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Parton distributions and the W mass measurement

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Abstract

We examine the sources of parton distribution errors in the W mass measurement, and point out shortcomings in the existing literature. Optimistic assumptions about strategies to reduce the error by normalizing to Z observables are examined and found to rely too heavily on assumptions about the parametrization and degrees of freedom of the parton distribution functions (PDFs). We devise a strategy to combine measurements as efficiently as possible using error correlations to reduce the overall uncertainty of the measurement, including Z data, and estimate a PDF error of $^{+10}_{-12}$ MeV is achievable in a W mass measurement at the LHC. Further reductions of the W mass uncertainty will require improved fits to the parton distribution functions.

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I. INTRODUCTION

As the Large Hadron Collider (LHC) at CERN begins its next run, there is significant interest in reducing the uncertainty in the measured value of the W boson mass [1]. The current best measurement of the W boson mass of 80.385$\pm$15 GeV is based on combined data from the CDF and D0 Collaborations [2] taken at the Fermilab Tevatron. The uncertainty in the experimental analyses is dominated by a theoretical uncertainty generically called "parton distribution function (PDF) errors." Looking forward to a measurement at the LHC, we clarify how "PDF errors" affect the measurements, assess their contribution to the uncertainty of the W boson mass using current PDF sets, and propose a series of steps to reduce that uncertainty by at least a factor of three at the LHC.

Since the discovery of the W boson in 1983 by the UA1 Collaboration [3], the W boson mass has played a central role in precision electroweak measurements and in constraints on the standard model through global fits. For many years the uncertainty in the measurement of the W boson mass was one of the main limits to the indirect prediction of the standard model Higgs boson mass [4]. With the recent discovery of a Higgs-like boson [5, 6] consistent with the standard model global fits, the need for a higher precision measurement W boson shifts to physics beyond the standard model. Models with enhanced symmetries, such as supersymmetry, predict shifts of 2–20 MeV [1, 7], hence the mass of the W boson is an important constraint on these models.

Current theoretical predictions of the W boson mass in the standard model include the full two-loop corrections [8] and leading 3- and 4-loop corrections [9–13]. The current standard model uncertainty is estimated to be $\sim$4 MeV, however inclusion of full 3-loop self-energies should reduce this to $\sim$1 MeV [1]. In supersymmetry, the shifts in mass can be large, but the additional uncertainty due to higher order effects tends to be small [7], hence an experimental precision of 5 MeV or better is desirable to constrain the supersymmetric parameter space [1, 14, 15].

The experimental uncertainty W mass measurement from the Tevatron was well balanced between systematic errors, predominantly lepton energy scale and recoil energy resolutions, and uncertainties due to proton structure through the PDFs [2, 16]. After combining CDF and D0 data sets, the PDF uncertainties remained the largest single uncertainty at $\pm$10 MeV on their own. At the LHC the statistical errors will be negligible, and the systematics are predicted to be well under control [17, 18] — at the 2–4% level that ATLAS found in measuring the W transverse momentum spectrum [19]. However, current experimental estimates of the uncertainty due to PDFs at the LHC utilized by the ATLAS and CMS Collaborations are ± 25 MeV [17]. Hence, without improvement in the PDF uncertainties, a measurement at the LHC will not contribute significantly to the world average.

Recently there have been claims that the errors are severely overestimated, and are closer to ± 10 MeV at the LHC using current techniques [21, 22]. Further, there are predictions that the uncertainty will reach ± 5 MeV at the LHC with expected improvements in the measurement of the PDFs [1]. We demonstrate below the current uncertainty predictions are actually slightly underestimated, but we provide a method to reach ± 10 MeV using current PDF uncertainties and 7 TeV or 13 TeV data with 90% confidence level. As PDF uncertainties improve, the goal of ± 5 MeV may still be in reach.

We begin our exploration of PDF errors in Sec. II by describing the methods used to determine the W boson mass, and our simulation of these methods and the PDF uncertainty. In Sec. III we first demonstrate that uncertainties in proton structure are important at both a hard scattering level and in their contribution to soft showering. Hence, error estimates that ignore some of these effects are too small. The most recent Tevatron analyses have taken the sum of these effects into account, and so we reproduce the CDF analysis [16] as a check on our calculations.

We use our full analysis in Sec. IV to determine the current PDF error contribution to the W boson mass uncertainty at the LHC for 7 and 13 TeV. We find the current uncertainty is at least ± 30 MeV, far above the desired range, and hence examine a few strategies in Sec. V that can be used to reduce the PDF contribution to ± 10 MeV or below. Finally, we conclude in Sec. VI with a discussion of where improvements are needed to reach ± 5 MeV.

II. DETERMINING THE W MASS

The W boson is an unstable particle, decaying either into jets or a charged lepton and neutrino. At hadron colliders, observation of W decay into jets is extremely difficult near threshold due to the resolution of reconstructing jets. The lepton channel ($l = e, \mu$) is much cleaner in this environment. In contrast to the Z, one of the decay products in this channel is invisible: the neutrino. Therefore, one cannot directly reconstruct the mass of the decay pair. However, distributions in the observable charged lepton show kinematic edges sensitive to the W mass. Typically experiments fit the full shaped of the observed distributions to templates to determine the best fit mass. Three such observables are commonly used:

- Transverse momentum (p_T^l) of the charged lepton. This is the simplest distribution to reconstruct, but suffers from the disadvantage that it is sensitive to the underlying p_T distribution of the W itself. Modeling this distribution requires careful attention to nonperturbative, resummed perturbative, and fixed-order corrections depending on the p_T^W regime. In practice, an experiment will restrict to small p_T^W to avoid the poorer resolution at higher p_T. This means that, in principle, a resummed generator with nonperturbative effects like ResBos [23–25] is ideal. In our analysis we are only interested in the relative shift in shape due to changes in PDF assumptions. Therefore, we use a matched fixed-order plus parton shower, MadGraph [26] plus PYTHIA [27], which is sufficient to describe the general features of the W mass analysis, and offers the advantage of generating events for analysis in a convenient format. We will discuss the details of our modeling in a later section.

- Missing transverse energy ($E_T^{miss} = p_T^\nu$). The neutrino transverse momentum is reconstructed by adding all other transverse energy in the detector and requiring the total transverse momentum to sum to zero, i.e.,

$$\vec{E}_T^{miss} = -\sum(\vec{E}_T). \tag{1}$$

  This observable suffers from the disadvantages of p_T^l, and additionally, any error in the measurement of every other particle in the detector, hadrons in particular. As such, it is poor variable to fit, particularly in an active environment such as the LHC, and will not be considered further.

