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Evidence for the decay $B^0 \to \eta \pi^0$

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The branching fraction can be used to constrain isospin-violating parameters ($\alpha$) measured in $B \to \pi\pi$ decays [3, 4]. It can also be used to constrain CP-violating parameters ($C_{\eta'K}$ and $S_{\eta'K}$) governing the time-dependence of $B^0 \to \eta' K^0$ decays [5]. The branching fraction is estimated using QCD factorization [6], soft collinear effective field theory [7], and flavor SU(3) symmetry [8] and is found to be in the range $(2 - 12) \times 10^{-7}$. Several experiments [9–13], including Belle, have searched for this decay mode. The current most stringent limit on the branching fraction is $B(B^0 \to \eta \pi^0) < 1.5 \times 10^{-6}$ at 90% confidence level (CL) [13]. Here we update our previous result [12] using the full data set of the Belle experiment running on the $Y(4S)$ resonance at the KEKB asymmetric-energy $e^+ e^-$ collider [14]. This data set corresponds to $753 \times 10^6$ $B \bar{B}$ pairs, which is a factor of five larger than that used previously. The analysis presented here also uses improved tracking, photon
reconstruction, and continuum suppression algorithms.

The Belle detector is a large-solid-angle magnetic spectrometer consisting of a silicon vertex detector (SVD), a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters (ACC), a barrel-like arrangement of time-of-flight scintillation counters (TOF), and an electromagnetic calorimeter (ECL) comprising CsI(Tl) crystals. These detector components are located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux-return located outside the coil (KLM) is instrumented to detect \( K^0_S \) mesons and to identify muons. Two inner detector configurations were used: a 2.0 cm beampipe and a three-layer SVD were used for the first 123 fb\(^{-1} \) of data, while a 1.5 cm beampipe, a four-layer SVD, and a small-cell inner drift chamber were used for the remaining 571 fb\(^{-1} \) of data. The detector is described in detail elsewhere [15, 16].

An ECL cluster not matched to any track is identified as a photon candidate. The timing of the energy deposited in the ECL must be consistent with the beam time of flight. All photon candidates are required to have an energy greater than 50 MeV. We reconstruct \( \pi^0 \to \gamma \gamma \) decays by pairing together photon candidates and requiring that the \( \gamma \gamma \) invariant mass be in the range 0.115–0.155 GeV/c\(^2 \). This corresponds to \( \pm 3.5 \sigma \) around the nominal \( \pi^0 \) mass [17]. Photon candidates in the endcap regions are required to have an energy greater than 100 MeV for \( \pi^0 \) reconstruction. To improve the \( \pi^0 \) momentum resolution, we perform a mass-constrained fit and require that the resulting \( \chi^2 \) be less than 50.

Candidate \( \eta \) mesons are reconstructed via \( \eta \to \gamma \gamma \) (\( \eta_{\gamma \gamma} \)) and \( \eta \to \pi^+ \pi^- \pi^0 \) (\( \eta_{3\pi} \)) decays. At least one of the photons in an \( \eta_{\gamma \gamma} \) candidate must have an energy greater than 100 MeV. To reduce combinatorial background from low-energy photons, \( \eta_{\gamma \gamma} \) candidates are required to satisfy \( |E_1 - E_2|/(E_1 + E_2) < 0.9 \), where \( E_1 \) and \( E_2 \) are the two photon energies. Photons used for \( \eta_{\gamma \gamma} \) reconstruction are not allowed to pair with any other photon to form a \( \pi^0 \) candidate. Candidate \( \eta_{3\pi} \) mesons are reconstructed by combining a \( \pi^0 \) with a pair of oppositely charged tracks. These tracks are required to have a distance of closest approach with respect to the interaction point along the \( z \) axis (antiparallel to the \( e^+ \) beam) of \( |dz| < 3.0 \) cm, and in the transverse plane of \( |dr| < 0.3 \) cm. Pions are identified using information obtained from the CDC \((dE/dx)\), the TOF, and the ACC. This information is combined to form a likelihood (\( L \)) for hadron identification. We require that charged tracks satisfy \( E_{\text{beam}}/|\vec{p}_B|^2 c^2 < 0.4 \), where \( E_{\text{beam}} \) is the likelihood of the track being a kaon (pion). We reject tracks whose response in the ECL and KLM are consistent with that of an electron or muon. The pion identification efficiency is 91.2\% and the probability for a kaon to be misidentified as a pion is 5.4\%.

