Relativistic gravity and parity-violating nonrelativistic effective field theories

Chaolun Wu and Shao-Feng Wu

Phys. Rev. D **91**, 126004 — Published 11 June 2015

DOI: 10.1103/PhysRevD.91.126004

Relativistic Gravity and Parity-Violating Non-Relativistic Effective Field Theories

Chaolun Wu[1] and Shao-Feng Wu[2,1]
[1] *Kadanoff Center for Theoretical Physics and Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637, USA and*
[2] *Department of Physics, Shanghai University, Shanghai 200444, China*

We show that the relativistic gravity theory can offer a framework to formulate the non-relativistic effective field theory in a general coordinate invariant way. We focus on the parity violating case in 2+1 dimensions which is particularly appropriate for the study on quantum Hall effects and chiral superfluids. We discuss how the non-relativistic spacetime structure emerges from relativistic gravity. We present covariant maps and constraints that relate the field contents in the two theories, which also serve as the holographic dictionary in context of gauge/gravity duality. A low energy effective action for fractional quantum Hall states is constructed, which captures universal geometric properties and generates non-universal corrections systematically. We give another holographic example with dyonic black brane background to calculate thermodynamic and transport properties of strongly coupled non-relativistic fluids in magnetic field. In particular, by identifying the shift function in the gravity as minus of guiding center velocity, we obtain the Hall viscosity with its relation to Landau orbital angular momentum density proportional to Wen-Zee shift. Our formalism has a good projection to lowest Landau level.

PACS numbers: 73.43.Cd, 11.25.Tq

Introduction Combined with effective field theory (EFT) techniques, symmetry plays an important role for understanding strongly correlated systems from high energy to condensed matter and atomic physics. One preeminent example is the fractional quantum Hall (FQH) effect, where interactions are crucial and defy perturbative approaches. Based on the non-relativistic (NR) general coordinate invariance (GCI) introduced in his seminal paper [1], recently Son studied the system coupled to Newton-Cartan (NC) geometry [2] and constructed an EFT for FQH states. Of particular interest is that this theory has a good projection to the lowest Landau level (LLL) and encodes universal geometric properties such as Hall viscosity [3–7]. Further developments along this line can be found in [8–15]. It is worthy to note that speaking of EFTs, we also include holography [16], which can be viewed as EFTs constructed in the aid of higher dimensional geometries and holographic dictionaries.

For NR field theories, a fundamental feature is the existence of a global time, a requirement of the NR causality. Thus to build up NR GCI EFTs, it is natural to employ NR gravity theories such as NC geometry [2, 10, 12] and Hořava gravity [9, 17–19]. Relativistic gravity theories do not have a built-in notion of global time *a priori*. However, this does not exclude the possibility that they can be used as a framework to construct NR GCI EFTs, provided that the background isometry or certain imposed condition selects a preferred time foliation. The pioneering works of [20, 21] and [22] show that this can be achieved in relativistic gravity with Schrödinger and Lifshitz backgrounds. Furthermore, [23] shows that for Lifshitz holography, the boundary geometry is of various types of NC geometry, depending on the boundary condition the time-like vielbein satisfies. Actually such boundary condition may exist independently of the holographic bulk structure. Our first step in this letter is to argue that relativistic gravity can be used for general NR EFTs even without the aid of holography, provided that the time-like vielbein satisfies the hypersurface orthogonality condition.

A second common feature in the recent geometric formalism of NR EFTs is a velocity field. In NR field theories, the $U(1)$ gauge field A_μ transforms under diffeomorphism not just as a Lie derivative but with extra terms only dependent on metric. This is a consequence of Galilean boost invariance and gives rise to the well-known relation between momentum density and conserved current [24] and its variations [12]. In EFTs the velocity field is necessary to cancel the extra terms and covariantize the gauge field. In holography, the relation between A_μ and its covariantized version (called as covariant map) serves as part of the holographic dictionary [25]. The precise form of this map may vary depending on how the microscopic theory is coupled to curved space. In this letter, in addition to local $U(1)$ gauge symmetry and spatial diffeomorphism widely studied in NR EFTs, we also consider homogeneous time reparametrization and local anisotropic Weyl rescaling. These almost completely determine the covariant map. As an example, we show a single Chern-Simons term together with the covariant map can reproduce all correlation functions obtained in [2] using NC formalism.

