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# Quarkonium-Nucleus Bound States from Lattice QCD

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Quarkonium-nucleus systems are composed of two interacting hadronic states without common valence quarks, which interact primarily through multi-gluon exchanges, realizing a color van der Waals force. We present lattice QCD calculations of the interactions of strange and charm quarkonia with light nuclei. Both the strangeonium-nucleus and charmonium-nucleus systems are found to be relatively deeply bound when the masses of the three light quarks are set equal to that of the physical strange quark. Extrapolation of these results to the physical light-quark masses suggests that the binding energy of charmonium to nuclear matter is  $B_{\text{phys}}^{\text{NM}} \lesssim 40$  MeV.

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## I. INTRODUCTION

Since quantum chromodynamics (QCD) was first proposed as the underlying theory of the strong interactions, enormous progress has been made in the understanding of hadrons as composite objects formed from quarks and gluons. A particularly interesting consequence of the extended nature of hadrons is their susceptibility to chromo-polarization, which allows for hadronic interactions that are distinct from meson-exchanges which dominate the long-range forces between nucleons. The effects of color polarization can be isolated and explored by considering hadronic systems without shared valence quarks, thereby eliminating the possibility of quark-exchange interactions and Pauli blocking.

The significance of a color van der Waals force (so called by analogy to the electromagnetic effect) was first appreciated by Brodsky, Schmidt and de Teramond (BSdT) in 1990 [1]. They observed that the rapid variation in the spin-spin correlation in  $pp$  scattering at a scattering angle of  $\theta = 90^\circ$  near the open charm production threshold ( $\sqrt{s} \sim 5$  GeV) may be indicative of a strong attractive interaction between charmonium and the di-proton system. In terms of quarks and gluons, these systems interact through multi-gluon exchanges, which manifest themselves as two-pion exchange interactions at long distances, but which are not expected to generate repulsion at short distances. Using a Yukawa toy model to describe the charmonium-nucleus interactions, BSdT predicted bound states for nuclei with atomic numbers  $A \geq 3$ , with binding energies of  $B_{3\text{He}\eta_c} = 19$  MeV,  $B_{4\text{He}\eta_c} = 140$  MeV, and as deep as  $B_{9\text{Be}\eta_c} = 407$  MeV. Subsequent works have refined these calculations, starting with the observation by Wasson [2] that the extended volume of large nuclei must modify the form of the potential, which had been assumed to scale with  $A$  in Ref. [1].

This more realistic model suppresses the binding energies compared with those obtained in Ref. [1], leading to estimates of  $B_{3\text{He}\eta_c} = 0.8$  MeV,  $B_{4\text{He}\eta_c} = 5$  MeV, and which rapidly saturate to  $B^{\text{NM}} \lesssim 30$  MeV in nuclear matter (NM). The heavy-quark expansion, in which the binding energies have expansions in inverse powers of the heavy-quark mass,  $M_Q$ , and in the radius of the quarkonium,  $r_{\bar{Q}Q}$ , was applied to these systems in Ref. [3]. Using an operator product expansion, the dominant effects arise from matching to the leading dimension-seven operators involving the quarkonium and two gluons with coefficients that scale as  $r_{\bar{Q}Q}^3$ . At NM density, a binding of  $\sim 10$  MeV was found for the  $J/\psi$ . However, since the chromo-polarizability depends upon the radius of the charmonium, the excited state  $\psi'$ , which is loosely bound and has large radius (about 1.8 fm), may be more deeply bound to nuclei, although the techniques used for that analysis become unreliable for these larger systems. Non-perturbative modifications to the interactions and nuclear binding of quarkonia have been explored through the inclusion of hadronic-exchange effects, e.g. Ref. [4]. A summary of the predictions for charmonium binding to the lightest nuclei and NM is given in Table I.

In addition to charmonium interactions with nuclei, the interactions of bottomonium and strange-quarkonium with nuclei have also been considered. The heavy-quark expansion works well for bottom quarks [3], from which it is found that, because of its smaller radius, bottomonium is less bound to NM than charmonium, with an estimated binding energy of  $\sim 4$  MeV. Strange quarkonia binding to nuclei has also been considered previously, and in particular, the  $\phi$  has been predicted to have a binding energy of  $\sim 40$  MeV to NM [9]. Although the strange pseudoscalar,  $\eta_s$ , mixes strongly with the light-quark pseudoscalars to form the physical  $\eta$  and  $\eta'$ , for theoretical purposes it can be treated as a

Ref.	Binding Energy (MeV)			Binding Energy (MeV)	
	$^3\text{He } \eta_c$	$^4\text{He } \eta_c$	NM $\eta_c$	$^4\text{He } J/\psi$	NM $J/\psi$
[1]	19	140	407		
[2]	0.8	5	27		
[3]			10		10
[5]	*	*	9		
[6]					5
[7]				5	18
[8]				15.7	

TABLE I: Estimates for the binding energies of charmonium to light nuclei and nuclear matter (in MeV) from selected models. A “\*” indicates the system is predicted to be unbound, while blank entries indicate that the system was not addressed.

pure  $\bar{s}s$  state, in a manner analogous to the  $\eta_c$ . The work of Ref. [10] finds that the  $\eta_s$  does not bind to nuclei with  $A < 12$ , but does bind to NM with  $B^{\text{NM}} \sim 17$  MeV, while Ref. [7] finds the  $\eta_s$  binds to NM with  $B^{\text{NM}} \sim 90$  MeV.