- Transverse mass of the W. Transverse mass is defined as

$$M_T = \sqrt{2p_T^l E_T^{miss}(1 - \cos(\Delta\phi_{l,miss}))}. \tag{2}$$

  where $\Delta\phi_{l,miss}$ denotes the angular separation between the charged lepton and reconstructed neutrino in the transverse plane. M_T has a Jacobian peak sensitive to the mass of the W without being dependent on the unobserved longitudinal momentum of the neutrino, and is much less sensitive to the underlying W p_T distribution than the leptons themselves.

Each of these observables has a slightly different shape depending on the precise mass of the W. In particular, in the limit of a tree-dominated, zero width process with a perfect detector, the observables would have a steep drop-off at $p_T = M_W/2$ and $M_T = M_W$. In practice these distributions are much smoother (and easier to fit) once realistic effects are included.

The W mass is fit by histogramming one of these variables and comparing to a template, using a best-fit χ^2 or likelihood function. We use the χ^2 method, using the statistical error in each bin for a typical number of events as the measure of the fit.

A. Analysis setup

Since we are only interested in PDF errors and their origins, we only need enough realism to reproduce results seen in experimental analyses; we are not performing an actual mass fit to a template with full detector effects. To isolate the effect of parton distributions with as simple an analysis as possible, we do the following: We generate pseudodata with a similar number of events as existent or anticipated experimental analyses. In practice this finite data would be compared to a parametrized template fit from billions of simulated events, or equivalently to histograms of those billions of events themselves. We create templates out of the pseudodata, and reweight event-by-event for different hypothesis masses and PDFs. This forces the best fit for the central PDF (that used in the original generation) to occur at the generated mass, which we have chosen to be $M_W = 80.4$ GeV.

We can then find PDF errors by comparing the best fit for each PDF eigenvector to the closest matching mass template. The total asymmetric error is found by summing the positive and negative shifts in mass for each eigenvector in quadrature according to the standard CTEQ "modified tolerance method" [28, 29],

$$\delta M_W^{\pm} = \sqrt{\sum_{i=1}^{n} \Big(\max[\pm(M_W^i - M_W^0), \pm(M_W^{-i} - M_W^0), 0] \Big)^2}\,, \qquad (3)$$

where n is the number of eigenvectors in the error set, and M_W^i is the reconstructed mass assuming PDF error set i.

In order for the templates to not be sensitive to the statistical fluctuations of the pseudodata, the entire event must retain the same fluctuations, from shower history to detector smearing. Therefore we implement a custom detector simulation that captures most of the realism while allowing us to maintain control over the random numbers used to generate the smearing. Otherwise our PDF errors would include contributions proportional to the errors arising from statistics and detector systematics.

B. Detector simulation

Our "detector" consists of a set of calorimeters which smear the momenta of the particles of the simulated event. Parameters for the CDF EM calorimeter are taken from Ref. [16]. In comparing to their analysis (Sec. III) we model the reconstructed hadronic recoil according to parameters of that reference as well, including the min bias contribution. For the purposes of comparing to their analysis we estimated the PYTHIA parameter PARP(131) = 0.1 for the luminosity considered there, which gives approximately 3-4 interactions per crossing. Other details of the event generation can be found in Sec. II C. Our generated distributions reproduce those in Ref. [16] quite well. For the LHC, we use the parameters of the simulator DELPHES [30] for ATLAS distributed with MadGraph.

C. Data generation

We generate 0–2 jet matched samples using MadGraph 5 [26] and shower using PYTHIA 6.420 [27] with a p_T-ordered shower. The samples are matched with the MLM scheme at a scale of 20 GeV. We find that the matched sample reproduces well the measured p_T distribution of the W measured by ATLAS [19], in contrast to a pure-showered 0-jet sample, though experiments usually put an upper cut p_T^W below the matching scale we have used. The matched sample up to two jets allows every type of parton to participate in W production. In principle, higher-orders in perturbation theory would improve the overall normalization, but we find it is less important once distributions are normalized. Resummation calculations such as ResBos use fewer arbitrary parameters than PYTHIA tunes in fitting very low p_T, but the W observables under consideration are not very sensitive to this region, M_T in particular. The shower does an excellent job of reproducing the W p_T data at moderate

(> 5 GeV) p_T, as expected, so the generated lepton p_T distribution should be suitable for the purposes of probing PDF sensitivity. Data binned below 5 GeV are unavailable, presumably due to the difficulty in measuring W recoil from a single lepton plus missing energy in this region. However, PYTHIA-showered predictions have been found to agree with data for the Z recoil down to 2 GeV [20].

As we will see in the next section, the M_T distribution is sensitive to detector effects through missing energy mismeasurement, the resolution of which is driven by hadron calorimetry. Since this effect is so important, a shower, ideally with hadronization, is needed for a study of M_T. ResBos provides a predicted p_T^W distribution and does not resolve individual partons in the shower. The p_T^l distribution is not sensitive to this issue, only to the underlying W transverse momentum, and therefore a careful resummation calculation would serve as a useful check on the shower evolution for the p_T^l fit. Unfortunately, PDF error eigenvector grids for ResBos are not available for the LHC and modern PDFs at this time.

Tevatron samples are generated using the set CTEQ 6.6 [31], and LHC samples with CT10 [32]. PYTHIA has been modified to use these sets via LHAPDF 5 [33]. The PYTHIA tunes used are D6 for the Tevatron and AMBT1 for the LHC. These tunes are paired with different PDF sets than those used in their calibration; while this may cause the low-energy physics to be somewhat different, we are most interested in the predicted errors for modern PDFs and use them in the shower for consistent reweighting.

The W events are decayed to either $e^-\bar{\nu}_e/e^+\nu_e$ for simplicity; the backgrounds in the electron channel are negligible in contrast to the muon channel. Parton uncertainties and strategies for dealing with them should be similar in the two channels.

III. SOURCES OF PARTON DISTRIBUTION ERROR

Here we reproduce the latest CDF W mass analysis [16] with simulated Tevatron data to test the rigor of our simulation. In so doing, we can illuminate the sources of PDF uncertainty in the W mass fit. We use all detector parameters, histogram bins, and cuts from that reference. In particular, we follow the CDF W recoil model in reconstructing the missing transverse energy. We get a good reproduction of their W recoil spectrum, with an average $u_T = 5.93 \pm 3.45$ GeV (c.f. 5.92 ± 3.52 GeV from Fig. 35 of Ref. [16]). After cuts, our sample contains approximately 440000 events, matching the sample used in the CDF analysis.

