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# Tuning, Ergodicity, Equilibrium and Cosmology

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I explore the possibility that the cosmos is fundamentally an equilibrium system, and review the attractive features of such theories. Equilibrium cosmologies are commonly thought to fail due to the “Boltzmann Brain” problem. I show that it is possible to evade the Boltzmann Brain problem if there is a suitable coarse grained relationship between the fundamental degrees of freedom and the cosmological observables. I make my main points with simple toy models, and then review the de Sitter equilibrium model as an illustration.

## I. INTRODUCTION

The relationship between cosmology and the thermodynamic arrow of time is a complex one. The idea first put forward by Penrose [1] that the “initial conditions” of the big bang fully account for the thermodynamic arrow of time we observe is, after some missteps<sup>1</sup>, now widely accepted among cosmologists (see for example [5–9]) although not necessarily by Penrose<sup>2</sup>.

Guth’s original paper on cosmic inflation [11] inspired many of us to believe that a full understanding of the cosmos should include an explanation of why the initial conditions for the big bang (in this case meaning the conditions in the radiation era after inflation) are “typical” or “natural” in some sense<sup>3</sup>. As has been emphasized in [13, 14], this expectation appears to be in conflict with the idea that those same conditions give the universe the low entropy start needed to explain the thermodynamic arrow of time. Basically, low entropy means “in an atypical part of phase space”, so how can one expect to argue that such a state is typical?

Although a finite period of “slow roll” inflation [15, 16] has been found to offer an extraordinarily successful picture of the origin of structure in the universe (see for example [17, 18]), Guth’s original idea of explaining that the initial state of the big bang was natural (or not “fine tuned”) has yet to be realized. The focus of this paper is the pursuit of this original goal. To achieve this goal, one has to offer a completion of the theory that describes what happened before the finite period of inflation (or your favorite alternative) that produced the observed structure. “Eternal inflation” [19–22] is one popular choice of a completion. It has the following straightforward motivation [23]: If one believes the universe had

the finite period of slow roll inflation needed to produce the observed cosmic structure, in many models a semi-classical extrapolation to earlier times would naturally take you back into the “self reproduction regime” that gives eternal inflation. However this picture has so far been plagued by technical problems that prevent it from having any real predictive power. Measure problems related to various infinities stemming from the “eternality” of the model are a major problem (see for example [24]). Potentially even more serious are the challenges to even defining probability at all in such a theory [25, 26], even if the regularization problems are solved (although recently I have taken a more hopeful view on this issue [26]) Also, as discussed below, the question of how much tuning may be involved in starting eternal inflation has not been resolved.

This tension between the need for low entropy and the wish for typicality has played out in a number of papers, including critiques of inflation by Penrose [27] and later by Coule [28]. Papers examining toy models of cosmological phase space give concrete illustrations of this point [29, 30]. Much of this later work takes the argument further, by pointing out that adding an inflationary phase to the story amounts to supposing an even lower entropy state prior to the radiation era, thus apparently making the problem worse: An inflationary state is clearly lower entropy than the subsequent radiation dominated phase, given the large entropy production during the reheating phase that connects the two. By comparison in the standard big bang (SBB), without inflation, there is essentially no entropy production in the radiation era all the way back to the singularity. Thus the SBB necessarily starts out in a higher entropy state, compared with the much lower entropy of the inflationary state.<sup>4</sup>

The same issue came up in work by Dyson et al. [31]

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<sup>1</sup> For example Hawking [2] originally took a very different view, which was corrected by Page [3] and Laflamme and Shellard [4])

<sup>2</sup> In later work (for example [10]) Penrose appears to have abandoned his original position in favor of a different explanation for the arrow of time that hypothesizes fundamentally irreversible physics.

<sup>3</sup> An earlier paper by Starobinsky [12] had many of the feature of inflation, but had the opposite goal of motivating a unique and atypical solution to describe the cosmos

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<sup>4</sup> In many (but not all) respects the low entropy associated with the inflationary state is tied with the assumption of the Bunch Davies vacuum for a great many field modes. The fact that this assumption is equivalent to one required in non-cosmological applications of field theory is interesting and non-trivial in my view. But our current level of understanding of the field theory vacuum is insufficient to use belief in the Bunch Davies vacuum as a “solution” to tuning problems.

