Axion induced oscillating electric dipole moments

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Axion Induced Oscillating Electric Dipole Moments

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The axion electromagnetic anomaly induces an oscillating electric dipole for *any* static magnetic dipole. Static electric dipoles do not produce oscillating magnetic moments. This is a low energy theorem which is a consequence of the space-time dependent cosmic background field of the axion in the limit that it is only locally time dependent ($\overrightarrow{v}/c = 0$). The electron will acquire an oscillating electric dipole of frequency m_a and strength $\sim 10^{-32}$ e-cm, two orders of magnitude above the nucleon, and within four orders of magnitude of the present standard model limit on a constant electron EDM. This may suggest sensitive new experimental venues for the axion dark matter search.

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I. INTRODUCTION

The axion is a hypothetical, low–mass pseudo-Nambu-Goldstone boson (PNGB) that offers a solution to the strong CP problem of the standard model, and simultaneously provides a compelling dark matter candidate. The expected mass scale of the axion is $m_a \approx m_\pi^2/f_A$ where typical expected values of the decay constant f_A range from $\sim 10^{10}$ GeV upwards [1–3].

The axion is expected to have an anomalous coupling to the electromagnetic field $\overrightarrow{E}\cdot\overrightarrow{B}$, taking the form:

$$\frac{g_{A\gamma\gamma}}{4}\left(\frac{a}{f_A}\right)F_{\mu\nu}\widetilde{F}^{\mu\nu} = -g_{A\gamma\gamma}\left(\frac{a}{f_A}\right)\overrightarrow{E}\cdot\overrightarrow{B} \qquad (1)$$

where $\widetilde{F}_{\mu\nu} = (1/2)\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$, and $g_{A\gamma\gamma}$ is the dimensionless anomaly coefficient. In various models we have [4]:

$$\begin{aligned} g_{A\gamma\gamma} &\approx 8.3\times 10^{-4} && \text{DFSZ [5]} \\ g_{A\gamma\gamma} &\approx -2.3\times 10^{-3} && \text{KSVZ [6]} \end{aligned} \qquad (2)$$

We will quote results below, scaled by $g_{A\gamma\gamma}/10^{-3}$.

Most strategies for detecting the cosmic axion exploit the electromagnetic anomaly [7–9] together with the assumption of a coherent galactic dark-matter background field [10], $a/f_A \equiv \theta(t) = \theta_0\cos(m_a t)$. Saturating the local galactic dark-matter density of $\sim 0.3\,\mathrm{GeV/cm^3}$ yields $\theta_0 \approx 3.7\times 10^{-19}$ [9]. In typical RF cavity experiments such as ADMX, one applies a large external constant magnetic field to the cavity, $\overrightarrow{B}_0$ and the anomalous coupling to $\theta(t)$ induces an oscillating electromagnetic response field, $\overrightarrow{E}_r$ and $\overrightarrow{B}_r$. Combined with the conducting boundary conditions of the detector, the "cavity modes" can become excited, which can generate a resonant signal in the cavity. This offers the possibility of both detecting the existence of the axion and simultaneously establishing that it is a significant component of dark-matter.

Recently several authors have considered the possibility of observing an *oscillating electric dipole moment for the nucleons* [11, 12]. Indeed, it was the original problematic strong CP-violating nucleon electric dipole moment, arising from the θ-term in QCD, that the axion was designed to cure. Nonetheless, the relic small cosmological oscillations of the axion in its potential about zero, $\theta(t)$, will induce a small oscillating electric dipole moment for the nucleons with a frequency m_a given by $d_N \sim 10^{-16}\theta(t) \approx 3.67\times 10^{-35}\cos(m_a t)$ e-cm [11]. Thus far, this effect has only been considered to be specific to baryons. It arises, not by the electromagnetic anomaly $\propto g_{A\gamma\gamma}$, but rather directly via the QCD-induced axion potential.

In the present paper we show that the axion electromagnetic anomaly, together with cosmic axions in the $v/c = \beta = 0$ limit (*i.e.*, the cosmic frame in which the oscillating axion field has negligible momentum), leads to a "low energy theorem:" *any magnetic dipole source (eg, magnetization, spin magnetic moments, orbital magnetic moment, etc.) will develop a small, time-oscillating, CP-violating, effective electric dipole.* Remarkably, the dual is not true: A static electric electric polarization will *not* lead to a small time-oscillating effective dipole magnetic field.

