsim \pi$, by the time q reaches $\sim$ a few GeV. This is the typical kinematic domain of the proposed fixed target or beam dump experiments. These results suggest that values of $\alpha_d \gtrsim 0.6$, used to illustrate the phenomenology in some studies [7, 8, 15], can typically lead to unreliable predictions, unless those values correspond only to the upper kinematic range $q \gtrsim 1$ GeV. In general, numerical predictions are more stable when only dark scalars are present in the low energy theory. However, even in that case, corresponding to $(n_F, n_S) = (0, 2)$, we see that the change in α_d is not negligible for $\alpha_d(q_0) \gtrsim 0.6$.

In Fig.3, we also present the results for running of α_d in an extended model that has two dark matter states. See, for example, Ref. [7] where such models have been discussed as a viable setup for sub-GeV dark matter. As can be seen from the figure, perturbative analysis becomes unreliable for $\alpha_d(q_0) \gtrsim 0.4$, with $q_0 = 100$ MeV, if dark matter states are fermionic. However, 2 scalar dark matter states do not lead to a significant loss of perturbative validity. Again, the running over $q \lesssim$ few GeV is not negligible, $\sim 50\%$, even for the all-scalar case and can potentially have measurable effects. Overall, we see that the assumption of a constant α_d over the kinematic range of typical experiments can lead to underestimation of the predicted rates, even when α_d is not close to unity.

The preceding results point to an interesting possibility at fixed target or beam dump experiments, where a range of values for momentum transfer q are accessible. For $\alpha_d(q_0) \gtrsim 0.2-0.3$, measurements of processes at different q can probe the running of $\alpha_d(q)$, in combination with the induced running of $\varepsilon(q)$. As illustrated above, the running of modest-sized α_d is sensitive to the number and type - *i.e.* fermion versus scalar - of dark states, as long as they are well below the typical scale of momentum transfer in the measured processes.

Definitive statements regarding the measurement of the running with q^2 in the above scenarios depend on the specifics, such as the production and detection processes, and the energy spectrum of the light dark matter produced by a beam dump or another intense source. Nonetheless, key general features of the dark matter scattering through γ_d exchange can be used to outline a potential path toward such measurements, as explained below. Of course, much more detailed studies based on specific theoretical and experimental parameters would be warranted in designing a dedicated experiment, or upon discovery of light dark matter models considered here.

Let us consider the on-shell production of γ_d, whose rate is proportional to $\varepsilon^2(m_{\gamma_d})$, a constant set by $q^2 = m_{\gamma_d}^2$. Here, γ_d primarily decays into dark matter which then scatters in the detector with a cross section $\sigma_{\rm DM} \propto \alpha_d(q)\varepsilon^2(q)$, where q has a distribution over some range. In typical scenarios, kinetic mixing is loop-induced and $\varepsilon^2(q) \propto \alpha_d(q)$, which gives $\sigma_{\rm DM} \propto \alpha_d^2(q)$.

At $q \gtrsim m_{\gamma_d}$, dark matter interactions with the nucleus are akin to electromagnetic interactions of a charged lepton with the nucleus, governed by quantum electrodynamics (QED). This correspondence can be used to obtain precise predictions for the dark matter scattering cross section $\sigma_{\rm DM}$. In particular, one could normalize $\sigma_{\rm DM}$ to the electron (or muon) electromagnetic cross section $\sigma_{\rm EM} \propto 1/q^2$, which is theoretically well-understood and can be precisely measured. Thus, modulo QED radiative corrections and non-zero m_{γ_d} propagator effects, we have (for $q \gtrsim m_{\gamma_d}$)

$$R \equiv \sigma_{\rm DM}/\sigma_{\rm EM} \simeq \alpha_d\, \varepsilon^2/\alpha \simeq \xi\, \alpha_d^2\,, \quad (3)$$

where ξ is approximately a constant (we have ignored the negligible running of the QED coupling α over the range of q^2 considered here). With the above assumptions, the value of ξ can be known with good accuracy,

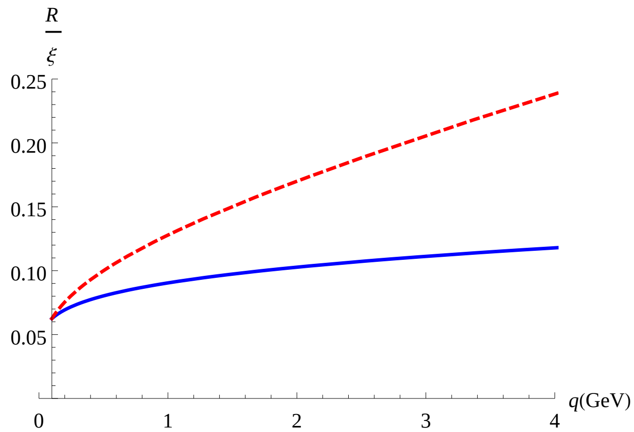

FIG. 4: The running of R/ξ as a function of momentum transfer q, assuming (A) one and (B) two light fermionic dark matter states, corresponding to the solid and dashed curves, respectively. We have set $\alpha_d(q_0) = 0.25$, $q_0 = 0.1$ GeV, and $m_{\gamma_d} \lesssim q_0$ is assumed. A scalar (dark Higgs boson) is included in the running for both cases.