(hereafter DKS). Those authors proposed a specific model in which the universe was fundamentally in equilibrium, and cosmology was obtained by fluctuations that were intrinsic to the equilibrium state. This picture gave further force to the tension between typicality and low entropy. For example, in the DKS scheme the low entropy of an early inflationary state fed in a quantitative way into an exponential disfavoring of cosmologies with inflation vs those without. Furthermore, the DKS result was a concrete example of a problem known for over a century [32–35], previously discussed in the context of modern cosmology in [13, 14, 28, 31, 36] and later dubbed the “Boltzmann Brain” problem [37].

The Boltzmann Brain problem basically is the observation that an equilibrium state strongly favors small fluctuations over large ones, so it seems obvious that a large out-of-equilibrium universe such as the one we observe would be highly disfavored in any model of the cosmos based on an equilibrium state<sup>5</sup>. This paper points out that this conclusion is based on simple assumptions about the relationship between the degrees of freedom that are in equilibrium and the cosmological observables. More subtle relationships between these two elements of the theory could result in a successful equilibrium cosmology that that is not undermined by the Boltzmann Brain problem.

There are good reasons to favor an equilibrium-based cosmological theory. Ultimately, an equilibrium theory may prove to be our only hope to “explain” the state of the universe using laws of physics. In a true equilibrium state, there is no notion of an “initial state”. The system simply exists eternally, fluctuating into one state or another with probabilities assigned by Boltzmann factors (or whatever formula correctly expresses the statistics of that system). For a finite system (likely to be the only situation in which such a picture is well-defined) the system will simply cycle through recurrences, reappearing in any given state within a finite recurrence time (phenomena discussed in a cosmological context in [31, 37]). One could imagine arbitrarily “launching” such a system with a particular initial state, but in the face of the recurrences there would be no real importance to an initial state defined in that way.<sup>6</sup>

However, a number of authors reject finite equilibrium cosmological models as certain to fail due to the Boltzmann Brain problem (for example see [31, 38]). According to their line of thinking our best hope is truly infinite theories (eternal inflation and variations such as [39] are

examples). Some go further and argue that only a true infinity can be counted on to evade the properties of finite systems that seem to generically lead to the Boltzmann Brain problem [40], and also are hopeful that infinite systems have a better chance of resolving the notorious measure problems [24]. I am skeptical of this line of reasoning, particularly because infinities can be used to hide finely tuned assumptions about initial conditions, as discussed at length in [41]. As the system becomes larger (on the way to infinity) the tuning of initial conditions can actually become worse. This effect was illustrated in [37], where an infinite volume universe was carefully regulated as a finite one in the limit of large volume. In that calculation the large volume limit reduced the impact of inflation because the probability of starting inflation (assembling all the degrees of freedom into an inflationary state) became exponentially smaller as the volume (and the total number of degrees of freedom) increased. This reduction was not sufficiently compensated by the increased total volume produced by inflation (even though that volume was probably factored in an overgenerous way in [37, 42, 43]). Thus I suspect that mathematical arguments such as those presented in [5, 40] about infinite theories will ultimately be seen as formal ways of re-casting high levels of tuning in ways where it is more difficult to identify.

I do not find my objections to infinite theories conclusive at this point, and I certainly find such theories a worthwhile avenue of investigation<sup>7</sup>. The point of this paper is not to rule out the infinite possibility, but to show that it is not so straightforward to rule out the possibility of a successful finite equilibrium cosmological model using arguments based on the Boltzmann Brain problem.

This paper is organized as follows: Section II introduces the “past hypothesis”, a concept that is important to my discussion. Section III makes the essential points of this paper by discussing two very simple “state machine” toy models. The first toy model has the essential features of a normal equilibrium system (which *would* have a Boltzmann Brain problem if interpreted cosmologically). The second toy model has the special features needed to avoid the Boltzmann Brain problem. Section IV discusses the de Sitter equilibrium cosmological model as a possible implementation of the ideas presented in Section III. My conclusions are summarized in Section V.

## II. THE PAST HYPOTHESIS

The “past hypothesis” (a term coined by Albert [44]) is entwined in an interesting way with the main points of this paper. This section provides a brief summary of

<sup>5</sup> Boltzmann Brains can be an issue for non-equilibrium models as well, but such models are not the focus of this paper.

<sup>6</sup> In everyday systems in equilibrium the temperature or total energy may seem like something equivalent to an initial condition. Different values of these quantities will correspond separate ergodic evolutions. For the cosmological systems we consider these parameters are interchangeable with the value of the cosmological constant, which at least in some respects appears to be a more fundamental thing

<sup>7</sup> In fact, I would not be surprised if at least some of these “infinite” theories will ultimately be found to have sufficient symmetry to be effectively finite in practice [42, 43].