In particular, the electron will develop an effective oscillating electric dipole moment proportional to the magnetic moment, $d_e \approx 2g_{A\gamma\gamma}\theta_0\cos(m_a t)\mu_{Bohr} \approx 1.4\times 10^{-32}(g_{A\gamma\gamma}/10^{-3})\cos(m_a t)$ e-cm. We use the term, "effective," because this arises at the level of a one-particle reducible Feynman diagram. The result is intrinsically non-relativistic.

The magnetic moment of the electron is much larger than that of the nucleon, and hence the axion-induced oscillating electric dipole moment is almost three orders of magnitude larger than that of the nucleon. Since the current best limit upon any constant elementary particle EDM is that of the electron, of order $d_e \leq 8.7\times 10^{-29}$ e-cm, [13], the electron may be a promising place to search for an oscillating EDM.

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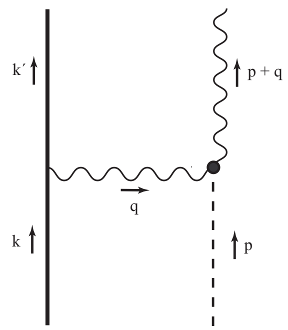

FIG. 1: Feynman diagram for axion induced electric dipole moment. Photon q emitted from electron magnetic moment. Solid dot is the axion anomaly interaction, $\theta\vec{E}\cdot\vec{B}$. Dashed line: incoming axion θ, p. Outgoing photon: $p+q$, ϵ_μ. Solid line: incoming electron k; recoil electron, k'.

II. FEYNMAN DIAGRAM ANALYSIS OF INDUCED ELECTRIC DIPOLE MOMENT

We consider the tree-level perturbative correction of the axion anomaly to the electron magnetic dipole operator. We begin by writing the axion anomaly in terms of vector potentials and integrating by parts:

$$\frac{1}{2}g_{A\gamma\gamma}\int d^4x\ \epsilon_{\mu\nu\rho\sigma}\partial^\mu\theta(x)A^\nu(x)\partial^\rho A^\sigma(x) \tag{3}$$

where $\theta(x) = a/f_A = \theta_0\cos(m_a t)$. Likewise, we have the Dirac operator of the magnetic moment of the electron:

$$\frac{ie}{2m_e}\int d^4x\ \overline{\psi}(x)\sigma_{\alpha\beta}\psi(x)\ \partial^\alpha A^\beta(x) \tag{4}$$

where $\sigma_{\alpha\beta} = (i/2)[\gamma_\alpha,\gamma_\beta]$ (we follow conventions of Bjorken and Drell [14]).

The Feynman amplitude for the first order $g_{A\gamma\gamma}\theta_0$ correction to the magnetic dipole moment follows from the time-ordered product of eqs.(3,4) and is the tree diagram of Fig(1). In momentum space we have:

$$\frac{ie}{2m_e}\ g_{A\gamma\gamma}\theta_0\ p^\mu\epsilon^\nu(p+q)^\rho\epsilon_{\mu\nu\rho\sigma}\ \times \\ \frac{1}{q^2}\left(g^{\sigma\tau}-\lambda\frac{q^\sigma q^\tau}{q^2}\right)q^\omega\overline{u}(k')\sigma_{\tau\omega}u(k) \tag{5}$$

where ϵ^ν is the outgoing photon polarization, $u(k)$ is an electron spinor, and momenta are as in Fig.(1). Note that a factor of 2 arises from the two possible photon field contractions. The gauge dependent terms $\propto\lambda$ vanish due to the antisymmetry of $\sigma_{\mu\nu}$.

The dipole moments are defined by going to the electron rest-frame $k = (m_e,\ \vec{0})$. The electron is very heavy compared to the axion, and is therefore essentially stationary, and absorbs 3-momentum but not energy (zero recoil). We assume in the electron rest frame that the axion field momentum is approximately pure timelike, $p_\mu = (m_a,\vec{0})$ and $|\vec{q}|\approx m_a << m_e$, $k_0\approx k_0'$, and the exchange photon momentum is therefore spacelike, $q = (0,\vec{q})$.