A NR EFT formalism built on relativistic gravity is particularly convenient for applications of holography, which is developed mostly within the frame of relativistic gravity theories. As a prerequisite for NR holography, a notion of global time must exist at the boundary. In [17, 19] the global time is extended to the whole bulk by employing Hořava gravity [27]. However, Hořava gravity is notorious for complications involving black hole event horizon [28] and difficulty of finding hyperbolic black hole solutions [29]. Moreover, it is not clear that whether

Hořava gravity accommodates the dynamical exponent $z \to 1$ since it corresponds to certain "unhealthy reduction" [30]. But some measurements on quantum Hall effects exhibit the isotropic $z = 1$ scaling indeed [31]. Thus, the holographic applications of Hořava gravity are limited. In this letter we offer an alternative: the bulk is still relativistic without a preferred time foliation. The global time is only realized at the boundary by imposing the hypersurface orthogonality condition for vielbein there. The holographic dictionary ensures the dual field theories are NR, while the relativistic bulk allows black hole solutions previously well studied in relativistic holography. This facilitates holographic study of thermal effects and phase transitions of NR systems. As an example, we employ dyonic black brane model of [32] to study NR Hall effects with finite temperature and magnetic field.

Notations: We use three types of spacetime indices $M, N, \cdots$, $\mu, \nu, \cdots$, $i, j, \cdots$, for (3+1)-, (2+1)- and 2-dimensional manifolds, respectively. We denote the tangent space index by $a, b, \cdots$ for the zweibein. Holographic radial direction is referred to r with boundary located at $r = 0$. "$\hat{}$" and "$\bar{}$" mark the bulk quantities and their boundary values after stripping off the asymptotic r-dependence.

NR Symmetries We consider a (2+1)-dimensional NR field theory described by the following microscopic action in curves spacetime

$$S = \int d^3x \sqrt{g} \frac{1}{2} \left(i\psi^{+} \overleftrightarrow{D}_t \psi - \frac{g^{ij} + i\varepsilon^{ij}}{me^{\Phi}} D_i \psi^{\dagger} D_j \psi + \dots \right)$$

where "..." denotes the interactions, g_{ij} is spatial metric, $g = \det(g_{ij})$ and

$$\begin{aligned} D_i &= \partial_i - i \left(A_i - s_0 \omega_i \right), \\ D_t &= \partial_t - i \left[A_t - s_0 \omega_t + \frac{1}{4me^{\Phi}} (\mathrm{g} - 2) B \right]. \end{aligned} \quad (1)$$

The magnetic field is $B = \varepsilon^{ij} \partial_i A_j$, $\varepsilon^{ij} = \epsilon^{ij}/\sqrt{g}$ with ϵ^{ij} the Levi-Civita symbol. g is the gyromagnetic factor. Our choice of D_t such that $(\mathrm{g} - 2)B/4m$ appears together with A_t and $g^{ij} + i\varepsilon^{ij}$ appears in a combination in the standard Pauli form has been employed in [33]. The scalar field Φ is introduced to source the energy density [12, 17]. The field ψ represents an underlying microscopic degree of freedom which is to be path-integrated out when computing the effective action. It can be the electrons as well as other composite particles. It has intrinsic spin s_0 and couples to curved space through spin connection: $\omega_t = \frac{1}{2} \epsilon_{ab} e^{aj} \partial_t e^b_j$, $\omega_i = \frac{1}{2} (\epsilon_{ab} e^{aj} \partial_i e^b_j - \varepsilon^{jk} \partial_j g_{ki})$, where e^a_i is the zweibein for metric g_{ij}. Note that the spin connection has been considered recently in [35] from flux attachment. The action is invariant under NR diffeomorphism and Weyl transformations (parameterized by ξ^μ and σ, with $\partial_i \xi^t = 0$) as shown in [19], with a slightly different transformation rule for A_t:

$$\begin{aligned} \delta A_t = {} & \xi^\mu \partial_\mu A_t + A_\mu \partial_t \xi^\mu - \frac{1}{2} (1 - s_0) \varepsilon^{ij} \partial_i (g_{jk} \partial_t \xi^k) \\ & - \frac{1}{4me^{\Phi}} (\mathrm{g} - 2) \left[\varepsilon^{ij} \partial_i \left(me^{\Phi} g_{jk} \partial_t \xi^k \right) + (1 - s_0) \nabla^2 \sigma \right], \end{aligned} \quad (2)$$

because we do not include Ricci scalar in D_t. The Ward identities resulting from the NR spacetime symmetries derived in [12] are still applicable here.