We require that the invariant mass of \( \eta_{\gamma \gamma} \) and \( \eta_{3\pi} \) candidates be in the ranges 0.500–0.575 GeV/c\(^2 \) and 0.538–0.557 GeV/c\(^2 \), respectively, corresponding to \( \pm 2.5 \sigma \) and \( \pm 3.0 \sigma \) in resolution around the nominal \( \eta \) mass [17]. For selected \( \eta \) candidates, mass-constrained fits are performed to improve the momentum resolution.

Candidate \( B \) mesons are identified using the beam-energy-constrained mass, \( M_{\text{bc}} = \sqrt{E_{\text{beam}}^2 - |\vec{p}_B|^2 c^2}/c^2 \), and the energy difference, \( \Delta E = E_B - E_{\text{beam}} \), where \( E_{\text{beam}} \) is the beam energy, and \( E_B \) and \( \vec{p}_B \) are the energy and momentum, respectively, of the \( B \) candidate. All quantities are evaluated in the \( \Upsilon(4S) \) center-of-mass (CM) frame. We require that events satisfy \( M_{\text{bc}} > 5.24 \) GeV/c\(^2 \) and \( -0.30 \) GeV < \( \Delta E < 0.25 \) GeV. We calculate signal yields in a smaller region \( M_{\text{bc}} > 5.27 \) GeV/c\(^2 \) and \( -0.21 \) GeV < \( \Delta E < 0.15 \) GeV.

Charmless hadronic decays suffer from large backgrounds arising from continuum \( e^+e^- \to q\bar{q} \) (\( q = u, d, s, c \)) production. To suppress this background, we use a multivariate analyzer based on a neural network (NN) [18]. The NN uses the event topology and \( B \)-flavor tagging information [19] to discriminate continuum events, which tend to be jet-like, from spherical \( BB \) events. The event shape variables include 16 modified Fox-Wolfram moments [20], the cosine of the angle between the \( z \) axis and the \( B \) flight direction, and the cosine of the angle between the thrust axis [21] of the \( B \) candidate and the thrust axis of the rest of the event. All quantities are evaluated in the \( \Upsilon(4S) \) CM frame.

The NN technique requires a training procedure and, after training, achieves excellent separation between signal and background from the output variable \( C_{NB} \). This variable ranges from -1 to +1: a value closer to -1 (+1) is more likely to identify a background (signal) event. The training samples used consist of Monte Carlo (MC) \( B^0 \to \eta \pi^0 \) events for signal and an \( M_{bc}-\Delta E \) sideband from data for background. The regions of \( 5.200 \) GeV/c\(^2 \) < \( M_{bc} < 5.265 \) GeV/c\(^2 \), \( \Delta E < -0.23 \) GeV and \( \Delta E > 0.17 \) GeV define the \( M_{bc}-\Delta E \) sideband. Independent samples are used to test the NN performance.


We require \( C_{NB} > -0.1 \), which rejects approximately 85\% of continuum background events while retaining 90\% of signal events. We subsequently translate \( C_{NB} \) to \( C'_{NB} \), defined as

\[
C'_{NB} = \ln \left( \frac{C_{NB} - C_{\text{min}}^{NB}}{C_{NB} - C_{\text{max}}^{NB}} \right) \tag{1}
\]

where \( C_{\text{min}}^{NB} = -0.1 \) and \( C_{\text{max}}^{NB} \) is the maximum value of
\( C_{NB} \) obtained from a large sample of signal MC decays. This translation is advantageous as the \( C_{NB} \) distribution for both signal and background is well-described by a sum of Gaussian functions.