Since p^μ is timelike, we have $p^\mu\epsilon_{\mu\nu\rho\sigma} = m_a\epsilon_{0\nu\rho\sigma}$ and we'll pass to $D = 3$ latin spatial indices, $\epsilon_{0\nu\rho\sigma}\to\epsilon_{ijk}$. Also, $\overline{u}(k')\sigma_{\tau\omega}u(k)\to\epsilon_{ijk}\chi^\dagger\sigma_k\chi$ ($(\tau\omega)\leftrightarrow(ij)$ are likewise spatial in the nonrelativistic limit). The Dirac four-component spinor ψ has been replaced by the two-component Pauli spinor, χ, with Pauli matrices σ^k. The amplitude becomes:

$$-\frac{ie}{2m_e}g_{A\gamma\gamma}\theta_0\ m_a\ \frac{1}{\vec{q}^2}\epsilon^i q^j q^l\epsilon_{ijk}\epsilon^{klm}\chi^\dagger\sigma_m\chi \\ = g_{A\gamma\gamma}\theta_0\ \mu_{Bohr}\chi^\dagger\sigma_i\chi\cdot m_a\epsilon^i \tag{6}$$

where $\mu_{Bohr} = e/2m_e$. The photon propagator pole, $1/\vec{q}^2$ has cancelled against $\vec{q}^2$ in the numerator, yielding a contact term. Here we've used the identity, $\epsilon_{ijk}\epsilon^{klm} = \delta^l_i\delta^m_j - \delta^l_j\delta^l_i$. The polarization of the outgoing photon is spacelike and transverse, $0 = (p+q)\cdot\epsilon = \vec{q}\cdot\vec{\epsilon}$. In Coulomb gauge with vector potential $\vec{A}$, the electric field is given by $\vec{E} = -\partial_t\vec{A} = m_a\vec{\epsilon}$. Our final result can be written as an effective interaction for the nonrelativistic electron as:

$$\int d^4x\ 2g_{A\gamma\gamma}\theta(t)\ \mu_{Bohr}\ \chi^\dagger\frac{\vec{\sigma}}{2}\chi(x)\cdot\vec{E}(x,t) \tag{7}$$

We thus see that the magnetic moment of the electron, in the presence of the axion, yields an oscillating electric dipole moment of frequency m_a. This result can be obtained in the Pauli-Schoedinger non-relativistic spin-1/2 theory, or in Georgi's heavy-quark formalism applied to the "heavy electron" [15][16].

The magnitude of the electric dipole moment is $g_{A\gamma\gamma}\theta_0\vec{m}$ where $\vec{m} = g\mu_{Bohr}\vec{s}$ for spin $\vec{s}$ and $g = 2$. In an applied oscillating, phase matched, external electric field, $\vec{E}(t)\propto cos(m_a t)$, the term eq.(7) in the action generates small energy shifts which may be in principle detectable, as in *e.g.*, [13].

In general, a nonrelativistic, static electric dipole moment $\vec{P}(x)\sim 2g_{A\gamma\gamma}\ \mu_{Bohr}\ \chi^\dagger\frac{\vec{\sigma}}{2}\chi$, modulated by $\theta(t)$, includes a nonlocal term:

$$S' = \int d^4x\ \theta(t)\left(\vec{P}\cdot\vec{E}+\vec{\nabla}\cdot\vec{P}\left(\frac{1}{\vec{\nabla}^2}\right)\vec{\nabla}\cdot\vec{E}\right) \tag{8}$$

In an arbitrary gauge, $\vec{E} = \vec{\nabla}\varphi - \partial_t\vec{A}$, we see that:

$$S' = \int d^4x\ \theta(t)\vec{\nabla}\cdot(\vec{P}\varphi) \\ +\int d^4x\ \partial_t\theta(t)\left(\vec{P}\cdot\vec{A}+\vec{\nabla}\cdot\vec{P}\left(\frac{1}{\vec{\nabla}^2}\right)\vec{\nabla}\cdot\vec{A}\right) \tag{9}$$

S' is a total divergence in the limit $\partial_t\theta(t)\to 0$. and is indistinguishable from eq.7 when $\vec{\nabla}\cdot\vec{E} = 0$.