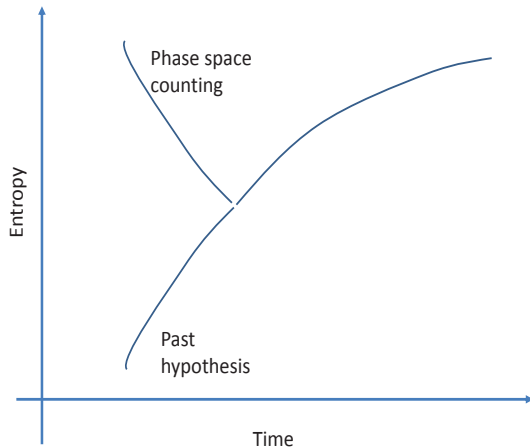


FIG. 1. Ergodic arguments allow us predict that entropy will increase into the future, but such state counting arguments would suggest that past also had higher entropy. Most physicists find it more compelling to assume entropy decreases in the past time direction. This is the “past hypothesis”.

the past hypothesis so it can be included in subsequent discussions. Carroll [38] gives a nice presentation with plenty of background<sup>8</sup>.

The usual statistical argument for entropy increase can be sketched like this: If a system is found in a state with sub-maximal entropy, there will be many more states available to it with higher entropy. Any kind of ergodic evolution that evenly explores phase space will favor (strongly so for large systems) evolution toward a higher entropy state.

One can then consider the past history of that same state. For usual physical laws that are time reversible, the phase space arguments might seem to apply just as well in reverse, strongly favoring a high entropy past as well as future. Thus one could argue that if one finds an ergodic system in a sub-maximal entropy state at time  $t_1$ , it is statistically by far most likely to be at an entropy minimum, with entropy heading up to a higher values in both past and future time directions. In such a picture the thermodynamic arrow of time, pointing toward higher entropy, points *away* from the low entropy state at  $t_1$  in both time directions. Figure 1 sketches this “double headed” arrow of time in the upper branch.

The “double headed” arrow of time describes behavior we simply do not believe to be true about our observed universe. We understand the observed universe to have a solid thermodynamic arrow of time that started billions of years ago and continued through to today without reversing (at least on macroscopic scales). This assumption

is a critical (although usually unmentioned) part of our modeling of observable processes in the early universe (from synthesis of nuclei to the production of the cosmic microwave background to the formation of cosmic structure). The success such modeling has had in matching cosmological data is evidence for the validity of assuming the 2nd law throughout. This belief, that the entropy was lower in the past, not higher as simple statistical arguments would suggest, is the “past hypothesis”. It means that while we might appeal to some notion of ergodicity to describe the future of a given system, we do not expect simple ergodic arguments to describe the past. Instead we believe that as we look further and further back in time the universe occupied a smaller and smaller region of phase space (or in other words, had a smaller and smaller entropy), as depicted in the lower branch of Fig. 1. The real universe appears to be only “partially ergodic”: It is ergodic into the future but not the past. This belief has proven itself many times over through the successes of the standard big bang cosmology.

The case for believing that the observed universe has a history of increasing entropy that spans billions of years is so compelling that it can be very difficult to imagine alternative physical situations that do not globally respect the past hypothesis (a phenomenon dubbed “temporal provincialism” in [31]). Even papers that discuss models in which the arrow of time is in some sense “emergent” [45–47] have very strong build-in assumptions about low entropy initial conditions (again, the past hypothesis) which are the main drivers of the arrow of time they find to be “emerging” [48]

Since the past hypothesis is deeply tied to the 2nd law of thermodynamics, it is also connected with the tension between our desire for “typicality” of the initial state of the cosmos and the belief that we have experienced a multi-billion year history of increasing entropy, requiring a finely tuned low entropy initial state.

### III. ERGODIC STATE MACHINES

Discussions of ergodic behavior are usually phrased in terms of some space of microstates. For finite systems with ergodic dynamics, each microstate is visited for the same amount of time, cycling through repeatedly. The observables typically live in a space of *macrostates* that are related to the microstates through coarse graining. As discussed above, the *past* behavior of the familiar microstates of particles and fields that we usually consider in cosmology clearly cannot be understood using ergodic state counting arguments. In this section I use a couple of toy model “state machines” to illustrate some key points.

