Grand unification and exotic fermions

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Grand Unification and Exotic Fermions

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We exploit the recently developed software package LieART to show that SU(N) grand unified theories with chiral fermions in mixed tensor irreducible representations can lead to standard model chiral fermions without additional light exotic chiral fermions, i.e., only standard model fermions are light in these models. Results are tabulated which may be of use to model builders in the future. An SU(6) toy model is given and model searches are discussed.

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I. INTRODUCTION

In the past, building grand unified theories (GUTs) with SU(N) gauge groups has nearly always been carried out using fermions in totally antisymmetric tensor irreducible representations (irreps). Choosing a chiral anomaly free set of these SU(N) irreps guarantees all fermions will continue to be anomaly free and in totally antisymmetric irreps when decomposed into regular SU(N') subgroups with $N' < N$. We will typically choose $N' = 5$. Hence, under the decomposition

$$\mathrm{SU}(N) \to \mathrm{SU}(5)$$

we have

$$\begin{aligned} &asym\ anomaly\ free\ SU(N)\ irreps \\ &\qquad \to n(\overline{\mathbf{5}}+\mathbf{10}) + \bar{n}(\mathbf{5}+\overline{\mathbf{10}}) + \text{singlets} \end{aligned} \tag{1}$$

so that $n_F = n - \bar{n}$ gives the number of families. There are only a few cases of studies of SU(N) models where other than totally antisymmetric irreps have been used. For example, single complex anomaly free irreps of SU(N) that contain chiral fermions have been searched for [1], and models with fermions in **6**s and **8**s of SU(3) color have been studied [2]. Here we ask if there are SU(N) models that start with fermions in complex mixed tensor irreps that lead to models with only standard model (SM) chiral fermions being light. The simplest way to explore such SU(N) models is to require that the only chiral fermions at the SU(5) level are in standard $(\overline{\mathbf{5}}+\mathbf{10})$s families which then lead to SU(3) × SU(2) × U(1) standard model families

$$\overline{\mathbf{5}}+\mathbf{10} \to (\mathbf{3},\mathbf{2})_{\frac{1}{6}} + (\overline{\mathbf{3}},\mathbf{1})_{\frac{1}{3}} + (\overline{\mathbf{3}},\mathbf{1})_{-\frac{2}{3}} + (\mathbf{1},\mathbf{2})_{-\frac{1}{2}} + (\mathbf{1},\mathbf{1})_{1} \tag{2}$$

However, GUT models [3] and partial gauge unifications [2, 4, 5] with exotic fermions are not unknown. Exotics from string theory [6, 7] and F-Theory [8, 9] have also been considered.

II. GENERAL SU(N) MODELS

Let us focus on the decomposition SU(N) → SU(5). A totally antisymmetric SU(N) tensor irrep corresponds to single column Young tableau. All SU(N) single column tableaux decompose to a single column SU(5) tableau under the regular embedding. In addition, if a set of SU(N) irreps is anomaly free, then so is the set of SU(5) irreps they decompose into. These two facts are the reason models can be successfully constructed in SU(N) gauge theories that reduce to exotic free models at the SM level.

Now we ask if it is still possible to build chiral SU(N) models that are both anomaly free and exotic free at the SU(5) and hence the SM level if we start with irreps that correspond to other than single column tableaux. We will begin with the case of models with fermions in irreps corresponding to two-column tableaux. These irreps can only decompose into irreps with two or fewer columns of SU(5). (More generally, for the standard embedding, an n column tableau of SU(N) can decompose into irreps with n or fewer columns in SU($N - k$).) Hence we would like to find a set of chiral anomaly free two-column SU(N) tableaux that decompose such that the resulting two-column set in SU(5) is vector-like, while at least part of the one column set remains chiral and anomaly free. These chiral fermions must then be in the form of standard $(\overline{\mathbf{5}}+\mathbf{10})$s families.

In the past this type of model has been difficult to explore, but we now have a tool in hand that makes the work quite easy. The software package LieART[1][10], written in Mathematica, can be used to project combinations of multicolumn SU(N) tableaux to SU(5) efficiently

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[1] LieART is hosted by Hepforge, IPPP Durham. The LieART project home page is http://lieart.hepforge.org and the LieART Mathematica application can be freely downloaded as a tar.gz archive from http://www.hepforge.org/downloads/lieart