Our result can be inferred from the classical Maxwell equations. For response fields, $\vec{E}_r$ and $\vec{B}_r$, in the presence of a constant-in-time external magnetic dipole field

$\overrightarrow{B}_0(\overrightarrow{x})$, we have $\overrightarrow{\nabla}\cdot\overrightarrow{B}_r = \overrightarrow{\nabla}\cdot\overrightarrow{E}_r = 0$, and:

$$\begin{aligned}
&\overrightarrow{\nabla}\times\overrightarrow{B}_r - \partial_t\overrightarrow{E}_r = -g_{a\gamma\gamma}\overrightarrow{B}_0(\overrightarrow{x})\,\partial_t\theta(t)\\
&\overrightarrow{\nabla}\times\overrightarrow{E}_r + \partial_t\overrightarrow{B}_r = 0
\end{aligned} \tag{10}$$

$\overrightarrow{E}_r(\overrightarrow{x},t) = g_{a\gamma\gamma}\theta(t)\overrightarrow{B}_0(\overrightarrow{x})$ and $\overrightarrow{B}_r(\overrightarrow{x},t) = 0$ would be exact if $\overrightarrow{\nabla}\times\overrightarrow{B}_0 = \overrightarrow{\nabla}\times\overrightarrow{m} = 0$. However, $\overrightarrow{B}_0(\overrightarrow{x},t)$ contains a nonzero δ-function singularity $\propto \overrightarrow{m}$. The full radiative solution is readily obtained with retarded Green's functions, and contains $\overrightarrow{E}_r(\overrightarrow{x},t) = g_{a\gamma\gamma}\theta(t)\overrightarrow{B}_0(\overrightarrow{x})$ in the near-zone. The radiated power, representing conversion of vacuum axions to photons by the oscillating electric dipole, is then of order $\sim (m_a^2 g_{a\gamma\gamma}\theta_0\mu_{Bohr})^2$ and is negligible [16].

III. AXIONIC ELECTROMAGNETIC DUALITY

The result of the previous section is a general low energy theorem. Consider an arbitrary localized magnetic dipole interaction in Coulomb gauge $\nabla\cdot\overrightarrow{A} = 0$, $A_0 = 0$:

$$\int d^4x\,\overrightarrow{M}(\overrightarrow{x})\cdot\overrightarrow{B} = \int d^4x\,\overrightarrow{M}\cdot\overrightarrow{\nabla}\times\overrightarrow{A} \tag{11}$$

The effect of the axion anomaly, to first order in perturbation theory as in the previous section, schematically produces a term,

$$\begin{aligned}
&-g_{a\gamma\gamma}\int d^4x\,\overrightarrow{M}(\overrightarrow{x})\cdot\overrightarrow{\nabla}\times\left(\frac{1}{\nabla^2}(\partial_t\theta(t))\overrightarrow{\nabla}\times\overrightarrow{A}\right)\\
&= g_{a\gamma\gamma}\int d^4x\,\theta(t)\,\overrightarrow{M}\cdot\overrightarrow{E}
\end{aligned} \tag{12}$$

where $1/\nabla^2$ is shorthand for the static potential (the time averaged Feynman propagator; presently we use the notation of ref.[17]). Throughout, we've assumed that $\theta(t)$ only has time dependence, *i.e.*, $\overrightarrow{\beta} = 0$. We've integrated by parts and used $\overrightarrow{E} = -\partial_t\overrightarrow{A}$ (in the static limit for $\overrightarrow{M}(\overrightarrow{x})$ the produced electric field $\overrightarrow{E}$ inherits the time dependence of $\theta(t)$). This establishes the result in general for any magnetic dipole $\overrightarrow{M}$ acquiring an electric dipole $g_{a\gamma\gamma}\theta_0\overrightarrow{M}$. The crucial element is the static Green's function $1/\nabla^2$ which is enforced by the zero recoil limit.