Despite the general agreement among theorists that charmonium-nucleus bound states should exist, the predictions for the binding energies are quite disparate, and such systems remain to be discovered experimentally despite many attempts to produce them. The latest experimental programs in this area include ATHENNA [11] as part of the 12-GeV program at Jefferson Lab, PANDA at FAIR [12, 13] and efforts at J-PARC [14]. A signal of  $^3\text{He } \eta$  was reported by MAMI [15] with  $B_{^3\text{He}\eta} \sim 4$  MeV, but the result could not be confirmed by COSY-GEM which, however, did report evidence for a bound  $^{25}\text{Mg } \eta$  system with  $B_{^{25}\text{Mg}\eta} \sim 12$  MeV [16].

In order to guide the present and future experimental programs aiming to discover and explore quarkonium-nucleus bound states, it is important to perform QCD calculations of these systems. Lattice QCD (LQCD) is currently the only reliable technique for such calculations in the nonperturbative regime, and exciting progress has been made in recent years applying LQCD to light nuclei [17–23]. In addition, an early calculation of the color polarizabilities of mesons was performed [24], in which it was found that Bose gases of pions or kaons become color-polarized when in the presence of static color sources. This has been extended to the case of charmonium and bottomonium interactions with many pion systems [25]. Lattice QCD calculations of the scattering of quarkonia and single nucleons have been previously performed [26–29]. Quenched calculations reveal a negative scattering length (with the nuclear physics convention), resulting from an attractive interaction, but the results are consistent with a volume-independent negative energy shift, as would arise from a bound state. Calculations with  $n_f = 2 + 1$  [29] at a pion mass of  $M_\pi \sim 640$  MeV yield a relatively small and negative scattering length, a large effective range, but not a bound state. The HAL QCD modeling method has been used to extract interpolating-operator- and energy-dependent

quarkonium–light-hadron potentials, e.g. Ref. [30].

In this work, we demonstrate the existence of quarkonium-nucleus bound states for  $A < 5$ , and calculate their binding energies, at the flavor SU(3)-symmetric point with unphysical values of the light-quark masses corresponding to that of the physical strange quark, resulting in a pion of mass  $M_\pi \sim 805$  MeV. The same lattice technology and parameters, with the addition of the charmed quark, are used as in the calculations of light nuclei presented in Refs. [21–23]. While calculations are performed in multiple lattice volumes, only one lattice spacing has been employed.

In Sec. II, the lattice QCD calculations performed in this work are described. The methods used to analyze the correlation functions, and the binding energies extracted from them, are presented in Sec. III. Boosted systems are found to present some unexpected challenges, which are discussed in Sec. IV. Finally, we present our conclusions and discuss the future lattice QCD prospects for quarkonium-nucleus systems in Sec. V.

## II. LATTICE QCD METHODOLOGY

Three ensembles of gauge-field configurations at the SU(3)-flavor symmetric point, where  $M_\pi = M_K \sim 805$  MeV, at a single lattice spacing of  $b = 0.145(2)$  fm (determined at this unphysical mass) were used in this work. The Lüscher-Weisz gauge [31] action was used with a clover-improved quark action [32] with one level of stout smearing ( $\rho = 0.125$ ) [33]. The clover coefficient was set equal to its tree-level tadpole-improved value,  $c_{\text{SW}} = 1.2493$ , a value that is consistent with an independent numerical study of the nonperturbative  $c_{\text{SW}}$  in Schrödinger functional scheme [34]. The ensembles have spatial extent  $L \sim 3.4, 4.5$  and  $6.7$  fm, and each consists of  $O(10^4)$  evolution trajectories. Large volumes are necessary for the study of bound states in lattice QCD even at heavy quark masses, and we have previously published results for the spectroscopy of light nuclei and hypernuclei [21], and for nucleon-nucleon scattering properties [22], obtained from them. The relevant features of these ensembles are given in Table II (further details can be found in Refs. [21, 22]). Somewhat fewer measurements are used in the present work than in Refs. [21, 22].

Multiple different correlation functions for the strangeonium- and charmonium-nucleus systems were calculated on the ensembles of lattice gauge-field configurations described above. The correlation functions of these systems are simply the product of the individual correlators of the component subsystems on each gauge field for a given source location. Consequently, the nuclear correlation functions previously calculated were reused, and additional computational resources were only expended on the quarkonium correlation functions. The nuclear correlation functions were produced using the recursive algorithm of Ref. [35], and a detailed study

$L^3 \times T$	$N_{\text{cfg}}$	$N_{\text{src}}$	$aM_\pi$	$M_\pi L$	$aM_N$
$24^3 \times 48$	1894	96	0.59388(14)	14.3	1.2042(5)
$32^3 \times 48$	3093	48	0.59451(08)	19.0	1.2046(8)
$48^3 \times 64$	614	64	0.59446(11)	28.5	1.2047(9)

TABLE II: Details of the ensembles of gauge-field configurations used in the present calculations, including the lattice dimensions, number of configurations per ensemble,  $N_{\text{cfg}}$ , number of sources used per configuration  $N_{\text{src}}$ , along with the pion and nucleon masses. [Note that as this involves only a subset of the number of sources used in our calculations of nuclear binding energies and nucleon-nucleon scattering [21, 22], the light-hadron masses in this table have somewhat larger uncertainties].