Global Time For the purpose toward NR EFTs, it is convenient to discuss geometry in term of vielbein rather than metric. For (2+1)-dimensional relativistic gravity [34], $ds^2 = -\tau^2 + \delta_{ab} e^a e^b$. The time-like vielbein is $\tau = e^{-\Phi}(dt - C_i dx^i)$ and space-like ones $e^a = e^a_i (dx^i + N^i dt)$. Notice $g_{ti} = e^{-2\Phi} C_i + N_i$ where i is lowered by g_{ij}. C_i is the source to NR energy flux [12, 17, 20], which can be seen by matching its diffeomorphism with that of the source. However, C_i does not appear in the above NR field theory because it is written in global time coordinates (GTC) where $C_i = 0$. The condition for existence of a global time as required by NR causality is $\tau \wedge d\tau = 0$ [12], which corresponds to the twistless torsion condition for NC geometry in [23]. For the application of relativistic gravity on NR EFTs, this hypersurface orthogonality condition for vielbein must be imposed. Then we can always work in GTC with $C_i = 0$. Under diffeomorphism, $\delta C_i = \xi^\mu \partial_\mu C_i + C_j (\partial_i + C_i \partial_t) \xi^j - (\partial_i + C_i \partial_t) \xi^t$. This implies the allowed diffeomorphism in GTC must satisfies $\partial_i \xi^t = 0$. This is called foliation preserving diffeomorphism and is exactly the assumption made to ensure the diffeomorphism invariance of the above NR field theory. Now Φ and e^a_i can be identified with their counterparts in NR field theory because they have the same symmetry transformations. To compute energy flux, C_i dependence has to be restored. This can be done by performing a $\partial_i \xi^t \neq 0$ diffeomorphism and going away from GTC. For the rest of this letter, we will work in GTC for simplicity.

Holography and Covariant Map For holography in 3+1 dimensions, the bulk theory includes relativistic graviton described by vielbein $(\hat{\tau}, \hat{e}^a, \hat{n})$ and a $U(1)$ gauge field $\hat{V} = \hat{V}_M dx^M$, among others. We assume the background near boundary $r \Rightarrow 0$ is asymptotic Lifshitz with AdS radius L:

$$ds^2 \Rightarrow -(L/r)^{2z} dt^2 + (L/r)^2 \left(d\vec{x}^2 + dr^2 \right), \quad (3)$$

and choose gauge condition for radial vielbein $\hat{n} = (L/r)dr$ and $\hat{\tau}_r = \hat{e}^a_r = V_r = 0$. These conditions do not completely fix the bulk gauge freedom. The residual diffeomorphism near boundary is $\hat{\xi}^\mu \Rightarrow \bar{\xi}^\mu$, $\hat{\xi}^r \Rightarrow -r\bar{\sigma}$. Then near boundary $\hat{\tau} \Rightarrow (L/r)^z \bar{\tau}$, $\hat{e}^a \Rightarrow (L/r) \bar{e}^a$ and $\hat{V}_\mu \Rightarrow \bar{V}_\mu$. Under $(\bar{\xi}^\mu, \bar{\sigma})$, $\bar{\tau}$, $\bar{e}^a$ and $\bar{V}_\mu$ transform in the same way as their counterparts in (2+1)-dimensional relativistic gravity, hence are identified with above τ, e^a and a V_μ. Now the global time condition becomes a boundary condition in holography: $\bar{\tau} \wedge d\bar{\tau} = 0$. We will work in the stronger condition $\bar{\tau}_i = 0$, which forces $\partial_i \bar{\xi}^t = 0$ at boundary but allows $\partial_i \hat{\xi}^t \neq 0$ in the bulk. The map for the $U(1)$ field $\bar{V}_\mu = V_\mu$ is non-trivial:

$$V_i = A_i - me^{\Phi} N_i + s'\omega_i + \frac{1-s_0-s'}{2}\varepsilon^{jk} g_{ki}\partial_j \log(me^{\Phi}),$$
$$V_t = A_t + \frac{\mathrm{g}-2}{4me^{\Phi}}B - \frac{1}{2}me^{\Phi}N_iN^i + s'\omega_t$$
$$+ \frac{1-s_0-s'}{2}\left[\varepsilon^{ij}\partial_i N_j + \varepsilon^{ij}N_j\partial_i \log(me^{\Phi})\right], \qquad (4)$$

where s' is an arbitrary constant. The additional structures in the map $\bar{V}_\mu = A_\mu + \ldots$ are built up to covariantize the NR gauge field, that is, to cancel the extra terms in δA_μ so that the symmetry transformations on both sides of the map are matched, see the detail properties of these structures in [19]. Being part of the holographic dictionary, this is an extension of that in [17] to parity violating case. It is also the covariant map for (2+1)-dimensional EFTs (independent of holography), an extension of [2, 12, 15]. A nice feature is for FQH effect, it has a good LLL projection when $\mathrm{g} = 2$ and $m \to 0$ (see how to give the usual LLL constraint on wave function in [12]). In holography the mass m is dual to a bulk scalar whose near-boundary behavior matches its Weyl transformation in NR field theory.

Shift Vector The only remaining problem in our formalism is the shift vector N^i, which has not been interpreted nor determined from NR field theory point of view. According to its symmetry transformations, it corresponds to the velocity field in NC formalism [2, 12]. In relativistic theories, it sources the momentum density. However, in the NR theories momentum density p^i is completely determined in terms of charge current J^μ provided that Galilean symmetry is respected and the particles have the same charge to mass ratio [24]. From the Ward identity in flat spacetime, one can read

$$p^i = mJ^i - \frac{\mathrm{g}-2s_0}{4}\epsilon^{ij}\partial_j J^t. \qquad (5)$$

Thus N_i does not source p^i [36]. To determine it in terms of other fields, a constraint has to be imposed. There is no universal prescription in the literature. [19] has a detailed discussion on how to impose a diffeomorphism invariant constraint with smooth LLL limit for FQH effect. Similarly here, there are two viable choices:

$$N_i\frac{\delta W}{\delta V_t} + g_{ij}\frac{\delta W}{\delta V_j} = 0 \quad \text{or} \quad V_{ti} + V_{ij}N^j = 0, \qquad (6)$$

where $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$. The former is essentially a path-integral in the spirit of [2] to integrate out the velocity field, while the latter also appears recently in [15]. For FQH effect, they yield the same universal features in Chern-Simons EFT [19].

Effective Action for FQH states As an example, we show how to build low energy effective action for FQH states using our formalism. We work in the limit when the magnetic field B is large compared to the electric field E_i and derivatives. At leading order in derivative expansion, the gauge Chern-Simons term encodes the universal properties. To build a NR GCI action, we start with a relativistic Chern-Simons term $S_{\text{CS}} = \frac{\nu}{4\pi}\int d^3x\, V \wedge dV$ and apply the map (4), then plug into the constraint (6) and solve for N^i. We get $N^i = -\epsilon^{ij}E_j/B + O(\partial B, \partial\Phi)$. For constant B and E_i, the final NR GCI Chern-Simons action has a simple form:

$$S_{\text{CS}} = \frac{\nu}{4\pi}\int d^3x \Big\{ (A + s'\omega)\wedge d\,(A+s'\omega) \qquad (7)$$
$$+ \sqrt{g}\left[\frac{mE^2}{B} + \frac{\mathrm{g}-2}{2m}B^2 + O(\partial_\mu)\right]\Big\}$$