First (Section III A) I will illustrate a simple system that has fully ergodic properties. It will be clear that that system does not observe the past hypothesis, and were it to be a cosmological model, it would definitely have a Boltzmann Brain problem.

<sup>8</sup> I recently learned in a private communication with R. Batterman that the term “past hypothesis” might have different meanings in different circles. The usage here is the same as that of Carroll.

The second state machine (Section IIIB) is also ergodic, but it has a very different relationship between the degrees of freedom undergoing ergodic behavior and the coarse-grained observables. The differences will allow the observables to respect the past hypothesis. When considered cosmologically, this toy model would not have a Boltzmann Brain problem.

I should emphasize up front that the toy models are nothing more than tables of numbers chosen “by hand” to illustrate certain very simple points. As I will discuss below, this utter simplicity reflects both a strength, and potentially also a weakness of the main points of this paper.

#### A. A state machine with “normal” fluctuations (Boltzmann Brains)

Table I illustrates a very simple “state machine” toy model that exhibits some properties of a typical equilibrium system. Each state is labeled by two numbers, “Micro” and “Macro”, shown one above the other. The lower “Micro” label assigns a unique label to each state, and this row lists the complete set of microscopic states that are accessible to the system and which are presumed in this model to be explored ergodically in the order listed. Since ergodicity implies the same amount of time is spent in each state, one can think of the “Micro” row as effectively a time variable. The system is assumed to be finite, with the time evolution looping back to microstate 1 after the last microstate is reached. (One could either imagine Table I shows a complete 95 state system or just the first 95 states of a larger system with overall similar properties.)

The “Macro” row represents a coarse-grained macrostate. The index “1” represents the equilibrium macrostate. As one would expect of an equilibrium state, the largest number of microstates coarse grain up to macrostate “1”. One can see this by inspection of Table I. Also, Table II shows the number of microstates that coarse grain up to each macrostate. Macrostates with labels  $> 1$  represent observable fluctuations from equilibrium. In this toy model there are only two possible fluctuation time sequences, a small one running up to 3 and back down to 1, and a large fluctuation running all the way up to 6 and back down to equilibrium. The large macro fluctuations resolve into fewer microstates (again, shown in Table II), and thus (thanks to ergodicity) appear less frequently than the small fluctuations. One can think of the large fluctuations up to 6 as cosmological fluctuations, and the smaller fluctuations, just up to 3 and down as the “Boltzmann Brains”. In this toy model a “3” is more likely to appear as part of a “Boltzmann Brain” than as part of a cosmological state, thus illustrating the usual problem with equilibrium-based cosmologies (small fluctuations are more likely than large ones). Generally, the features outlined in this paragraph correspond to features one

might expect in a realistic fluctuating equilibrium system.

Another (closely related) way this toy model is realistic for an equilibrium system is that it does *not* obey the past hypothesis. An expression of the past hypothesis in this simple toy model might be for example, if one finds the system in macrostate “3”, it is most likely to have been in state “2” one time step away in one time direction, but step 4 in the other. That is, it is part of a long-running thermodynamic arrow of time with entropy low at one end and high on the other. In this toy model smaller fluctuations (with 2’s on either side of a 3) are more favored. Thus, while realistic for an equilibrium system, this toy model is definitely not a good toy model for cosmology.

There are a number of ways this toy model is *not* realistic compared with everyday equilibrium physical systems. For one, there are only two possible fluctuations. Although the large fluctuation does appear less frequently than the small one, no effort is made to quantify the relative probability for the the two fluctuations in a standard way, such as with a Boltzmann factor. It is hard to imagine a realistic physical system that would have this rather odd space of states. Surely if one were found it would have to be carefully engineered by a human to have these properties, rather than being something easily found in nature. Still, thanks to the realistic features mentioned in the previous paragraph, this toy model captures enough realism for my purposes.

#### B. A state machine with fluctuations consistent with cosmology (suppressed Boltzmann Brains)

Table III shows a different toy model, with an interesting relationship to the first one. In this model, each state has three labels. “Macro” and “Micro” are familiar from the previous model, but now there is another label, “Trans-Micro”. “Trans” designates some extension of the picture beyond the original “Micro” level. In this toy model it is the Trans-Micro label that is assigned uniquely to each state, and Micro now itself reflects a level of coarse-graining. This toy model is designed to explore the possibility that the universe is ergodic only at a more fundamental level (the Trans-Micro level) which describes physics beyond the ordinary microphysics of particles and fields we usually consider when developing statistical arguments about the universe. Specifically, I would like to explore the possibility that ergodicity at a more fundamental level could actually be harnessed to make a realistic cosmology (long running arrow of time and all) “typical”, and specifically more favored than Boltzmann Brains.