and results for nuclear bindings and interactions can be found in Refs. [21, 22]. In the current study, we focus on the nucleon ( $J^\pi = \frac{1}{2}^+$ ), deuteron ( $J^\pi = 1^+$ ), di-neutron ( $J^\pi = 0^+$ ),  $^3\text{He}$  ( $J^\pi = \frac{1}{2}^+$ ) and  $^4\text{He}$  ( $J^\pi = 0^+$ ). We have previously calculated correlation functions of the strange mesons,  $\eta_s$  and  $\phi$ , for a range of momenta on the same ensembles. To calculate charmonium correlation functions, charm-quark propagators were produced using the relativistic heavy-quark (RHQ) action [36]:

$$S_Q = \sum_{x,x'} \bar{Q}_x \left( m_0 + \gamma_0 D_0 - \frac{a}{2} D_0^2 + \nu \left( \gamma_i D_i - \frac{a}{2} D_i^2 \right) - \frac{a}{4} c_B \sigma_{ij} G_{ij} - \frac{a}{2} c_E \sigma_{0i} G_{0i} \right)_{xx'} Q_{x'}, \quad (1)$$

where  $Q_x$  is the heavy-quark field at the site  $x$ ,  $\gamma_\mu$  are the Hermitian Dirac matrices,  $\sigma_{\mu\nu}$  is defined through  $i[\gamma_\mu, \gamma_\nu]/2$ ,  $D_\mu$  is the first-order lattice derivative, and  $G_{\mu\lambda} = \sum_a T^a G_{\mu\lambda}^a$  is the Yang-Mills field-strength tensor. The coefficients  $\nu = 1.295$  and  $m_0 = 0.1460$  were tuned to recover the spin-averaged  $\eta_c$  and  $J/\psi$  experimental masses and low-energy dispersion relations, while  $c_{E,B}$  were set to their tree-level tadpole-improved values,  $c_B = c_{\text{SW}}\nu = 2.24363524134292$  and  $c_E = c_{\text{SW}}(1 + \nu)/2 = 1.9880860536224$ . (For a more detailed discussion of this tuning, see Ref. [37] and references therein.) Analysis of the correlation functions, that give rise to the effective mass plots shown in Figure 1, including all statistical and systematic uncertainties, gives masses ( $M_i(L)$ ) of  $M_{\eta_c}(24) = 3012(33)$  MeV,  $M_{\eta_c}(32) = 3012(33)$  MeV,  $M_{J/\psi}(24) = 3105(34)$  MeV and  $M_{J/\psi}(32) = 3106(34)$  MeV, and mass splittings ( $\Delta M_i(L)$ ) of  $\Delta M(24) = 93(1)$  MeV and  $\Delta M(32) = 93(1)$  MeV, where the dominant uncertainty is that from the lattice spacing. Table III shows the “speed of light” for each hadron obtained from quadratic fits to the squared-energy versus squared-momentum for the chosen RHQ parameters. Figure 2 shows the calculated

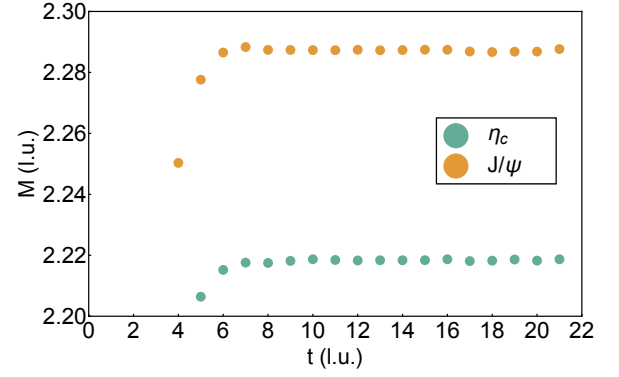


FIG. 1: The effective mass plots associated with the  $\eta_c$  and  $J/\psi$  formed from linear combinations of the smeared-point and smeared-smeared correlation functions.

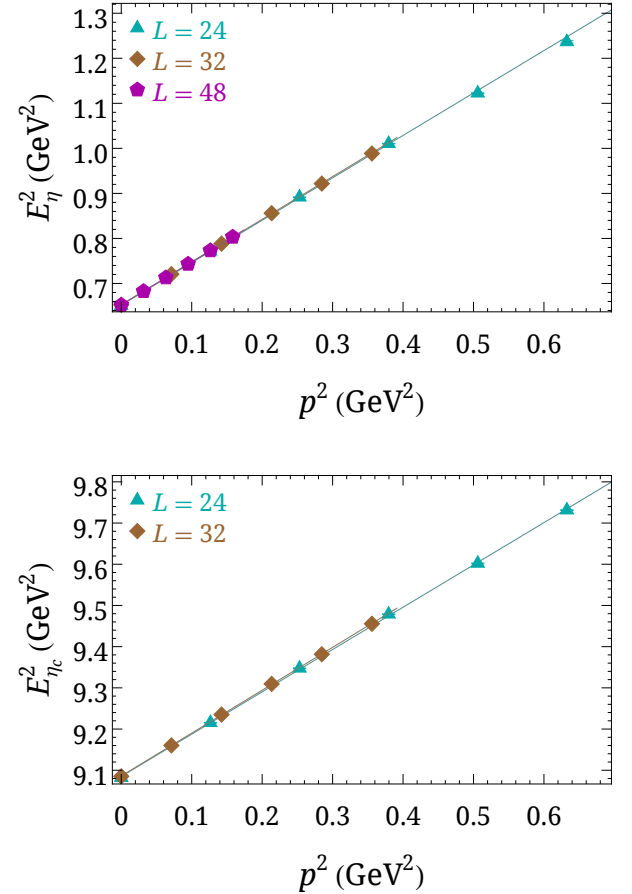


FIG. 2: The dispersion relations of the  $\eta_s$  and  $\eta_c$ . The blue triangles, brown diamonds and purple pentagons show results from the  $L = 24, 32$  and  $48$  ensembles, respectively. The curves correspond to linear fits to the  $p^2 \leq 0.4$   $\text{GeV}^2$ , and show small quadratic contributions at higher  $p^2$ .