In Fig. 1 we plot the transverse mass and transverse momentum distributions to be fit with increasing layers of realism. We see that the Jacobian peak present in both distributions at parton level is badly eroded by the transverse momentum of the W, p_T^e especially. M_T also suffers from the missing energy reconstruction once full detector effects are applied. Parton distributions affect M_T and p_T^e through acceptance effects. They alter the rapidity distribution of the produced W. The more central the W, the more likely the charged lepton is to decay in the detector acceptance. Low rapidity W bosons may decay perpendicular to the beam, allowing for a transverse mass/momentum near the Jacobian peak. Higher rapidity W bosons must have the charged lepton decay back toward the detector to be seen, biasing events away from the Jacobian peak.

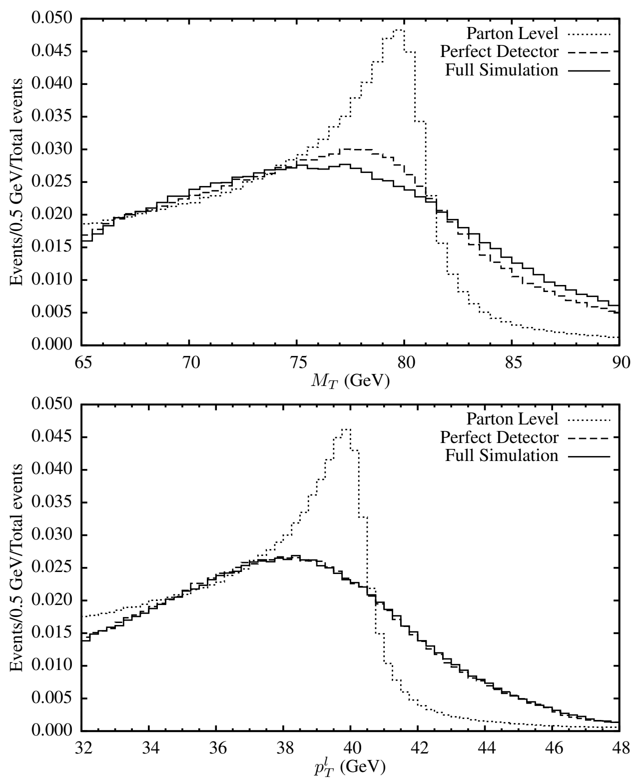

FIG. 1: Transverse mass and transverse momentum (lepton) distributions for the Tevatron at three levels of detail in simulation: parton level, hadron level (perfect detector), and full simulation including detector and reconstruction efficiencies.

We compare the PDF uncertainties of the full simulation to Ref. [16] using MSTW2008

NLO 68%CL [34] error distributions and find good agreement in both fits, 10 and 9 MeV, for M_T and p_T^e, respectively. From this point forward we use CTEQ distributions for consistency in comparison; we are interested in the differences between CT10 and CT10W [32] at the LHC in particular. Note, we make no attempt to rescale the CTEQ uncertainties in this paper to agree with the older PDF sets used above, as the PDF fits themselves are not statistical distributions, and are subject to important systematic shifts we examine in Sec. IV. Hence, while the numbers that follow may seem slightly larger, they are an accurate representation of the current status of the fits.

TABLE I: Predicted PDF errors in the Tevatron fit using CTEQ 6.6, with increasing levels of realism. All errors in MeV.

	Parton level	+ shower	+ detector
M_T error	$^{+8}_{-8}$	$^{+14}_{-13}$	$^{+18}_{-16}$
p_T^e error	$^{+8}_{-8}$	$^{+22}_{-20}$	$^{+22}_{-20}$

In Table I we see the effects on the uncertainty due to the layers of realism, using CTEQ 6.6 PDF uncertainty sets. There are two main drivers of the increase in PDF error. First, the smoothing of the distributions to be fit makes it easier to fake a different mass by shifting PDFs. A perfect Jacobian peak would be impossible to shift through acceptance effects. Second, PDFs affect the distribution of p_T^W. In fact, especially for the LHC analysis, we will see that a main contributor of the error in p_T^e comes through shifting p_T^W. To some extent this would be mitigated through modeling the W recoil by calibrating to the Z recoil, but it remains to be seen to what extent the PDF shifts in each distribution correlate.

We have seen that the shower contributes substantially to the PDF error. This occurs due to the: smoothing of the distributions; sensitivity to the distribution p_T^W; and probing of PDFs at high-x, low Q^2 in the shower. Care is taken in our reweighting procedure to correctly match the scale of the final, lowest p_T emission in the PYTHIA event history. Reweighting at the hard scale, especially using hard-scale partons, (or equivalently, generating hard-scale events with different PDF eigenvectors and showering with a fixed PDF) underestimates the PDF error. The p_T^W distribution is probing low-scale PDFs.

Some previous theory analyses have missed these effects by using fixed-order calculations without a detector, and as a result, dramatically underestimate the resulting errors. It is stated that only normalization of the distributions is needed to achieve small residual uncertainties. We point out that in practice one always normalizes distributions to the number of events measured in experimental analyses, cf. Ref. [16]. It is true, for the sharply peaked parton-level events, normalization results in a distribution that is insensitive to changes of the PDFs, but at the reconstruction level it is not sufficient to reduce uncertainties at the LHC to a level comparable to the Tevatron. In our analyses below we always normalize the distributions and focus on the sensitivity to shape.

IV. ERRORS AT THE LHC

In preparation for the next round of measurements at the LHC we turn to addressing two questions: what is a realistic estimate of current PDF uncertainties in a W mass measurement? And can we achieve the desired reach of 5 MeV or better? At the LHC, experimental systematic uncertainties on the W mass measurement are expected to be under good control, at around 7 MeV [18]; statistical errors are negligible for luminosities in the inverse femtobarn range. PDF errors are expected to be the dominant uncertainty, 25 MeV [17] for a M_T line shape analysis. Using correlations of the W and Z rapidity distributions, there are claims this can drop to as little as 1 MeV [18] at the matrix element level. In Sec. V we examine strategies to reduce the error on reconstructed events to a more realistic 10 MeV level.