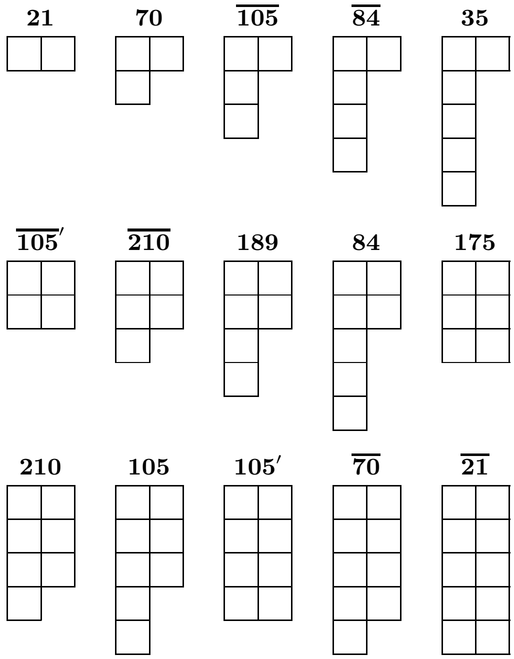

Table I. The two-column tableaux for SU(6). Note that the **35**, **189** and **175** are all real so will not contribute chiral fermions.

and keep track of the chirality in going from SU(N) to SU(5). Our results are displayed in the tables in the next section and other possible searches are discussed. An SU(6) toy model is given in section IV before we conclude in section V. Checking any of these results by hand will clearly demonstrate the power and flexibility of LieART.

III. RESULTS

Let us begin with the simplest example we have found—an SU(6) model with only two-column tableaux as displayed in Table I.

The non-conjugated, complex, two-column tableaux irreps of SU(6) decompose to SU(5) irreps as

$$\begin{array}{ll}\mathbf{21} & \to \mathbf{1}+\mathbf{5}+\mathbf{15}\\ \mathbf{70} & \to \mathbf{5}+\mathbf{10}+\mathbf{15}+\overline{\mathbf{40}}\\ \mathbf{84} & \to \mathbf{5}+\mathbf{10}+\mathbf{24}+\mathbf{45}\\ \mathbf{105} & \to \mathbf{10}+\overline{\mathbf{10}}+\mathbf{40}+\mathbf{45}\\ \mathbf{105'} & \to \overline{\mathbf{15}}+\mathbf{40}+\mathbf{50}\\ \mathbf{210} & \to \mathbf{40}+\mathbf{45}+\mathbf{50}+\mathbf{75}\end{array} \tag{3}$$

and the complex conjugated irreps decompose analogously. One then just has to find linear combinations of SU(6) irreps with three families that are free from exotics at the SU(5) level, which for SU(6) delivers the single example

$$6(\overline{\mathbf{21}})+9(\mathbf{70})+6(\overline{\mathbf{84}})+9(\mathbf{105})+3(\mathbf{105'})+3(\overline{\mathbf{210}}) \tag{4}$$

which when decomposed into SU(5) irreps reduces to

$$\begin{aligned}3(\mathbf{10}+\overline{\mathbf{5}})&+9(\mathbf{5}+\overline{\mathbf{5}})+15(\mathbf{10}+\overline{\mathbf{10}})\\ &+9(\mathbf{15}+\overline{\mathbf{15}})+12(\mathbf{40}+\overline{\mathbf{40}})\\ &+9(\mathbf{45}+\overline{\mathbf{45}})+3(\mathbf{50}+\overline{\mathbf{50}})\\ &+6(\mathbf{1})+6(\mathbf{24})+3(\mathbf{75})\end{aligned} \tag{5}$$

where all irreps not belonging to the three families come in conjugated pairs, thus being vector-like.

More generally we implemented an efficient determination of exotic-free combinations of mixed tensor irreps of SU(N) utilizing LieART. The requirement of three families and no chiral exotics at the SU(5) level leads to a system of linear equations which reduces the number of independent parameters being initially one per irrep type. To this end we introduce special multiplicities m_i coding the imbalance of complex-conjugated and non-conjugated irrep pairs, i.e., a positive multiplicity denotes an excess of non-conjugated irreps and a negative multiplicity an excess of conjugated irreps. For the SU(6) model with only two-column tableaux the ansatz for the determination of an exotic-free, three SM family model reads

$$\begin{aligned}&m_1\,\mathbf{21}+m_2\,\mathbf{70}+m_3\,\mathbf{84}+m_4\,\mathbf{105}+m_5\,\mathbf{105'}+m_6\,\mathbf{210}\\ &\quad\to -3(\mathbf{5})+3(\mathbf{10})+0(\mathbf{15})+0(\mathbf{40})+0(\mathbf{45})+0(\mathbf{50}).\end{aligned} \tag{6}$$