$L$	$\eta_s$	$\phi$	$\eta_c$	$J/\psi$
24	0.9705(6)	0.9471(11)	1.013(6)	0.989(5)
32	0.9737(5)	0.9536(11)	1.020(5)	0.996(6)
48	0.9774(8)	0.9597(22)	—	—

TABLE III: The calculated “speed of light” of the  $\eta_s$ ,  $\phi$ ,  $\eta_c$  and  $J/\psi$  extracted from each volume using a quadratic fit. The statistical and systematic uncertainties have been combined in quadrature.

dispersion relations for the  $\eta_s$  and  $\eta_c$ ,<sup>1</sup> which are representative of the dispersion relations for the quarkonia considered in this work, and demonstrate that the  $\mathcal{O}(am)$  effects in charmonium are well controlled.

As stated previously, the calculations have been performed at only one lattice spacing. Given that the clover action has been used, lattice-spacing artifacts are expected to be small, scaling as  $\mathcal{O}(a^2, \alpha_s a)$ . However, the uncertainties in the binding energies introduced by the discretization remain to be quantified, and calculations with other ensembles with smaller lattice spacings will be required in order to perform a continuum extrapolation.

### III. NUCLEUS-QUARKONIUM BINDING ENERGIES

The energies of quarkonium-nucleus systems may be extracted from two-point correlation functions with the appropriate quantum numbers. For the systems of interest, we considered the two-point functions

$$\begin{aligned}
\mathcal{C}_{AB}(t) &= \langle 0 | \chi_A(t) \tilde{\chi}_B^\dagger(0) | 0 \rangle, \\
\mathcal{C}_{AB}(t) &= \langle 0 | \chi_A(t) \tilde{\chi}_B^\dagger(0) | 0 \rangle, \\
\mathcal{C}_\Gamma(t) &= \langle 0 | \chi_{\overline{Q}\Gamma Q}(t) \tilde{\chi}_{\overline{Q}\Gamma Q}^\dagger(0) | 0 \rangle,
\end{aligned} \tag{2}$$

with  $\chi_A = \chi_A \chi_{\overline{Q}\Gamma Q}$  where  $\chi_A$  ( $\tilde{\chi}_A^\dagger$ ) and  $\chi_{\overline{Q}\Gamma Q}$  ( $\tilde{\chi}_{\overline{Q}\Gamma Q}^\dagger$ ) are interpolating operators that annihilate (create) states with the quantum numbers of the nucleus  $A$  and quarkonia  $\overline{Q}\Gamma Q$ , respectively (with  $\Gamma$  the relevant Dirac structure).<sup>2</sup> For brevity, the momentum labels on the correlation functions and interpolators are suppressed, however, correlation functions with zero total momentum, as well

as those with total momenta  $|\frac{L}{2\pi}\mathbf{P}_{\text{tot}}|^2 = 1, 2, 3$ , are considered. The correlation functions can be expanded over the complete set of lattice energy eigenstates with the appropriate quantum numbers,

$$\mathcal{C}_{AB}(t) = \sum_n Z_{n,A} Z_{n,B}^* e^{-E_n(t_f - t_i)}, \tag{3}$$

where the summation is over all eigenstates that couple to the operators  $\chi_A$ ,  $\chi_B$ , with amplitudes  $Z_{n,A}$ ,  $Z_{n,B}^*$ .

In extracting the quarkonium-nucleus binding energies from the correlation functions, it is helpful to consider both one-state and two-state fitting functions, truncating the sum in Eq. (3) to one or two terms. At short times, the correlation functions are contaminated by excited states, while at later times, the signal-to-noise ratio degrades exponentially. Two-state fits are applicable at earlier times (where the data are more precise) than one-state fits, but the latter serve as an important comparison to understand the systematic uncertainty induced by the choice of the fitting form. Performing two-state fits to the single hadron correlation functions yield energy splittings that are consistent with the lowest-lying excitation for each species. In addition to fits to the two-point correlation functions, the binding energy can be isolated by taking ratios of the two-point correlation functions of the system and its components (note that in this context, the entire nucleus is considered to be a single component of the system). In this latter case, the fitting function at large times (neglecting excited states) reduces to

$$\mathcal{R}(t) = \frac{\mathcal{C}_{AB}(t)}{\mathcal{C}_{AB}(t)\mathcal{C}_{\overline{Q}\Gamma Q}(t)} \rightarrow Z e^{-(E_{12} - (E_1 + E_2))(t_f - t_i)}, \tag{4}$$

where  $E_{12}$  is the total energy of the ground-state system,  $E_1$  and  $E_2$  are the energies of the system components, and  $Z$  is an overall normalization factor. The difference  $E_{12} - (E_1 + E_2)$  may be fit by a single parameter. The statistical quality of the calculations is illustrated in Figure 3, where the effective energy-shift plots associated with one of the correlation functions for each of the  $N\eta_c$ ,  $d\eta_c$  and  ${}^4\text{He}\eta_c$  are shown. These are derived from sets of correlation functions for which the nucleons are generated from Gaussian-smeared sources and sinks, and the  $\eta_c$  is also derived from a (different) Gaussian-smeared source and sink. The correlation functions of the quarkonium states have been translated back in time by a small number of time slices, as was used in Ref. [24], so that the start of the plateau regions of the nuclear and quarkonia correlation functions approximately coincide. While this does slightly degrade the uncertainty, the fact that the ground-state energies of the quarkonia are more than an order of magnitude more precise than those of the nuclei, this time translation has a minimal impact upon the analysis of binding energies. The fitting intervals used to extract the quarkonium-nucleus binding energies from the ratios of correlation functions corresponded approximately to those used to extract the binding energies of the nucleus, as detailed in Ref. [21]. For the two-state fits,