For the toy model in Table III the microstates and macrostates are the same as for first model described in Table I. That is to say, all microstates coarse grain up to the macro level in exactly the same way. The microstates and macrostates are intended to represent familiar “fun-





illustrate how one might realize a successful equilibrium cosmology. One important feature of this extended model is that the behavior of the macro and micro level states are counterintuitive when interpreted using the traditional intuition we have about familiar physical degrees of freedom (intuition which was successfully realized in the first toy model).

For example, consider the time sequences moving forward from trans-microstates with indices 19, 40, and 82. These represent the start of large fluctuations, which in the second toy model are more likely than small ones. In each of these cases the next step is assigned the (coarse grained) micro index 20. Using more traditional intuition, this would seem to be a “fine tuning”, since it violates state counting arguments made in the micro space (which would say, for example that micro index 20 should appear no more often than any other micro index, including micro indices 7, 53, 68 and 89, all of which correspond to small fluctuations). The point is that this is *not* a fine tuning in the 2nd toy model, because there state counting does not work in the micro space. The micro space is simply a coarse-graining up from the more fundamental trans-micro space and is not ergodic at all.

As I have already emphasized, by assuming the past hypothesis we have already accepted that state-counting arguments made about the particles and fields are not applicable to the universe as a whole (at least when working out our past history). The breakdown of counting arguments when applied to the microstates of the second toy model should not be seen as a problem with the toy model, but simply the way our (established) abandonment of counting arguments in cosmology shows up in this toy model. The novelty with the second toy model is that despite the breakdown of counting arguments at the micro level, ergodicity and state counting can be fully recovered at the more fundamental trans-micro level.

The triviality of the state machine toy models deserves further scrutiny, as it reflects both strong and weak aspects of the points I am making here. I have been explicit in the above discussion about how trivial these toy models are. They are literally just lists of numbers I have created “by hand”.

There is a widespread view that a profound conceptual barrier prevents successful equilibrium cosmological models from being built, due to issues discussed above such as the Boltzmann Brain problem [49]. The toy models presented here show, by trivial counterexamples, that no such conceptual barrier exists. Through their simplicity, the toy models illustrate how easy it is to circumvent the conceptual issues facing equilibrium cosmologies. All one needs is for our elementary particles and fields to not represent the fundamental physical phase space. Instead, they must represent coarse-graining up from a more fundamental phase space, and the coarse-graining relationship between the particle-field space and the fundamental space must embody key subtleties that let ergodicity in the fundamental space mimic tuning in the coarse-grained one.

The idea that fundamental physics should be expressed in a phase space much larger than that of the everyday particles and fields is hardly radical in contemporary physics (see for example discussions of the string theory landscape [50]). The more tricky issue here has to do with the particular coarse-graining properties required to make a successful equilibrium cosmology.

One idea might be to imagine how one might realize the required properties in “everyday” physical systems (boxes of gas with dividers, membranes, paddle wheels or whatnot) but so far my explorations have not yielded any nice illustration. Of course the toy models shown in this paper are easy enough to program on a computer, and would be just as straightforward to manufacture in a machine shop as a mechanical device, but the 2nd toy model has properties that seem difficult to find in more naturally occurring systems. Perhaps this difficulty is not a bad thing, but a feature. Perhaps the only possibility of realizing the special properties of the trans-macro space is exotic degrees of freedom that are not part of our everyday world. Nonetheless, the fact that I have not clearly established such an illustration means the relevance of the points illustrated by these toy models to realistic modeling of the cosmos remains a matter of speculation.

#### IV. DE SITTER EQUILIBRIUM AS AN ILLUSTRATION

The conclusion in the previous section about the speculative nature of equilibrium models of cosmology may seem disappointing, but it is the state of the art for any cosmological model building. The popular eternal inflation model discussed in the introduction requires speculation that the effective field theory of the inflaton is valid over a large [24] or perhaps even formally infinite [40] range of scales. And ideas about the dynamics of a string theory landscape, while influential, are even more speculative.