Note that real irreps such as **1**, **35**, **189**, **175** of SU(6) and **1**, **24** and **75** of SU(5) do not contribute chiral fermions and are disregarded here. Decomposing the SU(N) two-column tableaux irreps to SU(5) using (3) we obtain an inhomogeneous system of linear equations for the multiplicities m_i:

$$\left[\begin{array}{cccccc|c}1&1&1&0&0&0&-3\\0&1&1&0&0&0&3\\1&1&0&0&-1&0&0\\0&-1&0&1&1&1&0\\0&0&1&1&0&1&0\\0&0&0&0&1&1&0\end{array}\right] \tag{7}$$

Since the coefficient matrix is quadratic and of full rank the system has the unique solution given by $m_1{\to}-6$, $m_2{\to}9$, $m_3{\to}-6$, $m_4{\to}9$, $m_5{\to}3$, $m_6{\to}-3$ which translates to (4).

In SU(7) we have 9 complex, non-conjugated, two-column tableau irreps: **28**, **112**, **140**, **196**, **210**, **224**, **490**, **490′** and **588**. The system of equations for the corresponding multiplicities m_i, with $i=1,\ldots,9$, is underdetermined leading to solution sets with three independent coefficients, c_1, c_2 and c_3:

$$\begin{aligned}&m_1\to c_1,\ m_2\to c_2,\ m_3\to c_1+2c_3,\\ &m_4\to 3c_1+2c_2+2c_3+6,\\ &m_5\to -20c_1-8c_2-19c_3-51,\\ &m_6\to -16c_1-7c_2-16c_3-36,\\ &m_7\to 20c_1+8c_2+20c_3+51,\\ &m_8\to -28c_1-12c_2-27c_3-69,\\ &m_9\to 13c_1+6c_2+12c_3+30.\end{aligned} \tag{8}$$

28	**112**	**140**	**196**	**210**	**224**	**490**	**490′**	**588**
-2	1	-4	0	0	5	-1	2	-2
-1	1	-5	1	-1	5	-1	1	-1
0	1	-6	2	-2	5	-1	0	0
-7	4	-1	-1	0	0	3	-2	-1
-6	4	-2	0	-1	0	3	-3	0
-3	-1	-1	-3	-2	3	3	0	-3
-2	-1	-2	-2	-3	3	3	-1	-2
-1	-1	-3	-1	-4	3	3	-2	-1
0	-1	-4	0	-5	3	3	-3	0

Table II. Three family solutions for two-column tableau SU(7) irreps

36	**168**	**216**	**336**	**378**	**420**	**504**	**1008**	**1176**	**1344**	**1512**	**2352′**
-4	0	0	1	2	-1	0	1	0	0	1	-1
-2	1	-2	-1	1	1	1	0	-1	-1	0	1
0	-4	0	3	0	3	0	0	1	-2	0	0
1	-1	-3	0	2	0	4	0	1	1	-2	0
-4	0	-1	2	-1	1	1	-1	3	0	0	-2
-1	1	-4	0	1	0	-1	0	2	2	1	-3
-4	0	-2	3	-1	3	-2	2	0	0	-1	0
0	-1	-1	-1	1	1	-2	-2	1	-2	4	-2
-2	1	-3	0	-2	3	2	-2	2	-1	-1	0
2	-3	0	-1	2	1	3	-2	0	-3	1	1
3	-3	-3	1	2	2	-2	1	0	0	1	-1

Table III. Three family solutions for two-column tableau SU(8) irreps

For individual solutions the independent coefficients (c_js in general) take on positive and negative integer values. Simple solutions can be found by scanning through a limited range of integers for the c_js, which we choose to be $c_j = -20, \ldots, 20$, and we limit the total number of two-column tableau irreps to 19, i.e., $\sum_i |m_i| \leq 19$. With these self imposed limitations, we find 9 solutions for SU(7) displayed in a compact tabular form in terms of the multiplicities m_i in Table II, which translates to models with the following sets of SU(7) fermion irreps:

$$
\begin{array}{l}
2(\overline{\mathbf{28}})+\mathbf{112}+4(\overline{\mathbf{140}})+5(\mathbf{224})+\overline{\mathbf{490}}+2(\mathbf{490}')+2(\overline{\mathbf{588}})\\
\overline{\mathbf{28}}+\mathbf{112}+5(\overline{\mathbf{140}})+\mathbf{196}+\overline{\mathbf{210}}+5(\mathbf{224})+\overline{\mathbf{490}}+\mathbf{490}'+\overline{\mathbf{588}}\\
\mathbf{112}+6(\overline{\mathbf{140}})+2(\mathbf{196})+2(\overline{\mathbf{210}})+5(\mathbf{224})+\overline{\mathbf{490}}\\
7(\overline{\mathbf{28}})+4(\mathbf{112})+\overline{\mathbf{140}}+\overline{\mathbf{196}}+3(\mathbf{490})+2(\overline{\mathbf{490}'})+\overline{\mathbf{588}}\\
6(\overline{\mathbf{28}})+4(\mathbf{112})+2(\overline{\mathbf{140}})+\overline{\mathbf{210}}+3(\mathbf{490})+3(\overline{\mathbf{490}'})\\
3(\overline{\mathbf{28}})+\overline{\mathbf{112}}+\overline{\mathbf{140}}+3(\overline{\mathbf{196}})+2(\overline{\mathbf{210}})+3(\mathbf{224})+3(\mathbf{490})+3(\overline{\mathbf{588}})\\
2(\overline{\mathbf{28}})+\overline{\mathbf{112}}+2(\overline{\mathbf{140}})+2(\overline{\mathbf{196}})+3(\overline{\mathbf{210}})+3(\mathbf{224})+3(\mathbf{490})+\overline{\mathbf{490}'}+2(\overline{\mathbf{588}})\\
\overline{\mathbf{28}}+\overline{\mathbf{112}}+3(\overline{\mathbf{140}})+\overline{\mathbf{196}}+4(\overline{\mathbf{210}})+3(\mathbf{224})+3(\mathbf{490})+2(\overline{\mathbf{490}'})+\overline{\mathbf{588}}\\
\overline{\mathbf{112}}+4(\overline{\mathbf{140}})+5(\overline{\mathbf{210}})+3(\mathbf{224})+3(\mathbf{490})+3(\overline{\mathbf{490}'})
\end{array}
\tag{9}
$$

Moving on to SU(8) we have 12 complex, non-conjugated, two-column tableau irreps: **36**, **168**, **216**, **336**, **378**, **420**, **504**, **1008**, **1176**, **1344**, **1512** and **2352′** and the system of equations leads to solution sets with six independent coefficients c_j:

$$
\begin{aligned}
m_1 &\to c_1,\ m_2 \to c_2,\ m_3 \to c_3,\ m_4 \to c_4,\ m_5 \to c_5,\\
m_6 &\to -33c_1-48c_2-30c_3-28c_4-105,\\
m_7 &\to 4c_1+7c_2+3c_3+8c_5+28c_6,\\
m_8 &\to -33c_1-47c_2-30c_3-27c_4+2c_5+c_6-108,\\
m_9 &\to 60c_1+86c_2+54c_3+51c_4-3c_5-3c_6+195,\\
m_{10} &\to 24c_1+35c_2+21c_3+21c_4+75,\\
m_{11} &\to 30c_1+42c_2+28c_3+27c_4-7c_5-21c_6+108,\\
m_{12} &\to -63c_1-90c_2-57c_3-55c_4+6c_5+15c_6-210.
\end{aligned}
\tag{10}
$$

We find 11 solutions for a maximum of 19 two-column tableau irreps but with a smaller scan range for the six independent coefficients $c_j = -5, \ldots, 5$, with $j = 1, \ldots 6$ as displayed in Table III.

Finally, for SU(9) we obtain solution sets with 10 independent coefficients c_j for the multiplicities of the 16 complex, non-conjugated, two-column tableau irreps **45**, **240**, **315**, **540**, **630**, **720**, **1008**, **1050**, **1890**, **2520**, **2700**, **3402**, **3780**, **5292**, **6048** and **7560**:

$$
\begin{aligned}
m_1 &\to c_1, m_2 \to c_2, m_3 \to c_3, m_4 \to c_4, m_5 \to c_5,\\
m_6 &\to 2c_1+3c_2+2c_3+6c_6, m_7 \to 2c_3+2c_5+3c_7,\\
m_8 &\to 3c_1+3c_3+3c_5+4c_8, m_9 \to c_9,\\
m_{10} &\to 31c_1+33c_2+8c_3+43c_5+44c_6+19c_7\\
&\quad +10c_8+30c_9+57c_{10}+54,\\
m_{11} &\to 29c_1+27c_2+3c_3-4c_4+45c_5+38c_6\\
&\quad +21c_7+11c_8+36c_9+63c_{10}+56,\\
m_{12} &\to -263c_1-270c_2-58c_3+20c_4-378c_5-372c_6\\
&\quad -178c_7-86c_8-291c_9-518c_{10}-483,\\
m_{13} &\to -185c_1-185c_2-41c_3+15c_4-263c_5-258c_6\\
&\quad -119c_7-65c_8-200c_9-357c_{10}-329,\\
m_{14} &\to -773c_1-790c_2-167c_3+60c_4-1107c_5-1092c_6\\
&\quad -514c_7-256c_8-851c_9-1518c_{10}-1411,\\
m_{15} &\to 485c_1+495c_2+103c_3-40c_4+698c_5+685c_6\\
&\quad +327c_7+160c_8+540c_9+960c_{10}+892\\
m_{16} &\to 220c_1+224c_2+51c_3-15c_4+310c_5+310c_6\\
&\quad +140c_7+75c_8+234c_9+420c_{10}+390
\end{aligned}
\tag{11}
$$