<sup>1</sup> Unfortunately, charmonium correlation functions were not calculated in the  $L = 48$  volume. This was a consequence of this work occurring after the production of the nuclear correlation functions, and practical aspects associated therewith. The same is true for the  $N\eta_s$  correlation functions in this volume.

<sup>2</sup> The calculations presented here ignore the annihilation-type contractions in the quarkonium correlators as they are numerically expensive to evaluate. These effects are suppressed by the heavy quark mass and are found to be small for charmonium [38]. For the strange quarkonium, the effects may be slightly larger and remain to be quantified.

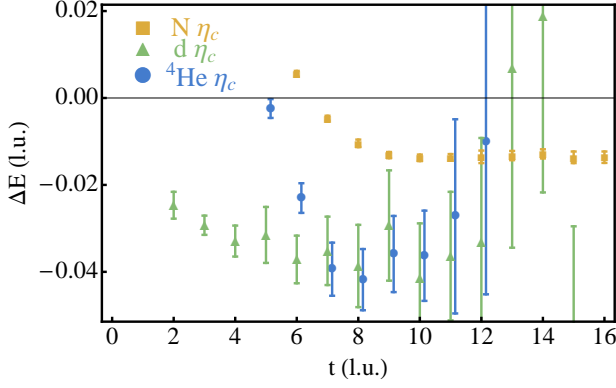


FIG. 3: Representative effective energy-shift plots associated with the  $N\eta_c$ ,  $d\eta_c$  and  ${}^4\text{He}\eta_c$  systems obtained from one set of correlation functions in the  $L = 32$  ensemble.

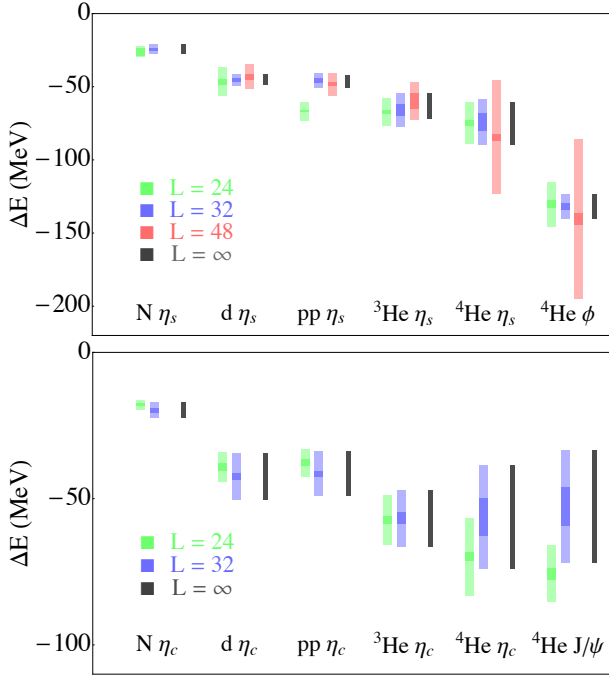


FIG. 4: Binding energies of strangeonium-nucleus (upper panel) and charmonium-nucleus (lower panel) systems from Table IV and Table V. The inner bands correspond to the statistical uncertainty, while the outer bands correspond to the statistical and systematic uncertainties combined in quadrature. The right most (gray) band for each system corresponds to the infinite-volume estimate, resulting from a weighted average of the  $L = 32$  and  $L = 48$  (where available) energies.

the intervals extend to shorter times by a number of time slices, dependent upon the goodness of fit. Variations of these fitting intervals are used to estimate the systematic uncertainties associated with extracted fit parameters.

The results of our calculations in the three volumes, combining the output from the three analysis methods outlined previously, are summarized in Figure 4 and in Table IV for the strangeonium-nucleus systems and in

Table V for charmonium-nucleus systems. The results obtained from one- and two-state fits to the correlation functions are consistent with those extracted from fitting to the effective mass at intermediate times, but are found to be more precise. A systematic fitting uncertainty is assessed based on the differences between the three methods.