The de Sitter equilibrium model (dSE) is constructed by deliberately shaping the speculation (inherent in any of the current attempts at a fundamental picture of cosmology) in a direction needed to construct a viable equilibrium theory. Whether or not these particular speculations turn out to be correct, I find the dSE model interesting as an illustration of what sort of behavior might be required of a fundamental theory to result in a successful equilibrium cosmology. The dSE model has been discussed at length elsewhere [22, 37, 51, 52]. Here I just provide a sketch, with the purpose of tying it in with the earlier parts of this paper.

The de Sitter equilibrium cosmology takes its inspiration from certain “holographic” ideas about de Sitter space, in the case where one assumes the cosmological constant is truly fundamental and not able to decay. As shown in [53] a pure de Sitter space has maximum entropy (vs states where other objects such as black holes

are added in). As discussed originally in [54, 55], one might be tempted to imagine the full quantum of theory de Sitter space can be expressed in a *finite* Hilbert space with dimension

$$N = e^{S_\Lambda}. \quad (1)$$

If you never observe an entropy larger than  $S_\Lambda$ , why would you need  $\ln(N) > S_\Lambda$ ? In such a picture the fundamental degrees of freedom would be sufficient only to describe a single horizon volume of space, ending at the “thermal” de Sitter horizon where fundamental quantum effects would be expected to be important. Following similar ideas from “black hole complementarity” [56], one might expect transformations to the frames of different observers to reorganize the physical degrees of freedom in a nonlocal manner to describe different-looking single horizon volumes, even if there are only sufficient degrees of freedom to describe one horizon volume at once<sup>9</sup>.

As discussed first by DKS [31], it seems natural to think that such a system would exist in (or be driven to) equilibrium, undergoing fluctuations around the equilibrium (de Sitter) state. Ergodic arguments suggest that localized fluctuations (that perturb space so that at a distance the perturbation looks Schwarzschild with ADM mass  $m_f$ ) would happen with probability

$$P_f = \frac{\exp(S_\Lambda - \sqrt{S_\Lambda S_m})}{\exp(S_\Lambda)} \quad (2)$$

$$= \exp(-\sqrt{S_\Lambda S_m}) \quad (3)$$

$$= \exp(-m_f/T_{GH}) \quad (4)$$

where  $S_m$  is the entropy of a black hole with mass  $m_f$  and

$$T_{GH} = \frac{1}{2\pi} \sqrt{\frac{3}{\Lambda}}. \quad (5)$$

Equation 4 is a result from Gibbons and Hawking [53] which comes from a careful analysis of the change in area of the de Sitter horizon when the localized perturbation is introduced. Rather remarkably, it winds up giving a simple exponential suppression of the Boltzmann form (Eqn. 4), which belies the general relativistic origin of this result.

In [51] I consider fluctuations which tunnel to an inflationary state with energy density  $\rho_I$  and estimate the mass of such fluctuations to be

$$m_s = 0.001 kg \left( \frac{(10^{16} GeV)^4}{\rho_I} \right)^{1/2}. \quad (6)$$

<sup>9</sup> It is important to remember when considering this somewhat odd picture that most of the degrees of freedom, from the point of view of any one observer, are tied up in the “thermal horizon”. Some of these can swap in and out of the “interior” region of semiclassical spacetime when transforming from one observer to another.

I also argue that the probability of tunneling to inflation is well approximated by Eqn. 4 with  $m_f = m_s$ . Thus, as long as the universe can not produce a “brain” fluctuation with mass of only a gram, it is more likely to fluctuate into a tunneling event that creates an entire inflationary universe large enough to encompass all that we see.

So far these quantitative results are fairly standard ones, and do not presuppose a finite system, with a finite Hilbert space. In [22, 51, 52] I consider the following “completion” of this picture (speculative of course, as is the case with any other fundamental picture of inflation)<sup>10</sup>.

I interpret the tunneling process as the evolution of the full quantum state (with some suppressed tunneling probability) from one that describes one semiclassical spacetime (the de Sitter space) to one that describes two semiclassical spacetimes: The perturbed de Sitter spacetime and the “baby universe” that has budded off (semiclassically speaking) and started inflating. As the baby universe inflates, reheats and evolves through a standard cosmology, the full state continues to describe both semiclassical spacetimes. During this period, some degrees of freedom are tied up describing the baby universe, and others are used to describe the perturbed de Sitter spacetime. These two sets of degrees of freedom are essentially decoupled during the baby universe phase. The time at which the baby universe finally approaches late time de Sitter behavior (its ultimate destiny if  $\Lambda$  is fundamental) is the time at which the degrees of freedom tied up in describing the baby universe re-couple with those describing the perturbed de Sitter space. Together they form the complete set of degrees of freedom, describing the equilibrium de Sitter state again.