where we have set $\Phi = 0$ and $O(\partial_\mu)$ denotes derivative corrections to the local Lagrangian. These corrections can be systematically calculated to any higher order using (4) and (6), and the first few are given in [19]. All correlators obtained from this action (for their explicit expressions see also [19]), including the derivative corrections omitted in the above expression, agree with those computed in [2]. From this action, we can easily recognize s' as half of the Wen-Zee shift [37] and Hall viscosity $\eta_H = (\nu B/4\pi)s'$. Furthermore, the total angular momentum density can be extracted from $\int d^2\vec{x}\epsilon_{ij}r^ip^j$ using (5), after integrating by parts:

$$l_{\text{tot}} = -\frac{\nu B}{2\pi}(1-s_0). \qquad (8)$$

There is another kind of angular momentum density related to the conjugate of the vorticity in grand potential density [38, 39]. By identifying $-N^i$ with the drift velocity, one can derive the so called guiding center angular momentum density [19]:

$$l_{\text{gc}} = -\frac{\nu B}{2\pi}(1-s_0-s'). \qquad (9)$$

Subtracting the latter from the former, we obtain the Landau orbital angular momentum density $\ell_{\text{orb}} = -(\nu B/2\pi)s'$. This justifies the relation $\eta_H = -\ell_{\text{orb}}/2$ in [5, 6, 40] and ensures the quantization of s'.

There are two more topological terms in (2+1)-dimensional relativistic theories beside $V \wedge dV$. One is the gravitational Chern-Simons term $\text{tr}\left(\tilde{\omega}\wedge d\tilde{\omega} + \frac{2}{3}\tilde{\omega}\wedge\tilde{\omega}\wedge\tilde{\omega}\right)$, where $\tilde{\omega}$ denotes the non-Abelian spin connection constructed from the full spacetime vielbein. Its contribution to the NR effective action after applying the covariant map has been calculated in [19]. Its primary role is to shift the coefficient of $\omega\wedge d\omega$ by a constant, which is related to the central charge of chiral conformal field theory on the boundary and the thermal Hall conductivity. A third relativistic topological term which mixes the $U(1)$ gauge field with spacetime curvature had recently been constructed in [41]. By applying our covariant map, its contributes to NR effective action is equivalent to a shift of the constant s'. Local terms in the relativistic parent theory will generate non-universal features related to the interactions of the microscopic theory. We will not discuss these terms here.

A Holographic Model We now give another example in term of relativistic holography [32], which is dual to strongly coupled quantum fluids in external magnetic fields. The bulk action includes Einstein-Maxwell terms

$$\hat{S}_{\mathrm{EM}} = \frac{-2}{\kappa_4^2}\int d^4x\sqrt{-\hat{G}}\left(\frac{1}{4}\hat{R} + \frac{3}{2L^2} - \frac{L^2}{4}\hat{F}_{MN}\hat{F}^{MN}\right)$$

with a non-dynamical Chern-Simons term $\hat{S}_{\mathrm{CS}} = \frac{\nu}{16\pi}\int \epsilon^{MNPQ}\hat{F}_{MN}\hat{F}_{PQ}$. Here $\hat{F}_{MN} = \partial_M \hat{V}_N - \partial_N \hat{V}_M$. The background metric is a dyonic black brane

$$\frac{1}{L^2}ds^2 = \frac{\alpha^2}{r^2}\left[-f(r)\,dt^2 + d\vec{x}^2\right] + \frac{dr^2}{r^2 f(r)}, \tag{10}$$

with $\hat{V}_t = -q\alpha(1-r)$, $\hat{V}_y = h\alpha^2 x$, where q and h are electric and magnetic charges. h is related to a constant magnetic field $B_0 = h\alpha^2$ on the boundary. The blackening function is $f(r) = 1 - (1+h^2+q^2)r^3 + \left(h^2+q^2\right)r^4$, with the horizon located at $r = 1$. The mass parameter α is related to the Hawking temperature by $4\pi T = \alpha\left(3 - h^2 - q^2\right)$. The renormalized action is given by subtracting the Gibbons-Hawking term and a counterterm of the boundary volume. To calculate correlation functions, we solve all the nine metric and gauge fluctuations in the bulk up to linear order in momentum $(\omega, \vec{k})$. By rotational symmetry we can set $k_x = k$, $k_y = 0$. We will not list the full expressions of the solutions nor the action here, but only give results of correlators computed from them. The procedure is similar to that in [32]. After obtaining the on-shell boundary action, still in relativistic form, we solve the constraint equation (6) (we use the first one) and apply the holographic dictionary (4) to calculate the NR GCI effective action.