Compared to earlier conditions at the Tevatron, the LHC is a high pileup environment. The transverse mass measurement becomes increasingly difficult as pileup increases, as the missing energy resolution degrades roughly as the square root of the total hadronic energy in an event. While tracking may be able to improve on this, for the LHC we restrict ourselves to the limit where spectator events can be removed. Thus the 2011 7 TeV run is preferable to the 8 TeV run; we also propose that a low-luminosity data sample be acquired if a fit is to be done using 13 TeV data.

For our simulated W events we impose the selection cuts of Ref. [18]: $p_T^l > 20$ GeV, $p_T^{miss} > 20$ GeV, recoil < 30 GeV. For 7 TeV after cuts we have 1.4×10^6 W^+ and 7.15×10^5 W^- events, and for 13 TeV we have 1.1×10^6 W^+ and 8×10^5 W^- events, corresponding to an integrated luminosity of about 2.5 fb^{-1} at 7 TeV, and 0.6 fb^{-1} at 13 TeV, respectively.

In Tab. II we present the expected errors on the transverse mass and transverse momentum fits for both data sets for the CT10 and CT10W PDFs. These values are larger than the 23–25 MeV predicted for the M_T fits in Refs. [17, 18] because those predictions used an older CTEQ 6.1 PDF set [35]. With CTEQ 6.1 we find ±24 MeV as a baseline uncertainty in complete agreement with those older predictions. The larger uncertainties in newer PDFs are due to relaxation of an artificial restriction on the function form of the strange quark PDF in the older fits [31]. Roughly 30% of the cross section is directly proportional to the sc initial state. Hence, this increase is simply an effect of a better estimate of the s PDF uncertainty.

TABLE II: PDF errors in the LHC fit, with or without intrinsic charm. All errors in MeV.

	7 TeV			13 TeV		
	CT10	CT10W	CT10+IC	CT10	CT10W	CT10+IC
m_T error	$^{+39}_{-39}$	$^{+27}_{-27}$	$^{+39}_{-40}$	$^{+30}_{-27}$	$^{+25}_{-24}$	$^{+30}_{-31}$
p^e_T error	$^{+59}_{-54}$	$^{+46}_{-45}$	$^{+59}_{-65}$	$^{+54}_{-52}$	$^{+48}_{-50}$	$^{+54}_{-65}$

In addition, we estimate the resulting contribution to the error if the input scale charm PDF assumption $c = \bar{c} = 0$ is relaxed by computing the difference between the intrinsic charm (IC) PDF CTEQ 6.6C2 [31] and the CTEQ 6.6 central set, and adding it in quadrature with the other errors, in effect treating it as an additional eigenvector. While we do not advocate for the existence of intrinsic charm, we point out the functional form of the c PDF is not relaxed in CT10 like it is for s. Hence, we caution that charm contributions to the uncertainty are not entirely accounted for. Without the improvements to W mass extraction we propose below, this could add another ±10 MeV error at the LHC.

We note that expected errors on p^e_T are quite high at the LHC. The PDFs can induce large shifts in the W recoil, which directly impacts the p^e_T distribution. To see how this drives the error, we plot the best-fit W mass versus the average reconstructed p^W_T for each CT10 eigenvector in Fig. 2. The mass shifts are strongly correlated with the W kick, coming from differences in the shower and hard emission with PDFs. If the W recoil can be modeled in some other way (it is usually fit to Z data), this part of the PDF error can be reduced.

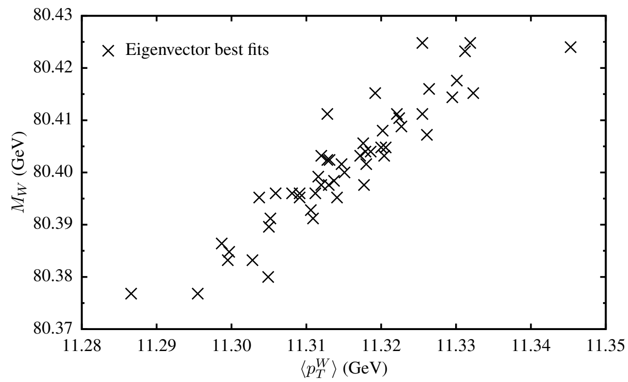

FIG. 2: The strong correlation between the best fit mass vs. average W transverse momentum.

V. IMPROVING ERRORS

Ultimately the parton distributions will be better determined in the relevant region of x and Q^2 using LHC data. Yet the W mass analysis contains the very same observables that could be used to constrain those PDFs. We propose to break the analysis into sub-analyses, each measuring M_W. At worst the sub-analysis with the least sensitivity to PDF uncertainty can be used. As long as there is not a perfect correlation in the PDF uncertainties among the sub-analyses, the PDF error will further be improved by combining the semi-independent analyses. There will still be shared systematic errors, but the statistical errors are negligible with so many events.

First, suppose we identify the charged lepton, positron or electron, and independently measure the mass of W^+ and W^-. These are of course equal, but for the purposes of experiment we are measuring two different quantities with some, but not all, shared errors. What we are determining is the average

$$M_W = \frac{M_W^+ + M_W^-}{2}. \tag{4}$$

If these are uncorrelated, the error in the average will be better than the average error, since the errors would be added in quadrature. At the LHC, W^+ and W^- are sensitive to different PDFs; in addition to sea-produced events, W^+ can be produced through valence u quarks and W^- through valence d quarks at tree level. These measurements should be at least somewhat uncorrelated.

We can do better by using multiple measurements and taking a weighted average. Suppose a sub-experiment i measures a W mass M_W^i. Construct an optimized measurement

$$M_W = \sum_i \alpha_i M_W^i\,, \tag{5}$$

with error

$$\delta M_W = \sqrt{Var(M_W)} = \sqrt{\sum_i \alpha_i^2 Var(M_W^i) + \sum_{i\neq j} \alpha_i \alpha_j Cov(M_W^i, M_W^j)}\,, \tag{6}$$

where Var and Cov refer to variance and covariance of the errors (PDF uncertainties in our case) treated as random variables, and α_i are the weighting coefficients. The overall measurement is subject to the constraint

$$\sum_i \alpha_i = 1. \tag{7}$$

For sets distributed with independent eigenvectors computed using the Hessian method, such as CTEQ, one could estimate the covariance using simple error propagation:

$$Cov(M_W^i, M_W^j) \approx \sum_k \delta M_W^{i,k} \delta M_W^{j,k},$$

with $\delta M_W^{i,k}$ the deviation of the ith measurement due to eigenvector k. However, we choose to use the relation

$$Var(M_W^i + M_W^j) = Var(M_W^i) + Var(M_W^j) + 2Cov(M_W^i, M_W^j), \tag{8}$$

and solve for the covariance, with all variances computed in accordance with the square of Eq. 3: $Var(M_W^i) = (\delta M_W^i)^2$. This procedure avoids approximations beyond the use of the tolerance method, but is complicated by the fact that the PDF errors are asymmetric. For the purpose of estimating the covariance matrix we use the average of the $+/-$ error found using the tolerance method. In the case of PDF error sets that use a sampling method, one could compute the variance and covariance directly using expectation values.