During this same period, the perturbed de Sitter space will have the following behavior: The initial fluctuation that tunnels off the baby universe appears, at least far away in the de Sitter space to be a small black hole with mass  $m_s$ . This small black hole will rapidly decay, but it will take a time of order  $H_\Lambda^{-1}$  for radiation from the decay to reach the de Sitter horizon. During most of that time, the decay products will appear localized, at least compared with the size of the de Sitter horizon, and thus de Sitter horizon (and the corresponding entropy) will remain reduced according to the Gibbons and Hawking formula. Only after a time  $H_\Lambda^{-1}$  will these quantities return to their equilibrium values. Interestingly, this is essentially the same time the baby universe will take (according to its own cosmic time) to return to equilibrium. The above process is sketched pictorially in Fig. 1<sup>11</sup>.

<sup>10</sup> This picture is identical to that in [39] except for the completion I describe next. Carroll and Chen complete their picture in the form of an infinite theory

<sup>11</sup> This picture, with its repeating cosmological cycles might invite comparison with the “cyclic cosmologies” described in [57]. However those models are envisioned as infinite systems, with

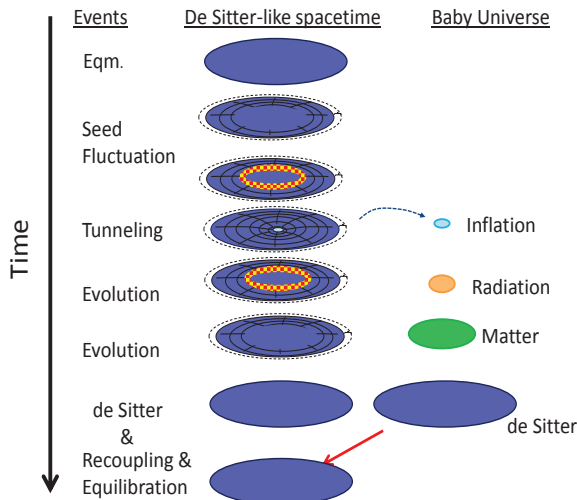


FIG. 2. In the dSE model, the equilibrium state fluctuates off a baby universe which inflates, reheats and undergoes standard cosmological evolution eventually re-equilibrating. The quantum state describes two semiclassical spacetimes between the moment of tunneling until the time of re-equilibration. The probability of producing this fluctuation is competitive with the production of an other 1 gram localized fluctuation in the matter.

I now further consider the parallels between this picture and the 2nd toy model (with the trans-macrostates) considered in Sect. IIIB. The part of Fig. 1 that gives the cosmological evolution (in one of the semiclassical spacetimes) corresponds to the large “cosmological” fluctuations in that toy model. The apparent time asymmetry of Fig. 1 is an artifact of the usual way of discussing quantum tunneling, and is not an actual property of the full quantum state which includes superpositions of the process going in both directions, as discussed in [51]. (The discussion in [59] also appears to be related to this point).

Consider the following properties of the standard cosmological evolution (with a period of early inflation): Take a snapshot today of the microstate of all the elementary particles and fields. We believe that microstate contains enough information that if it were (rigorously) evolved it back in time, the high temperature early universe state would “de-heat” back into an inflaton rolling back up the hill. This is simply a feature that everyone believes to be true about the current state of the universe, as long as one believes in an early period of inflation. One can argue that such a state is finely tuned in that, for example, the thermal state of the radiation era apparently corresponds to many more microstates that will *not* de-heat into cosmic inflation when time reversed. On the other hand, imposing the past hypothesis seems

a straightforward path to admitting such a “tuning” into one’s theory. So far this is just a reiteration of points reviewed earlier in this paper. The novelty comes when one extends this discussion more fully to the dSE model.

The time reverse of the full dSE picture (which can be seen by reading the sketch in Fig. 2 from bottom to top) involves the de Sitter horizon emitting a “shell” of black hole decay products with special (highly coherent) properties that the propagate inward and form the hole<sup>12</sup>. Another special property of that event is that some degrees of freedom decouple, producing a decoupled cosmological spacetime that is evolving according to a time-reversed standard inflationary cosmology. When thought of in terms of elementary particles and fields, this process certainly seems like a finely tuned one. One could compare it, for example, with the time reverse of a (non-inflationary) standard big bang cosmology relaxing toward de Sitter space (as was done by DKS [31]). That would involve no splitting into two spacetimes, and no special phase information necessary for the “de-heating” into inflation discussed above. When viewed from the point of view of elementary particles and fields, the time reverse of Fig. 2 certainly suggest that the fluctuation shown is much less likely than one corresponding to a non-inflationary standard big bang. (This, by the way, was the conclusion of DKS.)