After applying all selection criteria, 2\% (7\%) of events have more than one \( B^0 \rightarrow \eta \gamma\pi^0 (B^0 \rightarrow \eta_{3\pi}\pi^0) \) candidate. For these events, we retain the \( B^0 \rightarrow \eta \pi^0 \) candidate with the smallest \( \chi^2 \) value resulting from the \( \pi^0 \) and \( \eta \) mass-constrained fits. According to MC simulations, this criterion chooses the correct \( B \) candidate 63\% (77\%) of the time for \( B^0 \rightarrow \eta \gamma\pi^0 (B^0 \rightarrow \eta_{3\pi}\pi^0) \).

We calculate signal yields using an unbinned extended maximum likelihood fit to the variables \( M_{bc}, \Delta E, \) and \( C_{NB}' \). The likelihood function is defined as

\[
\mathcal{L} = e^{-\sum_{j} Y_j} \prod_{i}^{N} \left( \sum_{j} Y_j \mathcal{P}_j (M_{bc}, \Delta E, C_{NB}') \right),
\]

where \( N \) is the total number of events, \( \mathcal{P}_j (M_{bc}, \Delta E, C_{NB}') \) is the probability density function (PDF) of signal or background component \( j \) for event \( i \), and \( j \) runs over all signal and background components. \( Y_j \) is the yield for component \( j \). The background components include \( e^+e^- \rightarrow q\bar{q} \) continuum events, generic \( b \rightarrow c \) processes, and charmless rare processes. The latter two backgrounds are small compared to the \( q\bar{q} \) continuum events and are studied using MC simulations. We find that no \( b \rightarrow c \) events pass our selection criteria. The charmless rare background, however, shows peaking structure in the \( M_{bc} \) distribution, most of which arises from \( B^+ \rightarrow \eta \rho^0 \) decays.

Correlations among the fit variables are found to be small, and thus we factorize the PDFs as

\[
\mathcal{P}_j (M_{bc}, \Delta E, C_{NB}') = \mathcal{P}_j (M_{bc}) \cdot \mathcal{P}_j (\Delta E) \cdot \mathcal{P}_j (C_{NB}').
\]

All PDFs for \( C_{NB}' \) are modeled with the sum of two Gaussian functions. The \( M_{bc} \) and \( \Delta E \) PDFs for signal events are modeled with “Crystal Ball” (CB) functions [25]. The peak positions and resolutions in the signal \( M_{bc}, \Delta E, \) and \( C_{NB}' \) are adjusted according to data-MC differences observed in a high statistics control sample of \( B^0 \rightarrow D^0 (\rightarrow K^+\pi^-\pi^0)\pi^0 \) decays. This decay has four photons, as do signal decays, and its topology is identical to that of \( B^0 \rightarrow \eta_{3\pi}\pi^0 \). The \( C_{NB}' \) PDF of the continuum background is also adjusted by comparing data and continuum MC samples in the \( M_{bc} \) sideband (5.200–5.265 GeV/c\(^2\)). The \( \Delta E \) PDF for continuum background is modeled with a second-order polynomial, while the \( M_{bc} \) PDF is modeled with an ARGUS function [9]. The \( M_{bc} \) and \( \Delta E \) PDFs for charmless rare background are modeled with one-dimensional non-parametric PDFs based on kernel estimation [26]. In addition to the fitted yields \( Y_j \), the \( M_{bc} \) and \( \Delta E \) PDF parameters for continuum background are also floated, except for the end-point of the ARGUS function. All other parameters are fixed to the corresponding MC values. To test the stability of the fitting procedure, numerous fits are performed to large ensembles of MC events.