It is straightforward to find the optimum weights to minimize the error. First, the covariance of two sub-experiments is determined by adding the shifts of each pair and finding the overall error using Eq. 3. We can then solve for the covariance using Eq. 8. If the eigenvector shifts tend to be in opposite directions for each sub-experiment, they partially cancel, the error is smaller, and the covariance is negative. If they tend to be in the same direction, the error adds and the covariance is positive. Once we know how pairs of sub-experiments correlate, we can minimize the error δM_W over all weights α subject to the constraint in Eq. 7 by adding a Lagrange multiplier λ:

$$Var(M_W) = \sum_i \alpha_i^2 Var(M_W^i) + \sum_{i\neq j} \alpha_i\alpha_j Cov(M_W^i, M_W^j) - 2\lambda(\sum_i \alpha_i - 1), \quad (9)$$

$$\frac{\partial Var(M_W)}{\partial \alpha_j} = 2\alpha_j Var(M_W^j) + 2\sum_{i\neq j} \alpha_i Cov(M_W^i, M_W^j) - 2\lambda = 0, \quad (10)$$

$$\frac{\partial}{\partial \lambda}(\lambda(\sum_i \alpha_i - 1)) = 0. \quad (11)$$

Then, using $Var(M_W^j) = Cov(M_W^j, M_W^j)$ and folding this term with the $i \neq j$ sum, the optimized α_j obey the system of equations

$$\sum_i \alpha_j Cov(M_W^i, M_W^j) = \lambda, \quad (12)$$

for all j.

Besides separating electron and positron events, there is another way to get a handle on PDF errors. The pseudorapidity distributions of the leptons are PDF-dependent. Sea quarks have a bias toward more central W bosons due the initial state symmetry. Valence quarks tend to produce more forward events because the high-x peak in their distribution has to hit a low-x sea quark to produce a W. Furthermore, the d valence distributions peak at lower x than the u valence.

We therefore propose to split the analysis into low- and high-pseudorapidity regions. The ATLAS crack at $1.3 < |\eta| < 1.6$ is a good place to split regions for study. We are left with four sub-analyses to measure the W mass:

- W^+, $|\eta_{e^+}| < 1.3$
- W^+, $|\eta_{e^+}| > 1.6$
- W^-, $|\eta_{e^-}| < 1.3$
- W^-, $|\eta_{e^-}| > 1.6$

We refer to the lower pseudorapidity events as central, with a subscript c, and the higher as forward, with a subscript f.

A. Transverse mass W subanalysis errors

After breaking the W mass measurement into four sub-analyses, we see in Tab. III the PDF errors for each sub-analysis in the transverse mass fit, W_c^+, W_f^+, W_c^-, and W_f^-. Errors are larger for the forward events compared to central, and W^+ events vs. W^-. PDFs seem to be under best control for sea quarks at moderate x, though the high/low pseudorapidity difference is less pronounced at 13 TeV. The W asymmetry data in the CT10W sets predicts markedly reduced errors, especially for forward events. The CT10W PDFs correspond to inclusion of W asymmetry data incompatible with data samples included in CT10. Therefore, the spread of CT10 vs. CT10W should be taken as an indication of the range of uncertainty in current PDF estimates. It is encouraging, that the CT10W values predict generally smaller errors in the W mass measurement, and suggest that improvements in W asymmetry data may have a positive impact on the overall W mass measurement in the future.

TABLE III: PDF errors on each sub-analysis, in MeV. Refer to Tab. II for errors on the naive analysis using all events.

	7 TeV		13 TeV	
	CT10	CT10W	CT10	CT10W
W_c^+	$^{+46}_{-32}$	$^{+39}_{-28}$	$^{+41}_{-30}$	$^{+36}_{-30}$
W_f^+	$^{+98}_{-102}$	$^{+68}_{-78}$	$^{+52}_{-52}$	$^{+41}_{-42}$
W_c^-	$^{+20}_{-14}$	$^{+17}_{-13}$	$^{+29}_{-23}$	$^{+27}_{-21}$
W_f^-	$^{+49}_{-57}$	$^{+37}_{-50}$	$^{+24}_{-35}$	$^{+19}_{-32}$

Already we see that merely restricting to the best-known PDF regions, by cutting forward and positron events, would improve the error notably. Each sub-analysis in this case is weakly (anti-)correlated. In the case of CT10 at 7 TeV:

$$Corr(M_W^i, M_W^j) = \begin{pmatrix} 1 & 0.218 & 0.255 & -0.44 \\ 0.218 & 1 & 0.0279 & -0.104 \\ 0.255 & 0.0279 & 1 & 0.0439 \\ -0.44 & -0.104 & 0.0439 & 1 \end{pmatrix},$$

for $i = 1, 2, 3, 4$, corresponding to W_c^+, W_f^+, W_c^-, W_f^-. The optimal combination using our method above is

$$\alpha = \begin{pmatrix} 0.144 \\ 0.015 \\ 0.716 \\ 0.125 \end{pmatrix},$$

which yields an error on the W mass of +19/-12 MeV, an improvement of about 60%. Most of the improvement could be obtained by simply taking the best sub-measurement, W_c^-. For 13 TeV, the optimal weights are more even, but still dominated by W_c^-. Table IV summarizes improvements for 7 and 13 TeV for both CT10 and CT10W.

TABLE IV: Resulting error on the W mass after optimal sub-experiment weighting, in MeV.

	CT10	CT10W
7 TeV	$^{+19}_{-12}$	$^{+15}_{-11}$
13 TeV	$^{+20}_{-22}$	$^{+17}_{-21}$

The analysis at 7 TeV appears easier to improve than at 13 TeV; the optimal pseudorapidity cut may be lower for 13 TeV, but further pseudorapidity binning did not substantially improve the results, while degrading the statistics to the point where a coarser template binning was needed. If a low pileup 13 TeV run can be accomplished with a few inverse femtobarns of luminosity, a slightly better result might be obtained.