This apparent fine tuning corresponds exactly to the apparent tuning in the second toy model, where the large fluctuations occur more frequently than the small ones. In the case of the second toy model this phenomenon is actually not tuned at all, whereas it would require tuning if this phenomenon were observed in the first toy model. The difference lies in the fact that the equilibrium and ergodic behavior are taking place in a larger set of more fundamental (trans-macro) degrees of freedom in the second toy model. For that model both the macro and micro observables are coarse grainings of the fundamental states and their behavior is dictated only indirectly by the ergodic properties, via the special nature of the coarse-grained relationship to the fundamental degrees of freedom.

For the dSE model to work, with no tunings, the system would need to have its own equivalent to the trans-macro space that would play a similar role. The tuning discussed above in terms of the particles and fields could be undone in a similar way if the particles and fields where themselves suitably coarse grained from a more fundamental set of degrees of freedom. The idea that the particles and fields may not be fundamental is hardly radical these days, with the widespread hopes that ideas such as string theory offer a better fundamental picture. The key to the success of the dSE model though, lies in the special nature of the coarse graining required. Ba-

infinitely increasing entropy [58]. Thus they would seem to have an infinitely bad fine tuning problem for the initial state that starts the cycles.

<sup>12</sup> As discussed in [48] this seems to be the most likely way to seed perturbation in the “forward” direction as well, a feature I have tried to represent in Fig. 2

sically the coarse graining must be conceptually equivalent to that displayed in the second toy model for the cosmological behavior not to be finely tuned. One feature required of these special coarse grainings is that it would have to be more likely for the de Sitter horizon degrees of freedom to emit particles and fields that would exhibit time reversed cosmological evolution *including* de-heating to an inflationary state, vs standard big bang evolution with no inflation. The counterintuitive nature of this requirement can be thought of as either reflecting the exotic behavior required from the fundamental physics, or more prosaically, might simply be taken as a reason to be discouraged about the dSE picture. At this point I cannot offer a specific fundamental picture which exhibits the very special coarse graining relationship between the particles and fields and the fundamental degrees of freedom needed to support the dSE model. Thus, the viability of the dSE picture remains a matter of speculation. One hopeful note is that this speculation focuses on the behavior of degrees of freedom at the de Sitter horizon, a place where as with the black hole horizon, there seems to be a lot we don't yet understand.

## V. CONCLUSIONS

The question of whether it is possible to construct a theory of cosmology that is not finely tuned remains an open one. In this article I have argued that cosmological theories based on equilibrium have certain attractive features, including the fact that there are no “initial conditions” at all. The probabilities of a given fluctuation (cosmological or otherwise) are given by the laws of physics, through the Hamiltonian as it appears in the Boltzmann factors or the equivalent. However, equilibrium theories are notorious for the “Boltzmann Brain” problem, basically because they strongly favor small fluctuations over large ones. Here I have argued on the basis of extremely

simple toy models that the Boltzmann Brain problem is not insurmountable for equilibrium cosmological theories. What one needs to get around it is a special coarse graining relationship between the familiar particles and fields and another (larger) set of more fundamental degrees of freedom. I have illustrated via the de Sitter equilibrium model how such an equilibrium cosmological model might look, but recognize that the dSE model has yet to be realized in a fundamental theory.<sup>13</sup>

The approach I have explored here simply takes the very special properties we know our early universe must have and maps them onto very special properties of the coarse graining relationship between the fundamental degrees of freedom and the familiar particles and fields. This may seem like simply exchanging one exotic feature for another, but the fact is that one way or another, nature *has* chosen the observed universe to have some exotic properties that look naively like tuned initial conditions. Our job as cosmologists is to learn how nature has chosen to realize these features. In my view, the idea of a fundamentally equilibrium cosmos remains an attractive contender, and may in fact be the only alternative to simply accepting finely tuned initial conditions as an unexplained feature of our universe.

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<sup>13</sup> Given the exotic relationship between the fundamental degrees of freedom and the observable particles and fields, it seems unlikely that the fundamental degrees of freedom could themselves be described by a field theory.

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