given the input parameters ε and α_d at a reference value of momentum transfer, which we can naturally take to be $q = m_{\gamma_d}$. In turn, for a given value of ξ, one could predict the scattering cross section $\sigma_{\rm DM}$ and its dependence on q^2 quite well, in our assumed framework.

The combined effect of $\alpha_d(q)$ and $\varepsilon^2(q)$ running $\propto \alpha_d^2(q)$ can be significant. For an illustrative numerical example, let us consider the case with $\alpha_d(q_0) = 0.25$ (assuming $m_{\gamma_d} \lesssim q_0$) and $q_0 = 100$ MeV. With these parameters, the effect of running with q^2 on the event rate can be significant, depending on the value of the β function in Eq. (2). To show this, we consider the cases of (A) one dark fermion and (B) two dark fermions contributing to the running (a dark Higgs scalar is included in each case). We illustrate the running of $R/\xi = \alpha_d^2$, for the two cases A and B above in Fig.4. One can see that the running is significant, over the range $q \in [0.1, 4]$ GeV, in both cases: R/ξ changes by a factor of ~ 2 for case A (solid curve) and a factor of ~ 4 for case B (dashed curve). Furthermore, if sufficient statistics are available one can easily distinguish between the two cases, as the figure shows, which can potentially probe the dark sector spectrum over the measured q^2.

Here, we would like to add a comment. The rise of the ratio R with q only encodes the relative increase in $\sigma_{\rm DM}$ compared to the case with constant couplings. However, $\sigma_{\rm DM}$ falls like $1/q^2$, for $q \gtrsim m_{\gamma_d}$ and modulo α_d running, and hence it is expected that the dark matter scattering signal would be stronger for lower values of q^2, whereas potential backgrounds from neutrino-nucleus scattering become more suppressed. The optimal range of q^2 for detecting the dark matter scattering signals depends on the details of the experimental setup. However, as long as that range is moderately broad, the q^2 running could in principle be measurable, as implied by the plot in Fig.4.

So far, we have limited our discussion to the running of α_d over the GeV-scale values of q, relevant to predictions for proposed fixed target (beam dump) experiments. In practice, one may not worry if α_d becomes too large and approaches a Landau pole at $q^* \gg 1$ GeV, as far as those predictions are concerned. However, we will argue below that non-perturbative values of α_d should typically be postponed to q^* above the weak scale. To see this, note that a straightforward way to resolve the problem of an ever-growing α_d is to assume the appearance of a new non-Abelian gauge interaction G_d at $q > q^*$, whose breaking yields $U(1)_d$ at lower energies. In typical scenarios, the kinetic mixing parameter ε vanishes at q^*, as required by gauge invariance [25]. However, to observe any events in fixed target experiments, often $\varepsilon \gtrsim 10^{-4}$ is required. This implies that ε must run to non-zero values *below* q^*, due to the effects of new states, denoted by $F_{d,Y}$ in Fig.1, which generate $\varepsilon \neq 0$ at the quantum level [1]. These new states must be charged under both $U(1)_d$ and hypercharge $U(1)_Y$, which means that they cannot be lighter than ~ 100 GeV, or else they would have been discovered in high energy experiments [26].

In light of the above theoretical consistency conditions for $\varepsilon \neq 0$ at low energies, we find it well-motivated to require that α_d should remain perturbative up to $q^* \gtrsim 100$ GeV. This requirement implies an upper bound on the low energy value of the $U(1)_d$ coupling $\alpha_d(q_0)$. One can derive an estimate of this upper bound, using the 1-loop running equation

$$\alpha_d(q_0) = \frac{\alpha_d(q^*)}{1 + \frac{2}{3\pi}\alpha_d(q^*)(n_F + n_S/4)\ln(q^*/q_0)}. \qquad (4)$$

The value of $\alpha_d(q_0)$ becomes insensitive to the high scale value $\alpha_d(q^*) \gtrsim 1$, the onset of strong coupling, as long as $\ln(q^*/q_0) \gg 1$, and we get

$$\alpha_d(q_0) \approx \frac{3\pi}{(2n_F + n_S/2)\ln(q^*/q_0)}. \qquad (5)$$

In the above, we have implicitly assumed that $m_{\gamma_d} \lesssim q_0$ and we will consider, as before, that $q_0 = 0.1$ GeV, a typical value in the lower kinematic range for fixed target experiments.