In contrast to Sec. IV, the possibility of underestimated uncertainty due to, e.g. intrinsic charm, does not greatly alter this process, since the dominant source of error reduction is choosing a large weight for the lowest error piece, and the shifts due to charm are usually no more than a few MeV in this region.

B. Incorporating Z data

It is expected that incorporating Z measurements will reduce the PDF error on the W mass measurement, either directly, through refitting the PDFs, or indirectly, through normalizing W observables to the Z [17, 18, 36]. We estimate the efficacy of PDF error reduction by using a variant of the latter method. Specifically, we want to extend our procedure above adding additional observables to Eq. 5. For instance, if the Z mass were correlated to the W mass, we might want to measure $\Delta M_{W,Z} = M_W - M_Z$ instead of measuring the W mass directly. This would offer experimental advantages as well, since the systematic errors would likely correlate.

We generate Z samples in the same fashion as our W samples in Sec. II A, except that we restrict the other lepton, in this case charged and observable, to the detector acceptance of $-4.9 < \eta < 4.9$, and an invariant mass near the Z mass, 66 MeV $< M_{ll} <$ 116 MeV. The number of Z events is chosen to approximate the luminosity of the W events, 3×10^5 for 7 TeV, and 2.2×10^5 for 13 TeV.

We follow the same procedure of fitting transverse mass or lepton transverse momentum from the Z decays to templates. However, when we compare the Z fits to the W fits, we rescale the mass or lepton momentum from the Z measurements by $1/\cos\theta_W = M_Z/M_W$, whose value should be taken as a prior for the LHC W mass measurement. Any value will do as long as the histogram windows are roughly compatible taking the scaling into account. We plot example W and Z transverse mass distributions in Fig. 3. The chosen histogram window and binning for the Z is the same as the W but scaled by $1/\cos\theta_W$, hence the similar range on the x-axes. We now have in effect another W mass measurement, $M_W^i = \cos\theta_W M_Z^i$, for some measurement M_Z^i of the Z mass done in the same manner as the W.

We extend our optimization method using these new measurements M_W^i, which are in fact rescaled measurements of the Z mass. The true Z mass, whose error is nearly negligible, is used as an input in the sum $\sum_i \alpha_i M_W^i$ for the terms corresponding to Z events. In other words, we are measuring the error on the combination

$$M_W + \sum_{i\in Z} \alpha_i M_W^i = \sum_{i\in W,Z} \alpha_i M_W^i, \tag{13}$$

where the goal is to minimize the r.h.s. error, and use M_Z as an input on the l.h.s., whose error is subdominant. The α_i are unconstrained if i corresponds to a Z measurement, since these terms appear on both sides of Eq. 13. We can add or subtract as much or as little of the Z mass as needed to minimize the combination. Our example $\Delta M_{W,Z}$ above would correspond to a choice where $\sum_{i\in Z} \alpha_i = -1$. Thus, our minimization condition Eq. 12 is modified:

$$\sum_i \alpha_j Cov(M_W^i, M_W^j) = 0, j \in Z; \tag{14}$$

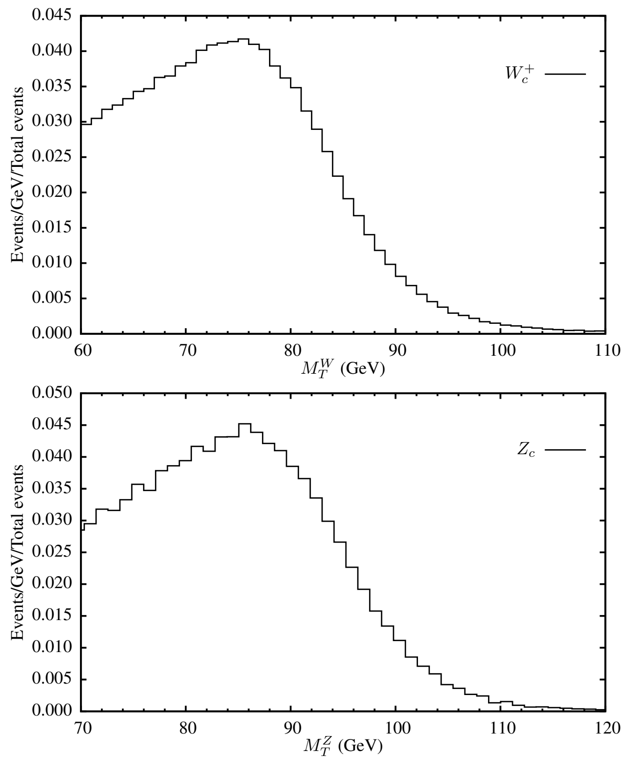

FIG. 3: Transverse mass distributions for central W^+ and Z events at the LHC. The histogram windows have been chosen so that the W and Z analyses are similar.

the derivative of the lambda term with respect to α_j is zero if $j \notin W$.

There is no difference between this procedure and the recommended method of finding the error of the ratio M_W/M_Z, since

$$\left[\delta\left(\frac{M_W}{M_Z}\right)M_Z\right]^2 \approx (\delta M_W)^2 + \left(\frac{M_W}{M_Z}\delta M_Z\right)^2 - 2Cov\left(M_W, \frac{M_W}{M_Z}M_Z\right),$$

except that our method maintains a simple linear system throughout, and additional flexibility to modify the contribution of various Z measurements.

Due to the Z vector-axial coupling asymmetry, the positron and electron can be correlated to different sides of the detector depending on the overall Z boost, and their distributions would be sensitive to different PDFs. Therefore we choose to split the Z events into W^--like events, with an electron in the relevant pseudorapidity region, and W^+-like events, with a positron with the relevant pseudorapidity. The other charged lepton is merely observed in the detector acceptance for Z identification. We now have four Z sub-measurements, Z_c^+, Z_f^+, Z_c^-, and Z_f^-, one for each of our W sub-measurements.

It is instructive to examine an older set of PDFs, CTEQ 6.1 first, on which the studies

Refs.[17, 18] are based. It is speculated there that Z measurements can nearly eliminate PDF error due to the especially strong correlations of a parameter describing the W and Z rapidity distributions; those rapidity distributions affect the W observables through acceptance effects as described above.