and find 11 solutions for a maximum of 26 two-column tableau irreps and $c_j = -1, \ldots, 1$, with $j = 1, \ldots, 10$ displayed in Table IV.

We do not need to limit ourselves to two-column tableaux. For instance we can search for three column sets that are exotic free, anomaly free and have three families. Here we conclude with the two- and three-column SU(6) case, where we find solution sets with six independent coefficients c_j for the multiplicities of the 22 complex, non-conjugated, two- and three-column tableau irreps **21**, **56**, **70**, **84**, **105**, **105′**, **120**, **210**, **210′**, **280**, **336**, **384**, **420**, **490**, **560**, **840**, **840′**, **896**, **1050**, **1176**,

45	**240**	**315**	**540**	**630**	**720**	**1008**	**1050**	**1890**	**2520**	**2700**	**3402**	**3780**	**5292**	**6048**	**7560**
0	-1	1	-1	0	-1	2	3	-1	-1	0	0	0	3	0	-2
-1	1	-1	0	0	-1	-2	-2	0	1	-1	0	4	1	-1	-2
1	-1	1	-1	0	1	-1	2	-1	1	-3	1	-1	0	-2	3
1	-1	1	-1	-1	1	-3	3	0	-2	-1	2	-3	0	0	2
0	0	0	1	0	-6	-3	4	0	1	4	1	-2	-1	0	0
0	0	0	1	1	-6	-1	3	-1	4	2	0	0	-1	-2	1
0	0	-1	1	0	-2	-5	-3	1	0	1	0	3	-3	2	-2
0	1	-1	0	-1	1	-7	-2	1	0	-2	2	1	-2	-1	2
1	0	-1	0	1	-6	-3	3	0	0	5	2	-2	0	0	-1
0	-1	1	-1	-1	-1	0	4	0	-4	2	1	-2	3	2	-3
0	0	-1	1	1	-2	-3	-4	0	3	-1	-1	5	-3	0	-1

Table IV. Three family solutions for two-column tableau SU(9) irreps

$\mathbf{1176}'$ and $\mathbf{1470}$:

$$\begin{aligned}
&m_1 \to c_1, m_2 \to -c_1{-}6, m_3 \to c_2, m_4 \to c_3,\\
&m_5 \to c_4, m_6 \to c_5, m_7 \to c_6, m_8 \to 6{-}c_2,\\
&m_9 \to c_2{+}c_6{-}9, m_{10} \to -c_6, m_{11} \to -c_3{-}c_4{-}c_6{+}3,\\
&m_{12} \to -c_3{-}6, m_{13} \to -c_1{-}c_5{-}c_6{-}3, m_{14} \to c_2{-}c_5{-}6,\\
&m_{15} \to c_1{-}c_3{+}c_6, m_{16} \to 9{-}c_4,\\
&m_{17} \to -c_1{+}c_3{-}c_5{-}c_6{+}3, m_{18} \to -c_3{-}6,\\
&m_{19} \to c_2{-}9, m_{20} \to c_1{+}c_4{+}c_6{-}3,\\
&m_{21} \to -c_1{-}c_4{-}c_6{+}3, m_{22} \to c_1{-}c_2{+}c_4{+}c_5{+}c_6{+}3.
\end{aligned} \tag{12}$$

With a maximum of 61 two- and three-column tableau irreps and $c_j = -2, \ldots, 2$, with $j = 1, \ldots 6$ we find 17 solutions displayed in Table V.

Other cases are also easily explored. For instance we could consider combinations of one and two-columns tableau, or just three column tableau, etc. We could also redo the above analysis for four families. Alternatively, we could study anomaly free three family models with a specific set of exotics. All these possibilities as well as other types of model scans (See e.g., [11].) can be easily handled with LieART [10].