The signal yields obtained from the fits are listed in Table I. The resulting branching fractions are calculated as

\[
B(B^0 \rightarrow \eta \pi^0) = \frac{Y_{\text{sig}}}{N_{BB} \times \epsilon \times B_\eta},
\]

where \( Y_{\text{sig}} \) is the fitted signal yield, \( N_{BB} = (753 \pm 10) \times 10^6 \) is the number of \( BB \) events, \( \epsilon \) is the signal efficiency as obtained from MC simulations, and \( B_\eta \) is the branching fraction for \( \eta \rightarrow \gamma\gamma \) or \( \eta \rightarrow \pi^+\pi^-\pi^0 \) [17]. The result is estimated from a sample of \( D^{*+} \rightarrow D^0 (\rightarrow K^-\pi^+)\pi^+ \) decays. In Eq. (4), we assume equal production of \( B^0\overline{B} \) and \( B^+\overline{B} \) pairs at the \( \Upsilon(4S) \) resonance. The combined branching fraction is determined by simultaneously fitting both \( B^0 \rightarrow \eta \gamma\pi^0 \) and \( B^0 \rightarrow \eta_{3\pi}\pi^0 \) samples for a common \( B(B^0 \rightarrow \eta \pi^0) \). Projections of the simultaneous fit are shown in Fig. 2.

### Table I. Fitted signal yield \( Y_{\text{sig}}, \) reconstruction efficiency \( \epsilon, \), \( \eta \) decay branching fraction \( B_\eta, \) signal significance, and \( B^0 \) branching fraction \( B. \) The errors listed are statistical only. The significance includes both statistical and systematic uncertainties (see text).

<table>
<thead>
<tr>
<th>Mode</th>
<th>( Y_{\text{sig}} ) (%)</th>
<th>( B_\eta ) (%)</th>
<th>Significance</th>
<th>( B(10^{-7}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B^0 \rightarrow \eta \gamma\pi^0 )</td>
<td>30.6( \pm )12.2</td>
<td>18.4</td>
<td>39.41</td>
<td>3.1</td>
</tr>
<tr>
<td>( B^0 \rightarrow \eta_{3\pi}\pi^0 )</td>
<td>0.5( \pm )0.3</td>
<td>14.2</td>
<td>22.92</td>
<td>0.1</td>
</tr>
<tr>
<td>Combined</td>
<td>3.0</td>
<td>4.1( \pm )1.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The signal significance is calculated as \( \sqrt{-2 \ln (\mathcal{L}_0 / \mathcal{L}_{\text{max}})} \), where \( \mathcal{L}_0 \) is the likelihood value when the signal yield is fixed to zero, and \( \mathcal{L}_{\text{max}} \) is the likelihood value of the nominal fit. To include systematic uncertainties in the significance, we convolve the likelihood distribution with a Gaussian function whose width is set to the total systematic uncertainty that affects the signal yield. The resulting significance is 3.0 standard deviations; thus, our measurement constitutes the first evidence for this decay mode. A Bayesian upper limit on the branching fraction is obtained by integrating the likelihood function from zero to infinity; the value that corresponds to 90\% of this total area is taken as the 90\% CL upper limit. The result is \( B(B^0 \rightarrow \eta \pi^0) < 6.5 \times 10^{-7} \).

The systematic uncertainty on the branching fraction has several contributions, as listed in Table II. The systematic uncertainty due to the fixed parameters in the PDF is estimated by varying them individually according to their statistical uncertainties. The resulting changes in the branching fraction are added in quadrature and the result is taken as the systematic uncertainty. We evaluate in a similar manner the uncertainty due to errors in the calibration factors. The sum in quadrature of these two contributions constitutes the uncertainty due to PDF.
uncertainties due to MC statistics are 0.4%. The total systematic uncertainty is obtained by summing in quadrature all individual contributions.