The non-vanishing 1-point functions are the charge density ρ, energy density ϵ and internal pressure P:

$$\rho = \frac{\nu B_0}{2\pi} - \frac{2L^2}{\kappa_4^2}q\alpha^2, \tag{11}$$

$$\epsilon = -\frac{(\mathrm{g}-2)\,\rho B_0}{4m} + \frac{L^2}{\kappa_4^2}(1+h^2+q^2)\alpha^3,$$

$$P = -\frac{(\mathrm{g}-2)\,\rho B_0}{4m} + \frac{1}{2}\frac{L^2}{\kappa_4^2}(1+h^2+q^2)\alpha^3.$$

Thermodynamic pressure (i.e. the grand potential density) can be obtained from the background action:

$$P_{\mathrm{thm}} = \frac{L^2}{2\kappa_4^2}(1-3h^2+q^2)\alpha^3 - \frac{\nu B_0}{2\pi}q\alpha. \tag{12}$$

The chemical potential $\mu = -q\alpha - (\mathrm{g}-2)\,B_0/(4m)$ is identified with the background of A_t in the NR field theory. The magnetization density is defined as $M = \partial P_{\mathrm{thm}}/\partial B_0|_{T,\mu}$. Bekenstein-Hawking law gives entropy density $s = 2\pi L^2\alpha^2/\kappa_4^2$. All these thermodynamic quantities satisfy the following fundamental relation:

$$\epsilon + P - Ts - \mu\rho + B_0 M = 0. \tag{13}$$

The system has local thermodynamic stability:

$$\det\left[\partial_\rho\partial_s\epsilon\,(\rho,s)\right] = \frac{12s^2 + 48\pi^2 B_0^2 + (2\pi\rho - \nu B_0)^2}{64\pi s^2} > 0.$$

Some of the 2-point functions are

$$G_{\mathrm{ra}}^{1,1} = G_{\mathrm{ra}}^{2,2} = 0, \quad G_{\mathrm{ra}}^{1,2} = -G_{\mathrm{ra}}^{2,1} = \frac{i\omega\rho}{B_0}, \tag{14}$$

$$G_{\mathrm{ra}}^{1,\Phi} = 0, \quad G_{\mathrm{ra}}^{2,\Phi} = -ik\frac{\epsilon+P}{B_0},$$

$$G_{\mathrm{ra}}^{11,2} = \frac{-ikq\alpha^2\,(\mathrm{g}-2)}{2m} + \frac{L^2}{\kappa_4^2}\frac{3ik}{8B_0}(1+h^2+q^2)\alpha^3,$$

where definitions of retarded correlators $G_{\mathrm{ra}}^{A,B}$ follow [12]. From $G_{\mathrm{ra}}^{i,j}$, the longitudinal conductivity vanishes and Hall conductivity equals ρ/B_0 as expected. $G_{\mathrm{ra}}^{i,\Phi}$ shows current response to inhomogeneous gravitational field $\partial_i\Phi$, with a transport coefficient $\sigma_H^G = (\epsilon+P)/B_0$ that agrees with [42] from hydrodynamic analysis on LLL. The above correlators satisfy Ward identities given in Eqs. (47) and (48) in [12].

The shear, bulk and Hall viscosities are

$$\eta = \frac{s}{4\pi}, \quad \zeta = \frac{\kappa_{\mathrm{int}}^{-1}}{i\omega}, \quad \eta_H = \frac{1}{2}\rho s'. \tag{15}$$

Some remarks are in order. First, the shear viscosity characterizes the fluid is strongly coupling. Second, [12, 43] pointed out that there is a zero-frequency divergent term proportional to the inverse internal compressibility $\kappa_{\mathrm{int}}^{-1}$ in the bulk viscosity. For our case, it is

$$\kappa_{\mathrm{int}}^{-1} = \frac{3}{4}(1+h^2+q^2)\alpha^3 - \frac{(\mathrm{g}-2)\,\rho B_0}{2m}. \tag{16}$$