We see that for $q^* = 100$ GeV, as a minimal requirement based on our preceding discussion, an upper bound $\alpha_d(q_0) \lesssim 0.68/(n_F + n_S/4)$ is obtained. Hence, for moderate values of n_F and n_S one can obtain interesting upper bounds. For instance, if $n_F = 1$ and $n_S = 1$ (corresponding to a dark Higgs), we find $\alpha_d(q_0) \lesssim 0.5$; this upper bound gets reduced to $\alpha_d(q_0) \lesssim 0.3$ for $n_F = 2$. One may entertain much larger values of q^*, potentially near the Planck scale $M_{\rm P} \approx 1.2 \times 10^{19}$ GeV. This could be motivated if one assumes that there is no new physical mass scale above the electroweak scale, which may possibly address the stability of the SM Higgs mass against large quantum corrections [27, 28]. In that case, one gets $\alpha_d(q_0) \lesssim 0.1/(n_F + n_S/4)$ which could place stringent upper bounds on $\alpha_d(q_0)$, in the scenarios considered in this work. In particular, a dark matter interpretation of the

γ_d invisible final state implies the relation among model parameters given in Eq. (1). For $\varepsilon \lesssim 10^{-4}$, accessible to future experiments, that relation would typically require $\alpha_d \gtrsim 0.1$, which would be in perturbative tension with $q^* \sim M_{\rm P}$.

In passing, we observe that based on constraints from Cosmic Microwave Background Radiation (CMBR) measurements [29], p-wave annihilation of GeV-scale thermal relic dark matter is favored, since it becomes less efficient at smaller velocities characteristic of the CMBR decoupling era [10]. This requirement points to $U(1)_d$-charged light scalar states as good thermal relic dark matter candidates [12, 30]. However, excess annihilation that could distort CMBR could also be avoided if dark matter density is given by an asymmetry at late times, which would preclude the possibility of conjugate-pair annihilation. We would also like to add that in the models considered in our work, dark matter scattering from itself or other dark states, mediated by γ_d, becomes stronger at earlier epochs with higher temperatures and larger characteristic values of q. In general, the running of dark sector couplings could lead to potentially interesting effects in early universe cosmology [16, 17], but we will not further speculate on this question here.

To summarize, we have considered the running of a dark sector $U(1)_d$ fine structure constant α_d due to the presence of light dark particles in quantum loop corrections. Invisible dark photons, assumed to decay on-shell to dark matter states, belong to this class of models. We observed that the running of $\alpha_d(q)$ can lead to a running kinetic mixing parameter $\varepsilon^2(q)$, for it is naturally loop-induced and therefore proportional to $\alpha_d(q)$. Under some minimal well-motivated assumptions about the dark sector content, we find that, roughly speaking, values of $\alpha_d \gtrsim 0.4$ can lead to departure from a perturbative analysis and unreliable predictions over the range of momentum transfer 10 MeV$\lesssim q \lesssim 1$ GeV in proposed fixed target or beam dump experiments. We note that α_d values not much smaller than ~ 1 are typical in scenarios with sub-GeV dark matter particles that are lighter than the dark photon. Light fermionic dark states lead to faster running of α_d compared to scalar states and can result in loss of perturbative control over larger regions of parameter space.

We pointed out that the dependence of α_d on q can be used to help probe the low-lying spectrum of the dark sector in fixed target or beam dump experiments. This can be done if measurement of the event rates as a function of q is feasible, over a moderately broad range of q^2. Those rates probe the combined running of $\alpha_d(q)$ and $\varepsilon^2(q) \propto \alpha_d(q)$ with momentum transfer, leading to a potentially significant sensitivity $\propto \alpha_d^2(q)$. We showed that for infrared values $\alpha_d(q_0) \gtrsim 0.2$, one could expect significant effects on the event rate from the running, in typical scenarios. A thorough discussion of q^2 running measurements would require input from specific experimental parameters and would certainly be warranted upon the discovery of a light dark matter signal from an intense source. However, on general grounds, we discussed how electromagnetic cross sections for electron (or muon) scattering from the nucleus can be used to obtain precise predictions for the corresponding dark matter scattering cross section, in the kinetic mixing scenario considered.

We also argued that theoretic considerations imply the perturbative range for α_d should extend to values of q at or above the weak scale, perhaps even the Planck scale, in which case tighter upper bounds on the low energy value of α_d and the dark sector spectrum can be obtained.

Acknowledgments

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