<table>
<thead>
<tr>
<th>Source</th>
<th>Uncertainty (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PDF parametrization</td>
<td>$^{+1.0}_{-0.4}$</td>
</tr>
<tr>
<td>Fit bias</td>
<td>$^{+0.0}_{-0.0}$</td>
</tr>
<tr>
<td>$\pi^0/\eta \rightarrow \gamma\gamma$ reconstruction</td>
<td>6.0</td>
</tr>
<tr>
<td>Tracking efficiency</td>
<td>0.3</td>
</tr>
<tr>
<td>PID efficiency</td>
<td>0.6</td>
</tr>
<tr>
<td>$C_{NB}$ selection efficiency</td>
<td>$^{+2.1}_{-2.2}$</td>
</tr>
<tr>
<td>MC statistics</td>
<td>0.4</td>
</tr>
<tr>
<td>Nonresonant contributions</td>
<td>$^{+0.0}_{-10.8}$</td>
</tr>
<tr>
<td>$B(\eta \rightarrow \gamma\gamma)$</td>
<td>0.5</td>
</tr>
<tr>
<td>$B(\eta \rightarrow \pi^+\pi^-\pi^0)$</td>
<td>1.2</td>
</tr>
<tr>
<td>Number of $B\bar{B}$ pairs</td>
<td>1.3</td>
</tr>
<tr>
<td>Total</td>
<td>$^{+12.2}_{-15.9}$</td>
</tr>
</tbody>
</table>

In order to check the reliability of the PDFs used for backgrounds, we fit the data in the $M_{bc}$ sideband 5.24–5.26 GeV/$c^2$, where the end-point of the ARGUS function used for the continuum $M_{bc}$ PDF is allowed to float. For all three distributions, $M_{bc}$, $\Delta E$, and $C_{NB}^\ast$, the MC PDFs give an excellent description of the data. We subsequently fit a sample of MC sideband events constructed with the same admixture of backgrounds as found in the data sideband and obtain signal yields consistent with zero.

To check for potential nonresonant $B^0 \rightarrow \gamma\gamma\pi^0$ and $B^0 \rightarrow \pi^+\pi^-\pi^0\pi^0$ decays, we relax the $\eta$ mass requirement and plot the $\gamma\gamma$ and $\pi^+\pi^-\pi^0$ invariant mass distributions (Fig. 3) for events in the $M_{bc}$–$\Delta E$ signal region. Significant peaks are observed for $M_{\gamma\gamma} \approx M_\pi^\gamma$ and $M_{\pi^+\pi^-\pi^0} \approx M_{\gamma\pi}$ as expected. The small sidebands indicate no significant contributions from nonresonant decays. We check this quantitatively by requiring that $M_{\gamma\gamma}$ in the sideband 0.45–0.50 GeV/$c^2$ (0.56–0.58 GeV/$c^2$) and repeat the fitting procedure. We find $2.2^{+4.8}_{-3.3} (-2.2^{+3.4}_{-2.5})$ signal decays, consistent with zero. To be conservative, we assign a systematic uncertainty due to $B^0 \rightarrow \gamma\gamma\pi^0$ nonresonant decays by appropriately scaling this fit result.

In summary, we report a measurement of the branching fraction for $B^0 \rightarrow \eta\pi^0$ decays. We obtain

$$B(B^0 \rightarrow \eta\pi^0) = (4.1^{+1.7}_{-1.5} + 0.5) \times 10^{-7},$$

where the first uncertainty is statistical and the second is systematic. This corresponds to a 90% CL upper limit of $B(B^0 \rightarrow \eta\pi^0) < 6.5 \times 10^{-7}$. The significance of this result is 3.0 standard deviations, and thus this measurement constitutes the first evidence for this decay. The measured branching fraction is in good agreement with...
FIG. 3. Distributions of (a) $M_{\gamma\gamma}$ and (b) $M_{\pi^+\pi^-\pi^0}$ invariant masses for events passing all selection requirements, except those for $M_{\gamma\gamma}$ or $M_{\pi^+\pi^-\pi^0}$.

Theoretical expectations [6–8]. Inserting our measured value into Eq. (19) of Ref. [3] gives the result that the isospin-breaking correction to the weak phase $\phi_2$ measured in $B \to \pi\pi$ decays due to $\pi^0-\eta-\eta'$ mixing is less than $0.97^\circ$ at 90% CL.

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[2] Charge conjugate modes are implicitly included unless stated otherwise.