Most of the systems we have explored in this work have negligible finite volume (FV) effects. For the isolated nuclear systems, the FV effects, which depend upon the nuclear binding energies, were quantified for these ensembles by previous calculations [21], from which it was determined that such effects are negligible in the  $L = 32$  and  $L = 48$  ensembles. The volume effects are also negligible for the isolated mesons, as is clear by explicit comparison of the dispersion relations extracted from each ensemble, see Figure 2. Finally, the calculated binding energies are sufficiently deep that the energy gap to the nearest state above the quarkonium-nucleus ground state is large enough so that the FV modifications to the binding energy of the combined system are negligible in the  $L = 32$  and  $L = 48$  volumes, as can be seen from Figure 4 (the  $L = 24$  ensemble shows some small volume dependence in a few systems). As a result, the infinite-volume binding energy is taken to be the weighted average of the binding energy in the  $L = 32$  and the  $L = 48$  ensembles (the largest volume is not available for the charmonium-nucleus systems, but we assume volume effects in this case are not larger than those in the corresponding strangeonium-nucleus system and are thus negligible for the  $L = 32$  results). The exponential dependence upon the spatial extent of the lattice for bound systems, along with the measured energy scales, allow for an estimate of the infinite-volume binding energy while introducing a systematic uncertainty that is much smaller than the statistical and fitting systematic uncertainties. There is one caveat to this discussion of FV effects, that will be discussed in detail in Section IV. It is possible, due to the finite time extent of the plateaus, that the states we have identified are contaminated by low-lying scattering states at some level. While the uncertainties in the present results preclude a stable power-law extrapolation to infinite-volume, by making reasonable assumptions about the scattering parameters describing their interactions, our results indicate that such contaminations are small, providing energy shifts that are smaller than the quoted uncertainties.

In contrast to the charmonium-nucleus systems, the non-interacting  $\eta_s$ -nucleus systems are, up to nuclear binding energy contributions, degenerate with other states, such as  $K$ -hypernucleus states in the SU(3) limit. From the standpoint of SU(3) flavor symmetry, the charmonia are singlets (charmonia are also deeply bound relative to the  $c\bar{c}$  threshold), while the  $\eta_s$  is a combination of a singlet and an octet. In the latter case, this complicates the classification of the composite systems. For example, as the deuteron transforms in a  $\mathbf{\bar{10}}$  of SU(3), the charmonium-deuteron system is also in a  $\mathbf{\bar{10}}$  representa-

System	$24^3 \times 64$	$32^3 \times 64$	$48^3 \times 64$	$L = \infty$
$N\eta_s$	26.1(2.5)(2.5)	24.3(0.7)(3.2)	-	24.3(3.2)
$d\eta_s$	46.5(1.9)(9.7)	45.5(1.3)(3.6)	43.0(2.0)(8.2)	45.0(3.5)
$pp\eta_s$	66.9(0.7)(6.5)	45.8(1.4)(4.8)	48.3(1.1)(7.7)	46.5(4.2)
${}^3\text{He}\eta_s$	67.6(1.1)(9.4)	66(04)(11)	60(05)(12)	63.2(8.6)
${}^4\text{He}\eta_s$	75(02)(14)	74(06)(14)	85 (02)(39)	75(14)
${}^4\text{He}\phi$	130(03)(15)	132.0(2.1)(8.1)	140(04)(55)	132.1 (8.2)

TABLE IV: The binding energies (in MeV) of strangeonium-nucleus systems calculated on the  $L = 24, 32$  and  $48$  ensembles. The right most column shows the infinite-volume estimate given by the weighted average of the  $L = 32$  and  $L = 48$  binding energies. The first and second set of parentheses show the statistical and quadrature-combined statistical plus systematic uncertainties, respectively.

System	$24^3 \times 64$	$32^3 \times 64$	$L = \infty$
$N\eta_c$	17.9(0.4)(1.5)	19.8(0.7)(2.6)	19.8(2.6)
$d\eta_c$	39.3(1.3)(4.8)	42.4(1.1)(7.9)	42.4(7.9)
$pp\eta_c$	37.8(1.1)(4.5)	41.5(1.0)(7.5)	41.5(7.6)
${}^3\text{He}\eta_c$	57.2(1.3)(8.3)	56.7(2.0)(9.4)	56.7(9.6)
${}^4\text{He}\eta_c$	70(02)(13)	56(06)(17)	56(18)
${}^4\text{He}J/\psi$	75.7(1.9)(9.4)	53(07)(18)	53(19)

TABLE V: The binding energies (in MeV) of charmonium-nucleus systems calculated on the  $L = 24$  and  $32$  ensembles. The right most column shows the infinite-volume estimate, which, without results on the  $L = 48$  ensemble, is taken to be the binding calculated on the  $L = 32$  ensemble. The first and second set of parentheses shows the statistical and quadrature-combined statistical plus systematic uncertainties, respectively.

tion, while the strangeonium-deuteron system transforms as  $(\mathbf{1} \oplus \mathbf{8}) \otimes \mathbf{10} = \mathbf{8} \oplus 2 \cdot \mathbf{10} \oplus \mathbf{27} \oplus \mathbf{35}$ . Including interactions, the energy eigenvalues of the  $\eta_s$ -nucleus systems therefore result from diagonalizing a coupled channels system, and one may anticipate potential difficulties in extracting the binding energy because of nearby levels. *A posteriori*, we find that the correlators exhibit single exponential behavior (to the level at which we can resolve it) and the corresponding ground states are sufficiently isolated to permit their extraction. Physically, the binding of quarkonium to the nucleus introduces a relatively large energy scale into the coupled-channel system, leading to an isolated ground state.