With CTEQ 6.1 at 13 TeV, we find an especially strong correlation between certain W and Z sub-measurements:

$$Corr(M_W^i, M_W^j) = \begin{pmatrix} 1 & 0.468 & 0.884 & 0.151 & \mathbf{0.849} & -0.29 & \mathbf{0.848} & 0.591 \\ 0.468 & 1 & 0.36 & 0.249 & 0.501 & 0.186 & 0.375 & 0.642 \\ 0.884 & 0.36 & 1 & 0.237 & \mathbf{0.934} & -0.545 & \mathbf{0.88} & 0.681 \\ 0.151 & 0.249 & 0.237 & 1 & 0.064 & 0.343 & 0.097 & 0.286 \\ 0.849 & 0.501 & 0.934 & 0.064 & 1 & -0.626 & 0.866 & 0.805 \\ -0.29 & 0.186 & -0.545 & 0.343 & -0.626 & 1 & -0.637 & -0.369 \\ 0.848 & 0.375 & 0.88 & 0.097 & 0.866 & -0.637 & 1 & 0.627 \\ 0.591 & 0.642 & 0.681 & 0.286 & 0.805 & -0.369 & 0.627 & 1 \end{pmatrix},$$

where now the sub-measurements are extended, with the ordering corresponding to W_c^+, W_f^+, W_c^-, W_f^-, Z_c^+, Z_f^+, Z_c^-, Z_f^-. The correlations in bold correspond to the central region where strong correlations would be expected, since events are dominated by sea/gluon-initiated processes at the parton level. These distributions have a strongly constrained functional form, where all sea distributions are set equal (or zero) at the input scale and evolve from the gluon distributions beyond mass threshold.

The solution to the system Eqs. 12, 14 in this case is

$$\alpha = \begin{pmatrix} 0.646 \\ 0.197 \\ 0.067 \\ 0.090 \\ -0.705 \\ -0.391 \\ -0.506 \\ 0.026 \end{pmatrix}.$$

As expected, adding Z measurements with negative weight is optimal. The resulting error is ${}^{+6}_{-7}$ MeV, a factor of four improvement! As high as they are, the PDF correlations are not perfect in M_T, so the effect is not as strong as anticipated, however much the "spread" in boson rapidity might correlate.

Even this result is overly optimistic, however. Errors in more modern sets are much larger, partly a result of the relaxing of functional constraints such as strangeness starting in CTEQ 6.6. In fact, it is these very constraints that are causing an overestimate in the W/Z correlation in this and previous analyses.

For comparison, we present the correlations in CT10 at 13 TeV:

$$Corr(M_W^i, M_W^j) = \begin{pmatrix} 1 & 0.474 & 0.476 & 0.074 & \mathbf{0.876} & 0.27 & \mathbf{0.412} & -0.153 \\ 0.474 & 1 & 0.062 & -0.018 & 0.552 & 0.521 & 0.107 & -0.069 \\ 0.476 & 0.062 & 1 & 0.33 & \mathbf{0.501} & -0.426 & \mathbf{0.730} & 0.458 \\ 0.074 & -0.018 & 0.33 & 1 & -0.073 & -0.034 & 0.108 & 0.292 \\ 0.876 & 0.552 & 0.501 & -0.073 & 1 & 0.229 & 0.41 & -0.027 \\ 0.27 & 0.521 & -0.426 & -0.034 & 0.229 & 1 & -0.345 & -0.394 \\ 0.412 & 0.107 & 0.730 & 0.108 & 0.41 & -0.345 & 1 & 0.368 \\ -0.153 & -0.069 & 0.458 & 0.292 & -0.027 & -0.394 & 0.368 & 1 \end{pmatrix}.$$

The relevant correlations have dropped as low as 41%, compared to 85% and above for CTEQ 6.1. Now, the optimum solution is

$$\alpha = \begin{pmatrix} 0.401 \\ 0.117 \\ 0.560 \\ -0.078 \\ -0.648 \\ 0.014 \\ -0.503 \\ 0.018 \end{pmatrix}.$$

To good approximation, this is simply taking the central events for W and Z and subtracting the resulting mass measurements, which was anticipated above. Due to the imperfect correlations, the error is reduced only to $^{+10}_{-11}$ MeV; a factor of three improvement over the larger CT10 error, but not nearly the improvement hoped for.

In Tab. V we summarize the end result of our optimization procedure including Z fits. It appears that by using the Tevatron W asymmetry data, and finding the best combination of sub-experiments possible, including anticorrelations with the Z, the LHC can do no better than 8 MeV without constraining PDFs with other processes or colliders; this also assumes the current functional form of CTEQ (and most other distributions) is sufficiently general that no artificial correlations remain.

TABLE V: Resulting error on the W mass after optimal sub-experiment weighting, including Z measurements, in MeV.

	CT10	CT10W
7 TeV	$^{+11}_{-10}$	$^{+8}_{-8}$
13 TeV	$^{+10}_{-11}$	$^{+7}_{-11}$

To test this hypothesis, we again mimic the uncertainty due to constraints on the functional form of charm by adding intrinsic charm as in Sec. IV to CT10 at 13 TeV, and check the sub-measurement correlation matrix:

$$Corr(M_W^i, M_W^j) = \begin{pmatrix} 1 & 0.507 & 0.492 & 0.108 & \mathbf{0.889} & 0.127 & \mathbf{0.158} & 0.021 \\ 0.507 & 1 & 0.089 & 0.012 & 0.578 & 0.427 & -0.0056 & 0.0261 \\ 0.492 & 0.089 & 1 & 0.341 & \mathbf{0.518} & -0.443 & \mathbf{0.618} & 0.473 \\ 0.108 & 0.012 & 0.341 & 1 & -0.0346 & -0.075 & 0.036 & 0.307 \\ 0.889 & 0.578 & 0.518 & -0.035 & 1 & 0.121 & 0.21 & 0.127 \\ 0.127 & 0.427 & -0.443 & -0.075 & 0.121 & 1 & -0.18 & -0.453 \\ 0.158 & -0.005 & 0.618 & 0.036 & 0.21 & -0.18 & 1 & 0.142 \\ 0.021 & 0.026 & 0.473 & 0.307 & 0.127 & -0.453 & 0.142 & 1 \end{pmatrix}.$$

The correlations between W and Z have grown weaker still, and the solution

$$\alpha = \begin{pmatrix} 0.302 \\ 0.104 \\ 0.692 \\ -0.098 \\ -0.666 \\ 0.093 \\ -0.405 \\ 0.060 \end{pmatrix}$$

has shifted slightly. The error is now $^{+10}_{-12}$ MeV, a slightly worse result. The charm can be compensated for to some extent, but only if we allow for its existence. Using the same sub-measurement combination for the original CT10 error set would increase the error to 14 MeV. Using the Z to cancel the W errors is a bit of a balancing act and depends on the degrees of freedom present in the PDFs to begin with. Table VI shows the results of allowing for this additional degree of freedom in the first two columns, and the result of trying to minimize the error taking into account the new correlation in the last two columns.