IV. AN SU(6) EXAMPLE

Besides the three family exotic models discussed above, we should also display the simplest of all models found to date that starts with any number of multicolumn tableaux plus some single column tableaux that has three families. Since we already have three families in SU(6) for our two-column example in (4) and as all coefficients are a multiple of 3, we must have one family if we divide all coefficients by three. Hence we can add the single column irreps $4(\overline{\mathbf{6}}) + 2(\mathbf{15})$ to this set to get a three family model

$$\begin{aligned}
2(\overline{\mathbf{21}}) + 3(\mathbf{70}) + 2(\overline{\mathbf{84}}) + 3(\mathbf{105}) + \mathbf{105}' + \overline{\mathbf{210}}&\\
+4(\overline{\mathbf{6}}) + 2(\mathbf{15}) = 3(\overline{\mathbf{5}} + \mathbf{10}) + \text{real}&
\end{aligned} \tag{13}$$

It seems most natural to let the two lightest families be in the $4(\overline{\mathbf{6}}){+}2(\mathbf{15})$ and the third family to be the "exotic" family.

While this example may not be simple enough to be a useful physical model, it is still instructive to examine it further. For instance, if we break the symmetry along the path $\mathrm{SU}(6) \to \mathrm{SU}(5) \times \mathrm{U}(1)'$ then as long as the extra $\mathrm{U}(1)'$ is unbroken, some of the SU(5) conjugate pair exotics (as well as some $(\mathbf{5}+\overline{\mathbf{5}})$ and $(\mathbf{10}+\overline{\mathbf{10}})$ pairs) stay light, as long as their $\mathrm{U}(1)'$ charges are imbalanced. This remains true even if we break to $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{U}(1)'$, but when we break to the standard model $\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ gauge group all the exotics can finally acquire mass.

If we were to keep $\mathrm{U}(1)'$ unbroken until ~ 1 TeV, then we would predict very many light (TeV scale) exotic fermions. Since keeping the extra $\mathrm{U}(1)'$ does not directly lead to proton decay it is probably allowed to be unbroken down nearly to the electroweak scale. However, since this model leads to so many exotics, a low energy $\mathrm{U}(1)'$ would undoubtedly upset the renormalization group running and spoil unification. So we conjecture that the best we can do is bring the $\mathrm{U}(1)'$ scale down a few orders of magnitude from the GUT scale. This model is by no means compelling, but it is still interesting, as it is the first example of a type of model with exotic fermions that can exist well below the GUT scale. As we noted above, better would be a model with only a few light exotics and a low energy $\mathrm{U}(1)'$ where the exotics could even be within reach of the LHC.

Other one-family exotic models can be found directy with our algorithm by requiring the decomposition to only one set of $\overline{\mathbf{5}}+\mathbf{10}$ and all other fermions to be vector-like. In Table VI we list the one-family model equation systems and some solutions for two-column tableaux for SU(7), SU(8) and SU(9). We have three column examples but they are complicated and not very enlightening, so we have chosen not to display them.

V. CONCLUSIONS

We have explored $\mathrm{SU}(N)$ gauge theory examples that start with mixed tensor fermonic irreps that none the less have only three standard families of chiral fermions

21	**56**	**70**	**84**	**105**	**105′**	**120**	**210**	**210′**	**280**	**336**	**384**	**420**	**490**	**560**	**840**	**840′**	**896**	**1050**	**1176**	**1176′**	**1470**
-2	-4	2	-2	2	-2	1	4	-6	-1	2	-4	0	-2	1	7	4	-4	-7	-2	2	0
-2	-4	2	-2	2	-2	2	4	-5	-2	1	-4	-1	-2	2	7	3	-4	-7	-1	1	1
-2	-4	2	-1	2	-2	1	4	-6	-1	1	-5	0	-2	0	7	5	-5	-7	-2	2	0
-2	-4	2	-1	2	-2	2	4	-5	-2	0	-5	-1	-2	1	7	4	-5	-7	-1	1	1
-1	-5	2	-2	2	-2	0	4	-7	0	3	-4	0	-2	1	7	4	-4	-7	-2	2	0
-1	-5	2	-2	2	-2	1	4	-6	-1	2	-4	-1	-2	2	7	3	-4	-7	-1	1	1
-1	-5	2	-2	2	-2	2	4	-5	-2	1	-4	-2	-2	3	7	2	-4	-7	0	0	2
-1	-5	2	-1	2	-2	0	4	-7	0	2	-5	0	-2	0	7	5	-5	-7	-2	2	0
-1	-5	2	-1	2	-2	1	4	-6	-1	1	-5	-1	-2	1	7	4	-5	-7	-1	1	1
-1	-5	2	-1	2	-2	2	4	-5	-2	0	-5	-2	-2	2	7	3	-5	-7	0	0	2
-1	-5	2	0	2	-2	1	4	-6	-1	0	-6	-1	-2	0	7	5	-6	-7	-1	1	1
0	-6	2	-2	2	-2	0	4	-7	0	3	-4	-1	-2	2	7	3	-4	-7	-1	1	1
0	-6	2	-2	2	-2	1	4	-6	-1	2	-4	-2	-2	3	7	2	-4	-7	0	0	2
0	-6	2	-1	2	-2	0	4	-7	0	2	-5	-1	-2	1	7	4	-5	-7	-1	1	1
0	-6	2	-1	2	-2	1	4	-6	-1	1	-5	-2	-2	2	7	3	-5	-7	0	0	2
0	-6	2	0	2	-2	0	4	-7	0	1	-6	-1	-2	0	7	5	-6	-7	-1	1	1
0	-6	2	0	2	-2	1	4	-6	-1	0	-6	-2	-2	1	7	4	-6	-7	0	0	2