This result is formally related to duality in electromagnetic theory. Deser and Teitelboim [17] elegantly formulated the continuous electromagnetic dual tranformation, whereby $\overrightarrow{E}\leftrightarrow -\overrightarrow{B}$. This arises from an infinitesimal non-local transformation at the level of the vector potential. In Coulomb gauge the Deser-Teitelboim dual transformation is:

$$\delta\overrightarrow{A} = \frac{\epsilon}{\nabla^2}\overrightarrow{\nabla}\times\partial_t\overrightarrow{A} \tag{13}$$

which implies at the field strength level,

$$\delta\overrightarrow{E} = -\epsilon\delta\overrightarrow{B} \qquad \delta\overrightarrow{B} = \epsilon\delta\overrightarrow{B} \tag{14}$$

where $\overrightarrow{E} = -\partial_t\overrightarrow{A}$ and $\overrightarrow{B} = \overrightarrow{\nabla}\times\overrightarrow{A}$. The transformation on $\delta\overrightarrow{E}$ uses the on-shell condition, $\partial_t^2\overrightarrow{A} - \nabla^2\overrightarrow{A} = 0$.

We see that the transformation acting on the magnetic source term $\overrightarrow{M}\cdot\overrightarrow{\nabla}\times\overrightarrow{A}$ will produce a dual rotation of the magnetic field into the electric field, provided we can replace the dual rotation angle by $\epsilon\to g_{a\gamma\gamma}\theta(t)$.

$$\delta\overrightarrow{A} = \frac{g_{a\gamma\gamma}\theta(t)}{\nabla^2}\overrightarrow{\nabla}\times\partial_t\overrightarrow{A} \tag{15}$$

Can we promote the static Deser-Teitelboim transformation to a time dependent transformation?

We might worry that this affects the kinetic term of the electromagnetic theory (note that the $g_{a\gamma\gamma}\theta(t)\overrightarrow{E}\cdot\overrightarrow{B}$ term is already infinitesimal in this sense and does not transform). However, we can see that the electromagnetic action, $S = \int d^4x(\overrightarrow{E}^2 - \overrightarrow{B}^2)/2$, is invariant under the time dependent transformation. Define $\epsilon(t) = g_{a\gamma\gamma}\theta(t)$ and consider:

$$\begin{aligned}
\delta\overrightarrow{A} &= \frac{1}{\nabla^2}\epsilon(t)\left(\nabla\times\partial_t\overrightarrow{A}\right)\\
\delta\overrightarrow{E} &= -(\partial_t\epsilon)\frac{1}{\nabla^2}\left(\nabla\times\partial_t\overrightarrow{A}\right) - \epsilon\overrightarrow{B}\\
\delta\overrightarrow{B} &= \epsilon\frac{1}{\nabla^2}\overrightarrow{\nabla}\times\left(\nabla\times\partial_t\overrightarrow{A}\right) = \epsilon(t)\overrightarrow{E}
\end{aligned} \tag{16}$$

where we follow [17] and use the vector potential equation of motion, $\partial_t^2\overrightarrow{A} = \nabla^2\overrightarrow{A}$. We see that $\delta\overrightarrow{B}$ has the same form as that of Deser and Teitelboim. Hence the magnetic dipole moment will cleanly rotate into an oscillating electric dipole moment.

If we consider the action integral we find:

$$\begin{aligned}
\int\frac{1}{2}\delta\overrightarrow{E}^2 &= -\int\epsilon\overrightarrow{E}\cdot\overrightarrow{B} + \int\partial_t\epsilon\overrightarrow{A}\cdot\left(\nabla\times\overrightarrow{A}\right)\\
\int\frac{1}{2}\delta\overrightarrow{B}^2 &= \int\epsilon\overrightarrow{E}\cdot\overrightarrow{B}
\end{aligned} \tag{17}$$

where we've integrated by parts in space and time and discarded surface terms. Note that, with some manipulation, $\int\epsilon\overrightarrow{E}\cdot\overrightarrow{B} = \frac{1}{2}\int\partial_t\epsilon\;\overrightarrow{A}\cdot\nabla\times\overrightarrow{A}$ + total divergence. We thus find for the shift in the action:

$$\begin{aligned}
\delta S &= -2\int\epsilon\left(\overrightarrow{E}\cdot\overrightarrow{B}\right) + \int\partial_t\epsilon\overrightarrow{A}\cdot\left(\nabla\times\overrightarrow{A}\right)\\
&= 0
\end{aligned} \tag{18}$$

modulo surface terms.