TABLE VI: Resulting error on the W mass including an additional PDF degree of freedom in charm, with and without optimization for the new degree of freedom. The "opt" column corresponds to the result after re-optimization of the weights taking the new degree of freedom into account. Since we treat intrinsic charm as a single shift without a $+/-$ eigenvector pair, the increase shows up asymmetrically.

	CT10	CT10W	CT10, opt	CT10W, opt
7 TeV	$^{+11}_{-20}$	$^{+8}_{-24}$	$^{+13}_{-12}$	$^{+9}_{-11}$
13 TeV	$^{+10}_{-14}$	$^{+7}_{-12}$	$^{+10}_{-12}$	$^{+7}_{-11}$

Because of the bigger reliance on sea PDFs for the cancellation at 7 TeV, adding an additional degree of freedom seems to reduce the effectiveness of the error reduction there significantly. The large (± 10 MeV) additional uncertainty is a concern because it represents a new systematic uncertainty not currently accounted in other analyses. Fortunately, the effect almost disappears at 13 TeV, which is sensitive to a different x and Q^2 range, and evidenced by the small difference between any of the 13 TeV results in Tabs. V and VI. Hence, this suggests there is more control over PDF uncertainties at the higher LHC energy.

C. Lepton transverse momentum

In Tab. II we saw that PDF errors are expected to be considerably larger for lepton transverse momentum than transverse mass at the LHC, and Fig. 2 shows that this is due to the variation in W recoil spectrum. If this is the case, perhaps once again the Z can come to the rescue. It is usually assumed that the low-energy physics governing the p_T spectrum of the bosons is universal up to scale; therefore one should be able to fit the W

recoil spectrum, which requires the badly-measured missing energy, to that of the Z, where both well-measured leptons in the decay can be added to produce p_T^Z.

We examine this assumption by plotting the joint p_T shifts of the bosons due to each PDF eigenvector in Fig. 4. The PDF errors on the mass measurement may be strongly correlated with the recoil, but the W and Z recoil are certainly not completely correlated with each other. Fitting one to the other is an assumption that should be critically reevaluated in the context of PDFs. We use the predicted correlations in our procedure as before to reduce the PDF error. Since the W and Z recoil are correlated, the p_T mass fits should also be, and we can find the optimum combination as in the transverse mass case.

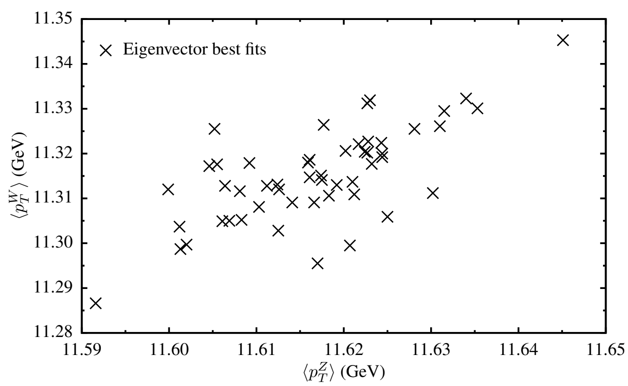

FIG. 4: Shifts in the W and Z recoil due to each PDF eigenvector.

Below in Tab. VII we present the results of our procedure for the p_T^e fit. There are generally stronger correlations between W and Z observables than in the transverse mass case, but not enough to overcome the larger inherent error. M_T appears to be the preferred variable at the LHC unless pileup becomes a problem.

TABLE VII: Resulting error on the W mass for p_T^e, in MeV, after optimizing with W and Z data.

	CT10	CT10W
7 TeV	$^{+17}_{-17}$	$^{+11}_{-14}$
13 TeV	$^{+18}_{-16}$	$^{+14}_{-15}$

VI. CONCLUSIONS

We have critically reevaluated the contribution of PDFs to the error on a potential mass measurement of the W boson at the LHC. Over-optimistic analyses have been shown to rely on unrealistic distributions in the observables used to constrain the W mass through template fitting. The softening of distributions through emission of additional partons and detector mismeasurement result in deviations that are much easier to reproduce by shifts in PDFs.

We have also devised strategies to combine sub-measurements to reduce the impact of PDF uncertainties, with and without the addition of Z boson observables. Since shifts in W and Z distributions are correlated in PDF space, this correlation can be exploited to reduce the error by also using the more directly measured Z mass as an input, a generalization of normalizing to the Z. However, past estimates of this effect have been greatly exaggerated due in part to artificial correlations induced by the assumed shared structure of sea partons in PDF fits. Adding freedom to the strange quark in more modern sets not only increases the expected PDF error directly, but reduces the correlations between W and Z due to the different parton flavors producing them at the high scale. New degrees of freedom such as looser restrictions on the fit of charm would increase the error further, but there do not appear to be strong artificial correlations remaining after strangeness is added.

In particular, we have shown that the transverse momentum distributions of W and Z do not entirely correlate in PDF space. If Z data is used to assume the underlying distribution of the W, care should be taken to propagate the additional PDF errors correctly; there is a PDF error component to the model of the W recoil. PDF errors in the p_T^e mass fit are large at the LHC due to PDF effects on the recoil. Transverse momentum should be used for mass fits to minimize PDF error unless missing energy resolution degrades too badly.

Having identified both the causes and a solution to the PDF portion of the W mass uncertainty, the next step should be to integrate these cuts into a fully resummed calculation including soft photon radiation effects. A promising framework has recently been proposed called DYRES, which adds resummation to the calculations of W and Z production [37]. As the performance of the LHC detectors is understood in Run II, we encourage an full analysis by the experimental collaborations using the latest tools available.

Our most optimistic estimate of the PDF error contribution on the W mass measurement at the LHC is 8 MeV, using CT10W PDFs, if our method of utilizing the correlation with Z mass fits is followed. It is encouraging that forward (large-x) W data can both improve and stabilize the uncertainties. However, we recommend a more conservative ±10–12 MeV as a fair estimate of what can be currently achieved. To reach the ultimate goal of ±5 MeV, further improvement will require additional data and PDF fits beyond W and Z production.

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