Table V. Three family solutions for two- and three-column tableau SU(6) irreps

SU(N)	**Equation system**	**One-family model solutions**
SU(7)	$\left[\begin{array}{ccccccccc\|c} 2 & 4 & 3 & -2 & -2 & 2 & -1 & 0 & 1 & -1 \\ 0 & 2 & 2 & -1 & -2 & 2 & -1 & 0 & 1 & 1 \\ 1 & 2 & 0 & -3 & -1 & 0 & -2 & -1 & 0 & 0 \\ 0 & -1 & 0 & 2 & 2 & 1 & 4 & 2 & 2 & 0 \\ 0 & 0 & 1 & 0 & 1 & 2 & 2 & 1 & 3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 2 & 2 & 1 & 0 \end{array}\right]$	$4(\overline{\mathbf{28}})+3(\mathbf{112})+\mathbf{210}+\overline{\mathbf{224}}+\mathbf{490}+\overline{\mathbf{490}}'$ $2(\overline{\mathbf{112}})+2(\overline{\mathbf{196}})+\overline{\mathbf{210}}+2(\mathbf{224})+\mathbf{490}+\mathbf{490}'+2(\overline{\mathbf{588}})$ $\mathbf{28}+2(\overline{\mathbf{112}})+\overline{\mathbf{140}}+\overline{\mathbf{196}}+2(\overline{\mathbf{210}})+2(\mathbf{224})+\mathbf{490}+\overline{\mathbf{588}}$ $3(\overline{\mathbf{28}})+3(\mathbf{112})+\overline{\mathbf{140}}+\mathbf{196}+\overline{\mathbf{224}}+\mathbf{490}+2(\overline{\mathbf{490}}')+\mathbf{588}$ $2(\mathbf{28})+2(\overline{\mathbf{112}})+2(\overline{\mathbf{140}})+3(\overline{\mathbf{210}})+2(\mathbf{224})+\mathbf{490}+\overline{\mathbf{490}}'$ $5(\overline{\mathbf{28}})+3(\mathbf{112})+\mathbf{140}+\overline{\mathbf{196}}+2(\mathbf{210})+\overline{\mathbf{224}}+\mathbf{490}+\overline{\mathbf{588}}$
SU(8)	$\left[\begin{array}{cccccccccccc\|c} 3 & 9 & 6 & 8 & 9 & 8 & 0 & -9 & -3 & 6 & 0 & 0 & -1 \\ 0 & 3 & 3 & 3 & 6 & 6 & 0 & -6 & -2 & 5 & 0 & 0 & 1 \\ 1 & 3 & 0 & 6 & 3 & 0 & -1 & -8 & -6 & 0 & -3 & -3 & 0 \\ 0 & -1 & 0 & -3 & -3 & 1 & 3 & 9 & 8 & 3 & 9 & 8 & 0 \\ 0 & 0 & 1 & 0 & -1 & 3 & 3 & 3 & 3 & 6 & 8 & 6 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 3 & 5 & 1 & 3 & 5 & 0 \end{array}\right]$	$\overline{\mathbf{36}}+\overline{\mathbf{216}}+\mathbf{336}+\mathbf{378}+\mathbf{504}+2(\mathbf{1008})+\overline{\mathbf{1176}}+\mathbf{1344}+2(\overline{\mathbf{1512}})+\mathbf{2352}'$ $\mathbf{36}+2(\overline{\mathbf{168}})+\mathbf{336}+\mathbf{378}+2(\overline{\mathbf{504}})+\mathbf{1176}+2(\mathbf{1512})+2(\overline{\mathbf{2352}}')$ $4(\overline{\mathbf{36}})+2(\mathbf{168})+\mathbf{216}+\overline{\mathbf{336}}+\overline{\mathbf{420}}+\mathbf{504}+\overline{\mathbf{1008}}+\overline{\mathbf{1344}}+\mathbf{1512}$ $3(\mathbf{36})+\overline{\mathbf{168}}+2(\overline{\mathbf{216}})+\overline{\mathbf{336}}+2(\mathbf{420})+\overline{\mathbf{504}}+\overline{\mathbf{1008}}+\overline{\mathbf{1344}}+\mathbf{1512}$ $\overline{\mathbf{36}}+\mathbf{378}+2(\overline{\mathbf{420}})+4(\mathbf{504})+\overline{\mathbf{1008}}+2(\mathbf{1176})+\mathbf{1344}+\overline{\mathbf{1512}}+\overline{\mathbf{2352}}'$ $2(\overline{\mathbf{168}})+2(\mathbf{216})+\mathbf{378}+\mathbf{420}+2(\overline{\mathbf{1176}})+3(\overline{\mathbf{1344}})+\mathbf{1512}+2(\mathbf{2352}')$