The remarkable identical cancellation in eq.(18) occurs because of the time dependent $\epsilon(t)$. Essentially the $\epsilon(t)\overrightarrow{E}\cdot\overrightarrow{B}$ term becomes physical when $\epsilon(t)$ has time dependence (*i.e.*, if ϵ is a constant then the Deser-Teitelboim transformation applied to the action is $\delta S = -2\int\epsilon\overrightarrow{E}\cdot\overrightarrow{B}$ which is a total divergence). The physical axion-$\gamma\gamma$ interaction cannot be rotated away by the dual transformation. Hence the physical dual rotation induced by the axion in Fig.(1) affects only the source terms. We can view the induced electric dipole moment as a physical, duality induced rotation of the magnetic source, via the axion.

However, static electric dipole moments *will not* acquire oscillating magnetic moments. The effect of the additional nonlocal term in eq.(16) of $\delta\overrightarrow{E}$ is nontrivial. Given an electric dipole moment term in the action, $\int d^4x\ \overrightarrow{P}\cdot\overrightarrow{E}$ where $\overrightarrow{P}$ is time independent, we find $\delta\int d^4x\ \overrightarrow{P}\cdot\overrightarrow{E}=0$ upon integrating the nonlocal term in $\delta\overrightarrow{E}$ by parts and using $\partial_t^2\overrightarrow{A}-\nabla^2\overrightarrow{A}=0$. The asymmetry between magnetic and electric dipoles in axion electrodynamics is a consequence of the exclusive time dependence in $\epsilon(t)$ (this is modified if $\beta_{axion}\neq 0$).

If we introduce large classical magnetic background fields, the physical dual rotation induced by the axion on these fields generates the solutions to Maxwell's equations. In an RF cavity experiment with a large applied constant magnetic field $\overrightarrow{B}_0 = B_0\hat{z}$ the Maxwell equations take the form of eq.(10) and above. The rhs of eq.(10) is just the time derivative of the dual rotation of the large applied field B_0. The particular solution of is likewise the infinitesimal dual tranformation of $\overrightarrow{B}_0$, $\overrightarrow{E}_r = -g_{a\gamma\gamma}\theta_0(t)\overrightarrow{B}_0$.

In a cylindrical cavity we will also have the homogeneous solution, *e.g.*, the lowest transverse magnetic mode with frequency m_a: $\overrightarrow{E}_h = k\theta_0(t)J_0(m_ar)\hat{z}$ and $\overrightarrow{B}_h = k\theta_0(t)J_1(m_ar)\hat{z}$. The conducting boundary condition at the cavity wall, $r=R$, implies $\overrightarrow{E}(R)=\overrightarrow{E}_r+\overrightarrow{E}_h(R)=0$. This condition locks the arbitrary constant, k, to $g_{a\gamma\gamma}$. This, in turn, leads to resonance if the cavity is designed so that $J_0(m_aR)\to 0$.

IV. CONCLUSIONS

We have obtained an induced oscillating electric dipole moment for the electron, proportional to the magnetic moment, $2g_{A\gamma\gamma}\theta_0\cos(m_at)\mu_{Bohr}$. The result is quantitatively $\approx 1.4\times 10^{-32}(g_{A\gamma\gamma}/10^{-3})\cos(m_at)$ e-cm. The result is two orders of magnitude greater than the typical result expected for the nucleon, $d_N \sim 3.67\times 10^{-35}\cos(m_at)$ e-cm [11], and within four orders of magnitude of the limit on the EDM of the electron, $d_e\leq 8.7\times 10^{-29}$ e-cm, [13].

The result is a general low energy theorem and applies to any static magnetic system. Axion electromagnetic anomaly effects are essentially local oscillating dual rotations that lead to potentially observable signals. The axion anomaly perturbs the system by locally producing a physical, infinitesimal, time dependent dual rotation of $\delta\overrightarrow{E} = g_{a\gamma\gamma}\theta_0(t)\overrightarrow{B}$, effectively rotating a magnetic moment to an oscillating electric dipole moment. The duality of axion electrodynamics also implies the absence of induced oscillating magnetic moments from static electric dipoles. Further technical details will be presented elsewhere [16].

The existence of such phenomena may imply a number of potentially sensitive venues. Existing sensitive EDM experiments utilizing molecular beams, such as the ACME experiment, might be adapted to search for AC dipole moments [13]. The anomalous induced electric dipole moment applies as well to the nucleon, but there we expect it is only comparable to the direct effect from nonperturbative QCD [11].

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