All of the quarkonium-nucleus systems that we have explored are found to have binding energies that differ significantly from zero, and the results are summarized in Figure 5. These binding energies are quite large when compared to typical nuclear binding energies at the physical point ( $\sim 8$  MeV per nucleon in NM), but similar in size to the nuclear bindings found at these unphysically heavy quark masses [21]. In analogy with the liquid-drop model description of nuclei, where binding energies per nucleon are of the form  $B/A \sim \alpha_V - \alpha_S A^{-1/3}$  (we keep only the volume and surface terms with coefficients  $\alpha_{V,S}$ , respectively), the binding between quarkonia and nuclei is expected to have a similar classical expansion of the form  $B_{A\bar{Q}Q} \sim \alpha_V \bar{Q}Q - \alpha_S \bar{Q}Q A^{-1/3}$ . As the long range component of the interaction between quarkonia and the nucleons scales as  $V(r) \sim e^{-2M_\pi r}/r^\alpha$  (with some positive constant  $\alpha$ ), the force is expected to saturate more

rapidly with increasing nuclear size than for pure nuclear bindings. Within significant uncertainties, we find the  $\eta_c$  to have equal binding to  ${}^3\text{He}$  and  ${}^4\text{He}$ , the weighted average of which yields an estimate of the nuclear matter binding energy of  $B^{\text{NM}} \sim 60$  MeV at this heavy pion mass. However, we have an insufficient range of nuclei to determine if, in fact, the  $A = 4$  system is at saturation, so this value is speculative.

The leading behavior of the binding to nuclear matter in the heavy-quark limit [3] is linear in the mass density of the nuclear system, which itself depends approximately upon the nucleon mass and baryon number density. Using the experimental nucleon mass, and assuming the number density is either constant or decreases towards the physical quark mass, this yields an upper bound on the  $\eta_c$  binding energy of  $B_{\text{phys}}^{\text{NM}} \lesssim 40$  MeV, but without a full quantification of uncertainties.

#### IV. BOOSTED SYSTEMS

As discussed previously, quarkonium-nucleus correlation functions associated with a given total three momentum were constructed by multiplying the appropriate correlation functions. In our calculations, at least one of the component systems was at rest in the lattice volume. For systems with total center-of-mass (CoM) momentum,  $\mathbf{P}_{\text{tot}} \neq 0$ , the total energy of the ground state was translated to the CoM energy, and then to the binding energy of the system by removing the rest

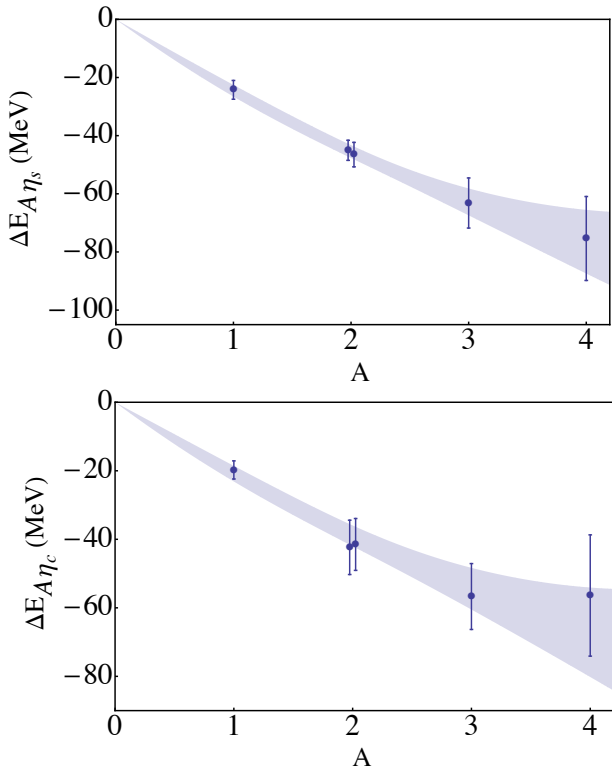


FIG. 5: Binding energies of the  $A \eta_s$  (upper) and  $A \eta_c$  (lower) systems as functions of atomic number. For  $A = 2$ , we display both the deuteron and  $nn$  results. The shaded region corresponds to a phenomenological quadratic fit to the results.

masses of the constituents. An example of the energy shifts for the charmonium-nucleus systems in the CoM frame is shown in Figure 6 as a function of relative rapidity,  $\eta = \tanh^{-1} \beta$ , where  $\beta$  is the velocity of the boosted hadron. Similar dependence is seen for all of the quarkonium-nucleus systems that we have studied. Naïvely, one expects that the CoM energy should be independent of the relative velocity, however, this is not what we find in our results. Instead, there is a trend for the extracted total energy to increase approximately quadratically with the relative rapidity. We speculate that this behavior arises because the overlap of the momentum projected sink interpolators onto a bound state is suppressed at non-zero relative momentum, while the overlap onto the continuum states remains of order unity, dictated by the lattice volume. While the bound state dominates the correlation functions for  $\beta \sim 0$ , its contribution will be suppressed for interpolating operators with relative momenta that are of order or greater than the binding momentum of the state. At intermediate times from the source, the effective mass plots associated with such systems may exhibit a “plateau” with an energy that exceeds the actual energy of the bound state. Toy models of such systems, with two or more nearby states, can be readily constructed that exhibit such behavior, and there are sets of natural-sized parameters that are