Other than this contact term, the bulk viscosity is vanishing as required by Weyl invariance. Third, the covariant map (4) is crucial to the above non-vanishing Hall viscosity, whose form in terms of charge density and the shift agrees with [5, 6]. Moreover, by calculating the total angular momentum density

$$l_{\mathrm{tot}} = \frac{(\mathrm{g}-2)}{8\pi}\left(\nu B_0 + 4\pi\rho\right) - \frac{\mathrm{g}-2s_0}{2}\rho \tag{17}$$

and the guiding center one

$$l_{\mathrm{gc}} = \frac{(\mathrm{g}-2)}{8\pi}\nu B_0 - (1-s_0-s')\rho, \tag{18}$$

we can check $\ell_{\mathrm{orb}} = -\rho s'$, which indicates $\eta_H = -\ell_{\mathrm{orb}}/2$ and the quantization of s' in the strongly coupling model.

At the end we have two comments on the mass m. (1) The dyonic black brane background is asymptotic AdS with dynamical exponent $z = 1$. From our holographic dictionary, for $z \neq 2$, m is dual to a bulk scalar with non-trivial profile and r^{z-2} asymptote. Here we do not consider this profile explicitly because we work in the probe limit where this scalar sector can be engineered such that it decouples, similar as in [17]. (2) Instead of working in the probe limit, we can also project to LLL, where $m \to 0$ and g $= 2$. In this limit the cyclotron frequency $\omega_c = B/m$ diverges which forbids higher Landau level mixing. In this case all $(\mathrm{g}-2)/m$ terms in the above expressions drop off, and this holographic model becomes one for LLL.

Conclusions We have shown that relativistic gravity theories can be used as a framework to build effective theories for NR systems that respect all NR spacetime symmetries, holographically or not, for any dynamical exponent z. In order to adapt to the global time, the time-like vielbein must satisfy hypersurface orthogonality condition. Under this condition, we present a covariant map that relates the relativistic gauge field to the NR one, which can also serve as part of the holographic dictionary. Additional constraints are given to fix the shift vector. Our formalism is particular suitable for spin-polarized NR particles, including the FQH fluids and chiral superfluids [44]. Low energy effective actions for these systems are then constructed from purely (2+1)-dimensional Chern-Simons field theory and from (3+1)-dimensional holographic theory with a dyonic black brane background. They have a good LLL projection and capture the linear response properties of these systems.

Acknowledgments We are very grateful to Dam Thanh Son for constant support and inspiring discussions throughout the progress of this work. We thank Andreas Karch for reading the draft and Michael Geracie for useful discussions. C. Wu is supported by the US DOE grant No. DE-FG02-13ER41958 and S.-F. Wu was supported by NNSFC No. 11275120 and the China Scholarship Council.