SU(9)	$\left[\begin{array}{cccccccccccccccc\|c} 4 & 16 & 10 & 20 & 24 & 20 & -12 & 11 & -35 & -20 & 20 & -18 & 14 & -3 & -12 & 6 & -1 \\ 0 & 4 & 4 & 6 & 12 & 12 & -8 & 8 & -18 & -11 & 14 & -12 & 11 & -2 & -8 & 5 & 1 \\ 1 & 4 & 0 & 10 & 6 & 0 & -4 & -1 & -20 & -20 & 0 & -15 & -4 & -9 & -20 & -6 & 0 \\ 0 & -1 & 0 & -4 & -4 & 1 & 6 & 4 & 16 & 20 & 4 & 24 & 16 & 16 & 34 & 20 & 0 \\ 0 & 0 & 1 & 0 & -1 & 4 & 4 & 6 & 4 & 6 & 10 & 15 & 20 & 9 & 20 & 20 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 4 & 9 & 1 & 6 & 4 & 10 & 16 & 9 & 0 \end{array}\right]$	$\mathbf{45}+\overline{\mathbf{240}}+\overline{\mathbf{315}}+\mathbf{720}+\overline{\mathbf{1890}}+\mathbf{2520}+\mathbf{3402}+\overline{\mathbf{3780}}+\overline{\mathbf{6048}}+\mathbf{7560}$ $\overline{\mathbf{240}}+\mathbf{540}+\overline{\mathbf{630}}+\mathbf{720}+\mathbf{1050}+\overline{\mathbf{1890}}+2(\mathbf{2520})+\overline{\mathbf{2700}}+\overline{\mathbf{3780}}$ $+\overline{\mathbf{5292}}+\overline{\mathbf{6048}}+2(\mathbf{7560})$ $\overline{\mathbf{45}}+\overline{\mathbf{315}}+\overline{\mathbf{540}}+\mathbf{630}+2(\mathbf{1008})+\mathbf{1050}+2(\overline{\mathbf{2520}})+2(\mathbf{2700})$ $+2(\overline{\mathbf{3780}})+2(\mathbf{6048})+\overline{\mathbf{7560}}$ $\overline{\mathbf{315}}+\overline{\mathbf{540}}+\mathbf{630}+2(\mathbf{720})+\overline{\mathbf{1008}}+\overline{\mathbf{2700}}+\mathbf{3402}+3(\overline{\mathbf{3780}})$ $+3(\overline{\mathbf{5292}})+4(\mathbf{7560})$

Table VI. One family equation systems and solutions for two-column tableau irreps

at the SU(5) level. These results have been obtained with LieART, which is a programmable group theory software package capable of handling such complicated tasks. If we relax the constraint of starting with 20 irreps and a limited scan range for the independent coefficients, then there is an arbitrarily large class of models that start with chiral exotic fermions (i.e., fermions in multicolumn tableaux) at the SU(N) level, but where there are only standard chiral families at the SU(5) and SM level. While so far none of these models are particularly compelling, the results do demonstrate a new avenue for model building. It is conceivable that a model like one of these could describe the UV completion of the SM. Although at present we do not have an example, that such models could arise remains a logical possibility. We plan to search for such models.

NOTE ADDED IN PROOF

The chirality and fermionic particle content of the SM coming from grand unified theories has been investigated from a somewhat different point of view in [12]. Where results overlap with our work they agree.

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