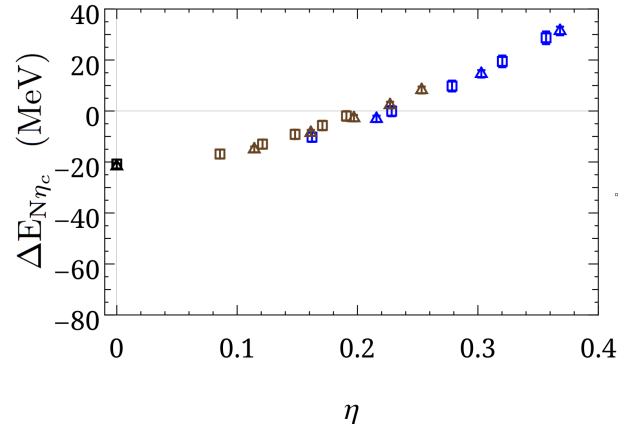


FIG. 6: An example of the energy differences (in MeV) for charmonium-nucleus systems,  $N \eta_c$ , versus the rapidity of the boosted hadron. The brown points show the extracted energies of systems produced from sinks for which the quarkonium is boosted and the nucleon is at rest, while the blue points show the extracted energies of systems produced from sinks for which the nucleon is boosted and the quarkonium is at rest. The black point corresponds to the system produced at rest. Triangles (squares) denote results from lattice volumes with spatial extent  $L = 24$  ( $L = 32$ ).

consistent with the behavior seen in the numerical results. Only at very large times can the true ground state be extracted, but at these times the signal-to-noise ratio has degraded to the point where the energy cannot be usefully constrained at the current (and foreseeable) statistical precision. The observed approximate linearity in  $\beta^2$  is consistent with this scenario, but our argument remains a conjecture at this point. In order to convincingly diagnose the origin of this momentum dependence, a more extensive set of calculations are required, involving single- and multi-hadron sources and sinks, and utilizing the full machinery of the variational method [39, 40].

Our current understanding of the observed relative-velocity dependence of the extracted binding energies of the quarkonium-nucleus systems remains incomplete and it is possible that these concerns also effect the zero velocity systems. The associated systematic uncertainties must be more concretely quantified in future calculations, however the relatively weak dependence on  $\beta$  near  $\beta = 0$ , and the lack of volume dependence, suggests that the ground states of these systems are bound states rather than scattering states. From the energies extracted at non-zero relative velocity, we expect that removing this systematic will lead to a deeper binding energy than we have estimated, but within the quoted uncertainties. To demonstrate the validity of this statement we consider  $N\eta_c$  system. With binding energies in only two volumes, a generic extrapolation of the form  $B(L) = B_0 + \beta/L^3$ , that would describe such contamination from the lowest-lying continuum state (with an admixture  $\beta$ ), is unstable when fit to the results, due to the relative size of the



uncertainties in each. However, assuming that the scattering parameters of the system are of natural size, and that the extracted energies are perturbatively close to the true binding energy, the scattering length of this system is found to be  $a \sim 1$  fm when higher order terms in the effective range expansion are ignored. This value then yields an expected energy difference between the lowest-lying continuum states in the  $L = 24$  and 32 volumes of  $\delta E \sim 0.005$  l.u.  $\sim 7$  MeV (using Lüscher's method). This is larger than the difference in ground state energies extracted from the two volumes,  $\sim 2$  MeV, indicating that the admixture of scattering state in the observed bound state is small. Taking central values to constrain the scattering state contamination, the binding energy is  $\sim 1.5$  MeV deeper than shown in Table V. This value is within the uncertainty associated with this binding. However, the contamination is consistent with zero in all systems we have calculated, and this effect should be considered as an uncertainty, smaller than those from other sources, as opposed to an energy shift. Further, it can only lead to the extrapolated binding energies being deeper than shown in Table V. Only higher precision calculations in additional volumes can further address this issue.

## V. CONCLUSIONS

In this study, we have performed lattice QCD calculations that demonstrate the existence of bound quarkonium-nucleus systems in QCD at the flavor-symmetric SU(3) point. Calculations were performed in multiple lattice volumes to enable an exploration of volumes effects, in particular to distinguish between scattering states and bound states. Only one lattice spacing was used in this work, and so the continuum limit could not be taken, however, given the  $\mathcal{O}(a)$  improvement of the lattice action, we expect lattice artifacts to be smaller than the other uncertainties in our calculation. For all of the strangeonium-nucleus and charmonium-nucleus systems that we study (atomic numbers  $A = 1, \dots, 4$ ), we find significant binding at light quark masses corresponding to  $M_\pi = M_K \sim 805$  MeV. Assuming the consistency of the bindings for  $A = 3$  and 4 is indicative of saturation of the interactions, we infer a charmonium-nuclear matter binding energy of  $B^{\text{NM}} \sim 60$  MeV at this heavy pion mass, although further studies are required to confirm saturation.

As the quark masses decrease towards their physical values, the nucleon mass decreases and it is also expected that the energy density of a nucleus will decrease [21].

Quarkonium-nucleus systems are therefore likely to be less bound at lighter quark masses and it is possible that the systems involving the lightest nuclei will be unbound at the physical point. Additional lattice QCD calculations at smaller light-quark masses will be necessary to investigate whether this is the case. The clean signals found in this study at the SU(3) point, suggest that such studies will be able to conclusively resolve the nature of a range of quarkonium-nucleus systems. For the case of nuclear matter, assuming our numerical results for the charmonium-nucleus binding energies indicate saturation, the leading order extrapolation to the physical quark masses results in an estimated binding energy of  $B_{\text{phys}}^{\text{NM}} \lesssim 40$  MeV, although the uncertainties in this result are not yet fully quantified. With greater computational resources becoming available, future calculations will be more precise, extended to larger nuclei, and be will performed at smaller lattice spacings, which will ultimately lead to predictions for the binding of quarkonium to nuclei that can guide, and be directly compared with, ongoing and future experiments.

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