[1] D. T. Son and M. Wingate, Annals Phys. **321,** 197 (2006).
[2] D. T. Son, arXiv:1306.0638.
[3] J. E. Avron, R. Seiler, P. G. Zograf, Phys. Rev. Lett. **75**, 697 (1995).
[4] I. V. Tokatly and G. Vignale, Phys. Rev. **B 76**, 161305(R) (2007); ibid. **79**, 199903(E) (2009).
[5] N. Read, Phys. Rev. **B 79**, 045308 (2009).
[6] N. Read, and E. H. Rezayi, Phys. Rev. **B 84**, 085316 (2011).
[7] C. Hoyos and D. T. Son, Phys. Rev. Lett. **108**, 066805 (2012)
[8] S. Golkar, D. X. Nguyen, D. T. Son, arXiv:1309.2638.
[9] O. Andreev, M. Haack and S. Hofmann, Phys. Rev. **D 89**, 064012 (2014).
[10] A. Gromov and A. G. Abanov, arXiv:1401.3703; arXiv:1403.5809; arXiv:1407.2908.
[11] R. Banerjee, A. Mitra and P. Mukherjee, Phys. Lett. B **737**, 369 (2014); arXiv:1407.3617.
[12] M. Geracie, D. T. Son, C. Wu, and S. F. Wu, arXiv:1407.1252.
[13] T. Brauner, S. Endlich, A. Monin and R. Penco, arXiv:1407.7730.
[14] K. Jensen, arXiv:1408.6855.
[15] O. Andreev, arXiv:1408.7031.
[16] O. Aharony, S. S. Gubser, J. M. Maldacena, H. Ooguri and Y. Oz, Phys. Rept. **323**, 183 (2000).
[17] S. Janiszewski and A. Karch, JHEP **1302**, 123 (2013).
[18] S. Janiszewski and A. Karch, Phys. Rev. Lett. **110**, 081601 (2013).
[19] C. Wu and S. F. Wu, arXiv:1409.1178.
[20] D. T. Son, Phys. Rev. **D 78**, 046003 (2008).
[21] K. Balasubramanian and J. McGreevy, Phys. Rev. Lett. **101,** 061601 (2008).
[22] S. Kachru, X. Liu and M. Mulligan, Phys. Rev. D **78**, 106005 (2008).
[23] M. H. Christensen, J. Hartong, N. A. Obers and B. Rollier, Phys. Rev. D **89**, 061901 (2014); JHEP **1401**, 057 (2014); J. Hartong, E. Kiritsis and N. A. Obers, arXiv:1409.1519; arXiv:1409.1522; E. A. Bergshoeff, J. Hartong and J. Rosseel, arXiv:1409.5555.
[24] M. Greiter, F. Wilczek, E. Witten, Mod. Phys. Lett. **B 3,** 903 (1989).
[25] A general algorithm for constructing parity-preserving Lifshitz holographic dictionary was discussed in [26].
[26] W. Chemissany and I. Papadimitriou, arXiv:1405.3965; arXiv:1408.0795.
[27] P. Horava, JHEP **0903**, 020 (2009); Phys. Rev. **D 79**, 084008 (2009).
[28] C. Eling, T. Jacobson, Class. Quant. Grav. **23**, 5643 (2006) [Erratum-ibid. 27, 049802 (2010)]; E. Barausse, T. Jacobson and T. P. Sotiriou, Phys. Rev. **D 83**, 124043 (2011).
[29] S. Janiszewski, arXiv:1401.1463; S. Janiszewski, A. Karch, B. Robinson and D. Sommer, JHEP **1404**, 163 (2014).
[30] P. Horava, Class. Quant. Grav. **28,** 114012 (2011).
[31] L. W. Engel, D. Shahar, C. Kurdak, and D. C. Tsui, Phys. Rev. Lett. **71**, 2638 (1993); W. Li et al., Phys. Rev. Lett, **102**, 216801 (2009).
[32] S. A. Hartnoll and P. Kovtun, Phys. Rev. **D 76**, 066001 (2007).
[33] R. Alicki, Phys. Rev. Lett. **72**, 2620 (1994).
[34] In non-holographic context, the "relativistic gravity" means the relativistic background geometry while not involving the spacetime dynamics.
[35] G. Y. Cho, Y. You and E. Fradkin, arXiv:1406.2700.
[36] Alternatively, one would introduce the shift vector as the source of momentum density in the microscopic action. In a Galilean invariant theory, it will appear only in a specific combination with the source of number current [13]. Consequently, the form of the effective action will be constrained. For instance, the holographic model that we will consider below is not applicable to provide the desired action.
[37] X. G. Wen and A. Zee, Phys. Rev. Lett. **69**, 953 (1992).
[38] K. Jensen, M. Kaminski, P. Kovtun, R. Meyer, A. Ritz and A. Yarom, JHEP **1205**, 102 (2012).
[39] C. Hoyos, B. S. Kim and Y. Oz, arXiv:1407.2616.
[40] A. Nicolis and D. T. Son, arXiv:1103.2137.
[41] S. Golkar, M. M. Roberts, D. T. Son, arXiv:1403.4279; arXiv:1407.7540.

[42] M. Geracie and D. T. Son, arXiv:1407.4460; arXiv:1408.6843.

[43] B. Bradlyn, M. Goldstein, and N. Read, Phys. Rev. **B 86**, 245309 (2012).

[44] S. Moroz, C. Hoyos, D. T. Son, Phys. Rev. **B 89**, 174507 (2014); S. Moroz and C. Hoyos, arXiv:1408.5911.